Numerical validation of the Generalized Equivalent Spectral Model through crowd dynamics simulations

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NUMERICAL VALIDATION OF THE GENERALIZED EQUIVALENT SPECTRAL MODEL THROUGH CROWD DYNAMICS SIMULATIONS

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Abstract. *This paper aims to provide a validation of the Generalized Equivalent Spectral Model of pedestrian-induced loading through numerical simulations of crowd dynamics. Pedestrian flows with varying density are numerically simulated based on the social force model. The instantaneous step frequency of each pedestrian is obtained from his instantaneous velocity. Results obtained from numerical simulations are statistically analyzed to obtain probability density functions of the step frequency. Then, each pedestrian is modelled as a moving harmonic load, whose trajectory and velocity are obtained from crowd simulations. The power spectral density function of the modal force obtained from numerical simulations is then compared with the analytical expression provided by the Generalized Equivalent Spectral Model. Possible modifications of the original formulation are analyzed, in order to fit numerical results.*

1 INTRODUCTION

Vibration serviceability assessment of footbridges under human-induced excitation requires the availability of suitable and reliable load models. Several walking load models have been proposed in the last decades, but so far none of them has been fully validated and recognized as the most reliable. Recently, increasing attention has been devoted to stochastic load models, which allow considering the inherent randomness of walking parameters, also known as inter-subject variability. A possible approach is the modelling of pedestrian loading as a stationary random process through the definition of a suitable spectral model (e.g. [1], [2]). Most spectral models proposed so far refer to unrestricted pedestrian traffic, i.e. to very low pedestrian density, so that pedestrians are free to walk at their desired speed. To the authors' knowledge, the only extension to crowded conditions has been proposed by Ferrarotti and Tubino [3]. In the Generalized Equivalent Spectral Model (GESM), the effects of human-human interaction are taken into account in two ways: (1) the mean step frequency of pedestrians is expressed as a function of the crowd density; (2) a coherence function is introduced to model the increasing correlation of the loading at different locations for increasing crowd density. Due to the lack of experimental data, the coherence function has been defined in a qualitative physically-based way. Thus, a numerical and/or experimental validation of the model is needed.

The aim of the proposed work is to numerically validate the GESM through crowd dynamics simulations. These are carried out with the commercial software MassMotion [4], which is based on a microscopic description of pedestrian dynamics, i.e. each pedestrian is modelled as a single agent, whose velocity is determined by the interactions with the environment and the surrounding pedestrians. The simulations are performed on an ideal footbridge, crossed by unidirectional flow of pedestrians with crowd densities varying in the range 0.3-1.5 ped/m^2. The pedestrian trajectories and velocities obtained from crowd simulations are used to derive the pedestrian-induced forces. The power spectral density function of the modal force obtained from numerical simulations is then compared with the analytical expression provided by the GESM to evaluate whether the proposed coherence function is suitable to correctly model human-human interaction effects. A critical analysis is provided, and possible modifications of the original formulation are proposed in order to fit numerical results.

2 PEDESTRIAN-INDUCED FORCES: ANALYTICAL FORMULATION

In this Section, the Generalized Equivalent Spectral Model (Section 2.1) and the time-domain force model (Section 2.2) are introduced.

2.1 Generalized Equivalent Spectral Model

The Generalized Equivalent Spectral Model (GESM) [3] of pedestrian-induced forces is based on the definition of the cross-power spectral density function (cpsdf) of the force per-unit-length $S_{ff}(x,x',n)$, which, under the assumption of uniform equivalent loading, is given by:

$$S_{ff}\left(x,x';n\right)=S_f\left(n\right)Coh_{ff}\left(x,x';n\right) \tag{1}$$

where $S_f(n)$ is the power spectral density function (psdf) of the force per-unit-length, and $Coh_{ff}(x,x',n)$ is its coherence function, given by:

$$S_f\left(n\right)=\frac{\left(\alpha_m G_m\right)^2}{\varepsilon L}\frac{N_p}{4}p_N\left(n\right) \tag{2}$$

$$Coh_{ff}\left(x,x';n\right)=\begin{cases}1 & \text{if } \left|x-x'\right|<\varepsilon \\ \exp\left[-C\left(\left|x-x'\right|-\varepsilon\right)\right] & \text{otherwise}\end{cases} \tag{3}$$

In Eq. (2), α_m and G_m are, respectively, the mean value of the DLF and of pedestrian weight, N_p is the mean number of pedestrians on the footbridge, $p_N(n)$ is the probability density function (pdf) of the step frequency n, ε is interpreted as the separation distance that each pedestrian interposes with others to avoid contact, assumed as ε= 4 m. The exponential decay coefficient in Eq. (3), is expressed as a function of the mean step frequency n_m, $C = \exp(C_1 n_m + C_2)$, (C_1 = 22.7, C_2=-41). The mean step frequency n_m can be obtained as a function of the mean walking velocity v_m [5]:

$$n_m = 0.35v_m^3 - 1.59v_m^2 + 2.93v_m \tag{4}$$

The mean step velocity v_m is related to pedestrian density ρ through the fundamental law:

$$v_m\left(\rho\right)=v_{\max}\left\{1-\exp\left[-\gamma\rho_{\max}\left(\frac{1}{\rho}-\frac{1}{\rho_{\max}}\right)\right]\right\} \tag{5}$$

with $\rho_{\max}$=5.4 ped/m^2, γ=0.354, v_{max}=1.34 m/s according to Buchmueller and Weidmann [6].

Based on the GESM, the psdf of the modal force for a generic mode shape $\varphi_j(x)$ is given by:

$$S_{F_j}\left(n\right)=S_f\left(n\right)\chi_j\left(n\right) \tag{6}$$

where $\chi_j(n)$ is the admittance function, defined as follows:

$$\chi_j\left(n\right)=\int_0^L\int_0^L Coh_{ff}\left(x,x';n\right)\varphi_j\left(x\right)\varphi_j\left(x'\right)dxdx' \tag{7}$$

being L the length of the structure. Under the assumption of unrestricted traffic, and considering the first mode shape of a simply-supported beam $\varphi_j(x)=\sin(\pi x/L)$, the psdf of the modal force is given by [3]:

$$S_{F_j\text{unr}}\left(n\right)=\left(\alpha_m G_m\right)^2\frac{N_p}{4}p_N\left(n\right) \tag{8}$$

2.2 Time-domain force model

Pedestrian-induced force is commonly schematized as a moving load. The force per unit length exerted by N_p pedestrians can be expressed as the sum of the forces exerted by each single pedestrian as follows:

$$f\left(x,t\right)=\sum_{i=1}^{N_p} f_i\left(t\right)\delta\left[x-x_i\left(t\right)\right] \tag{9}$$

where t and x are the time and the abscissa along the structure, $x_i(t)$ and $f_i(t)$ are, respectively, the instantaneous position and the force exerted by the i-th pedestrian, and δ(•) is the Dirac delta function.

If pedestrian traffic is simulated numerically in order to take into account interaction among pedestrians, then the instantaneous position x_i and velocity v_i of the i-th pedestrian are the output of the simulation, and the step frequency n_i can be derived from the step velocity v_i through a suitable law (e.g. Eq. (4)). Since the pedestrian step frequency $n_i(t)$ is time variant during footbridge crossing, the pedestrian force exerted by the i-th pedestrian can be modelled as a sinusoidal carrier signal, whose base frequency $n_{m,i}$ is modulated, as follows:

$$f_i(t) = \alpha_i G_i \sin\left[2\pi n_{m,i} t + 2\pi \int_0^{T_i} \left(n_i(t) - n_{m,i}\right) \mathrm{d}t\right] \tag{10}$$

where $n_{m,i}$ is the mean value of the instantaneous step frequency during footbridge crossing time T_i. Furthermore, α_i and G_i are the dynamic load factor (DLF) and the weight of the i-th pedestrian, respectively.

Starting from Eq. (9), the modal force for a generic mode shape $\varphi_j(x)$ can be expressed as:

$$F_j(t) = \int_0^L f(x,t)\varphi_j(x)\,\mathrm{d}x = \sum_{i=1}^{N_p} f_i(t)\varphi_j\left[x_i(t)\right] \tag{11}$$

3 CROWD DYNAMICS NUMERICAL SIMULATION

Numerical simulations of crowd dynamics are carried out on an ideal footbridge, whose dimensions in plan recall the ones of the De Gasperi footbridge in Milan: length L=60 m, width B=2.7 m [7]. Pedestrian initial positions are randomly distributed in a “starting area” before the footbridge entrance, ten times longer than the footbridge length: this assures that pedestrians are initially uniformly distributed with a crowd density ρ. Hence, the total number of generated pedestrians N is set equal to $10\rho BL$. Four values of crowd density, [0.3 0.7 1.1 1.5] ped/m^2, from unrestricted to extremely dense pedestrian traffic, are considered. For each crowd density, 100 simulations are performed to obtain statistical reliability.

For each simulation, mean values of the pedestrian density and velocity are calculated during the period of full occupancy T_{full}. The lower and upper boundaries of T_{full} are determined as the first and last time instant when the 95% of the mean number of pedestrians N_p are on the footbridge [8]. The obtained mean values are then averaged over 100 simulations.

Figure 1 plots the v_m-ρ relation resulting from crowd simulations (MM), together with a fitting law and the fundamental law in Eq. (5). It can be observed that for pedestrian densities below around 1.7 ped/m^2 the mean walking velocities obtained from numerical simulations are lower than those estimated with the Weidmann law in Eq. (5).

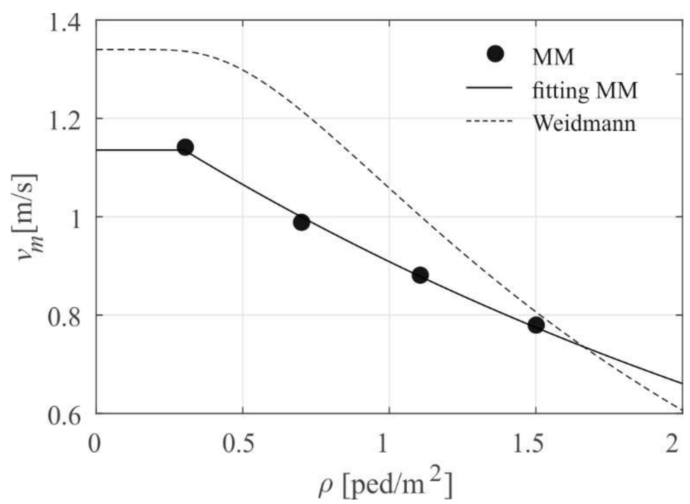

Figure 1: Comparison between v_m-ρ relation obtained with MM and proposed by Weidmann.

Time histories of step frequencies are derived from walking velocities through Eq. (4). Moreover, in order to account for the reaction time needed by pedestrians to adapt their step frequency to variations of the walking velocity, the step frequency is estimated as the 5s-moving average of the instantaneous values.

Table 1 reports the obtained mean value and standard deviation of step frequencies for the four crowd densities.

ρ [ped/m^2]	0.3	0.7	1.1	1.5
n_m [Hz]	1.786	1.676	1.588	1.492
n_{std} [Hz]	0.1083	0.0962	0.0752	0.0598

Table 1: Mean and standard deviation of step frequencies obtained from numerical simulations.

Pdfs of the mean step frequencies during footbridge crossing are estimated from each simulation. Figure 2 plots the obtained numerical mean pdfs and their Gaussian fitting, together with the model adopted in [3]. The numerical pdf is well approximated by a Gaussian fitting, especially for densities below 1.5 ped/m^2, while the pdf in [3], assuming n_{std}=0.18 Hz, does not fit the simulation results.

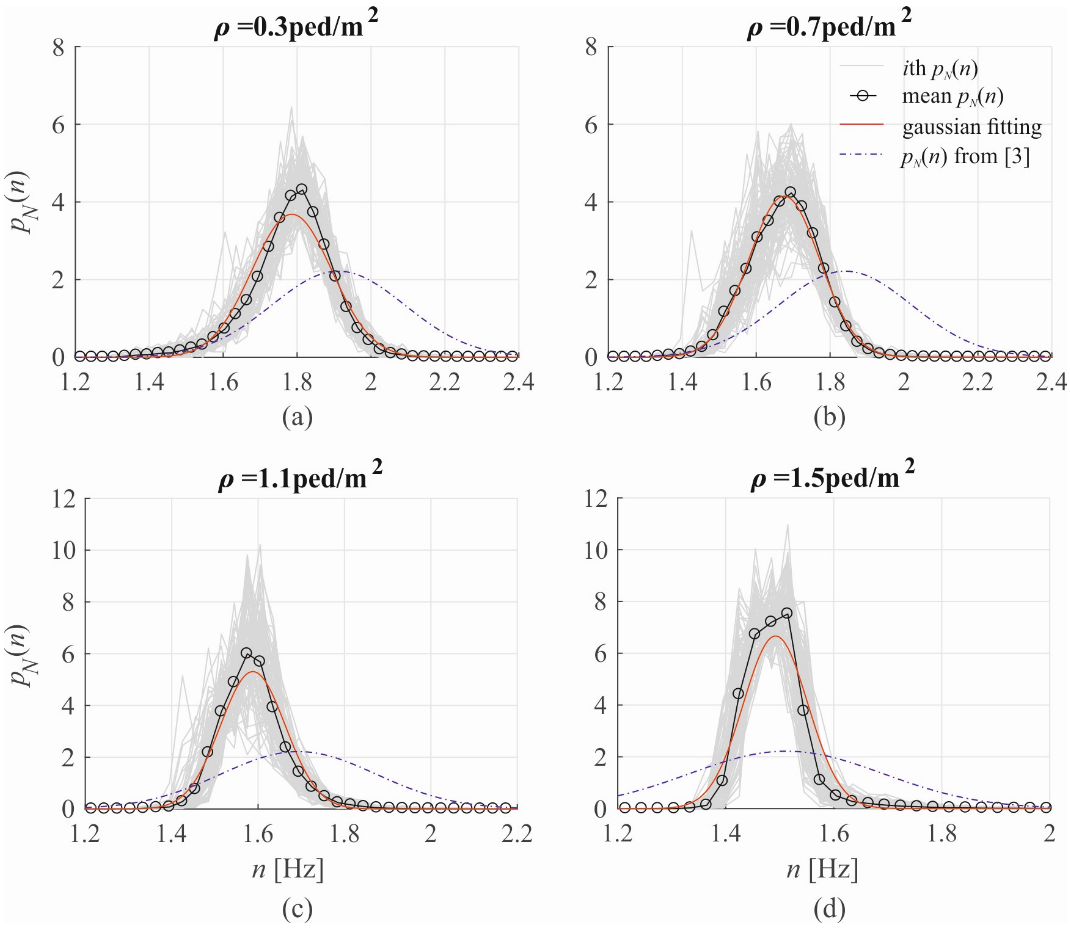

Figure 2: Numerical PDFs of the mean step frequencies and Gaussian fitting.

Figure 3 shows the mean and standard deviation (std) of step frequencies obtained from the fitting of numerical pdfs for each crowd density, compared with the mean step frequencies estimated through Eqs (4), (5). In line with the v_m-ρ trend in Figure 1, numerical mean step frequencies decrease on increasing crowd density and numerical values are lower than those estimated from Eqs. (4), (5). Standard deviation values also show a decreasing trend for increasing crowd density, meaning that in crowded conditions pedestrians tend to walk at step frequencies closer to the mean value.

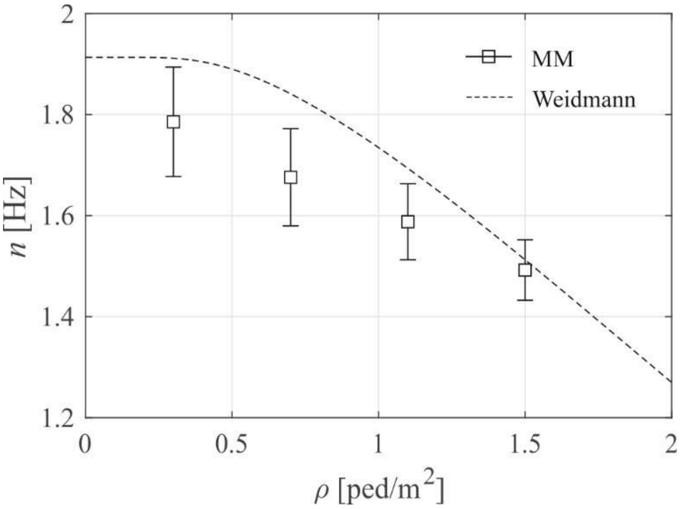

Figure 3: Numerical mean and std of step frequencies in comparison with those estimated from Eqs (4), (5).

The forces induced by each pedestrian are then calculated according to Eq. (9), with each contribution given by Eq. (10), and the modal force is calculated as in Eq. (11), assuming $\alpha_i G_i = \alpha_m G_m = 280$ N. Figure 4 plots an example of the time history of the modal force obtained from a simulation.

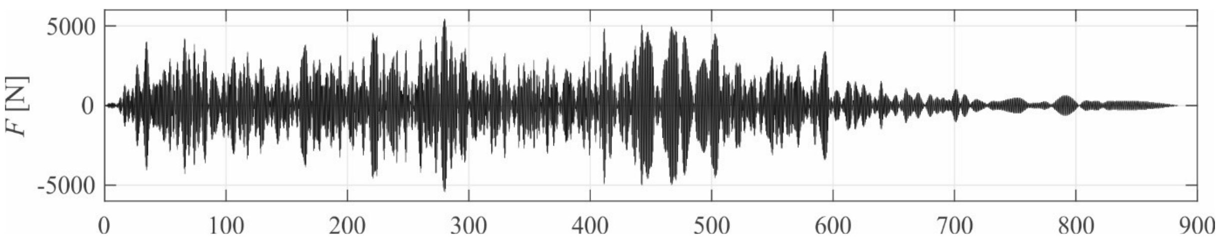

Figure 4: Time history modal force for a simulation with 0.7 ped/m^2.

4 PSDF OF THE MODAL LOAD AND GESM VALIDATION

This section is devoted to the comparison between the numerical psdfs and the analytical expressions provided by the GESM. For each value of the crowd density, the numerical psdfs of the modal force are calculated for each simulation and then averaged over 100 simulations. Concerning GESM, three different psdfs are considered:

- the psdf obtained from Eq. (6), assuming $p_N(n)$ in Eq. (2) as a normal distribution, with mean value from Eq. (4) and std corresponding to 0.18 Hz [3] (GESM);
- the psdf obtained from Eq. (6), assuming $p_N(n)$ as a normal distribution, with mean value and std obtained from numerical simulations in Table 1 (GESM$_{sim}$);
- the psdf obtained from Eq. (8), valid for unrestricted pedestrian traffic, assuming $p_N(n)$ as a normal distribution, with mean value and std obtained from numerical simulations in Table 1 (GESM$_{unr}$).

Figure 5 plots the comparison between the modal psdfs obtained from numerical simulations and the analytical estimates obtained from the GESM. From Figure 5, it can be deduced that the original formulation (GESM) provides a psdf of the modal force which is far from the one obtained numerically from crowd simulations: the difference comes mainly from the significant difference between the mean step frequency obtained from crowd numerical simulations and the ones provided by Eq. (4), already remarked in Figure 4; furthermore, Table 1 shows that the numerically obtained standard deviation of the step frequency is much smaller than the value assumed in the GESM. The analytical prediction provided by the GESM with mean and std of the step frequency obtained from numerical simulations ($GESM_{sim}$) is in accordance with the numerical result for low crowd density (unrestricted pedestrian traffic), while it greatly overestimates the numerical results for higher crowd densities. The analytical solution that provides results in better accordance with numerical simulations is the one obtained from GESM with numerically estimated mean and std of the step frequency under the hypothesis of unrestricted pedestrian traffic ($GESM_{unr}$). Thus, it seems that interaction among pedestrians can be globally modelled assuming a psdf of the modal load provided by the GESM for unrestricted traffic, adopting a pdf of the step frequency coherent with simulations, i.e. taking into account the reduction of its mean value and std on increasing crowd density.

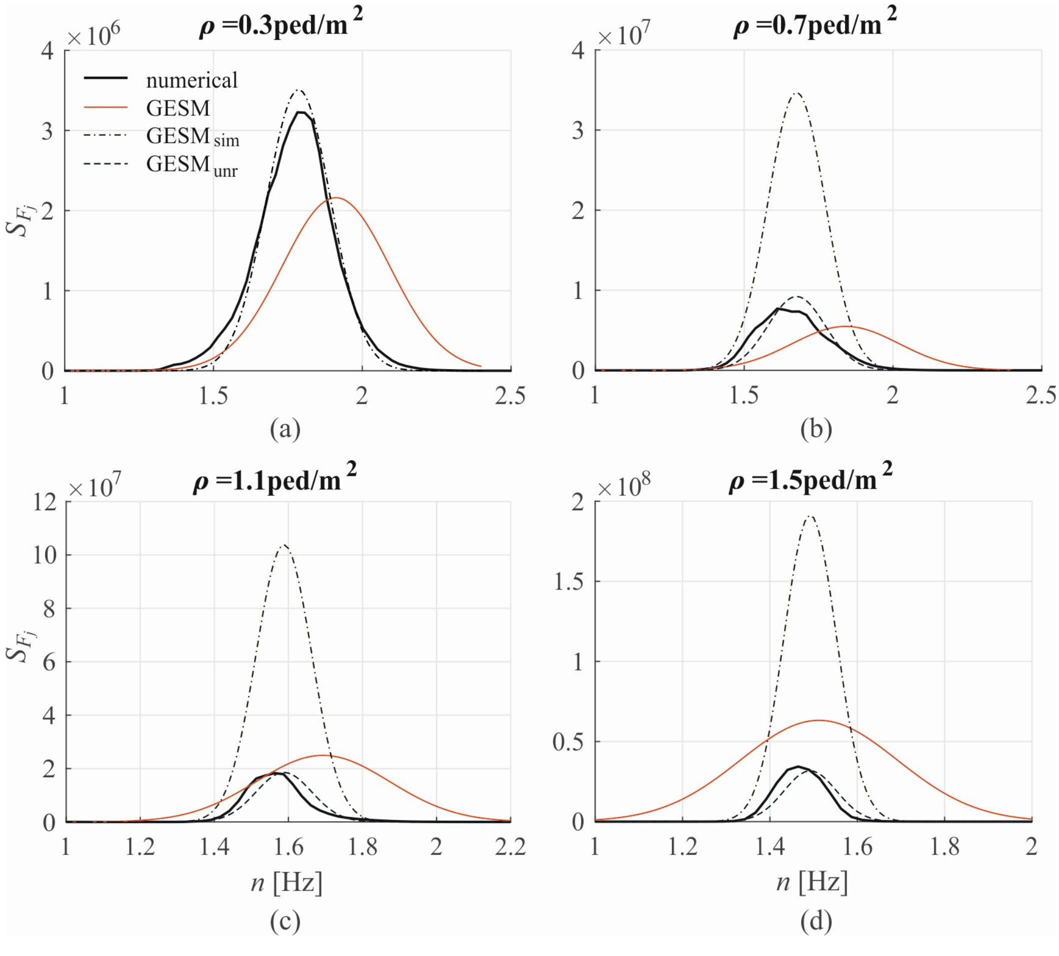

Figure 5: Comparison between psdfs of the modal force obtained from numerical simulations and GESM predictions.

5 CONCLUSIONS AND PROSPECTS

This paper has studied pedestrian-induced forces on footbridges in different traffic conditions. Pedestrian flows have been simulated numerically with the commercial software MassMotion, based on the social force model, and the instantaneous step frequency of pedestrians has been derived from their velocity. The probabilistic analysis of the step frequency has revealed that, in accordance with the Weidmann law, the mean step frequency tends to decrease on increasing crowd density but numerically obtained walking velocities are lower. Furthermore, also the standard deviation of the step frequency tends to decrease in crowded conditions.

The comparison between the power spectral density function of the modal force obtained from numerical simulations and the analytical expression provided by the GESM has shown that the correlation among pedestrians can be modelled through the GESM including density-dependent values of the mean and standard deviation of the step frequency. The coherence model originally proposed highly overestimates the correlation among pedestrians and provides very large values of the modal force, if compared with numerical simulations. Assuming the coherence model proposed by the GESM for unrestricted pedestrian traffic provides results in accordance with numerical simulations.

The results here obtained are referred to a single deck geometry. Further analyses investigating the influence of the deck width on the equivalent loading spectral properties are necessary. Furthermore, the attention is here focused on the spectral properties of the modal load: a strategy to directly characterize the coherence function of the equivalent distributed loading is under investigation.

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