Optimization of diagrid geometry based on the desirability function approach

Research Article

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Abstract: Diagrids represent one of the emerging structural systems employed worldwide for the construction of high-rise buildings. Their potential relies on the peculiar architectural effect and their great lateral stiffness. Because of the modular nature of the diagrid triangular element, optimization processes are usually carried out to assess the best arrangement of the external diagonals in order to enhance the structural performance while using the lowest amount of structural material. In this contribution, we make use for the first time of the desirability function approach to investigate the optimal geometry of the diagrid system. A 168-meter tall building, with four different floor shapes, is analyzed, and the inclination of the external diagonals is varied between 35° and 84°. The desirability function approach is applied to find the most desirable geometry to limit both the lateral and torsional deformability, the amount of employed material as well as the construction complexity of the building. A sensitivity analysis is also carried out to investigate the influence of the individual desirability weight on the obtained optimal geometry. The effect of the building height is finally evaluated, through the investigation of sets of 124-, 210- and 252-meter tall diagrid structures.

1 Introduction

In the last decades, the realization of tall buildings around the world has experienced an intense growth. Without neglecting the importance of economic issues, attention should be paid to the sustainability related to such a persistent construction process [1]. From a structural viewpoint, the need for sustainability has led designers and researchers to a deep investigation of the most suitable solutions and the most recent developments in the field of tall buildings [2, 3]. One of the most efficient system for the realization of tall buildings up to ~150 stories has relied on the diagrid tube. It is a tubular structure, placed over the exterior of the building, made up of diagonals which are designed to carry both the horizontal and gravity loads [4–7]. The diagonals can span across several floors and their spatial arrangement allows the realization of complex-shaped structures with remarkable architectural effects.

Based on the pioneering work of Moon *et al.* [8], the most common procedure for the preliminary design of diagrid systems has usually followed a stiffness-based approach. The sizing process of the diagonals is carried out by minimizing the horizontal displacement of the building when subjected to lateral loads, in order to fulfill the requirements of international codes (typically, the maximum lateral deflection at the top of the building should be lower than $H/500$, being H the total height of the structure). Zhang *et al.* [9] made use of the stiffness-based methodology for the analysis of diagrid tubes composed of straight diagonals with gradually varying angles, finding the optimal inclination. The same approach was also used to investigate the structural performance of diagrid tubes made up of curved diagonals by Zhao and Zhang [10]. The stiffness-based method was also applied by Liu and Ma [11], who proposed a modular method (MM) for the calculation of the bending and shear stiffness of polygonal diagrid tubes. More recently, Lacidogna *et al.* [12] developed a matrix-based method (MBM) for the structural analysis of diagrid structures, which allows to take into account general geometries. Moreover, the MBM provides information regarding both the bending, shear, torsional and axial deformability of the whole diagrid building.

Being composed of periodic modular units, *i.e.* the basic triangular module, diagrid systems are suitable for optimization procedures that aim at limiting the lateral building deflection as well as the amount of employed material [8–10, 13, 14]. Several researchers have investigated different geometrical configurations in order to optimize the structural performance of the diagrid. These proce-

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dures have been typically carried out by means of Finite Element (FE) calculations, by varying the arrangement of the diagonals along the height of the building. Montuori *et al.* [15] analyzed the structural performance of diagrid square tubes with different diagonal patterns along the building, namely uniform-angle, variable-angle and variable-density patterns. Tomei *et al.* [16] also examined other pattern configurations, like the double-density pattern and the diagrid-like pattern, where the diagonals follow the principal stress lines obtained from an equivalent cantilever building. Angelucci and Mollaioli [17] investigated the response of diagrid structures with non-uniform pattern configurations, also simulating the presence of outriggers inducing a local increase in the density of the diagonals. Mirniazmandan *et al.* [18] used Genetic Algorithms coupled with FE modeling to explore the optimal geometrical solution of diagrid systems, when changing both the diagonal inclination and the floor shape. Mele *et al.* [19] investigated the effect of the diagrid slenderness on the structural behavior and the optimal design parameters. More recently, Lacidogna *et al.* [20] made use of the previously developed MBM in order to explore both the lateral and torsional behavior of diagrid structures, by changing the floor shape and the inclination of the external diagonals. The MBM was also recently used to analyze the effect that an internal closed- or open-section concrete core has on the global building response [21].

Most of the optimization procedures carried out in the literature rely on posing the constraint on the lateral deflection (typically $H/500$) and obtaining the optimal solution as the one that minimizes the structural weight. However, in recent studies it was shown that other responses characterize the structural behavior of the diagrid and might then influence the choice of the optimal solution. For example, Lacidogna *et al.* [20, 21] recently showed that the optimal diagrid geometries that minimize the lateral deflection of the building are not the same that allow to minimize the torsional rotations. The latter is minimum when the diagonals are very shallow, whereas the former gets minimized when the diagonal inclination lies in an intermediate range that depends on the building aspect ratio [20]. Furthermore, Tomei *et al.* [16] pointed out that each diagrid pattern has its own complexity, that needs to be minimized in order for the structure to be achievable from a construction perspective. Based on these considerations, it follows that multiple responses (lateral deflection, torsional rotation, structural mass and construction complexity) need to be minimized simultaneously in order to reach the optimal stiff, light and feasible diagrid geometry.

The desirability function approach is one of the most widely used methodology in multi-response optimization due to its simplicity. Firstly formulated by Harrington in 1965 [22], it found extensive use in multi-response problems in the form proposed by Derringer and Suich in 1980 [23], with applications ranging from industrial engineering to applied science. The desirability function approach is based on the assignment of a score between 0 and 1, called the individual desirability $d_{i,p}$, to the i^{th} combination of input parameters with respect to the p^{th} response variable. Then, an overall desirability (*OD*) is assigned to each i^{th} combination of variables based on the calculated individual desirability values. The *OD* values finally allow to select the optimal solution among the sample.

In this paper, we apply for the first time the desirability function approach to the problem of finding the optimal geometry of diagrid systems. Different diagonal inclinations and floor shapes are considered for the diagrid tall building, that represent the different combination of input parameters, and four response variables are obtained for each geometry, namely the lateral deflection and torsional rotation at the top of the building under horizontal loads, the mass of the external diagrid tube and the diagrid complexity as suggested by Tomei *et al.* [16]. Based on these four response variables, each diagrid geometry is assigned an individual desirability value based on the minimization of each response. The final *OD* is then calculated for each diagrid solution, allowing to discuss the optimal shape leading to the stiffest, lightest and least complex geometry. Eventually, the influence of the weight of the different responses is also investigated on the obtained results, as well as the influence of the building aspect ratio.

2 Methods

Here we investigate the optimal diagrid geometry (diagonal inclination and floor shape) for the 168-meter tall building considered in [20]. In particular four floor shapes (square, hexagon, octagon, circle) and six different diagonal inclinations are analyzed (Figure 1). The diagonal inclinations are related to the different number of floors that lie within the diagrid module, *i.e.* 1, 2, 3, 4, 6 and 12. Note that, in this analysis, the diagrid module corresponds to the triangular unit. Based on the different combination of the number of intra-module floors and floor shape, twenty-four different diagrid geometries are generated as reported in Table 1.

The plan dimensions of the external diagrid tubes are reported in Figure 1b, while the total height and inter-story height of the building are 168 m and 3.5 m respectively. The diagrid structure is made of steel, with an elastic modu-

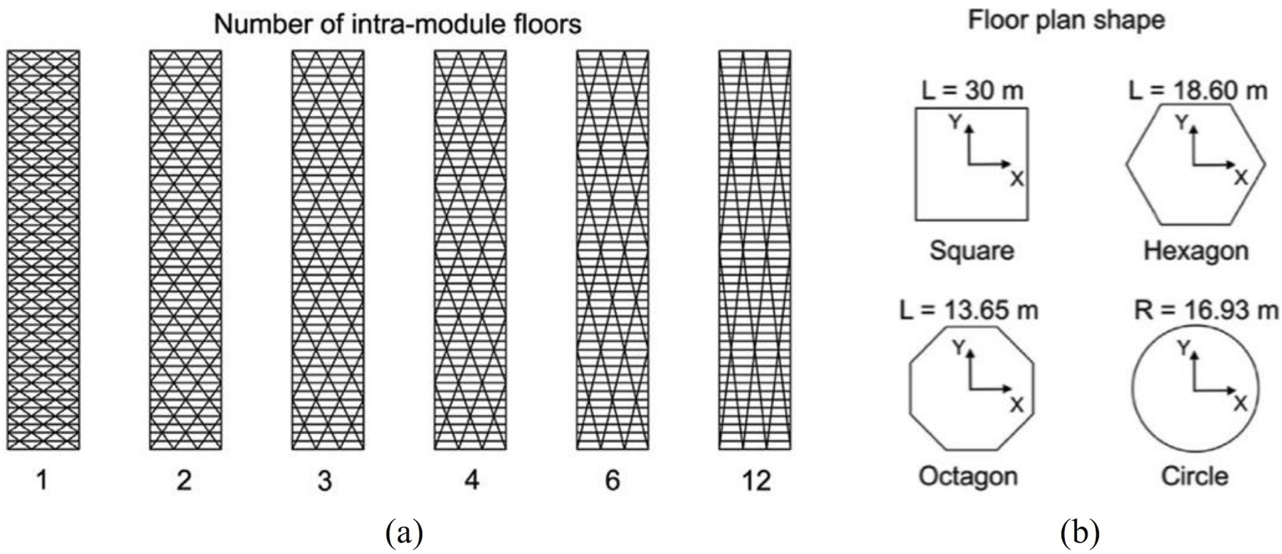


Figure 1: Diagrid geometries with different (a) diagonal inclination and (b) floor shape [20]

Table 1: Twenty-four diagrid structures with different diagonal inclination and floor shape

Number of intra-module floors Floor shape	1	2	3	4	6	12
Square	S1	S2	S3	S4	S6	S12
Hexagon	H1	H2	H3	H4	H6	H12
Octagon	O1	O2	O3	O4	O6	O12
Circle	C1	C2	C3	C4	C6	C12

lus of 210 GPa and mass density of 7.8 ton/m^3. The cross-sectional areas of the external diagonals are linearly variable along the height of the building, with a maximum value of 1000 cm^2 at the ground module and 100 cm^2 at the upper module (see Appendix A1 in [20]). The building is subjected to a uniform horizontal load of 30 kN/m along the X axis and a uniform torque load of 70 kNm/m. Based on the different diagrid geometry, these distributed loads are converted into concentrated horizontal forces and in-plane torque moments acting at the level of the floor centroids.

As mentioned in the Introduction, four response variables are considered in this study to seek the optimal diagrid geometry, namely the top lateral deflection, top torsional rotation, total mass of the diagrid and a synthetic index that is related to the construction complexity. The lateral deflection and torsional rotation are computed by means of the MBM, previously developed by the Authors [12, 20]. The total mass of the diagrid is simply calculated based on the steel unit density and the dimensions and geometrical arrangement of the diagonals. Finally, the diagrid complexity is evaluated according to the complexity index (*CI*) proposed by Tomei *et al.* [16]. For each diagrid geometry, the *CI* is computed based on five metrics, *i.e.* N_1, N_2, N_3, N_4 and N_5. These are related to the construction complexity of the structure and are defined by Tomei *et al.* [16] as follows: N_1 is the weighted number of nodes, *i.e.* the number of joints of the pattern multiplied by a numerical coefficient, differently attributed on the basis of the joint connectivity (number of connecting members); N_2 is the number of different cross-sections utilized for the diagonals in the pattern; N_3 represents the number of splices required for the diagonals in the pattern, calculated assuming a maximum member length of 12 m; N_4 is the number of diagonals of the pattern; N_5 is the number of different lengths of diagonal members in the pattern.

In this work we applied the same definition of the five metrics above with one minor difference regarding N_1. Instead of considering the weighted number of nodes based on the joint connectivity, due to the fact that we do not necessarily know the connectivity degree of the nodes that connect the diagonal to the intra-module floors, we just considered the total number of diagrid panel nodes, *i.e.* only the nodes connecting the diagonals. After the five metrics defined above have been computed for each diagrid geometry, each metric is normalized to the maximum value among all the different geometries. Finally, the sum of the normalized parameters gives the *CI* of each geometry [16], *i.e.*:

$$CI_i = \sum_{j=1}^{5} \frac{N_j}{\max_i N_j}. \quad (1)$$

It is clear that high values of the five metrics involve greater values of the *CI*, meaning higher construction complexity.

Once the four response variables (lateral deflection δ, torsional rotation ϕ, mass M and *CI*) are computed for each diagrid geometry, the goal is to find the solution that minimizes them all. As a matter of fact, minimizing lateral deflections and torsional rotations is important for the building safety and serviceability, limiting the amount of structural material is pivotal for sustainability purposes, while minimizing the *CI* leads to an easier and faster construction process. Hence, the multi-response optimization is carried out by means of the desirability function approach [22, 23].

The desirability function approach is widely used in industrial engineering and other fields for the optimization of multi-response processes and is based on finding the conditions that lead to the most desirable responses. In this case, the optimal diagrid solution should involve the lowest values of the four response variables (δ, ϕ, M and *CI*). The desirability function approach yields the definition of the individual desirability value $d_{i,p}$ associated to the i^{th} geometrical solution, *i.e.* S1, S2, …, C12 (Table 1), with respect to the p^{th} response variable, *i.e.* $p = \delta$, ϕ, M, CI. The individual desirability $d_{i,p}$ can be expressed as follows:

$$d_{i,p} = \left(\frac{\max_i p_i - p_i}{\max_i p_i - \min_i p_i} \right)^{r_p}, \quad (2)$$

being p_i the value of the p^{th} response variable for the i^{th} geometrical solution, $\max_i p_i$and $\min_i p_i$ the maximum and minimum values of the p^{th} response variable across all the geometrical solutions, and r_p the exponent of the individual desirability $d_{i,p}$ related to the p^{th} response variable.

Based on Equation (2), it can be inferred that the geometrical solution that provides the minimum value of the p^{th} response variable across all the solutions is assigned an individual desirability $d_{i,p}$ equal to 1, whereas the solution that provides the maximum value of the p^{th} response variable exhibits an individual desirability $d_{i,p}$ equal to 0. This means that we assign an individual score of 1 to the diagrid geometry that minimizes the particular response variable (δ, ϕ, M, *CI*), while we assign a score of 0 to the solution that provides the highest value of the response variable. Note that these extreme solutions do not depend on the exponent r_p. Conversely, all the other diagrid geometries are assigned an individual desirability score between 0 and 1, whose value also depends on r_p. The higher the individual desirability $d_{i,p}$, the better the performance of the i^{th} geometry to minimize the p^{th} response variable.

Once the individual desirability values have been computed for each geometrical solution and response variable, the overall desirability of the i^{th} diagrid geometry, OD_i, can be calculated as:

$$OD_i = \prod_{p=1}^{k} \left(d_{i,p}\right)^{1/k} = \left(d_{i,\delta} d_{i,\phi} d_{i,M} d_{i,CI}\right)^{1/4}, \quad (3)$$

being k the total number of response variables considered ($k = 4$ in this study). The application of Equation (3) directly provides a global score for each i^{th} diagrid geometry that depends on the individual desirability values obtained from Equation (2). High values of OD_i imply high performance of the i^{th} solution with respect to all the considered variables, whereas low values of OD_i imply low performance. Note that, according to Equation (3), if one individual desirability $d_{i,p}$ is 0, $OD_i$is directly equal to 0, no matter the value of the other individual desirability values. Conversely, to obtain OD_i equal to 1, it is necessary that all the individual desirability values $d_{i,p}$ reach 1, meaning that the i^{th} solution is the optimal one with respect to the all the response variables. Usually, the values of OD_i lie in between. In the following Section, the results that arise from Equations (2) and (3) are reported for the 168-meter diagrid tall building, in order to find the optimal geometry among the ones considered in Figure 1 and Table 1.

For sake of simplicity, the analysis has been initially carried out by considering a unit value for each exponent r_p, *i.e.* $r_\delta = r_\phi = r_M = r_{CI} = 1$, thus assuming a linear distribution of the individual desirability with respect to the response variables and assigning the same weight for the different response variables. However, a sensitivity analysis has also been carried out afterwards to investigate the influence of these exponents, *i.e.* $r_\delta \neq r_\phi \neq r_M \neq r_{CI} \neq 1$, on the obtained optimal geometries. Finally, the analysis has also been extended to 126-, 210- and 252-meter tall buildings, in order to investigate the influence of the building aspect ratio on the results.

3 Results and discussion

Considering the twenty-four diagrid geometries shown in Table 1 and applying the procedure presented in the previous Section, the four response variables (δ, ϕ, M, *CI*) were obtained as reported in Table 2.

Table 2: Response variables (δ, ϕ, M, CI) for the twenty-four diagrid geometries (minimum values for each floor shape are in *italic*, absolute minimum values are in ***bold italic***)

i^{th} solution	δ [cm]	ϕ [10^{-5} rad]	M [ton]	N_1 [-]	N_2 [-]	N_3 [-]	N_4 [-]	N_5 [-]	CI [-]
S1	28.1	*3.8*	3016	576	48	***0***	1152	***1***	4.00
S2	10.5	5.3	2126	288	24	***0***	576	***1***	2.50
S3	***8.6***	8.8	1916	192	16	***0***	384	***1***	***2.00***
S4	8.7	13.8	1837	144	12	288	288	***1***	2.75
S6	11.0	28.9	1778	96	8	192	192	***1***	2.17
S12	29.0	124.2	*1742*	***48***	***4***	288	***96***	***1***	2.25
H1	25.3	*3.3*	2876	576	48	***0***	1152	***1***	4.00
H2	10.3	4.9	2077	288	24	***0***	576	***1***	2.50
H3	*8.8*	8.5	1892	192	16	***0***	384	***1***	***2.00***
H4	9.2	13.5	1823	144	12	288	288	***1***	2.75
H6	11.9	28.6	1772	96	8	192	192	***1***	2.17
H12	32.8	123.9	*1740*	***48***	***4***	288	***96***	***1***	2.25
O1	24.0	*3.1*	2837	576	48	***0***	1152	***1***	4.00
O2	10.0	4.8	2063	288	24	***0***	576	***1***	2.50
O3	*8.7*	8.4	1885	192	16	***0***	384	***1***	***2.00***
O4	9.2	13.4	1819	144	12	288	288	***1***	2.75
O6	12.2	28.5	1770	96	8	192	192	***1***	2.17
O12	34.0	123.7	*1740*	***48***	***4***	288	***96***	***1***	2.25
C1	23.0	***3.0***	2790	576	48	***0***	1152	***1***	4.00
C2	10.0	4.8	2047	288	24	***0***	576	***1***	2.50
C3	*8.8*	8.4	1877	192	16	***0***	384	***1***	***2.00***
C4	9.4	13.6	1814	144	12	288	288	***1***	2.75
C6	12.6	29.0	1768	96	8	192	192	***1***	2.17
C12	35.8	126.3	***1739***	***48***	***4***	288	***96***	***1***	2.25

The second column shows the obtained lateral displacements at the top of the building due to the lateral load, based on the MBM. As can be seen, the top lateral deflection of the diagrid is strongly dependent on the number of intra-module floors, *i.e.* on the diagonal inclination. The influence of the floor shape is less important. Based on the obtained results, it is found that the geometrical solutions that minimize the lateral displacements are always the ones with three intra-module floors (S3, H3, O3, C3), that correspond to a diagonal inclination of 64°–67°. Among these, the stiffest solution that minimizes the lateral deflection corresponds to the S3. Conversely, the solutions with twelve intra-modules floors (S12, H12, O12, C12), *i.e.* diagonal angles of 83°–84° are the ones providing the highest lateral deflection. Among these, the most flexible one is C12. As will be seen below, based on Equation (2), the geometrical solution S3 will have the highest individual desirability value with respect to the lateral displacement ($d_{S3,\delta} = 1$), whereas the solution C12 will exhibit a null individual desirability value ($d_{C12,\delta} = 0$). The other solutions will be assigned an individual desirability lying between these values according to Equation (2).

Similarly, the third column of Table 2 reports the computed torsional rotation at the top of the building due to the external torque moments, as obtained from the MBM. From the results, it can be inferred that the lowest torsional rotation is always provided by the geometrical solutions with the lowest number of intra-module floors (S1, H1, O1, C1), thus corresponding to the shallowest diagonal inclination (35°–38°). Among these, the stiffest solution is the circular diagrid tube C1, which provides the highest torsional rigidity. Conversely, the highest torsional rotations are obtained for the diagrid structures with the highest number of intra-module floors (S12, H12, O12, C12), the maximum one obtained with the solution C12. Accordingly, based on Equation (3), we will obtain the highest individual desirability value for the solution C1 ($d_{C1,\phi} = 1$) and the lowest value for the geometry C12 ($d_{C12,\phi} = 0$). Again, the other solutions will exhibit desirability values in between according to Equation (2).

From the results obtained in these first two columns, it can be inferred that the different flexibilities (lateral and torsional) are minimized by different geometrical solutions. This point has already been addressed by the Authors in [20]. The lateral deflection is usually minimized by intermediate values of the diagonal angle, due to the competition between shear and bending rigidity. The former is maximum for shallow angles (around 35°) whereas the latter is maximum for diagonal inclinations of 90°. Due to the fact that the lateral deformability of the diagrid building is governed by both the shear and bending deformation of the diagrid module, an intermediate angle between these two is often found to provide the maximum lateral rigidity. The optimal angle is also shown to depend on the building aspect ratio, since this one governs the different involvement of shear over bending rigidity, the former being more involved in shorter buildings, the latter in taller buildings. Conversely, the torsional rigidity of the building only depends on the shear rigidity of the diagrid module, therefore it is maximum for very shallow diagonals [20]. These considerations make the choice of the optimal geometry difficult, as one should limit both the lateral and torsional flexibility of the structure. To this purpose, the desirability function approach seems an effective yet simple way to tackle this problem.

The fourth column of Table 2 reports the total steel mass of the external diagrid tube, which is directly calculated based on the steel density and the actual diagrid geometry. The solutions with higher numbers of intra-module floors (S12, H12, O12, C12) involve the lowest amount of employed material. This is simply due to the fact that, when the diagonal inclination is very steep, the density of the diagonals in the patterns gets remarkably lower, as can be appreciated by Figure 1a. Based on the mass response, the highest individual desirability score is assigned to the solution C12 ($d_{C12,M}$ = 1), whereas the lowest one to the solution S1 ($d_{S1,M}$ = 0).

Finally, the last columns of Table 2 report the five metrics N_1, N_2, N_3, N_4 and N_5 that are used to calculate the complexity index. Note that, although in the previous cases the variation of the first three responses (δ, ϕ, M) among the different floor shapes was not so evident, in this case the five metrics N_1, N_2, N_3, N_4 and N_5 do not vary at all with respect to the floor shape, being only dependent on the diagonal inclination. N_1 represents the total number of diagrid nodes, therefore it is minimum for the solutions S12, H12, O12 and C12, while it is maximum for S1, H1, O1 and C1. Similarly, N_2 is the number of different diagonal cross-sections used in the pattern, thus in this case it corresponds to the number of diagrid modules, as each module has its own cross-sectional area. Therefore, it is minimum for S12, H12, O12 and C12, while it is maximum for S1, H1, O1 and C1. N_3 takes into account the maximum diagonal length of 12 meters for transportability issues, and it is found to be minimum for all the solutions with one, two and three intra-module floors, while it is higher for steeper diagonals. N_4 represents the number of diagonals in the pattern and it is found to be minimum for the solutions S12, H12, O12 and C12, while it is maximum for S1, H1, O1 and C1. Finally, N_5 takes into account the different lengths of the diagonals in the pattern. In this case, it is equal to one for each solution, as each pattern has all the diagonals with the same length, having a constant inclination across the building height.

Based on the evaluation of N_1, N_2, N_3, N_4 and N_5, Equation (1) is applied to compute the *CI* of each geometrical solution, obtaining the results reported in the last column of Table 2. According to what already reported above, no variation is found for this response variable across the different floor shapes. Conversely, it can be seen that the diagrid solutions that minimize the *CI* are the ones with three intra-module floors ($d_{S3,CI} = d_{H3,CI} = d_{O3,CI} = d_{C3,CI} = 1$), whereas the ones that maximize the construction complexity are the ones with one intra-module floor ($d_{S1,CI} = d_{H1,CI} = d_{O1,CI} = d_{C1,CI} = 0$). The other geometrical solutions exhibit *CI*s that lie in between these values.

Based on the response variables reported in Table 2, Equation (2) has been applied to calculate the individual desirability value for each geometrical solution referred to each response variable. The results are shown in Table 3, calculated by adopting a unit value of the exponent r_p for all the responses, *i.e.* $r_\delta = r_\phi = r_M = r_{CI} = 1$. The obtained individual desirability values are also represented in graphical form in Figure 2a. As can be seen, the influence of the floor shape is negligible, whereas the diagonal inclination has a strong influence on the individual desirability values for each given floor shape.

Finally, the individual desirability values are combined together to obtain the *OD* according to Equation (3). The results are reported in the last column of Table 3 and are represented graphically in Figure 2b. As can be seen from the obtained *OD* values, the most desirable solution ($OD_{max} = OD_{C3} = 95.94\%$) is C3, thus the circular diagrid building with three intra-module floors, corresponding to a diagonal inclination of 67°. This result arises from the fact that the solution C3 is indeed one of the best performing with respect to all the four response variables. As a matter of fact, this geometrical solution allows to reach very low lateral deflections ($d_{C3,\delta} = 99.32\%$) and torsional rotations ($d_{C3,\phi} = 95.62\%$), it is also highly desirable with respect to the minimization of the structural weight

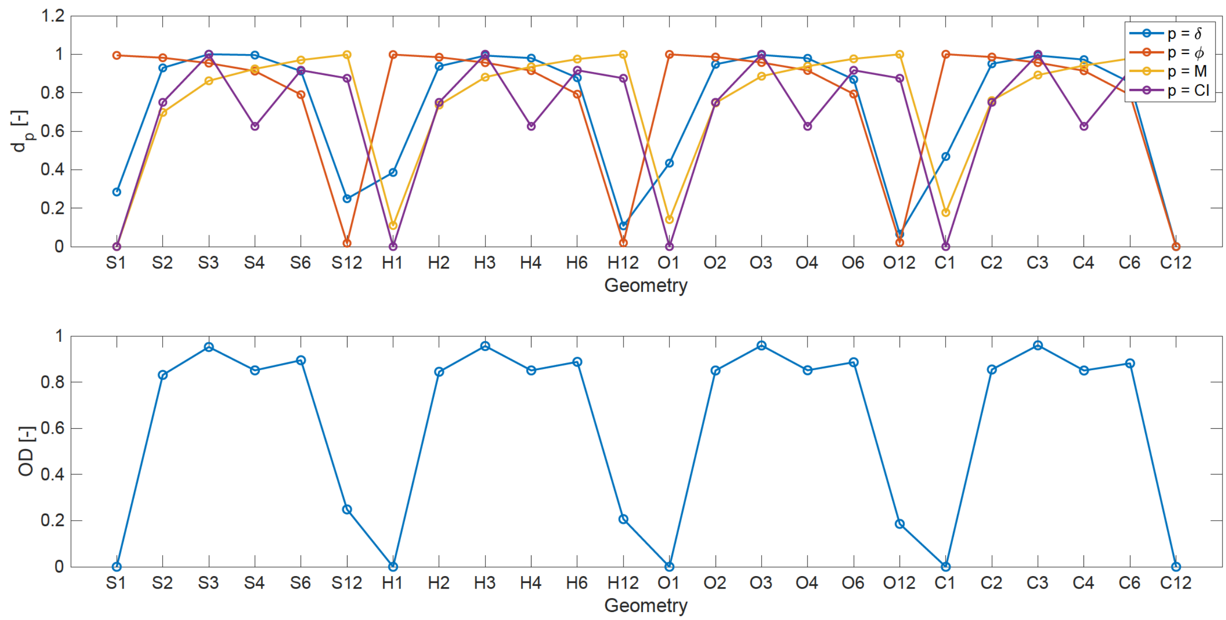


Figure 2: (a) Individual desirability values for the four response variables (r_p = 1) and (b) OD values

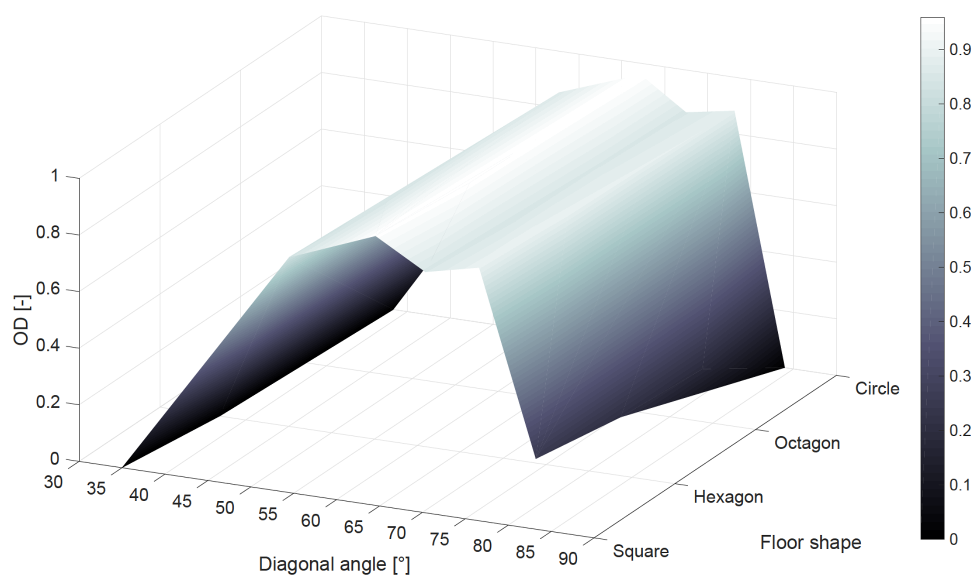


Figure 3: Surface representation of the $OD$with respect to the diagrid geometrical parameters (diagonal angle and floor shape). ODvalues are reported in the vertical axis and represented by means of color shades

($d_{C3,M}$ = 89.19%) and it is one of the best structures from a construction complexity perspective ($d_{C3,CI}$ = 100.00%).

Note that the other solutions with three intra-module floors and different floor shapes, *i.e.* S3, H3 and O3, provide similar values of OD: OD_{S3} = 95.21%, OD_{H3} = 95.62% and OD_{O3} = 95.86%. This confirms what already reported above, *i.e.* the influence of the floor shape on the optimal diagrid geometry is less important. This can also be seen from Figure 2b, where the OD graph shows a similar trend for the different floor shape. Moreover, Figure 3 reports a surface representation of the OD values with respect to the diagonal inclinations and floor shapes. From the figure, it is evident that most of the OD variation occurs with respect to the diagonal inclination, whereas the surface is almost cylindrical in the direction of the floor shape axis.

The ODdrops to lower values for different number of intra-module floors. The solutions with one intra-module floor (S1, H1, O1, C1) have always an OD equal to 0, due to the fact that, despite their high torsional rigidity ($d_{i,\phi}$ ∼ 99–100%), they are quite flexible under lateral

Table 3: Individual and overall desirability values for the four response variables ($r_p = 1$)

i^{th} solution	$d_{i,\delta}$ [-]	$d_{i,\phi}$ [-]	$d_{i,M}$ [-]	$d_{i,CI}$ [-]	OD_i [-]
S1	0.2840	0.9941	0.0000	0.0000	0.0000
S2	0.9292	0.9817	0.6975	0.7500	0.8311
S3	**1.0000**	0.9535	0.8618	**1.0000**	0.9521
S4	0.9958	0.9123	0.9237	0.6250	0.8510
S6	0.9124	0.7899	0.9696	0.9167	0.8946
S12	0.2490	0.0176	0.9979	0.8750	0.2486
H1	0.3839	0.9981	0.1096	0.0000	0.0000
H2	0.9375	0.9845	0.7359	0.7500	0.8448
H3	0.9931	0.9560	0.8806	**1.0000**	0.9562
H4	0.9799	0.9147	0.9348	0.6250	0.8507
H6	0.8776	0.7922	0.9747	0.9167	0.8878
H12	0.1078	0.0193	0.9992	0.8750	0.2065
O1	0.4338	0.9992	0.1404	0.0000	0.0000
O2	0.9477	0.9853	0.7465	0.7500	0.8503
O3	0.9964	0.9568	0.8858	**1.0000**	0.9586
O4	0.9791	0.9155	0.9378	0.6250	0.8514
O6	0.8689	0.7931	0.9761	0.9167	0.8861
O12	0.0650	0.0210	0.9996	0.8750	0.1859
C1	0.4686	**1.0000**	0.1771	0.0000	0.0000
C2	0.9504	0.9855	0.7590	0.7500	0.8545
C3	0.9932	0.9562	0.8919	**1.0000**	**0.9594**
C4	0.9721	0.9140	0.9413	0.6250	0.8503
C6	0.8533	0.7889	0.9777	0.9167	0.8813
C12	0.0000	0.0000	**1.0000**	0.8750	0.0000

loads ($d_{i,\delta} \sim$ 28–47%), quite heavy ($d_{i,M} \sim$ 0–18%) and very complex ($d_{i,CI}$ = 0%). The solutions with two intra-module floors (S2, H2, O2, C2) show *OD*s in the range 83–85%: their lateral and torsional rigidity is quite high ($d_{i,\delta}$~ 92–95 %, $d_{i,\phi}$ ~ 98–99%), but they are still not so light ($d_{i,M} \sim$ 70–76%) and easily constructible ($d_{i,CI}$ = 75%). The solutions with four intra-module floors (S4, H4, O4, C4) provide *OD*s around 85 %: they exhibit a great lateral rigidity ($d_{i,\delta} \sim$ 97–99%), a good torsional behavior ($d_{i,\phi} \sim$ 91%), quite low values of structural weight ($d_{i,M} \sim$ 92–94%), but they are still quite complex ($d_{i,\delta}$ = 62%). The solutions with six floors per module (S6, H6, O6, C6) show *OD*s in the range 88–89%: they show a lower lateral and torsional rigidity ($d_{i,\delta} \sim$ 85–91%, $d_{i,\phi} \sim$ 79%), although their mass and complexity responses show satisfactory desirability values ($d_{i,M} \sim$ 97–98%, $d_{i,CI}$ = 92%). Finally, the solutions with twelve intra-module floors lead to low *OD*s in the range 0–24%: despite their low structural weight and satisfactory complexity ($d_{i,M} \sim$ 100%, $d_{i,CI}$ = 87%), they are extremely flexible under lateral and torque actions ($d_{i,\delta} \sim$ 0–25%, $d_{i,\phi} \sim$ 0–2%).

Therefore, based on the results reported in Table 3 and Figures 2 and 3, the optimal diagrid solutions that simultaneously minimize the lateral and torsional flexibility, as well as the diagrid structural weight and the construction complexity can be selected. This approach, based on the desirability function, seems to be a powerful yet very simple tool to select the optimal geometry of the diagrid among a set of solutions and based on different responses.

However, the previous analysis was quite arbitrary as we chose $r_\delta = r_\phi = r_M = r_{CI} = 1$. This implicitly means assigning the four response variables the same importance in the definition of the optimal shape. For this reason, a parametric analysis has been also carried out by considering $r_\delta \neq r_\phi \neq r_M \neq r_{CI} \neq 1$, in order to investigate how the optimal diagrid geometry is influenced by the different weights assigned to the different response variables (δ, ϕ, *M*, *CI*).

In order to carry out the sensitivity analysis based on the weights r_p, eight values of r_p have been considered for each variable, namely 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00. Then, we obtain 8^4 = 4096 combinations of exponents, as synthetically shown in Table 4. Based on the

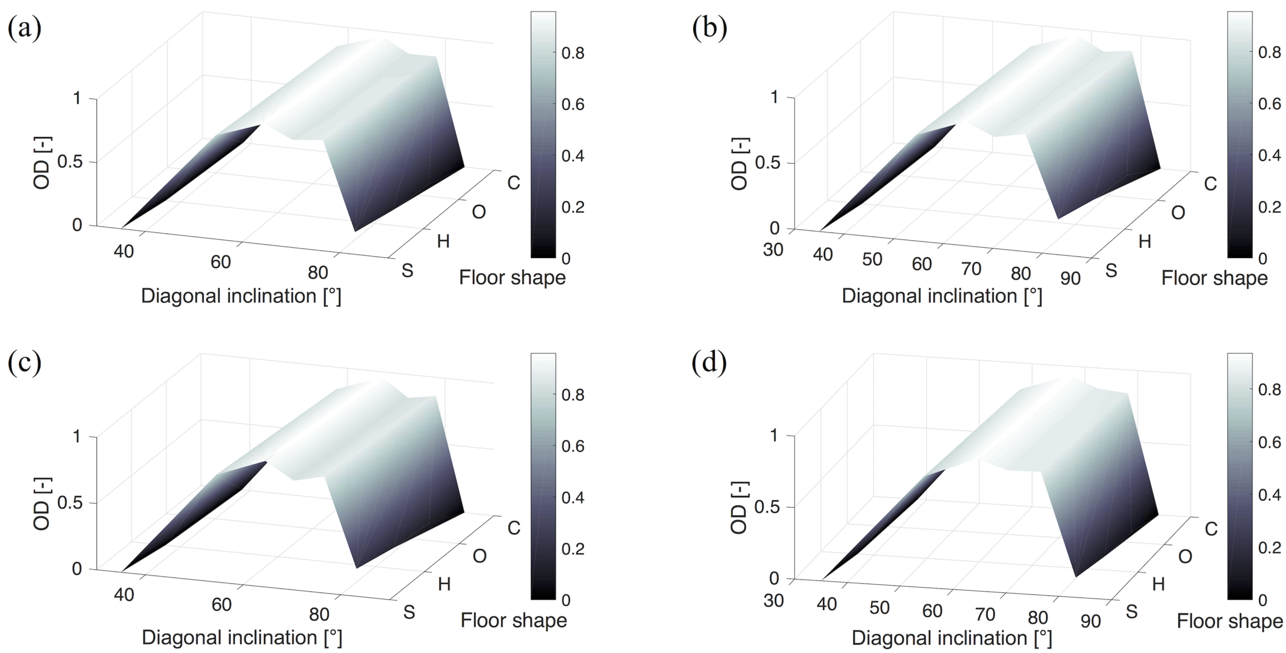


Figure 4: Surface representation of the OD with respect to the diagrid geometrical parameters obtained with different combinations of the individual desirability exponents: (a) $r_\delta = 2.00$, $r_\phi = 1.00$, $r_M = 1.00$, $r_{CI} = 1.00$; (b) $r_\delta = 1.50$, $r_\phi = 0.75$, $r_M = 1.25$, $r_{CI} = 1.00$; (c) $r_\delta = 1.50$, $r_\phi = 1.00$, $r_M = 1.00$, $r_{CI} = 1.25$; (b) $r_\delta = 2.00$, $r_\phi = 1.25$, $r_M = 1.75$, $r_{CI} = 0.75$

Table 4: The possible combinations by considering eight different exponents r_p for each response variable

Combination	r_δ [-]	r_ϕ [-]	r_M [-]	r_{CI} [-]
1	0.25	0.25	0.25	0.25
2	0.25	0.25	0.25	0.50
3	0.25	0.25	0.25	0.75
...	...	...	...	...
1755	1.00	1.00	1.00	0.75
1756	1.00	1.00	1.00	1.00
1757	1.00	1.00	1.00	1.25
...	...	...	...	...
4094	2.00	2.00	2.00	1.50
4095	2.00	2.00	2.00	1.75
4096	2.00	2.00	2.00	2.00

response variables obtained in Table 2, for each combination of exponents r_δ, r_ϕ, r_M and r_{CI}, the same analysis presented above can be carried out by applying Equations (2) and (3). Eventually, the optimal geometry can be found out, based on the maximum value of the obtained OD values.

As an example, Figure 4 shows four different OD surfaces based on four different sets of the exponents r_p. As can be seen, the four surfaces share many common features, meaning that the influence of the exponent r_p might not be so relevant for the purpose of determining the most desirable diagrid geometry. As a matter of fact, in all the cases, the optimal solution is associated with three intra-module floors, whereas the specific floor shape still has lower influence.

Figure 5 reports the obtained optimal geometry, expressed as relative frequency of occurrence out of the 4096 simulations. From the outcomes, it was obtained that the solution C3, which was assessed as the optimal geometry in the previous analysis (with $r_\delta = r_\phi = r_M = r_{CI} = 1$), is found as the optimal one for 3072 exponent combinations (75.00 % of the total cases). It was also found that, in 1000 simulations (24.41% of the total), the optimal geometry is the solution O3, which is the octagonal diagrid with three intra-module floors. This should not surprise as we have already seen in the previous analysis (with $r_\delta = r_\phi = r_M = r_{CI} = 1$) that the solution O3 ($OD_{O3} = 95.86\%$) was not so different from the C3 ($OD_{O3} = 95.94\%$). Therefore, out of 4096 combinations of exponents, 4072 cases (99.41% of the total) provided O3 or C3 as the optimal diagrid geometry, based on their lateral and torsional flexibility, structural mass and construction complexity.

The remaining 24 combinations (0.59% of the total) assigned the optimal geometry to the solution S6 (14 cases – 0.34%) and O6 (10 cases – 0.25%), which correspond to the square and octagonal geometry with six intra-module floors, respectively. However, these rare cases were found

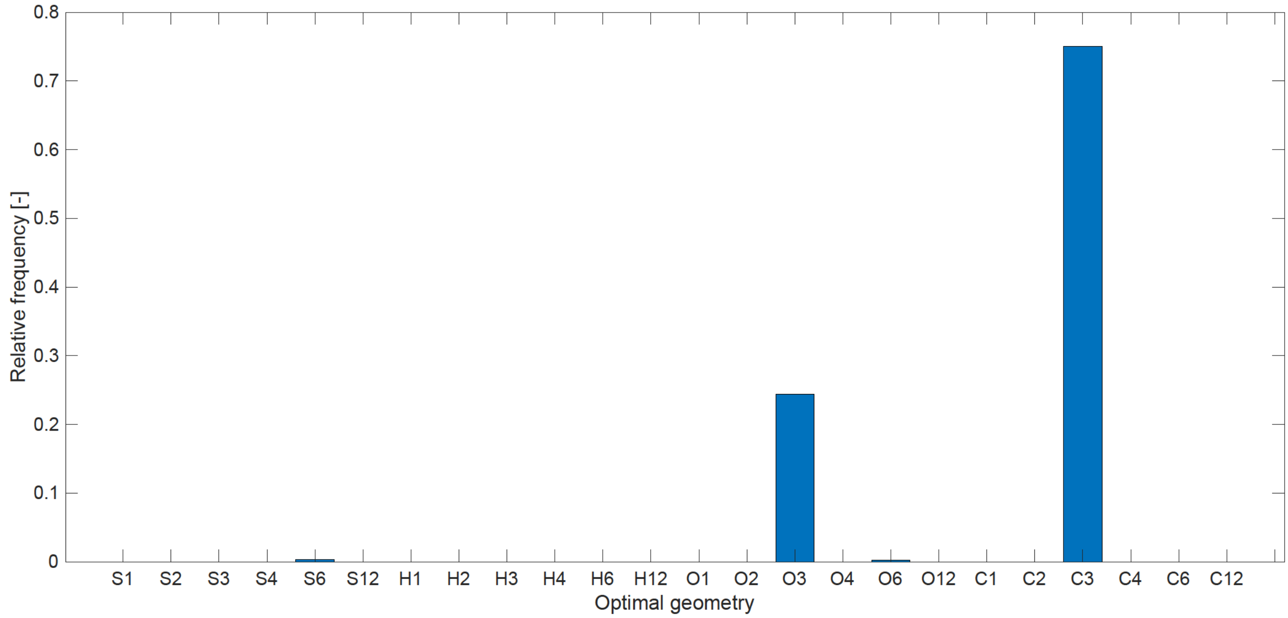


Figure 5: Optimal diagrid geometry based on 4096 simulations with different exponents of the individual desirability values

to correspond to highly unbalanced exponents, where the top lateral deflection and torsional rotation, *i.e.* the structural responses, were much underweighted than the diagrid mass and construction complexity, *i.e.* the geometrical responses.

In conclusion, the sensitivity analysis carried out here demonstrates that, for the investigated 168-meter tall diagrid building, the optimal diagonal inclination should always correspond to three intra-module floors in order to minimize both the lateral deflection, torsional rotation, structural weight and construction complexity. The floor shape seems to be less important, as already shown by the Authors in [20], although a slight bias towards curved floor shapes, *i.e.* circular and octagonal, has been obtained here.

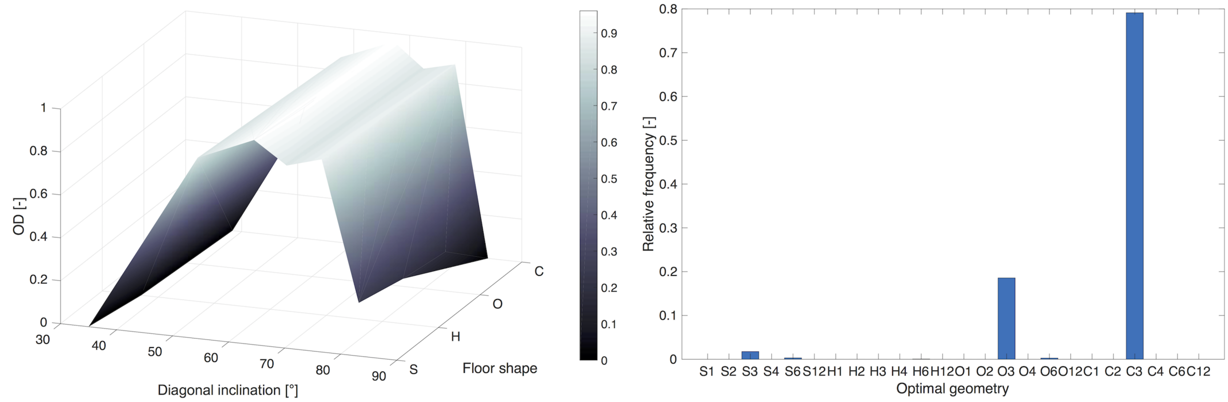
Based on the analysis carried out for the 168-meter tall building, the optimal diagrid geometry has also been investigated for the other buildings with different heights considered in [20], *i.e.* 126-, 210- and 252-meter tall structures. The geometrical features of these buildings are the same reported in [20], with six different diagonal inclinations and four floor shapes, for a total of twenty-four geometrical solution per building.

Figures 6a, 7a and 8a show the *OD* surface for the three buildings, obtained by applying Equations (2) and (3) and by considering $r_\delta = r_\phi = r_M = r_{CI} = 1$. In both cases, the optimal diagrid geometry is found to be associated with the solution C3, with OD_{C3} values of 96.04% for the 126-, 95.68% for the 210- and 95.52% for the 252-meter building. Also in these cases, the influence of the specific floor shape is found to be almost negligible, the diagonal inclination being the only parameter affecting the variation of the individual and overall desirability values.

The sensitivity analysis by varying the exponents r_p has been carried out as well, and the results are shown in Figures 6b, 7b and 8b for the three building heights. Similarly to Figure 5, these graphs report the obtained optimal diagrid geometry expressed as relative frequency of occurrence out of the 4096 combinations from Table 4. The results are similar to what already found for the 168-meter building investigated above.

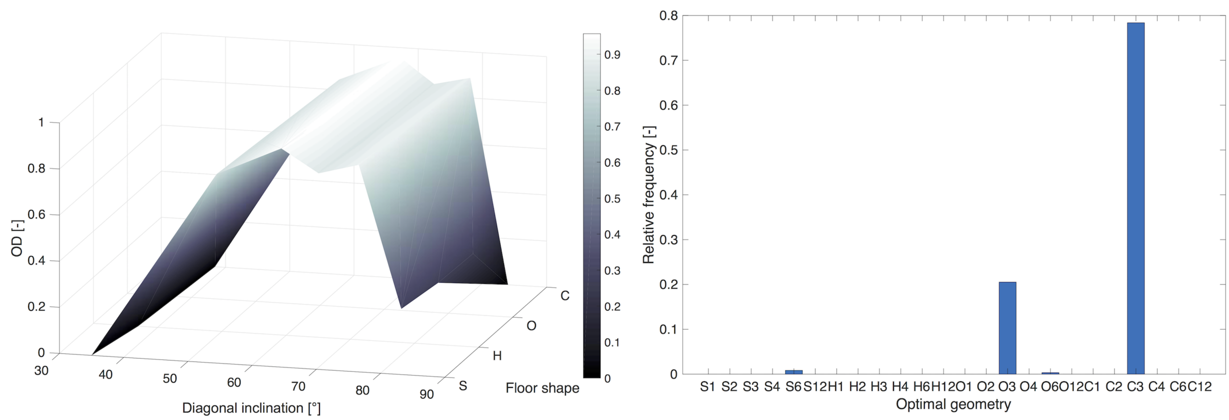
Specifically, for the 126-meter tall building (Figure 6b), the C3 solution is found to be the optimal one for 3240 combinations of the exponents r_p (79.10% of the total), the O3 solution is the optimal one for 760 cases (18.55% of the total), whereas the S3, S6, O6 and H6 geometries are assigned the highest overall desirability in 72 (1.76%), 12 (0.29%), 10 (0.25%) and 2 cases (0.05%), respectively. As can be seen, in 99.41% of the combinations the optimal solutions still refer to three intra-module floors, with a preference towards more curved floor shapes, whereas the solutions with six intra-module floors are to be preferred only in 0.59% of the cases. Similarly to what already reported above, these few cases often refer to very unbalanced combinations of the weight exponents, where the importance of the construction complexity and diagrid mass largely prevails over the minimization of the lateral and torsional deformability.

As for the 210-meter tall building, the results are shown in Figure 7b. Again, the C3 solution is found to be



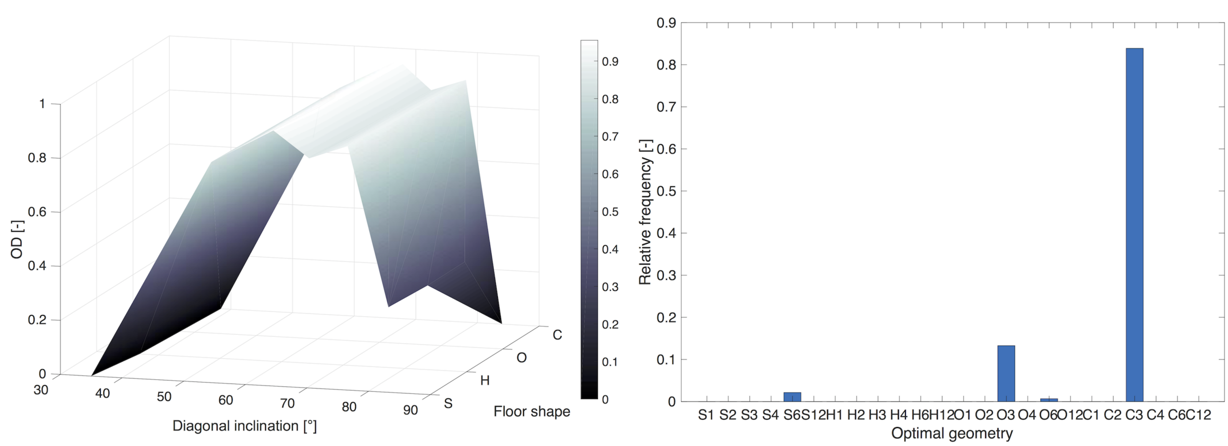
(a) (b)

Figure 6: Results for the 126-meter tall building: (a) surface representation of the $OD$with respect to the diagrid geometrical parameters obtained with $r_\delta = r_\phi = r_M = r_{CI} = 1.00$; (b) optimal diagrid geometry based on 4096 simulations with different exponents r_p



(a) (b)

Figure 7: Results for the 210-meter tall building: (a) surface representation of the OD with respect to the diagrid geometrical parameters obtained with $r_\delta = r_\phi = r_M = r_{CI} = 1.00$; (b) optimal diagrid geometry based on 4096 simulations with different exponents r_p



(a) (b)

Figure 8: Results for the 252-meter tall building: (a) surface representation of the OD with respect to the diagrid geometrical parameters obtained with $r_\delta = r_\phi = r_M = r_{CI} = 1.00$; (b) optimal diagrid geometry based on 4096 simulations with different exponents r_p

the most desirable one for 3209 combinations (78.34% of the total), the O3 solution is the optimal one for 840 cases (20.51% of the total), whereas the S6 and O6 geometries are assigned the highest overall desirability in 34 (0.83%) and 13 combinations (0.32%), respectively. In this case, 98.85% of the combinations lead to the optimal solutions with three intra-module floors, again with a preference towards more curved floor shapes, whereas the solutions with six intra-module floors are to be preferred only in 1.15% of the cases.

Finally, Figure 8b shows the outcomes related to the 252-meter tall building. Once again, the C3 solution is found to be the most desirable one for 3436 combinations (83.89% of the total), the O3 solution is the optimal one for 544 cases (13.28% of the total), whereas the S6 and O6 geometries are assigned the highest overall desirability in 88 (2.15%) and 28 combinations (0.68%), respectively. In this case, 97.17% of the combinations lead to the optimal solutions with three intra-module floors, whereas the solutions with six intra-module floors are to be preferred only in 2.83% of the cases.

From the results shown above, it is evident how the optimal geometry is only slightly affected by the specific set of weight exponent for the different response variables. Moreover, for the investigated buildings, having aspect ratios in the range 4.1–8.4, the optimal geometry is also found to be slightly affected by the building height, being the solution C3 always the prevailing one. From previous studies [13, 14], we know that for higher aspect ratios the bending behavior prevails over the shear deformation mode, thus the diagonal angle that minimizes the lateral diagrid deflection increases with the building height. In this case the optimal diagonal inclination does not increase as we need to minimize multiple responses simultaneously, not only the lateral deflection.

As already shown by the Authors in [20], increasing the building height leads to greater diagonal inclinations needed to minimize the lateral displacement. However, higher diagonal inclinations also lead to higher torsional rotations, thus worsening the torsional behavior. The *CI* also varies when modifying the diagonal inclination and, specifically, it is found to increase when moving from the solution with three intra-module floors to the one with four intra-module floors (Table 3). Therefore, although the solutions with four intra-module floors might be better candidates to minimize the lateral deflections and the structural mass for higher buildings [20], their higher torsional flexibility and construction complexity prevent their suitability as optimal geometries.

In conclusion, due to its inherent simplicity and its ability to consider the simultaneous optimization of several responses, the desirability function approach is a good candidate to assist the designer through the preliminary design stages in assessing the optimal diagrid geometries. Note that here we decided to take into account the lateral deflection, torsional rotation, diagrid mass and construction complexity as response variables. Obviously, this choice is not unique. Other response variables might also be selected, such as the maximum inter-story drift under lateral loads, the maximum axial stress (both tensile and compressive stress), some feature about the dynamic/seismic behavior, etc. Ultimately, it is the designer's choice to select the specific response variables to be included into the analysis as well as define their importance, through the individual desirability weight, to finally carry out the optimization of the diagrid.

4 Conclusions

In this paper, we apply for the first time the desirability function approach for the optimization of the diagrid geometry based on multiple responses. In particular, a set of twenty-four geometries has been considered, by varying the diagonal inclination and floor shape. Four response variables have been taken into account, namely the lateral deflection at the top of the building due to horizontal loads, the torsional rotation at the top due to torque actions, the total mass of the external diagrid tube and its construction complexity, which is measured through the complexity index (*CI*). Based on the value of each response variable, the desirability function approach yields the definition of an individual desirability score for each geometrical solution associated to the specific response variable. Finally, the individual desirability values are combined together to compute the overall desirability (OD) of each diagrid geometry. The optimal solution is the one that leads to the highest value of the OD.

The methodology has been firstly applied to a 168-meter tall diagrid building, by considering the same weight for the four response variables. From the results, it is found that the circular building with three intra-module floors (corresponding to a diagonal inclination of 67°) is the optimal one to simultaneously minimize the lateral deflection, torsional rotation, structural mass and construction complexity. Also, in line with previous findings, it is obtained that the diagonal inclination has an important influence on the overall performance of the building, whereas the floor shape has minor effect. A sensitivity analysis has been also carried out to investigate the role of the weight parameters that modify the relative importance assigned to the response variables. The results have shown that the optimal solution remains the same for the large majority of the weight combinations, with a minority of cases where the optimal diagrid is the one with the octagonal shape and three intra-module floors. The same analyses carried out for the 168-meter tall building have also been applied to 124-, 210- and 252-meter tall diagrid structures.

The methodology presented here has the advantage to be simple, fast and easily implementable for the analysis of large sets of structures. Moreover, it enables the designer to take simultaneously into account several response variables, and not only the lateral deflection and unit structural weight. As already specified above, the choice of the response variables to be optimized is not unique and might also include the inter-story drifts, axial stresses in the diagonals and dynamic/seismic performance factors. The selected response variables to be optimized obviously depend on the specific needs for the analyzed building (withstanding strong lateral loads, necessity to limit the amount of employed material, reaching an easily constructible solution, minimizing the building vibrations, etc.). In any case, the desirability function approach can be easily applied to find out the best solution, or the best set of solutions, that provide the greatest performance.

It has also to be noted that the proposed approach carries out the choice of the optimal solution based on the comparison between all the selected solutions, through the definition of the individual desirability scores. Therefore, the desirability function approach relies on *a posteriori* optimization, which processes the obtained results in comparative terms and then selects the optimal geometry. Conversely, other optimization approaches usually implemented in commercial codes, such as Genetic Algorithms, start from a population of individuals and operate on those with specific actions (slight changes in the input parameters, cross-overs, etc.) to obtain a new population which is potentially better performing. Both these approaches have advantages and limitations, and might be used in synergy as well. However, the application of the desirability function approach for the optimization of diagrid geometry in the preliminary design stages is highly recommended when the designer needs to have individual metrics of comparison among multiple solutions, based on the specific responses.

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