Fast and Accurate Estimation of Fuel-Optimal Trajectories to Near-Earth Asteroids

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Abstract

This paper proposes an improved method for the preliminary evaluation of minimum-propellant trajectories to Near-Earth Asteroids (NEAs). The method applies to missions from Earth to asteroids with small eccentricity and inclination. A planar and a plane-change problem can be distinguished. In the planar problem, the solution assumes that multiple burn arcs are performed in correspondence of the apsides of the target asteroid in order to change the initial spacecraft orbit (i.e., Earth's orbit) into the target one. The number of arcs is established once the time of flight is given (1 burn at each apsis per revolution, 1 revolution per year can be assumed). The length and propellant consumption of each arc to attain the required changes of semi-major axis and eccentricity are computed by a procedure based on Edelbaum's approximation, which is well-suited to the problem at hand, as eccentricity changes are expected to be small for feasible missions. No numerical integration is required, but only the numerical solution of a three-unknown algebraic system is needed, making the procedure extremely fast. Plane change is taken into account assuming a constant out-of-plane thrust angle during each burn. A previous simple formulation used an averaged thrust effect over one revolution and neglected the fact that plane changes are more effective at the nodes. Several improvements are here introduced, which greatly increase the method accuracy. The influences of the eccentricity change, the angle between the asteroid line of nodes and line of apsides, and the expected length of the arc are considered: In fact, when the eccentricity is small, the thrust arc can be performed at the nodes where the inclination is efficiently changed, with little penalty in the planar maneuver. An efficient plane change is also performed when the angle between the asteroid line of nodes and line of apsides is small and/or the length of the arc is large, because, in this case, the node is comprised in the apsidal burn. A simple corrective formula accounts for this effect. The new method shows remarkable accuracy. The results comparison with solutions obtained with an indirect optimization method for a set of more than 60 NEAs shows a 0.95 correlation coefficient in the propellant masses. The estimation error is below 10% for 75% of the targets, below 15% for 95% of the targets, and always below 20%.
Keywords: Near-Earth Asteroids, Trajectory Optimization, Edelbaum Approximation, Electric Propulsion

Nomenclature

a	semi-major axis, AU
c	effective exhaust velocity, m/s
e	eccentricity
e_x e_y	eccentricity vector components
i	inclination, deg
K	correction factor
$k_{0,..3}$	correction parameters
m	mass, kg
m_p	propellant mass, kg
n	number of revolutions
r	radius, AU
r_a	aphelion radius, AU
r_p	perihelion radius, AU
Ω	right ascension of the asc. node, deg
T	thrust, N
α	in-plane thrust angle, deg
β	out-of-plane thrust angle, deg
$\Delta i_{2\pi}$	1-rev. inclination change, deg
Δm	propellant consumption error, kg
ΔPA	total propulsive effort at perihelion
ΔAP	total propulsive effort at aphelion
Δt	burn arc time length, s
ΔV	velocity increment, m/s
$\Delta\vartheta$	burn arc angular length, deg
Λ	control law coefficient
ϑ	right ascension, deg
ϑ_e	reference right ascension, deg
ω	argument of periapsis, deg

Subscripts

0	initial
AP	aphelion maneuver
avg	average

E	Earth
PA	perihelion maneuver
Ref.7	results of Ref. 7
ref	reference value
T	target

1. Introduction

Electric Propulsion (EP) is a realistic and serious alternative to chemical propulsion. The large specific impulse of EP enables missions that would require prohibitive amounts of propellant with chemical propulsion. EP is limited by the available electric power onboard the spacecraft; hence it is suitable for low-thrust long-duration missions. For these reasons, the use of EP, in particular ion propulsion, has already been adopted for different missions aimed at Near-Earth asteroids (NEAs), such as Hayabusa to (25143) Itokawa [1] and Hayabusa 2 to (162173) Ryugu [2], and at main-belt asteroids, such as DAWN [3], which reached (4) Vesta and (1) Ceres. The use of ion propulsion for NEAs exploration is the object of this paper.

NEAs are asteroids with a perihelion distance of less than 1.3 AU. Their scientific interest lies in their status as unchanged remnant debris from the solar system formation process. Furthermore, they give the possibility to observe specific features not accessible in planets and, in the future, they could also be exploited for their raw materials [4,5]. Given their proximity with the Earth, missions to many NEAs require low ΔV and propellant consumption. Up to date, a large number of NEAs are already known, and this set is constantly growing as a result of new objects' discoveries. Such a broad set of targets provides opportunities for different scientific missions to new objects.

Usually, direct and indirect methods are employed for the optimization of low-thrust space trajectories [6], whereas evolutionary algorithms are less used and less viable methods for these problems. The main issue, when a large set of targets is considered, is that the trajectory optimization for the entire set could be extremely demanding and time-consuming. Preliminary estimation methods can be employed for an initial selection of the most promising targets and reduce the computational burden related to the optimization process. The availability of an accurate estimation of the transfer cost enables the identification of the most attractive targets (in terms of dynamics), and an exact optimization can be performed only for the most promising ones.

In a recent paper, Mereta and Izzo [7] compared the accuracy of several estimation techniques. The authors used the European Space Agency mission M-ARGO, aimed at rendezvous with a NEA with the use of EP, as a case study for these comparisons. In particular, they considered exact solutions obtained with a direct optimization method based on a nonlinear programming approach as a benchmark. Two different propulsion systems (i.e., nuclear and solar EP) were considered. These results were compared to solutions obtained considering a two-impulse transfer (Lambert's problem solution) and three-impulse trajectories. Results show a good general accuracy of the different approximations, but also a certain degree of unpredictability, with some estimation errors up to 30%.

Several preliminary estimation methods have been presented in the literature: low-fidelity models with closed-form integration and state variables representation with Chebychev polynomials [8], shape-based methods [9], and semi-analytical methods based on large databases of pre-computed trajectories to several targets [10,11]. However, these methods are not specifically developed for NEAs.

A novel method based on Edelbaum approximation, and therefore specifically tailored to the analysis of NEAs minimum-fuel trajectories with EP, was recently presented [12]. This method applies to small eccentricity and inclination changes with the use of low thrust, as in Edelbaum's approximation [13]. No integration of the equations of motion is required since the method produces a set of algebraic equations, which is numerically solved to determine the mission cost. The computational time is drastically reduced and the preliminary evaluation of large sets of possible targets is quick and accurate. The solution accuracy was similar to the other approximate methods treated in [7].

In the present paper, the estimation accuracy is greatly improved, compared to the previous work [12], with the introduction of correction factors that take the geometry of the transfer problem into account. Section 2 presents the basic guidelines to define the sequence of burns. Section 3 presents the solution of the planar problem, and Section 4 integrates the out-of-plane problem into the planar solution. Section 5 presents and discusses the results.

2. Trajectories to NEAs

Both the ΔV and propellant cost of a mission to reach a NEA are strongly related to the required changes in the orbital parameters from the starting orbit to the target one. Asteroids that require significant changes of eccentricity and inclination with respect to the departure orbit, e.g., Earth's one, are selected as mission objectives only when they present unique characteristics to justify the significant cost, such as (433) Eros reached by the NEAR spacecraft [14]. With few exceptions, the vast majority of asteroids considered for science missions are those characterized by small changes of eccentricity and inclination with respect to Earth's reference orbit. Indeed, favorable targets have semi-major axes close to 1 AU, and small to null eccentricities and inclinations. Therefore, the present research develops a method to approximate the cost of transfers to the most easily reachable NEAs, namely those characterized by almost circular orbits with little inclination.

A preliminary analysis is carried out to define which kind of orbit modifications are required and, consequently, a suitable sequence of burns. The planar problem is considered in this phase, whereas inclination changes will be addressed in Section 4.

Earth is the departure planet and a circular orbit with 1-AU radius is initially assumed ($a_E = 1$ and $e_E = 0$). The most straightforward transfer is accomplished with two in-plane impulsive maneuvers that modify perihelion and aphelion to attain the target values. A perihelion impulse provides $r_a = a_T(1 + e_T)$, whereas an aphelion impulse produces $r_p = a_T(1 - e_T)$. These two impulses, in opposition to one another and aligned with the line of apsides of the target, accomplish the required orbit changes by causing specific semi-major axis and eccentricity variations, Δa and Δe . One can easily obtain the required orbital elements changes at the perihelion to change the aphelion radius (PA) and at the aphelion to change the perihelion radius (AP), which are

$$\Delta a_{PA} = \Delta e_{PA} = [(a_T - 1) + e_T]/2 \qquad (1)$$
$$\Delta a_{AP} = -\Delta e_{AP} = [(a_T - 1) - e_T]/2 \qquad (2)$$

It is worth noting that Eqs. (1) and (2) assume that the impulses are both applied at 1 AU from the Sun. In the real case, the second impulse is applied at a different radius, as the first one has modified the apsidal distance. These effects are neglected here, and the cost of each maneuver is instead not influenced by the previous ones.

A chemical propulsion system can perform the impulses required to achieve the target orbit. Given that EP is used here, the low thrust provided by the propulsion system could achieve the required orbital changes only with long thrusting arcs. In this case, however, the thrust effect is spread among different orbital positions and becomes less effective than the impulse placed at the most favorable one (i.e., the apsis). A single burn may not even be capable of providing the required changes. This problem is solved if sufficient time is available and more revolutions are performed: Each maneuver is equally split into smaller burns with a shorter duration, conveniently placed close to the line of apsides.

The targets considered in this paper have revolution periods of about 1 year. In this case, a mission with a trip time close to *n* years usually has *n* passages through the apsides, allowing for *n* equal perihelion burns and *n* equal aphelion burns. Each burn must therefore provide a fraction 1/*n* of the total semi-major axis and eccentricity change. Again, the effect of previous burns is neglected.

The eccentricity vector, with e_x and e_y components on the ecliptic plane, is introduced to assure the proper alignment of the line of apsides. In particular, by projecting along reference directions x and y the eccentricity vectors of Earth and target, one has

$$\Delta e_x = e_T \cos(\Omega_T + \omega_T) - e_E \cos(\Omega_E + \omega_E) \qquad (3)$$
$$\Delta e_y = e_T \sin(\Omega_T + \omega_T) - e_E \sin(\Omega_E + \omega_E) \qquad (4)$$

The values Δa and Δe are introduced taking the Earth's actual orbit into account

$$\Delta a = a_T - a_E \qquad (5)$$
$$\Delta e = \left(\Delta e_x^2 + \Delta e_y^2\right)^{1/2} \qquad (6)$$

and, in the *n* burns scenario, the following changes of orbital elements must be attained: For each perihelion maneuver, one has

$$\Delta a_{PA} = (\Delta a + \Delta e)/(2n) \qquad (7)$$
$$\Delta e_{PAx} = \Delta a_{PA}\,(\Delta e_x/\Delta e) \qquad (8)$$
$$\Delta e_{PAy} = \Delta a_{PA}\left(\Delta e_y/\Delta e\right) \qquad (9)$$

and, for aphelion maneuvers,

$$\Delta a_{AP} = (\Delta a - \Delta e)/(2n) \qquad (10)$$
$$\Delta e_{APx} = -\Delta a_{AP}\,(\Delta e_x/\Delta e) \qquad (11)$$
$$\Delta e_{APy} = -\Delta a_{AP}\left(\Delta e_y/\Delta e\right) \qquad (12)$$

The necessary contributions to change the eccentricity vector and achieve the proper direction, at the perihelion or at the aphelion, are proportional to the semi-major axis variations imposed at the same locations.

3. Planar problem

Section 2 has defined the orbital changes that must be provided by each burn. The thrust direction must be properly controlled during each burn to minimize the propellant consumption while performing the required changes. The in-plane thrust angle α and the out-of-plane angle β are introduced to express the in-plane thrust component along the radial direction as $T \sin\alpha \cos\beta$, along the velocity direction as $T \cos\alpha \cos\beta$, and the out-of-plane component as $T \sin\beta$. A linear control law [15] that approximates the optimal solution in Edelbaum's approximation [13] is adopted here.

Edelbaum's approximation considers almost circular orbits and small inclination changes and is perfectly suited to the targets of interest. With Edelbaum's assumption, the planar problem can be numerically solved, but an analytical solution for the generic change of semi-major axis and eccentricity does not exist.

Typical shapes of the optimal control law show that a linear control law for the in-plane thrust angle α can accurately approximate the optimal one while allowing for analytical integration [15]. In particular, either $\alpha = \Lambda(\vartheta - \vartheta_e)$ or $\alpha = \pi + \Lambda(\vartheta - \vartheta_e)$, depending on the sign of Δa, provide a good approximation by properly selecting Λ and ϑ_e . The angle $\vartheta = \vartheta_e$ corresponds to the point where the thrust is either concurrent with (for $\Delta a > 0$) or opposite to (for $\Delta a < 0$) the spacecraft velocity, with $\alpha = 0$ or π, respectively. Thrust remains close to the velocity direction when Λ, which varies between 0 and 1, is small. Large values of Λ see more significant variations of the thrust direction.

The evaluation of the transfer cost is made by varying the initial angle ϑ_0 at 5-degree steps over a complete revolution. Tentative values are assumed for Λ, ϑ_e, and $\Delta\vartheta$, i.e., the burn angular length, and the corresponding $\Delta\alpha$, Δe_x, and Δe_y are analytically determined [15]. Tentative values are corrected according to a Newton's scheme to achieve the required orbital changes, given by Eqs (7)-(9) or (10)-(12). The solution requires the value of the out-of-plane thrust angle, which is kept constant during each burn and progressively adjusted to achieve the required plane change, as shown in Section 4. Tentative values for unknown quantities can be easily guessed to start the procedure using averaged effects of thrust components along a complete revolution [13,15].

After each iteration, the average mass is computed and used to evaluate the thrust acceleration for the following iteration. From $\Delta\vartheta$, the time duration $\Delta t = \Delta\vartheta/(\mathrm{d}\vartheta/\mathrm{dt})$ is computed assuming the circular angular velocity at the average radius between Earth and asteroid perihelion for the perihelion maneuver, and between Earth and asteroid aphelion for the corresponding burn. The propellant consumption of each burn is then determined as $\Delta t\, T/c$. The consumption influence at one apsis is neglected when evaluating the average mass for the burns at the opposite apsis.

The best initial angle for each maneuver is easily selected by a direct comparison of the solutions at 5-degree steps. All the burns are summed up to eventually evaluate the estimation of the optimal overall transfer consumption. The overall propellant mass is therefore $m_p = n\,(\Delta t_{PA} + \Delta t_{AP})\,T/c$.

4. Plane change

Maneuver combination is beneficial; in-plane and out-of-plane thrust components add up vectorially so the thrust effort for a combined maneuver is lower than the sum of the individual efforts. Therefore, the approximation assumes that inclination is adjusted during the apsidal burns necessary for the in-plane problem. A suitable out-of-plane thrust angle, which is assumed to be constant during each burn, is adopted (more properly, β assumes a constant positive value during the half revolution centered at the ascending node and the opposite negative value during the half revolution centered at the descending node). The required overall inclination change is $\Delta i = i_T$, since $i_E \approx 0$. The total change is split between the burns assuming that each maneuver provides an inclination change proportional to the change of the in-plane orbital elements, that is, $\Delta PA = |\Delta a_{PA}| + |\Delta e_{PA}|$ or $\Delta AP = |\Delta a_{AP}| + |\Delta e_{AP}|$ for the two apsides. One has

$$\Delta i_{PA} = \frac{\Delta PA}{\Delta PA + \Delta AP}\frac{\Delta i}{n} \tag{13}$$

$$\Delta i_{AP} = \frac{\Delta AP}{\Delta PA + \Delta AP}\frac{\Delta i}{n} \tag{14}$$

From Edelbaum's approximation, the differential equation that expresses the inclination change with respect to the angular position is

$$di = (T/m)\sin\beta\cos\vartheta\; d\vartheta \tag{15}$$

where $\vartheta = 0$ corresponds to the ascending node. The advantage of thrusting at the nodes, where $di = (T/m)\sin\beta d\vartheta$ is clearly visible. However, a generic burn spans a finite angle $\Delta\vartheta$ and the evaluation of its effect is difficult, even for constant thrust angle. A simple expression for Δi can be obtained for constant-β over 1 revolution ($\Delta\vartheta = 2\pi$), that is, $\Delta i_{2\pi} = 4\,(T/m)\sin\beta\Delta\vartheta$. The average thrusting effect is therefore

$$\Delta i = \Delta i_{2\pi}/(2\pi) = (2/\pi)\,(T/m)\sin\beta\Delta\vartheta \tag{16}$$

and was used in Ref. [12] to update the required out-of-plane angle β, given the required inclination change of each burn. The average rate of inclination change does not account for the position of the burn with respect to the nodes and affects the estimation in many cases. In particular, consumptions are underestimated when the line of apsides is close to the line of nodes, while overestimated when lines of nodes and apsides are in quadrature [12].

The assumption of a constant out-of-plane angle and the position of the apsides with respect to the nodes' location are indeed critical for the estimation accuracy. For this reason, a correction factor K is introduced here to evaluate more consistently the effect of the out-of-plane thrusting, namely by assuming

$$\Delta i = \frac{(2/\pi)\,(T/m)\,\sin\,\beta\Delta\vartheta}{K} \tag{17}$$

$$K = k_0 + k_1 k_2 k_3 \tag{18}$$

The terms k_0, k_1, k_2, k_3 are functions of the different orbital parameters or factors that influence the propellant consumption. In particular, three effects are considered: the angle ω_T between the target line of nodes and apsides, the target eccentricity, and the angular length of the burns. The adopted values are here explained.

The value $k_0 = 0.6$ corresponds to the cost of a plane change precisely at the node. The exact value would be $2/\pi$, which is slightly larger, but the lower value is preferred. The use of an approximate control law usually overestimates the consumption, and the use of a lower value for k_0 partially offsets this fact by reducing the propellant estimation.

The factor $k_1 = 1 - \cos(2\omega_T)$ penalizes the thrusting effect when apsides and nodes are in quadrature ($\omega_T = \{\pi/2, 3\pi/2\}$), whereas no penalty is added when nodes are close to apsides ($\omega_T = \{0, \pi, 2\pi\}$).

The factor $k_2 = 1.5e_T$ reduces the penalty for small eccentricities and causes a large penalty for large eccentricity changes. In fact, when the eccentricity is small, the burns to change perihelion and aphelion can be moved away from the lines of apsides and close to the line of nodes with little penalty on the consumption. For instance, if the target eccentricity is zero, the problem concerns the transfer between circular orbits, and the burns to change aphelion and perihelion can be placed at arbitrary angular positions. The burn location close to the line of apsides is instead crucial to minimize cost when the eccentricity is large.

Finally, the factor $k_3 = (3 + \cos\Delta\vartheta)/4$ accounts for the burn angular length and reduces the penalty for large arcs. In fact, burns that extend for more than 90 degrees always comprise both the line of nodes and the line of apsides. In this case, a variable out-of-plane angle can be adopted in the optimal solution, to increase the out-of-plane thrust component at the nodes, where plane change is more efficiently obtained, thus reducing its cost. The coefficients are selected in order to have K between 0.6 and 1.35 (for a 0.25 target eccentricity). The minimum value corresponds to plane change at the nodes; the maximum value corresponds to plane change performed at about 60 degrees from the line of nodes.

Equation (17) is used in this paper to evaluate the cost of the out-of-plane maneuver and results compared to the estimations without corrections of Ref. [12]. The approximate solution relies on analytical expressions and is extremely fast; the complete analysis for each asteroid requires about 1 second on a standard 64-bit Intel i7 3.6 GHz processor.

5. Results

The present work considers the same study-case shown in Ref. [7]; in particular, the focus is here on the 75 NEAs that presented the smallest propellant consumption. A 20 kg spacecraft with solar EP system (3100 s specific impulse, 1.74 mN thrust at 1 AU), starting its mission between 2020 and 2023 (included), travels to the selected targets with a maximum 3-year time of flight.

Reference solutions are evaluated with an indirect optimization method [16,17]. The heliocentric transfer from Earth to the asteroid in the two-body problem model is optimized to minimize the propellant consumption. It is worth noting that the dynamical and propulsion system models used here are not exactly the ones used in Ref. [7]. Furthermore, this work considers optimal phasing instead of rendezvous transfer. Consequently, a direct comparison between the results with Ref. [7] is not possible, but they still are significantly consistent.

The estimations presented here concern optimal phasing conditions. Favorable rendezvous opportunities require proper phasing between Earth and asteroid and a procedure to evaluate mission opportunities is also derived. Both the overall angular length and time of flight can be easily estimated: the initial and final positions of each burn are known, so departure and final points are given, depending on the preferred sequence (perihelion or aphelion burn first, both options are readily evaluated). The initial position provides available departure dates, that is, once a year, when the Earth is at the departure point. The time of flight is estimated from the transfer angular length using the average angular velocity between Earth and asteroid. The position of the asteroid at the corresponding arrival date can be compared to the expected arrival position, and favorable rendezvous opportunities happen when the angle between them is sufficiently small. The procedure proves to be effective and also capable of estimating the penalty of rendezvous trajectories with respect to optimal phasing solutions [12].

In the present section, results from the indirect optimization method are compared to the proposed approximate solutions based on Edelbaum's approximation from Ref. [12], and to the new results after the introduction of the correction factors presented in Section 4. In most cases, the solutions based on Edelbaum's approximation present starting points slightly before either perihelion or aphelion, with an almost tangential thrusting (relatively small values of Λ). These solutions remarkably replicate the efficient impulsive scenario used to define the burn sequence.

Among the 75 targets mentioned above, 13 asteroids were excluded because of their large inclination (greater than 5 degrees), large change in semi-major axis (greater than 0.2 AU), or large eccentricity (greater than 0.25). These values seem to be beyond the applicability limit of Edelbaum's approximation, but tweaks to the algorithm will be tested in the future to allow for the treatment of these cases. Relevant orbital elements and results are shown in Table 1 for the remaining 62 targets. In particular, the reference propellant mass m_p provided by the indirect method and the errors of the estimations with respect to this value in Ref. [12] ($\Delta m_{Ref.7}$) and after the introduction of the correction factor (Δm and its percentage of the propellant mass with the corresponding rank) are presented.

It is evident from the results that the introduction of the correction factor K remarkably improves the estimation. The average magnitude of the error is down to 0.12 kg from 0.18 kg, respectively for $< |\Delta m| >$ and $< |\Delta m_{Ref.7}| >$. The average error is down to 0.09 kg from 0.12 kg. The correlation coefficient between reference and estimated propellant mass is 0.96. Despite the average overestimating of the propellant consumption by about 5%, results are still extremely satisfactory, given the approximate method's simplicity

Table 1. Orbital elements, propellant consumption and estimation errors for the selected asteroid set

	asteroid	a AU	e	i deg	ω deg	r_p AU	r_a AU	m_p kg	$\Delta m_{Ref.7}$ kg	Δm kg	rank	Δm %	Rank
1	2013 RV9	1.167	0.200	3.511	108.8	0.93	1.40	2.534	-0.307	0.151	40	5.96	31
2	2012 UW68	1.136	0.155	2.472	102.4	0.96	1.31	1.910	-0.249	-0.018	14	-0.92	14
3	2008 TX3	1.179	0.187	2.382	249.9	0.96	1.40	2.136	-0.221	0.158	42	7.37	39
4	2014 QH33	1.085	0.185	2.832	264.4	0.88	1.29	2.497	-0.212	0.016	22	0.66	22
5	2006 QV89	1.192	0.224	1.071	236.7	0.92	1.46	2.234	-0.204	0.245	56	10.98	48
6	2012 UY68	1.175	0.228	2.901	35.8	0.91	1.44	2.466	-0.185	0.156	41	6.34	32
7	2016 TP11	1.037	0.178	1.538	108.2	0.85	1.22	2.167	-0.167	0.001	18	0.06	18
8	2012 HK31	1.074	0.121	2.205	96.7	0.94	1.20	1.818	-0.165	-0.115	3	-6.35	3
9	2015 PL57	1.121	0.144	1.631	115.3	0.96	1.28	1.788	-0.149	-0.024	12	-1.35	12
10	2011 CG2	1.178	0.159	2.757	283.9	0.99	1.36	2.184	-0.145	0.148	38	6.77	34
11	2017 EB3	1.039	0.153	2.840	247.5	0.88	1.20	2.242	-0.132	-0.078	6	-3.50	8
12	2016 UE	1.057	0.152	1.088	296.5	0.90	1.22	1.517	-0.122	-0.009	15	-0.62	15
13	2015 FG36	1.100	0.168	3.514	300.6	0.91	1.29	2.481	-0.119	-0.008	16	-0.32	16
14	2012 UV136	1.007	0.139	2.213	288.6	0.87	1.15	1.737	-0.096	-0.039	9	-2.26	10
15	2014 UY	1.174	0.173	3.565	245.6	0.97	1.38	2.290	-0.087	0.195	47	8.51	43
16	2012 EC	1.152	0.137	0.914	333.9	0.99	1.31	1.568	-0.068	0.059	26	3.73	28
17	2007 UY1	0.951	0.175	1.019	273.6	0.78	1.12	2.043	-0.045	0.135	36	6.59	33
18	2016 TB57	1.102	0.123	0.298	147.8	0.97	1.24	1.056	-0.032	0.031	25	2.93	26
19	2016 CF137	1.090	0.100	2.445	301.5	0.98	1.20	1.434	-0.025	-0.089	5	-6.18	4
20	2009 CV	1.116	0.151	0.943	181.4	0.95	1.28	1.537	-0.016	0.105	32	6.80	35
21	2014 YD	1.072	0.087	1.736	34.1	0.98	1.16	1.107	-0.014	-0.131	2	-11.82	1
22	2017 BF30	1.045	0.130	3.624	256.1	0.91	1.18	2.117	-0.009	-0.031	11	-1.48	11
23	2007 DD	0.987	0.116	2.624	77.6	0.87	1.10	1.703	-0.008	-0.060	7	-3.52	6
24	2013 BS45	0.992	0.084	0.773	150.7	0.91	1.07	1.007	0.003	0.077	30	7.60	40
25	2010 WR7	1.046	0.235	1.563	159.1	0.80	1.29	2.334	0.008	0.195	48	8.36	42
26	2001 QJ142	1.062	0.086	3.104	64.0	0.97	1.15	1.738	0.013	-0.148	1	-8.52	2
27	2016 TB 18	1.078	0.084	1.527	305.6	0.99	1.17	1.023	0.015	-0.036	10	-3.51	7
28	2017 HK1	0.909	0.147	1.510	258.7	0.78	1.04	1.756	0.017	0.064	28	3.64	27
29	2009 HC	1.039	0.126	3.778	269.9	0.91	1.17	2.027	0.034	0.010	20	0.48	19
30	2004 JN1	1.085	0.176	1.500	2.1	0.89	1.28	1.651	0.047	0.133	34	8.05	41
31	2006 FH36	0.955	0.199	1.587	154.7	0.77	1.14	1.970	0.071	0.193	46	9.78	46
32	2004 VJ1	0.944	0.164	1.294	332.3	0.79	1.10	1.772	0.092	0.199	50	11.24	49
33	2003 SM84	1.125	0.082	2.796	87.4	1.03	1.22	1.675	0.096	0.008	19	0.49	20
34	2001 CQ36	0.938	0.177	1.258	344.4	0.77	1.10	1.856	0.101	0.215	51	11.60	52
35	2017 HZ24	0.908	0.215	1.812	312.2	0.71	1.10	2.429	0.112	0.171	43	7.05	37
36	2009 OS5	1.144	0.097	1.695	120.9	1.03	1.26	1.467	0.156	0.138	37	9.42	45
37	2015 BM510	0.947	0.122	1.589	357.3	0.83	1.06	1.452	0.190	0.178	45	12.23	54
38	2010 HA	0.960	0.196	2.183	185.7	0.77	1.15	1.931	0.215	0.221	52	11.45	51
39	2009 RT1	1.156	0.106	4.150	136.5	1.03	1.28	2.151	0.220	0.018	23	0.84	23
40	2014 YN	0.892	0.134	1.208	15.8	0.77	1.01	1.603	0.234	0.199	49	12.39	55
41	2005 TG50	0.923	0.135	2.401	200.9	0.80	1.05	1.683	0.247	0.064	29	3.81	29
42	2000 AE205	1.165	0.138	4.459	150.3	1.00	1.32	2.237	0.249	0.060	27	2.69	25
43	2013 WA44	1.101	0.061	2.302	176.7	1.03	1.17	1.197	0.275	0.026	24	2.20	24
44	2016 FY2	0.869	0.177	1.868	205.0	0.72	1.02	2.141	0.277	0.151	39	7.05	36
45	2006 XP4	0.873	0.215	0.515	346.0	0.69	1.06	2.419	0.309	0.333	62	13.78	57
46	2013 XY20	1.131	0.106	2.863	18.2	1.01	1.25	1.507	0.311	0.107	33	7.07	38
47	2013 HP11	1.185	0.125	4.156	9.5	1.04	1.33	2.206	0.333	0.084	31	3.82	30
48	2017 BF29	1.181	0.134	2.614	203.8	1.02	1.34	1.764	0.336	0.304	60	17.20	61
49	2015 VV	1.137	0.105	4.007	177.2	1.02	1.26	1.870	0.350	-0.018	13	-0.99	13
50	2003 LN6	0.856	0.210	0.660	211.6	0.68	1.04	2.105	0.355	0.307	61	14.58	58
51	2014 SD304	1.168	0.108	2.294	19.3	1.04	1.29	1.590	0.360	0.275	58	17.31	62
52	2013 EM89	1.178	0.117	2.411	189.9	1.04	1.32	1.684	0.367	0.281	59	16.72	60
53	2019 PA7	1.154	0.088	3.472	93.3	1.05	1.26	1.803	0.404	0.245	55	13.58	56
54	1999 AO10	0.912	0.111	2.623	8.0	0.81	1.01	1.537	0.436	0.134	35	8.69	44
55	2001 BB16	0.855	0.172	2.026	195.6	0.71	1.00	2.102	0.450	0.239	54	11.38	50
56	1996 XB27	1.189	0.058	2.465	58.2	1.12	1.26	1.767	0.468	0.266	57	15.04	59
57	2015 TZ24	1.192	0.100	3.350	3.2	1.07	1.31	1.950	0.472	0.229	53	11.77	53
58	2014 MF18	0.886	0.162	2.614	350.4	0.74	1.03	1.762	0.481	0.177	44	10.07	47
59	2007 TF 15	1.109	0.045	4.253	28.9	1.06	1.16	1.767	0.499	-0.091	4	-5.14	5
60	2014 EK24	1.008	0.070	4.805	63.7	0.94	1.08	1.857	0.543	0.012	21	0.64	21
61	2011 AA37	1.096	0.017	3.817	131.5	1.08	1.11	1.530	0.551	-0.048	8	-3.16	9
62	2012 WH	0.907	0.145	4.094	8.7	0.78	1.04	2.222	0.558	0.001	17	0.05	17

and speed. Note that an *a posteriori* tuning of k_0 could easily solve this issue.

When the correction is not applied, several asteroids showed either underestimation (-0.31 kg in the worst case) or overestimation (up to 0.56 kg) of the propellant consumption. Severe underestimation occurred for asteroids characterized by large values of eccentricity (above 0.15) and apsides-nodes quadrature, such as 2013 RV9, 2012 UW68, 2008 TX3. Asteroids 2014 QH33 and 2006 QV89 are farther from quadrature and closer to apsides-nodes alignment (36 and 45 degrees between them, respectively), but have eccentricity above 0.22. A large value of K penalizes these solutions and increases the propellant consumption; the error becomes positive but, as expected, smaller in absolute magnitude, except for 2006 QV89, which shows a slight increase to about 11% of the propellant mass. The large eccentricity excessively penalizes this case.

In Ref. [7], eighteen asteroids showed errors above $+0.3$ kg, while ten asteroids presented errors larger than $+0.4$ kg. Most of these asteroids are close to an apsides-nodes alignment (1999 AO10, 2001 BB16, 2015 TZ24, 2014 MF18, 2007 TF15, 2011 AA37, 2012 WH) and the optimal solution takes advantage of this favorable geometry. This effect is caught by the correction factor K, which becomes small due to k_2, properly reducing the consumption estimation when thrusting at the nodes becomes possible. Hence, an impressive reduction of the error magnitude is obtained. A similar improvement occurs for small-eccentricity large-inclination asteroids (2019 PA7, 1996 XB27, 2014 EK24) for the combined effect of small values of k_1 and k_3.

The corrected estimations still show some asteroids with negative errors, but the worst underestimation is now only -0.15 kg and is larger than 0.1 kg in absolute magnitude for only three cases. On the other side, an overestimation above $+0.2$ kg occurs for only twelve asteroids and above $+0.3$ kg only for three of them. Asteroids 2003 LN6 and 2006 XP4 have very small periapsis. The real optimal solution performs more than three revolutions in the 3-year time of flight; hence, with four passages at one of the apsis, it can take advantage of shorter and more efficient arcs. If the number of thrust arcs is constrained to 3 at each apsis by reducing the time of flight, the actual consumption is larger and the estimation error decreases to about $+0.2$ kg for both cases.

As regards asteroid 2017 BF29, the relatively large error has a less clear explanation, but, in general, large errors occur when the periapsis is either small or larger than 1. Figure 1 shows the shape of the asteroids' orbits that present the greatest values of error in the propellant estimation, together with Earth's orbit, assumed to be circular. For ease of comparison and simpler analysis, all orbits are rotated so that the perihelia lie on the positive X axis. Orbits are barely distinguishable, as two groups arise: a family of outer orbits with respect to the Earth, and a family of inner orbits, except for a minimum intersecting arc. These groups are both characterized by relatively large values of eccentricity, and Edelbaum's model starts to suffer from the hypothesis of almost circular orbits.

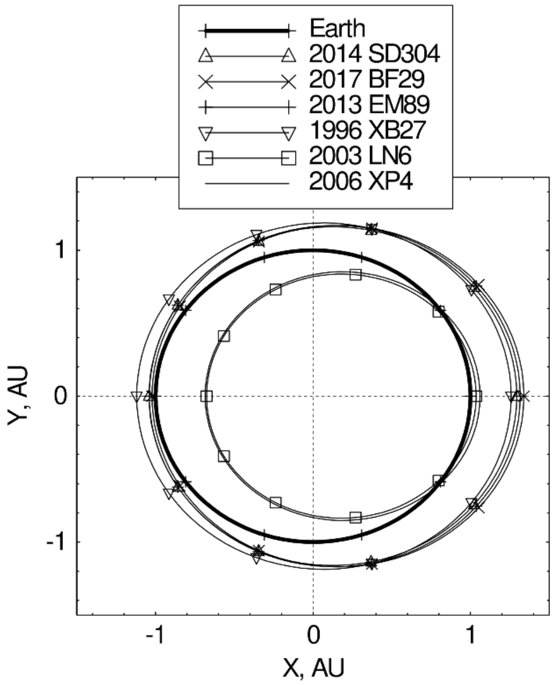

Fig. 1. Orbit shape for large-error asteroids.

The thrust angle and the orbit evolution for a transfer to 2001 QJ142 are shown in Figure 2. The spacecraft performs only two perigee burns at the first two passages to achieve most of the aphelion change (with α close to 0 and with the expected almost linear behavior). The small variation of r_p is reflected in the aphelion burn arcs, which are moved away from the aphelion and shifted towards the node; during these burns, the inclination is changed with large β and small adjustments of perihelion and aphelion (a similar effect will be seen for asteroid 2017 BF 29). In this case, the mission does not follow the strategy of uniform split between apsides burns, but, despite this, the estimation error remains sufficiently small, at about 10%.

Figure 3 presents the thrust angles and the orbit evolution over time for the asteroid 2006 XP4. This transfer has four apogee burns to reduce r_p due to the short revolution period. However, a four-month reduction of the time of flight can prevent the fourth burn, with a propellant consumption $m_p = 2.55$ kg. This value corresponds to an error of only 0.2 kg, or 7.5 % of the propellant mass. In this case, the correction has little effect on the results and actually worsen the error, since ω_T is close to 360 degrees, and apsis and nodes are close to each other. Shape and inclination are adjusted synergistically and the correction factor, which is close to the minimum value 0.6, has little effect because of the small inclination of the asteroid orbit. The large eccentricity is again the primary source of error.

Figure 4 shows the results for the transfer to asteroid 2017 BF29. In this case, regular perigee burns, are used to change the aphelion, even though they are not perfectly centered at the apsis. In similarity to 2001 QJ142, the required perihelion change is small, and aphelion burns are replaced by node burns in order to change the inclination more efficiently. This fact explains the overestimation of the approximate solution, as the real optimal trajectory can take advantage of both node vicinity and large radius and perform an efficient inclination change.

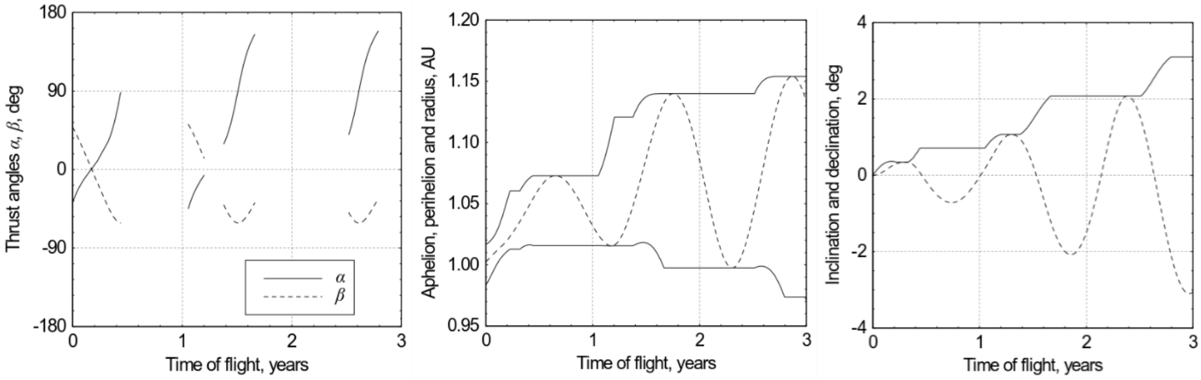

Fig. 2. Thrust angles and orbit evolution for optimal transfer to 2001 QJ142

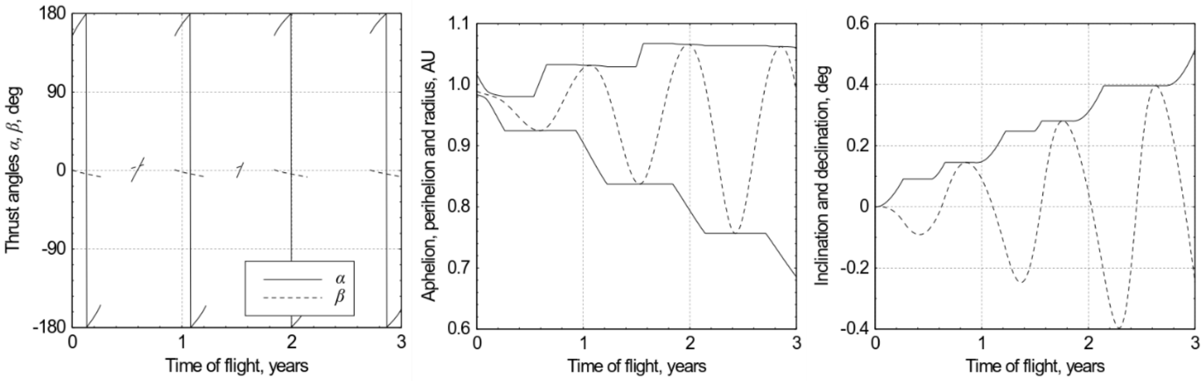

Fig. 3. Thrust angles and orbit evolution for optimal transfer to 2006 XP4

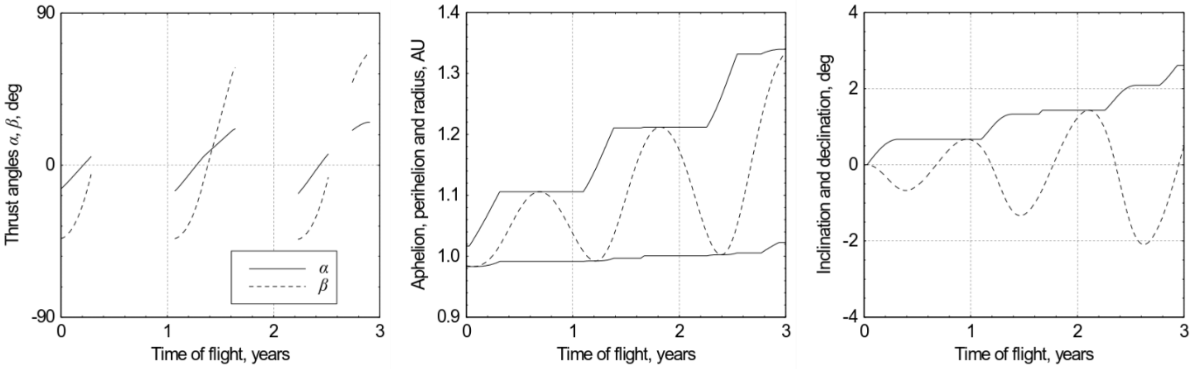

Fig. 4. Thrust angles and orbit evolution for optimal transfer to 2017 BF29

From these examples, which concern the worst estimations provided by the method, it is clear that the basic assumptions to define the burn sequence substantially replicate the real scenario. Alternation of burns at the apsides, linear variation of α in the in-plane control strategy, and effect of eccentricity and line of nodes position on the out-of-plane corrections substantially replicate the real scenario of the optimal solutions, and therefore explain the excellent performance of the estimation method.

6. Conclusions

An analytical method for the evaluation of orbital transfers to NEAs with small eccentricity and inclination has been improved and tested. The improvements introduced do not affect the previously presented strengths; the method does not require the integration of the equations of motion and is sufficiently fast and accurate for the analysis of large sets with hundreds of potential targets in a matter of minutes. An analysis of systematic errors has led to the modified method presented in this paper. Insight on the interaction between the plane-change and planar maneuvers defines a correction factor that significantly improves the estimations. All the targets have errors below 20% and 94% of them below 15 %. The method supposes optimal-phasing conditions but can be easily modified to consider the actual rendezvous conditions and optimal launch windows, while adding the estimation of the propellant mass necessary for the phasing.

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