dP-FMEA: An innovative Failure Mode and Effects Analysis for distributed manufacturing processes

dP-FMEA: an innovative *Failure Mode and Effects Analysis* for distributed manufacturing processes

Domenico A. Maisano[a], Fiorenzo Franceschini[a], Dario Antonelli[a]

[a]*Politecnico di Torino, DIGEP (Dept. of Management and Production Engineering), Corso Duca degli Abruzzi 24, 10129, Torino, Italy*

Abstract

The *Failure Mode and Effects Analysis* (FMEA) is a powerful tool to design and maintain reliable systems (products, services or manufacturing processes), investigating their potential failure modes from the threefold perspective of *severity*, *occurrence* and *detectability*. The *Process FMEA*, or more briefly P-FMEA, is a declination of the FMEA for manufacturing processes (or parts of them). Being progressively characterized by decentralized networks of flexible manufacturing facilities, the current scenario significantly hampers the implementation of the traditional P-FMEA, which requires the joint work of a group of experts formulating collective judgments. This paper revises the traditional P-FMEA approach and integrates it with the ZM_{II}-technique – i.e., a recent aggregation technique based on the combination of the Thurstone's *Law of Comparative Judgment* and the *Generalized Least Squares* method – allowing experts distributed through organizations to formulate their judgments individually. The revised approach – referred to as "*distributed-Process FMEA*" or more briefly dP-FMEA – allows to manage a number of experts, without requiring them to physically meet and formulate collective decisions, thus overcoming a relevant limitation of the traditional P-FMEA. The dP-FMEA approach also includes a relatively versatile response mode and overcomes several other limitations of the traditional approach, including but not limited to: (i) arbitrary formulation and aggregation of expert judgments, (ii) lack of consideration of the dispersion of these judgments, and (iii) lack of estimation of the uncertainty of results. The description is supported by a real-life application example concerning a plastic injection-moulding process.

Keywords: P-FMEA; Distributed manufacturing systems; Failure modes; Failure causes; ZM_{II}-technique; Risk Priority Number; Incomplete ranking.

Introduction and literature review

The *Failure Mode and Effects Analysis* (FMEA) is a very popular technique to improve the reliability of products, services and manufacturing processes, by analysing failure scenarios before they have occurred and preventing the occurrence of causes or mechanisms of failures (Stamatis, 2003; Tague, 2005; Liu et al., 2019a). When applied at the product/service design stage, FMEA helps to achieve reliability while reducing the amount of design corrections (Geramian et al. 2019); when applied to manufacturing processes, FMEA is very useful to improve reliability and safety and provides a useful basis for planning the corresponding predictive maintenance (Johnson and Khan, 2003).

The FMEA is generally carried out by a cross-functional and multidisciplinary team of experts – typically composed of engineers and technicians specialized in design, testing, reliability, quality, maintenance, manufacturing, safety, etc. – which is coordinated by a team leader in various activities (see the flowchart in Fig. 1). Many of these activities must be carried out collectively by the experts, trying to overcome conflicting situations and converging towards a unanimous agreement. The major collective activities are those concerned with the prioritisation of the so-called *failure modes*, based on the *Risk Priority Number* (*RPN*), which is a composite indicator given by the product of three so-called *risk factors*: *occurrence* (*O*), *severity* (*S*), and *detection* (*D*) (Yeh and Chen, 2014). Each of these risk factors is determined by collective judgments, using a conventional ordinal scale from 1 to 10 (Stamatis, 2003); see activities 7, 8 and 10 of the flowchart in Fig. 1. The failure modes with higher *RPN*s are considered more critical and deserve priority for the implementation of risk mitigation actions: since the resources available for corrective actions are limited, it is reasonable to concentrate them where they are most needed, tolerating the minor failure modes. This sort of criterion of economy/sustainability is represented by activities 12 and 13 in the flowchart in Fig. 1; the feedback loop related to activity 13 highlights the iterative nature of the entire FMEA procedure.

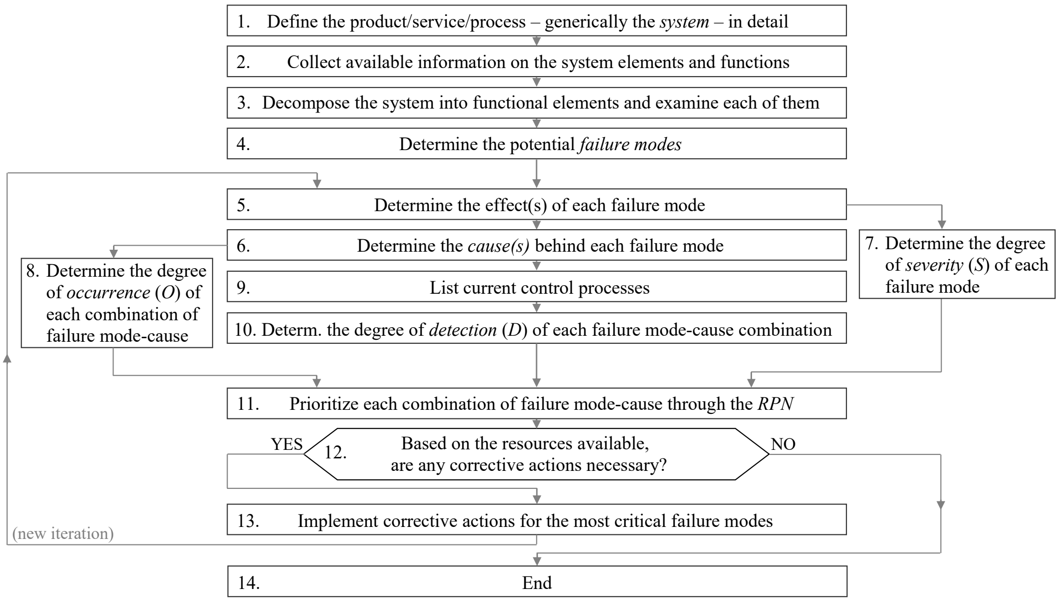

Fig. 1. Flowchart showing the main stages of a FMEA (Stamatis, 2003).

The traditional method for prioritizing failure modes shows important shortcomings, extensively debated in the scientific literature (Franceschini and Galetto, 2001; Das Adhikary et al., 2014; Zhou et al., 2016; Certa et al., 2017; Huang et al., 2019; Liu, 2019; Liu et al., 2019b; Geramian et al., 2020); including but not limited to:

- Use of arbitrary reference tables for assigning 1-to-10 scores to the three risk factors *S*, *O* and *D* (AIAG, 2019; Franceschini et al., 2019). As an example, Table 1 contains evaluation criteria suggested by the Automotive Industry Action Group (AIAG, 2019), in an endeavour to unify FMEA for automotive manufacturing processes.
- *S*, *O* and *D* are arbitrarily considered as equally important (Franceschini and Galetto, 2001).
- Since *S*, *O* and *D* are evaluated using ordinal scales, their product is not a meaningful measure according to the Measurement Theory (Roberts, 1979; Franceschini et al., 2019).
- The degree of (dis)agreement between the team members in formulating collective judgments is not taken into account.

Table 1. Severity/occurrence/detection evaluation criteria for P-FMEA suggested by the Automotive Industry Action Group (AIAG, 2019).

Rating	(*S*) Severity	(*O*) Occurr.	(*D*) Detection
10	May endanger (machine or assembly) operator, without warning.	Very high ($P \approx 10\%$)	No current process control; failure mode and/or cause cannot be detected or prevented.
9	May endanger (machine or assembly) operator, with warning.	High ($P \approx 5\%$)	Failure mode and/or cause is not easily detected (e.g., random audits).
8	100% of product may have to be scrapped; the production line may be shut down.	High ($P \approx 2\%$)	Failure-mode detection post-processing by operator through visual/tactile/audible means.
7	Product may have to be sorted and a portion (lower than 100%) scrapped; the production line is operational albeit at a reduced level of performance.	High ($P \approx 1\%$)	Failure-mode detection in-station by operator, trough visual/tactile/audible means, or post-processing, through use of *attribute* gauging (go/no-go, manual torque check/clicker wrench, etc.).
6	100% of the product may have to be reworked off-line and then accepted/rejected.	Moderate ($P \approx 0.2\%$)	Failure-mode detection post-processing by operator, through use of variable gauging, or in-station by operator, through use of *attribute* gauging (go/no-go, manual torque check/clicker wrench, etc.).
5	A portion (lower than 100%) of the product may have to be reworked off-line and then accepted/rejected.	Moderate ($P \approx 0.05\%$)	Failure-mode/cause detection in-station by operator, trough *variable* gauging, or by automated controls that will detect discrepant part and notify operator (light, buzzer, etc.). Gauging performed on setup and first-piece check (for set-up causes only).
4	100% of product may have to be reworked in-station before being processed.	Moderate ($P \approx 0.01\%$)	Failure-mode detection post-processing by automated controls that will detect discrepant part and lock part to prevent further processing.
3	A portion (lower than 100%) of product may have to be reworked in-station before being processed.	Low ($P \approx 0.001\%$)	Failure-mode detection in-station by automated controls that will detect discrepant part and automatically lock part in station to prevent further processing.
2	Slight inconvenience to process, operation, or operator.	Low ($P \leq 0.0001\%$)	Failure-cause detection in-station by automated controls that will detect error and prevent discrepant part from being made.
1	No discernible effect.	Very low ($P \approx 0$)	Failure-cause prevention as a result of fixture design, machine design or part design. Discrepant parts cannot be made because item has been error-proofed by process/product design.

From this point on, the attention will be focused on the *Process FMEA* (or more briefly P-FMEA), i.e., a declination of the FMEA approach for manufacturing processes, which can be carried out both (1) at the *design* stage, to consider potential failures prior to launching production, and (2) at the *operational* stage, to gradually improve the process reliability, in line with the concept of *continuous improvement* (Indrawati and Ridwansyah, 2015).

It is particularly interesting and challenging to assess the role of P-FMEA in the current globalised scenario, which is increasingly characterised by *distributed manufacturing processes* i.e., a form of decentralized manufacturing based on a network of geographically dispersed facilities that are supposed to be flexible, reconfigurable and coordinated through information technology (Matt et al., 2015; Srai et al., 2016). The use of local resources for customized products and the adoption of new production technologies in a digitalized environment (e.g., additive manufacturing, collaborative robots, etc.) make distributed manufacturing increasingly attractive for potential sustainability gains.

Unfortunately, decentralized production in some ways hampers the application of the traditional P-FMEA, which requires the joint work of several parties in formulating a number of collective judgments. Although it was proven that the effectiveness of the P-FMEA tends to improve for large groups of experts (Guerrero and Bradley, 2013), for decentralised processes the number of experts in each facility is generally small (e.g., five or less) (Liu et al., 2018; Liu et al., 2019a). The distinct application of P-FMEA to various decentralized processes is therefore ineffective in practice, being fragmented as well as unnecessarily repetitive.

In other practical situations, the P-FMEA is carried out only by the experts of the dominant production facility – e.g., the largest or the most central one – and then imposed on the satellite facilities – e.g., those of smaller size and/or more dispersed geographically. This other practice does not fully exploit the technical knowledge of the experts affiliated to the satellite facilities, which instead could deserve to be shared. The application of one-and-only-one P-FMEA involving the totality of the experts – although they are dispersed in a network of decentralized manufacturing facilities – would be much more effective.

Despite the great relevance of this problem, few existing studies focus on large-group decision making in the FMEA context (Geramian et al., 2019; Liu et al., 2019b). The purpose of this paper is to revise the traditional P-FMEA approach, making it effective also for distributed manufacturing environments. The revised approach – which will be referred to as "distributed P-FMEA" or, more briefly, "dP-FMEA" – replaces the collective judgments with the aggregation of individual judgments by experts, from the perspective of each of the three risk factors S, O and D.

The dP-FMEA approach is divided into two phases:

1. For each risk factor, expert judgments are fused through a recent aggregation technique – called ZM_{II} – which combines the Thurstone's *Law of Comparative Judgment* (LCJ) and the *Generalized Least Squares* (GLS) method (Thurstone, 1927; Kariya and Kurata, 2004). The ZM_{II}-technique can be applied in a variety of other decision-making contexts where experts make individual judgements on certain objects of interest (Franceschini and Maisano, 2019).
2. Further aggregation of the analysis results concerning S, O and D into a single composite indicator of criticality, with relative uncertainty estimation.

The new approach also includes a relatively versatile response mode based on the formulation of (incomplete) rankings and overcomes other limitations of the traditional P-FMEA, such as: (i) lack of consideration of the variability of expert judgments, (ii) arbitrary rating and questionable aggregation of expert judgments, (iii) lack of estimation of the uncertainty of results.

The remainder of the paper is organized into four sections. The section "ZM_{II}-technique" briefly recalls and exemplifies the ZM_{II}-technique, which will be applied in the first phase of the dP-FMEA. The section "Proposed methodology" illustrates the dP-FMEA methodology; the description is supported by a real-life case study concerning the processes of plastic injection-moulding, which are performed in several distributed manufacturing facilities of a worldwide company of thermal systems. The section "Conclusions" summarizes the original contributions of this paper, its practical implications, limitations and suggestions for future research. Further details on the ZM_{II}-technique and the dP-FMEA methodology are contained in the Appendix section.

ZM_{II}-technique

In general, the ZM_{II}-technique can be used for any group-decision problem in which a number of *experts* express their individual judgments on certain *objects*, based on the degree of a specific *attribute* (Franceschini and Maisano, 2019). This technique can be seen as a black box transforming some specific input data – i.e., expert judgments on the objects – into some specific output data – i.e., *ratio* scaling of the objects, with a relevant uncertainty estimation (Franceschini et al., 2019). Precisely, for each (i-th) object, the ZM_{II}-technique produces an estimate of (1) the (mean) ratio-scale value y_i and (2) the corresponding standard deviation σ_{y_i} (Franceschini and Maisano, 2019). The ZM_{II}-technique includes three fundamental phases, as summarized in the following three subsections. For details, see also the section "Detailed description of the ZM_{II}-technique" (in the Appendix).

Data collection (2nd level title)

A prerequisite of the ZM_{II}-technique is that each of the experts involved in the problem formulates a ranking of the objects – i.e., an ordered sequence including the objects with the highest grade of the attribute in the top positions and those with the lowest grade of the attribute in the bottom ones.

Apart from the *regular* objects (e.g., f_1 to f_{11} in the case study), experts may also include two (fictitious) *dummy* objects in their rankings: i.e., one (f_Z) corresponding to the *absence* of the attribute of interest, and one (f_M) corresponding to the *maximum-imaginable* degree of the attribute (Franceschini and Maisano, 2019). When dealing with these dummy objects, two important requirements should be considered:

- f_Z should be positioned at the bottom of a ranking, i.e., there should not be any other object with degree of the attribute lower than f_Z. In the case the attribute of another object is judged to be absent, that object will be considered indifferent to f_Z and positioned at the same hierarchical level.
- f_M should be positioned at the top of a ranking, i.e., there should not be any other object with degree of the attribute higher than f_M. In the case the attribute of another object is judged to be the maximum-imaginable, that object will be considered indifferent to f_M and positioned at the same hierarchical level.

In the best cases, experts formulate *complete* rankings, characterised by relationships of *strict dominance* (e.g., "$f_i > f_j$") or *indifference* (e.g., "$f_i \sim f_j$") among the possible pairs of objects. Unfortunately, the formulation of these rankings may be problematic for some experts, especially when the number of objects is large (Harzing et al., 2009). To overcome this obstacle, a flexible response mode that tolerates *incomplete* rankings is adopted. Below is a list of possible types of incomplete rankings.

- Rankings including only the objects with the higher degree of the attribute (or "t-objects", where "t" stands for "top") and those with the lower degree of the attribute (or "b-objects", where "b" stands for "bottom"); these rankings will be hereafter denominated "Type-t&b". The t parameter, which will be used below, is conventionally defined as the number of regular objects (i.e. excluding the two dummy objects) within the t-objects, while the b parameter is conventionally defined as the number of regular objects within b-objects. In the example in Fig. 2(a), t=b=2.
- Rankings including only the objects with the higher degree of the attribute (i.e., t-objects) among those available; see the example in Fig. 2 (b), in which t=2. From now on, these rankings will be denominated "Type-t".
- Type-t&b or Type-t rankings where t and b-objects are not ordered. These rankings reflect an even higher level of incompleteness, where experts simply indicate the t or b-objects of the ranking, without necessarily ordering them.
- Rankings not including the two dummy objects (f_Z and f_M), e.g., in the case experts find it difficult to envisage them. The rankings that do not include dummy objects but do include all regular ones will hereafter be referred to as "quasi-complete"; see the example in Fig. 2 (c).
- To contemplate the fact that experts may be unable to evaluate certain (regular) objects – e.g., those less familiar to them – they could formulate (incomplete) rankings that intentionally exclude some objects (see the example in Fig. 2(d)). Of course, there

cannot be relationships of strict dominance or indifference between the excluded objects and those included in the expert rankings, but only relationships of incomparability.

- Combining the previous three types of incomplete rankings, one can obtain Type-*t&b* or Type-*t* rankings that include or not the dummy objects and/or with ordered or unordered *t*/*b*-objects.

Fig. 2 also shows that a generic incomplete ranking can be transformed into a "reconstructed" ranking, including all the (dummy and regular) objects, with the addition of appropriate incomparability relationships. Borrowing the language of the Order Theory, the above reconstructed rankings may be referred to as *partial*: in addition to the relationships of strict dominance or indifference, these rankings also include some relationships of *incomparability* (e.g., "$f_i \| f_j$") among the possible pairs of objects.

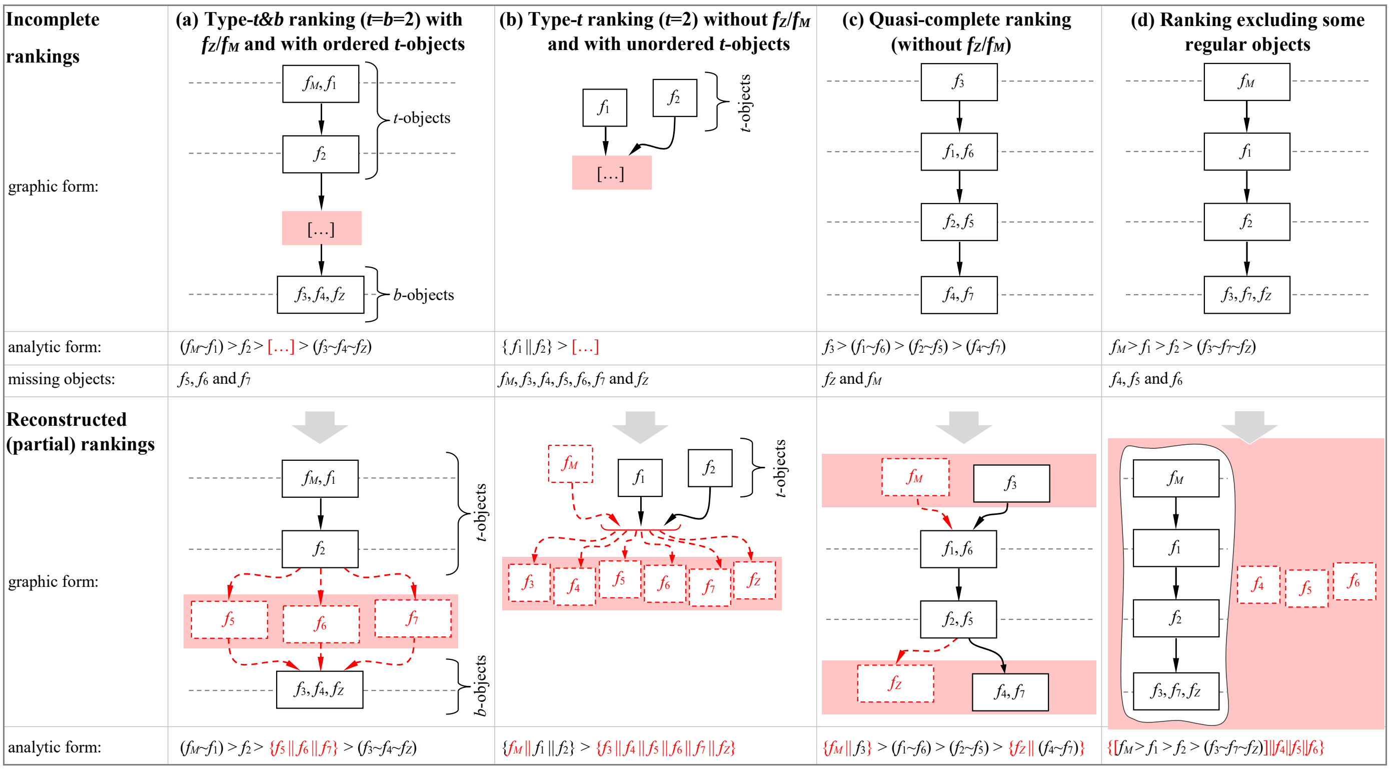

Fig. 2. Example of different types of incomplete rankings. Incomplete rankings can be turned into "reconstructed" (partial) rankings, including all the objects; reconstructed parts are marked in red.

The use of these rankings may also favour reliability of responses, since – in case of indecision – experts are not necessarily forced to provide complete and (illusorily) precise responses (Lagerspetz, 2016; Franceschini and Maisano, 2019). In general, the choice to provide more or less complete rankings may depend on several risk factors that affect experts, such as availability of time, level of education, willingness to collaborate, degree of experience, technical knowledge, etc.. To further improve the reliability of the data collected, it is possible to use common online procedures to support the generation of rankings, such as those briefly described below (Fabbris, 2013).

- *Numerical assignment*. In this technique, the respondent (expert) associates each object with a relevance order number, using a numerical drop-down box. The order of relevance of the objects and a respective ranking are then reconstructed.
- *Fixed total partitioning*. A respondent is assigned a fixed budget – say 100 points – and is asked to partition it over the objects, according to some attribute. This support technique is generally applied through a computer-assisted interviewing system that makes it possible to check the sum of points spent after each assignment (Conrad et al., 2005). Next, the ordinal hierarchies between the objects can be transformed into a ranking.
- *Drag and drop interface*. In this type of online supporting approach, respondents can drag and drop the objects to re-order them as they choose. This approach is interactive and helps the respondents to construct a ranking in a practical and intuitive way.

- *Picking the best/worst object*. Each respondent may be asked to pick the most relevant object and, in case, the least relevant one. Respondents are sometimes asked to pick k objects as the most/least relevant ones and – in some instances – to construct an order of this subset of objects. Once the most/less relevant objects have been isolated, the procedure could be iterated to the remaining objects.

In our case study, the latest support technique was mainly used, allowing experts to formulate complete/incomplete rankings, depending on their level of confidence and without forcing them to make uncertain judgments.

Data processing and solution (2nd level title)

The mathematical formalization of the problem relies on the postulates and simplifying assumptions of the *Law of Comparative Judgment* (LCJ) by Thurstone (1927), who postulated the existence of a *psychological/psychophysical continuum*, in which objects are positioned depending on the degree of a certain attribute. The position of a generic i-th object (f_i) is postulated to be distributed normally, in order to reflect the intrinsic expert-to-expert variability: $f_i \sim N(x_i, \sigma_{x_i}^2)$, where x_i and $\sigma_{x_i}^2$ are the unknown mean value and variance related to the degree of the attribute of that object. Considering two generic objects, f_i and f_j, and having introduced further simplifying hypotheses (see the section "Mathematical formulation of the problem", in the Appendix), it can be asserted that (Thurstone, 1927):

$$p_{ij} = P[(f_i - f_j) > 0] = 1 - \Phi[-(x_i - x_j)], \tag{1}$$

which expresses the probability (p_{ij}) that the position of f_i is higher than that of f_j, Φ being the cumulative distribution function of the standard normal distribution $z \sim N(0, 1)$.

Although p_{ij} is unknown, it can be estimated using the information contained in a set of judgments expressed by a number (m) of experts. The section "Mathematical formulation of the problem" (in the Appendix) explains how to estimate the p_{ij} values based on the positioning of the objects in the (reconstructed) rankings of experts (Franceschini and Maisano, 2019).
Extending the reasoning to all possible pairs of objects, an over-determined system of equations (like the equation in Eq. 1) can be obtained (Thurstone, 1927). Then, this system can be solved by applying the *Generalized Least Squares* (GLS) method (Kariya and Kurata, 2004), which allows to obtain an estimate of the mean degree of the attribute of each object: $\boldsymbol{X} = [\ldots, x_i, \ldots]^T$, expressed on an arbitrary *interval* scale with a relevant dispersion estimation (in the form of the standard deviations: $\sigma_{x_i}\ \forall i$). For details, see the section "Initial scaling" in the Appendix.

Transformation on a 0-to-10 scale (2nd level title)

Through the following transformation, the scale value of a generic i-th object (x_i) is transformed into a new scale value (y_i), which is defined in the conventional range [0, 10]:

$$\boldsymbol{Y} = \boldsymbol{Y}(\boldsymbol{X}) = [\ldots, y_i(\boldsymbol{X}), \ldots]^T = \left[\ldots, 10 \cdot \frac{x_i - x_Z}{x_M - x_Z}, \ldots\right]^T, \tag{2}$$

where: x_Z and x_M are the scale values of f_Z and f_M in the initial interval scale; x_i is the scale value of a generic i-th object in the initial interval scale; y_i is the scale value of a generic i-th object in the new scale (Franceschini and Maisano, 2019). Since scale y "inherits" the *interval* property from scale x and has a conventional zero point that corresponds to the absence of the attribute (i.e., $y_Z = 0$), it can be reasonably considered as a *ratio* scale, without any conceptually prohibited "promotion" (Franceschini et al., 2019). The section "Transformation of the initial scaling into the final one" (in the Appendix) illustrates how to determine the standard deviations (σ_{y_i}) of the y_i values, by "propagating" the uncertainty of the x_i values.

Proposed methodology

This section outlines the proposed methodology; the description will be followed by a practical application to a case study. In addition, a preliminary comparison with the results of the application of a traditional P-FMEA will be presented.

Data collection and processing (2nd level title)

Considering a generic P-FMEA, we can identify three separate decision-making problems in which:

- *experts* (e_1, e_2, …) are the engineers/technicians affiliated to different manufacturing plants of a company/organization of interest;
- *objects* (f_1, f_2, …) are the failure mode-cause combinations identified in the initial stages of the analysis; for the sake of simplicity, these objects will be hereafter referred to as "failure modes".
- *attributes* are respectively the risk factors: *S* for the first problem, *O* for the second problem, and *D* for the third problem.

In the proposed P-FMEA approach, for each of the three risk factors (*S*, *O* and *D*) each expert formulates his/her own three distinct (subjective) rankings of the failure modes. In line with what is explained in the "Data collection" section, these rankings may be *incomplete* and – in addition to the *regular* failure modes – they may include two *dummy* failure modes:

f_Z corresponding to a fictitious failure mode of absent severity/occurrence/detection (e.g., a failure mode associated with the rating S=1/O=1/D=1, according to the traditional FMEA reference tables, such as those in Table 1);

f_M corresponding to a fictitious failure mode of the maximum-imaginable severity/occurrence/detection (e.g., a failure mode associated with the rating S=10/O=10/D=10, according to the traditional P-FMEA reference tables, such as those in Table 1).

The rankings related to each risk factor are then aggregated through the application of the ZM_{II}-technique (see the section "ZM_{II}-technique"), resulting into a ratio scaling with a corresponding uncertainty estimation (Franceschini and Maisano, 2019). For the risk factors *S*, *O* and *D*, the resulting scale values related to a generic *i*-th failure mode will be conventionally referred to as S_i, O_i and D_i; the respective standard deviations will be referred to as σ_{S_i}, σ_{O_i} and σ_{D_i}. An important difference between the proposed methodology and the traditional P-FMEA is that for the former S_i, O_i, $D_i \in [1,10]$, while for the latter they $\in [0,10]$.

Aggregation of the three risk factors (2nd level title)

For a generic (*i*-th) failure mode, the aggregation of the scale values related to the three risk factors can be performed through the classic multiplicative model (Stamatis, 20013; Franceschini et al., 2019):

$$RPN_i = S_i \cdot O_i \cdot D_i. \quad (3)$$

This model implicitly assumes that the three risk factors are equally important. The section "Weighted additive aggregation model" (in the Appendix) presents an alternative (weighed) additive aggregation model, in which the three risk factors of interest are not necessarily equally important. In addition, it contains a sensitivity analysis aimed at showing the robustness of the alternative model, with respect to small variations in the weights.

Since the S_i, O_i and D_i values are defined on *ratio* scales, their product is a permissible operation; on the other hand, we remark that the traditional procedure unduly aggregates quantities defined on *ordinal* scales (Franceschini et al., 2019).

Uncertainty calculation (2nd level title)

The uncertainty related to the RPN_i values can be determined by applying the so-called *delta method*, also referred to as *law of propagation of uncertainty* or *error transmission* formula (JCGM 100:2008, 2008). It is thus obtained:

$$\sigma_{RPN_i} = \sqrt{(O_i \cdot D_i)^2 \cdot \sigma_{S_i}^2 + (S_i \cdot D_i)^2 \cdot \sigma_{O_i}^2 + (S_i \cdot O_i)^2 \cdot \sigma_{D_i}^2}\ , \tag{4}$$

where σ_{S_i}, σ_{O_i} and σ_{D_i} are the standard deviations associated with the S_i, O_i and the D_i values, for the i-th failure mode. The formula in Eq. 4 implicitly neglects the contributions of the correlations between the S_i, O_i and D_i values. This assumption seems reasonable, considering that the above values derive from three distinct scaling processes.

Assuming that the RPN_i values are approximately normally distributed, a 95% confidence interval related to each RPN_i value can be computed as:

$$RPN_i \pm U_{RPN_i} = RPN_i \pm 2 \cdot \sigma_{RPN_i} \quad \forall i\ , \tag{5}$$

U_{RPN_i} being the *expanded uncertainty* (JCGM 100:2008, 2008) of RPN_i, with a coverage factor $k = 2$.

The section "Weighted additive aggregation model" (in the Appendix) shows a similar uncertainty calculation for a (weighted) additive aggregation model.

Case study (2nd level title)

This subsection presents a case study, which exemplifies the application of the proposed procedure. An important worldwide supplier of thermal systems – which is kept anonymous for reasons of confidentiality – operates predominantly in the automotive sector. This company not only assembles electric compressors, heating-ventilating-and-air-conditioning (HVAC) units, radiators, etc., but also manufactures most of the corresponding components in-house. The focus of this case study is on the production of plastic pipes by injection moulding. Fig. 3 schematically illustrates (a) the main phases of this process, (b) the typical components manufactured for thermal systems, and (c) the structure of a generic injection-moulding press.

In Europe, the company of interest carries out this manufacturing process in four different plants located in four countries (i.e., Germany, Italy, Czech Republic and Spain), as illustrated in Fig. 4. The equipment used in the various plants is almost equivalent, as are the types of components manufactured; it is therefore reasonable to expect that equivalent injection-moulding processes are likely to be subject to the same failures. Following this reasoning, it would seem appropriate to share the experience accumulated in the various production facilities, in order to improve all processes in a comprehensive manner.

The above four processes are managed by twenty engineers/technicians overall, hereinafter referred to as "experts" (i.e., respectively seven for the German process, five for the Italian one, four for the Czech one and four for Spanish one, as shown in Fig. 4). Given the great difficulty in bringing together all the experts and making them interact to reach shared decisions, the traditional P-FMEA approach would be extremely difficult to manage, especially for activities 7, 8 and 10 in Fig. 1, which concern the formulation of collective judgments.

The initial activities of data collection, process description/analysis and determination of failures can be carried out quite easily. These activities are coordinated by a team leader (i.e. expert e_1, affiliated to process 1), who collects information and technical data from other experts, processing and organizing them appropriately (i.e., activities 1 to 6 and 9 in Fig. 1). The results of the initial activities are summarised (in a simplified way) in the P-FMEA table in Fig. 5, in which seven failure modes (A.1, A.2, B.1, etc.) and eleven relevant failure causes (A.1.1, A.1.2, A.2.1, etc.) have been determined. Table 2 shows the resulting failure mode-cause combinations (f_1 to f_{11}), which should be prioritized according to the three risk factors.

(a) Basic steps of a plastic injection-moulding process

1. *Material preparation*:
   - Determine the required amount of thermoplastic material;
   - Remove moisture using a dedicated dryer;
   - Mix the material with masterbatch (if required).
2. *Process set-up*:
   - Clamp the appropriate mould tool in the press;
   - Set the required process parameters (pressure, temperature, speed, …);
   - Load thermoplastic granules into the feeder hopper.
3. *Moulding*:
   - Material funnels from the hopper down into the (pre-heated) screw;
   - Barrel is pre-heated at (different) controlled temperatures along its length;
   - Screw rotates, moving the material to fill the mould cavity;
   - Material is injected into the feed channels of the mould tool;
   - Mould tool is pre-heated at a predefined temperature;
   - After a certain cooling time, the mould is opened and the workpiece ejected.

(b) Example of thermal-system components

(c) Scheme of a typical press

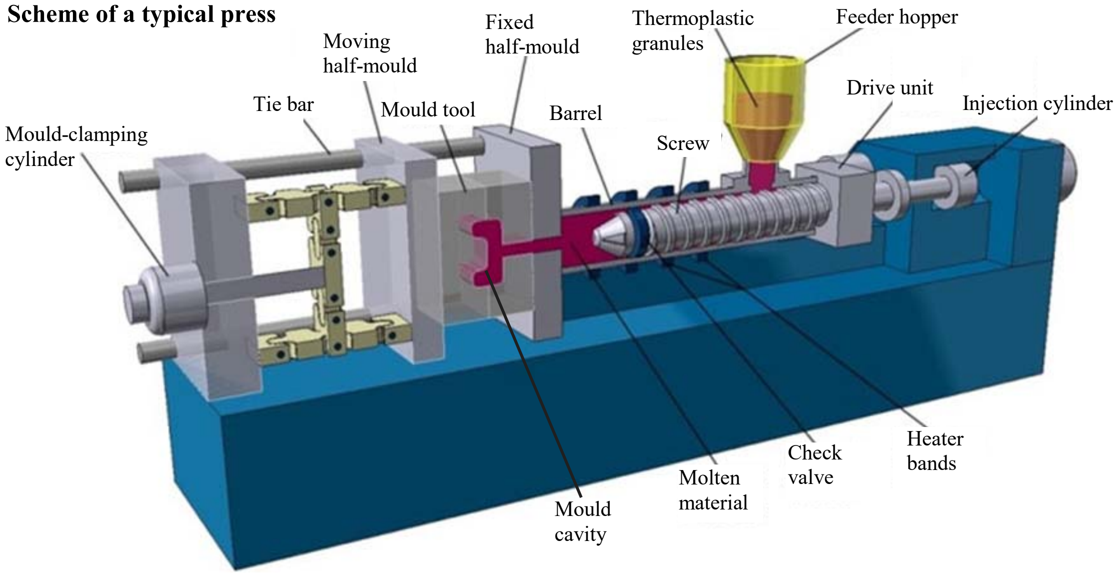

Fig. 3. Plastic injection-moulding process: (a) basic process steps, (b) example of finished components for thermal systems, and (c) scheme of a typical injection-moulding press.

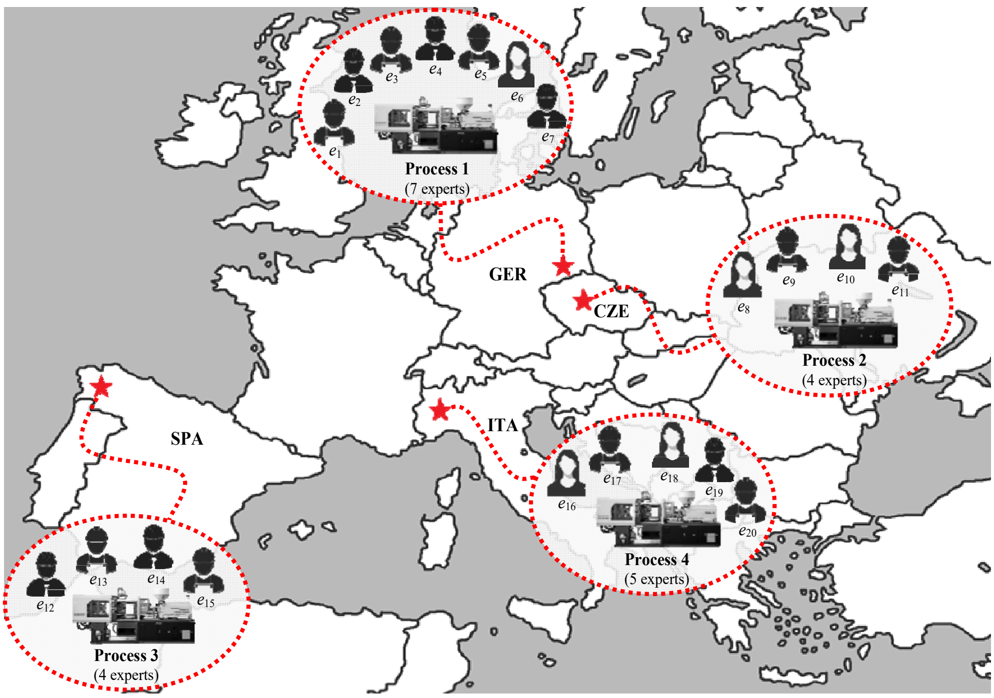

Fig. 4. Scheme of the four European injection-moulding processes of the company of interest, with the corresponding engineers/technicians, referred to as "experts", e_1 to e_{20}.

Function	Failure modes	Potential effects	(*S*) Sever.	Failure causes	(*O*) Occurr.	Current process control(s)	(*D*) Detect.	*RPN*
A - Material preparation	A.1 - Incorrect material	Some characteristics of the finished product may be (at least partially) compromised		A.1.1 - Error by the operator		Use of a check list		
				A.1.2 - Incorrect material tag		Initial weight check		
	A.2 - Wet material	More difficult moulding		A.2.1 - Malfunction of the dryer		None		
B - Material heating/filling	B.1 - Incorrect temperature of some parts of the mould	Material does not fill the mould cavity correctly, making moulding difficult		B.1.1 - Malfunction of the temperature-control system of the screw		Screw-mounted thermocouple		
				B.1.2 - Malfunction of the temperature-control system of the barrel		Barrel-mounted thermocouple		
C - Injection-parameter setting	C.1 - Inadequate pressure	Insufficient filling		C.1.1 - Incorrect setting of pressure parameters		Pressure gauge		
	C.2 - Exaggerated injection speed	Generation of so-called “silver streaks”		C.2.1 - Incorrect setting of speed parameters		Automatic parameter loading		
D - Mould closing	D.1 - Half moulds are not in perfect contact	Plastic threads through the separation plane, resulting in defects		D.1.1 - Insufficient force of the clamping cylinder		Visual inspection		
				D.1.2 - Deformation of the half-mould pins		Pressure gauge		
E - Workpiece rejection	E.1 - Malfunction of the ejection system	Process interruption due to failure to unload the workpiece		E.1.1 - Insufficient draft angles		None		
				E.1.2 - The moving part of the mould is blocked by the workpiece		None		

Fig. 5. Simplified P-FMEA table related to a plastic injection-moulding process.

Table 2. Failure mode-cause combinations that will be prioritized according to the risk factors *S*, *O*, and *D*.

Abbrev.	Failure mode	Failure cause
f_1	A.1 - Incorrect material	A.1.1 - Error by the operator
f_2	*Ibidem*	A.1.2 - Incorrect material tag
f_3	A.2 - Wet material	A.2.1 - Malfunction of the dryer
f_4	B.1 - Incorrect temperature of some parts of the mould	B.1.1 - Malfunction of the temperature-control system of the screw
f_5	*Ibidem*	B.1.2 - Malfunction of the temperature-control system of the barrel
f_6	C.1 - Inadequate pressure	C.1.1 - Incorrect setting of pressure parameters
f_7	C.2 - Exaggerated injection speed	C.2.1 - Incorrect setting of speed parameters
f_8	D.1 - Half moulds are not in perfect contact	D.1.1 - Insufficient force of the clamping cylinder
f_9	*Ibidem*	D.1.2 - Deformation of the half-mould pins
f_{10}	E.1 - Malfunction of the ejection system	E.1.1 - Insufficient draft angles
f_{11}	*Ibidem*	E.1.2 - The moving part of the mould is blocked by the workpiece

The parts concerning the collective assignments of the *S*, *O* and *D* scores and the aggregation of the aforesaid scores through *RPN* have been intentionally left incomplete. These parts are completed below using the new dP-FMEA approach, which replaces collective expert judgments with the aggregation of individual judgments.

The rankings formulated by the experts are shown in Fig. 6. It can be noted that most of the experts have opted for the formulation of incomplete rankings, since they are simpler and faster. For each ranking, it is specified: (i) the ranking type (complete, quasi-complete, Type-*t&b* or Type-*t*), (ii) whether or not the two dummy failure modes have been included by the expert (“Manage f_Z/f_M?”), (iii) the value of the parameters *t* and *b*, in the case of Type-*t&b* and Type-*t* rankings, and (iv) if the *t*/*b*-objects are ordered or not. Fig. 6 also shows that rankings related to the same risk factor can be very different from each other, denoting a certain inter-expert disagreement. For instance, while the majority of experts included the object f_8 among the top positions of their *O*-rankings, other experts – such as e_8, e_{10} and e_{12} – excluded it from the top positions. This makes us reflect on the actual difficulty of experts to converge towards collective judgements in the traditional P-FMEA; in addition, the opinion of younger and less experienced experts may not infrequently be inhibited/conditioned by that of senior experts. Being based on the formulation of individual judgments, the proposed response mode will avoid this.

Process	Expert	Ranking type	Manage f_Z/f_M?	t/b value	Order t/b-objects?	Dimension	Ranking
1. GER	e_1	Complete	Yes	N/A	N/A	S	$(f_M \sim f_1 \sim f_2) > f_4 > f_9 > (f_8 \sim f_{11}) > f_5 > f_3 > (f_Z \sim f_6 \sim f_{10} \sim f_7)$
						O	$(f_M \sim f_8) > (f_7 \sim f_5) > (f_3 \sim f_2) > f_1 > (f_6 \sim f_{10}) > f_{11} > f_9 > (f_4 \sim f_Z)$
						D	$f_M > f_9 > (f_6 \sim f_4 \sim f_{11}) > (f_8 \sim f_1 \sim f_3) > f_{10} > (f_5 \sim f_7) > (f_2 \sim f_Z)$
	e_2	Type-t	N/A	2	No	S	$\{f_M \Vert f_1 \Vert f_8 \Vert f_9\} > \{f_Z \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_8 \Vert f_{11}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10}\}$
						D	$\{f_M \Vert f_2 \Vert f_6\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
	e_3	Type-t&b	N/A	3	No	S	$\{f_M \Vert f_1 \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_7\} > \{f_2 \Vert f_8 \Vert f_9\} > \{f_Z \Vert f_6 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_5 \Vert f_8 \Vert f_{11}\} > \{f_2 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_{10}\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_9\}$
						D	$\{f_M \Vert f_3 \Vert f_6 \Vert f_9 \Vert f_{10}\} > \{f_1 \Vert f_5 \Vert f_8\} > \{f_Z \Vert f_2 \Vert f_4 \Vert f_7 \Vert f_{11}\}$
	e_4	Type-t&b	N/A	2	No	S	$\{f_M \Vert f_1 \Vert f_2 \Vert f_5\} > \{f_3 \Vert f_4 \Vert f_7 \Vert f_8 \Vert f_9\} > \{f_Z \Vert f_6 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_8 \Vert f_{11}\} > \{f_1 \Vert f_3 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10}\} > \{f_Z \Vert f_2 \Vert f_4\}$
						D	$\{f_M \Vert f_3 \Vert f_5 \Vert f_9\} > \{f_1 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_{10}\} > \{f_Z \Vert f_2 \Vert f_8 \Vert f_{11}\}$
	e_5	Quasi-complete	No	N/A	N/A	S	$\{f_M \Vert f_2\} > (f_5 \sim f_{10}) > f_9 > (f_7 \sim f_8 \sim f_4 \sim f_1 \sim f_{11}) > f_6 > \{f_Z \Vert f_3\}$
						O	$\{f_M \Vert f_8\} > (f_4 \sim f_9 \sim f_{11}) > (f_{10} \sim f_5) > f_3 > f_1 > f_6 > \{f_Z \Vert (f_2 \sim f_7)\}$
						D	$\{f_M \Vert f_3\} > (f_2 \sim f_8) > f_{11} > f_5 > f_{10} > (f_4 \sim f_1 \sim f_6) > \{f_Z \Vert (f_9 \sim f_7)\}$
	e_6	Type-t	N/A	3	No	S	$\{f_M \Vert f_1 \Vert f_3 \Vert f_4\} > \{f_Z \Vert f_2 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_4 \Vert f_5 \Vert f_{11}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10}\}$
						D	$\{f_M \Vert f_3 \Vert f_9 \Vert f_{11}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_{10}\}$
	e_7	Type-t&b	N/A	2	No	S	$\{f_M \Vert f_1 \Vert f_2\} > \{f_4 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10}\} > \{f_Z \Vert f_3 \Vert f_5 \Vert f_{11}\}$
						O	$\{f_M \Vert f_5 \Vert f_8 \Vert f_9\} > \{f_2 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_{11}\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_{10}\}$
						D	$\{f_M \Vert f_2 \Vert f_3\} > \{f_4 \Vert f_5 \Vert f_8 \Vert f_9 \Vert f_{10}\} > \{f_Z \Vert f_1 \Vert f_6 \Vert f_7 \Vert f_{11}\}$
2. CZE	e_8	Type-t	Yes	3	Yes	S	$(f_M \sim f_1 \sim f_2) > (f_3 \sim f_4 \sim f_9) > \{f_Z \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_{10} \Vert f_{11}\}$
						O	$(f_M \sim f_5) > f_{10} > f_4 > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{11}\}$
						D	$f_M > f_6 > f_3 > (f_1 \sim f_7 \sim f_{10}) > \{f_Z \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_8 \Vert f_9 \Vert f_{11}\}$
	e_9	Type-t&b	Yes	3	Yes	S	$f_M > (f_1 \sim f_3 \sim f_4) > \{f_2 \Vert f_5 \Vert f_6 \Vert f_8 \Vert f_9\} > f_{11} > f_{10} > (f_Z \sim f_7)$
						O	$(f_M \sim f_5) > f_7 > f_8 > \{f_6 \Vert f_9 \Vert f_{10} \Vert f_{11}\} > (f_2 \sim f_3 \sim f_4 \sim f_Z \sim f_1)$
						D	$f_M > f_2 > f_8 > f_4 > \{f_3 \Vert f_5 \Vert f_{11}\} > (f_6 \sim f_7 \sim f_Z \sim f_9 \sim f_{10} \sim f_1)$
	e_{10}	Type-t	N/A	2	No	S	$\{f_M \Vert f_1 \Vert f_2\} > \{f_Z \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_3 \Vert f_{10}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{11}\}$
						D	$\{f_M \Vert f_5 \Vert f_{10}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{11}\}$
	e_{11}	Type-t&b	N/A	3	No	S	$\{f_M \Vert f_3 \Vert f_5 \Vert f_8 \Vert f_9\} > \{f_1 \Vert f_{10}\} > \{f_Z \Vert f_2 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_{11}\}$
						O	$\{f_M \Vert f_5 \Vert f_8 \Vert f_9 \Vert f_{10}\} > \{f_2 \Vert f_6 \Vert f_7 \Vert f_{11}\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_4\}$
						D	$\{f_M \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_6 \Vert f_7 \Vert f_8\} > \{f_5 \Vert f_{11}\} > \{f_Z \Vert f_4 \Vert f_9 \Vert f_{10}\}$
3. SPA	e_{12}	Type-t	Yes	2	Yes	S	$f_M > (f_2 \sim f_3 \sim f_4 \sim f_9) > \{f_Z \Vert f_1 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_{10} \Vert f_{11}\}$
						O	$f_M > f_{10} > (f_6 \sim f_5) > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{11}\}$
						D	$f_M > f_3 > f_8 > \{f_Z \Vert f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
	e_{13}	Type-t&b	Yes	2	Yes	S	$(f_M \sim f_1) > (f_7 \sim f_9 \sim f_{10}) > \{f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_8\} > f_{11} > f_3 > f_Z$
						O	$f_M > (f_5 \sim f_{11}) > \{f_1 \Vert f_3 \Vert f_4 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10}\} > f_6 > (f_2 \sim f_Z)$
						D	$(f_M \sim f_3) > f_4 > \{f_2 \Vert f_5 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\} > (f_6 \sim f_Z \sim f_1)$
	e_{14}	Type-t	N/A	3	No	S	$\{f_M \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_8 \Vert f_9\} > \{f_Z \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_5 \Vert f_8 \Vert f_{10}\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{11}\}$
						D	$\{f_M \Vert f_2 \Vert f_4 \Vert f_8\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
	e_{15}	Type-t&b	No	2	Yes	S	$\{f_M \Vert f_2\} > (f_5 \sim f_6) > \{f_1 \Vert f_3 \Vert f_4 \Vert f_8 \Vert f_9 \Vert f_{11}\} > f_7 > \{f_Z \Vert f_{10}\}$
						O	$\{f_M \Vert f_5\} > f_6 > \{f_1 \Vert f_4 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\} > \{f_Z \Vert (f_2 \sim f_3 \sim f_7)\}$
						D	$\{f_M \Vert f_2\} > f_9 > \{f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_8 \Vert f_{10}\} > \{f_Z \Vert (f_1 \sim f_7 \sim f_{11})\}$
4. ITA	e_{16}	Type-t	Yes	1	Yes	S	$(f_M \sim f_1) > \{f_Z \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						O	$(f_M \sim f_5) > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						D	$f_M > f_3 > \{f_Z \Vert f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
	e_{17}	Type-t&b	No	3	Yes	S	$\{f_M \Vert f_7\} > f_5 > (f_4 \sim f_{10}) > \{f_1 \Vert f_2 \Vert f_3 \Vert f_8\} > (f_9 \sim f_6) > \{f_Z \Vert f_{11}\}$
						O	$\{f_M \Vert f_{10}\} > f_5 > f_6 > \{f_2 \Vert f_4 \Vert f_7 \Vert f_8 \Vert f_9\} > f_3 > \{f_Z \Vert (f_1 \sim f_{11})\}$
						D	$\{f_M \Vert f_3\} > f_2 > f_6 > \{f_8 \Vert f_9\} > \{f_Z \Vert (f_5 \sim f_7 \sim f_1 \sim f_4 \sim f_{10} \sim f_{11})\}$
	e_{18}	Type-t	No	2	Yes	S	$\{f_M \Vert (f_1 \sim f_5)\} > \{f_Z \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert (f_5 \sim f_8)\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
						D	$\{f_M \Vert (f_4 \sim f_8)\} > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$
	e_{19}	Type-t&b	Yes	1	Yes	S	$(f_M \sim f_1 \sim f_2) > \{f_3 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_9 \Vert f_{11}\} > f_{10} > f_Z$
						O	$f_M > f_6 > \{f_4 \Vert f_5 \Vert f_8 \Vert f_{10} \Vert f_{11}\} > (f_2 \sim f_7 \sim f_3 \sim f_9 \sim f_Z \sim f_1)$
						D	$(f_M \sim f_3) > \{f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_6 \Vert f_7 \Vert f_8 \Vert f_{10} \Vert f_{11}\} > (f_Z \sim f_9)$
	e_{20}	Type-t	No	3	Yes	S	$\{f_M \Vert (f_2 \sim f_5 \sim f_8 \sim f_9)\} > \{f_Z \Vert f_1 \Vert f_3 \Vert f_4 \Vert f_6 \Vert f_7 \Vert f_{10} \Vert f_{11}\}$
						O	$\{f_M \Vert f_5\} > (f_4 \sim f_7 \sim f_9) > \{f_Z \Vert f_1 \Vert f_2 \Vert f_3 \Vert f_6 \Vert f_8 \Vert f_{10} \Vert f_{11}\}$
						D	$\{f_M \Vert f_3\} > (f_6 \sim f_7 \sim f_8) > \{f_Z \Vert f_1 \Vert f_2 \Vert f_4 \Vert f_5 \Vert f_9 \Vert f_{10} \Vert f_{11}\}$

Fig. 6. (Incomplete) rankings of failure modes, formulated by the experts for each of the three risk factors. Referring to the rankings in the last column, the failure modes identified directly by the experts are marked in black, while the reconstructed parts are marked in red.

The rankings related to each risk factor are then aggregated through the application of the ZM_{II}-technique; results are reported in the first seven columns of Table 3.

Table 3. Results of the application of the proposed methodology to the case study, in terms of mean and standard deviation of the S_i, O_i, D_i values and corresponding RPN_i values.

	S_i values		O_i values		D_i values		RPN_i values		
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.	$U_{RPN_i} = 2 \cdot \sigma_{RPN_i}$
f_1	7.69	0.51	2.03	0.60	3.01	0.55	46.9	16.5	33.0
f_2	6.91	0.51	2.23	0.58	5.19	0.51	79.9	22.9	45.8
f_3	4.88	0.52	2.69	0.55	7.46	0.55	98.0	23.8	47.6
f_4	5.60	0.53	4.14	0.51	4.48	0.52	103.8	20.1	40.2
f_5	5.80	0.52	8.06	0.56	4.14	0.54	193.7	33.4	66.8
f_6	2.71	0.61	4.57	0.52	4.82	0.51	59.5	16.3	32.6
f_7	3.76	0.54	4.05	0.54	3.05	0.56	46.4	12.4	24.8
f_8	5.08	0.54	6.93	0.56	5.54	0.52	195.1	31.7	63.4
f_9	5.96	0.52	4.26	0.54	4.23	0.51	107.1	20.9	41.8
f_{10}	3.36	0.55	5.74	0.52	4.03	0.53	77.7	17.8	35.6
f_{11}	2.09	0.66	5.27	0.52	3.29	0.55	36.3	13.5	27.0

Next, the RPN_i values of the failure modes and the respective uncertainties are determined by applying Eqs. 3, 4 and 5; these results are contained in the last three columns of Table 3. It can be noticed that the expanded-uncertainty values of the failure modes are relatively large, due to the uncertainty propagation. The most critical failure modes – i.e., those deserving more attention when planning possible corrective actions – are those with higher RPN_i values (see also the Pareto chart in Fig. 7).

The relatively wide uncertainty bands indicate that the RPN_i alone is a "myopic" indicator, since it may perform differentiations that are unfounded from a statistical point of view. For instance, while it makes sense to say that f_8 is certainly more critical than f_2 or f_6 (being the uncertainty band of the former not superimposed on those of the latter two), it can not necessarily be said that f_2 deserves priority over f_6 (being the relevant uncertainty bands superimposed).

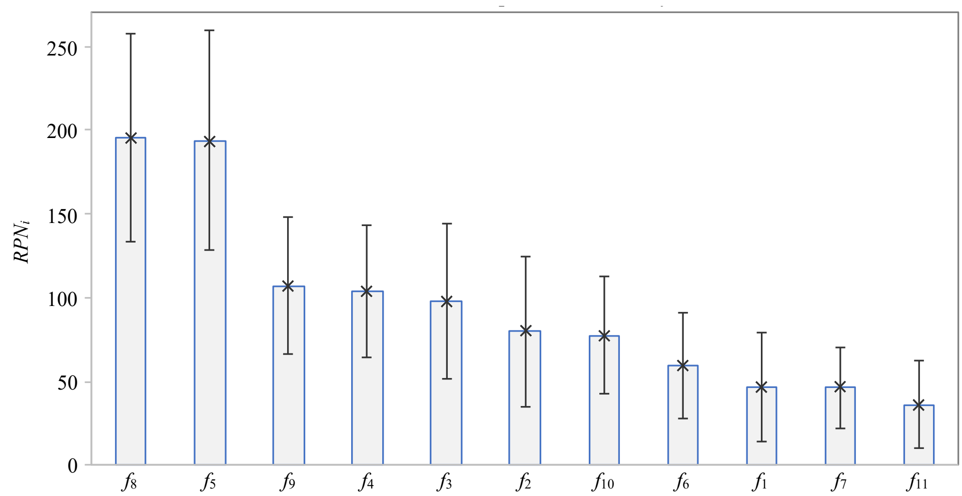

Fig. 7. Pareto chart of the failure modes based on their RPN_i values and relevant expanded-uncertainty (U_{RPN_i}) bands (data in Table 3).

These considerations give the team a few more degrees of freedom in the choice of corrective actions, perhaps taking into account other external constraints (such as cost, technical difficulty, time required, etc.).

Additionally, we note that failure modes with higher RPN_i values tend to have higher uncertainty. This sort of *heteroschedasticity* depends on the multiplicative aggregation model of S, O and D (Eq. 3) (Ross, 2014). Another limitation of this aggregative model is that it does not allow to weigh the contributions of S, O and D, which are actually considered as equally important. The section "Weighted additive aggregation model" (in the Appendix) exemplifies the application of a (weighted) additive aggregation model, showing the results with their respective uncertainty.

After prioritizing failure modes, P-FMEA continues with the iterative definition and implementation of corrective actions, as illustrated in Fig. 1. Like a traditional P-FMEA, this activity requires the coordination of the team leader who interacts with individual experts.

Comparison with traditional P-FMEA (2nd level title)

As a further verification of the validity of the results provided by the proposed technique, this section contains a preliminary comparison with the results deriving from a traditional P-FMEA. To do this, we reconsidered the case study, asking the experts to collectively assign *S*, *O* and *D* scores to each combination of failure mode-cause, in line with the traditional P-FMEA procedure (phases 7, 8 and 10 of the flowchart in Fig. 1). Unfortunately, due to the great difficulty in bringing together and coordinating a large number of experts affiliated in different and often remote production facilities, it was possible to bring together concurrently only four out of these twenty experts(!): e_1, e_2, e_8 and e_{16}. Despite its small size, this subset of experts consists of engineers with relatively high experience, who are responsible for guiding and coordinating manufacturing activities in three different European plants (see Fig. 4). Therefore, this subset can be considered as sufficiently representative of all experts.

Table 4 contains the S_i, O_i and D_i-scores collectively assigned to the eleven failure modes (f_1 to f_{11}, cf. Table 2). Subsequently, for each (i-th) failure mode, the corresponding scores were aggregated in the composite indicator RPN_i, according to the traditional multiplicative model in Eq. 3. We note that, being the above mentioned scores $\in [1, 10]$, the resulting RPN_i values $\in [1, 1000]$. On the other hand, being the S_i, O_i and D_i-scale values from the dP-FMEA approach $\in [0, 10]$, the resulting RPN_i values $\in [0, 1000]$. Despite this slight discrepancy, it is possible to make a quantitative comparison between the results deriving from the two approaches, as illustrated by the graphs in Fig. 8.

Precisely, there is a strong correlation between the data resulting from the two approaches; see respectively the graphs in Fig. 8(a), (b) and (c) (R^2 determination coefficients very close to 90%). An analogous correlation between the corresponding RPN_i values can be observed, denoting a sort of "convergent validity" between the new dP-FMEA approach and that of the traditional P-FMEA, keeping in mind the major practical advantages of the former with respect to the latter (Hair et al., 2017).

Table 4. S_i, O_i and D_i-scores from the implementation of the traditional P-FMEA to the case study. Subsequently, the above scores are aggregated in the RPN_i values (last column), using the multiplicative model of Eq. 3.

	S_i	O_i	D_i	RPN_i
f_1	7.69	2.03	3.01	46.9
f_2	6.91	2.23	5.19	79.9
f_3	4.88	2.69	7.46	98.0
f_4	5.60	4.14	4.48	103.8
f_5	5.80	8.06	4.14	193.7
f_6	2.71	4.57	4.82	59.5
f_7	3.76	4.05	3.05	46.4
f_8	5.08	6.93	5.54	195.1
f_9	5.96	4.26	4.23	107.1
f_{10}	3.36	5.74	4.03	77.7
f_{11}	2.09	5.27	3.29	36.3

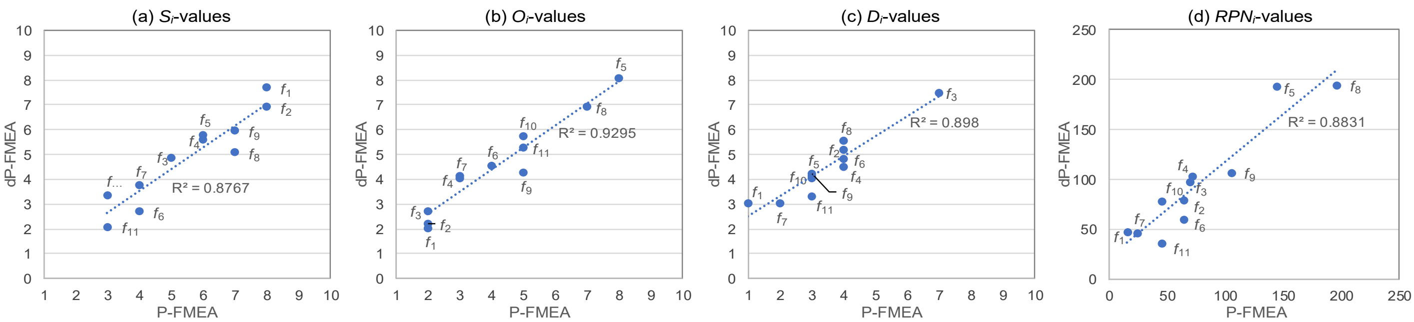

Fig. 8. Qualitative comparison between the results of the new dP-FMEA and the traditional P-FMEA, with reference to the case study. The comparison is made in terms of (a) S_i, (b) O_i, (c) D_i, and (d) RPN_i-values. In all cases there is a strong correlation (R^2 values very close to 90%).

Conclusions

This paper illustrated the innovative dP-FMEA approach, which can be applied to distributed manufacturing environments. This approach is potentially more suitable than the traditional P-FMEA, for several practical reasons:

- The procedure allows to manage dozens of experts, without requiring them to physically meet and make collective decisions. In addition, the procedure considers the precious contribution of all the experts – concept of "wisdom of crowds" (Cai et al., 2017) – even the younger/weaker ones, who are not infrequently inhibited in the traditional P-FMEA group sessions (Geramian et al., 2019).
- The method includes a flexible response mode, which does not force experts to make detailed judgments, even in case of hesitation.
- Unlike the traditional P-FMEA and other variants in the scientific literature, the proposed procedure provides an estimation of the uncertainty of the results obtained (Franceschini and Galetto, 2001; Das Adhikary et al., 2014; Liu et al., 2018; Li et al., 2019; Liu et al., 2019b). This aspect is far from being insignificant since it gives the expert team more freedom in planning possible corrective actions.
- The methodology can be easily implemented using an *ad hoc* software application developed by the authors (in MS Excel - VBA environment), which is available on request.
- The proposed methodology allows to overcome some widely debated shortcomings of the traditional P-FMEA, such as:
  (1) It does not require the use of arbitrary reference tables for the assignment of S, O and D scores (e.g., those exemplified in Table 1);
  (2) It does not introduce any unduly "promotion" of the scales on which expert judgments are defined (Franceschini et al., 2019).

Although there is no absolute reference ("gold standard") to evaluate the validity of the proposed procedure, a preliminary comparison with the traditional P-FMEA procedure shows a certain agreement between the results obtained (concept of "convergent validity"). It should also be noted that the ZM_{II}-technique – which is used in the first phase of the proposed procedure – is a widely validated and consolidated that is strictly related to the traditional LCJ (Thurstone, 1927; Edwards, 1957; Gulliksen, 1956; Franceschini and Maisano, 2019). This constitutes a certain guarantee of the soundness of the results provided.

The proposed procedure has some limitations:

- The way of determining the RPN_i values is more laborious than for the traditional P-FMEA.
- The proposed response mode, although being flexible, represents a novelty with respect to the traditional one, which is based on the use of reference tables. This could create some problems, especially for more experienced users that are accustomed to the traditional procedure.

Regarding the future, we plan to develop a variant of the dP-FMEA approach, in which experts are not equally important, but are characterized by a hierarchy of importance.

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Appendix

Detailed description of the ZM_{II}-technique (2nd level title)

This section provides a detailed description of the ZM_{II}-technique and is organized into three sub-sections, which respectively illustrate (i) the mathematical formulation of the decision-making problem of interest, (ii) the determination of an initial scaling, and (iii) the "transformation" of this initial scaling into a *ratio* scaling, with relevant uncertainty estimation.

Mathematical formulation of the problem (3rd level title)

The mathematical formalization of the problem relies on the postulates and simplifying assumptions of the *Law of Comparative Judgment* (LCJ) by Thurstone. Precisely, Thurstone (1927) postulated the existence of a *psychological continuum*, i.e., an abstract and unknown unidimensional scale, in which objects are positioned depending on the degree of a certain attribute – i.e., a specific feature of the objects, which evokes a subjective response in each expert. In the context of P-FMEA, possible attributes are the *severity*, *occurrence* and *detection* of the failure modes (objects).

The position of a generic *i*-th object is postulated to be distributed normally, in order to reflect the intrinsic expert-to-expert variability: $f_i \sim N(x_i, \sigma_{x_i}^2)$, where x_i and $\sigma_{x_i}^2$ are the unknown mean value and variance related to the degree of the attribute of that object. Considering two generic objects, f_i and f_j, it can therefore be asserted that:

$$f_i - f_j \sim N\left(x_i - x_j, \sigma_{x_i}^2 + \sigma_{y_i}^2 - 2 \cdot \rho_{x_i x_j} \cdot \sigma_{x_i} \cdot \sigma_{y_i}\right) \tag{A.1}$$

where $\rho_{x_i x_j}$ is the *Pearson coefficient*, denoting the correlation between the positions of objects f_i and f_j (Ross, 2014). The probability that the position of f_i is higher than that of f_j can be expressed as:

$$p_{ij} = P\left[\left(f_i - f_j\right) > 0\right] = 1 - \Phi\left[\frac{0 - \left(x_i - x_j\right)}{\sqrt{\sigma_{x_i}^2 + \sigma_{x_j}^2 - 2 \cdot \rho_{x_i x_j} \cdot \sigma_{x_i} \cdot \sigma_{x_j}}}\right], \tag{A.2}$$

Φ being the cumulative distribution function of the standard normal distribution $z \sim N(0, 1)$.

The LCJ (*case V*) includes the following additional simplifying assumptions (Thurstone, 1927; Edwards, 1957): $\sigma_{x_i}^2 = \sigma^2 \ \forall i$, $\rho_{x_i x_j} = \rho, \forall i, j$, and $2 \cdot \sigma^2 \cdot (1 - \rho) = 1$. Eq. A.2 can therefore be expressed as:

$$p_{ij} = 1 - \Phi[-(x_i - x_j)]. \tag{A.3}$$

Although p_{ij} is unknown, it can be estimated using the information contained in a set of (subjective) judgments by a number (m) of experts (e_1, e_2, …) (Thurstone, 1927). In fact, experts formulate rankings of the objects, which can be decomposed into paired-comparison relationships of *strict dominance* (e.g., "$f_i > f_j$" or "$f_i < f_j$"), *indifference* (e.g., "$f_1 \sim f_2$") or *incomparability* (e.g., "$f_1 \parallel f_2$") (Franceschini and Maisano, 2019). For the purpose of example, the four-object ranking "$(f_1 \parallel f_2) > (f_3 \sim f_4)$" can be decomposed into the following $C_2^4 = 6$ paired-comparison relationships: "$f_1 \parallel f_2$", "$f_1 > f_3$", "$f_1 > f_4$", "$f_2 > f_3$", "$f_2 > f_4$", and "$f_3 \sim f_4$".

Then, for each expert for which $f_i > f_j$, a frequency indicator k_{ij} is incremented by one unit. In the case the two objects are considered indifferent, k_{ij} is conventionally incremented by 0.5, so that:

$$k_{ij} = m_{ij} - k_{ji}, \tag{A.4}$$

m_{ij} being the total number of experts from which it is possible to obtain a relationship of strict dominance or indifference for the *i,j*-th paired comparison. These two types of relationships are called "usable" because they can contribute to the p_{ij} estimation, as shown below. In general, $m_{ij} \leq m$ since for some experts it is possible to obtain only a relationship of incomparability.

The observed proportion of experts for which the degree of the attribute of f_i is higher than that of f_j can be used to estimate the unknown probability p_{ij}:

$$\hat{p}_{ij} = \frac{k_{ij}}{m_{ij}}. \tag{A.5}$$

Of course, the relationship of complementarity $\hat{p}_{ij} = 1 - \hat{p}_{ji}$ holds.

Returning to Eq. A.3, it can be expressed as:

$$\hat{p}_{ij} = 1 - \Phi[-(x_i - x_j)], \tag{A.6}$$

from which:

$$x_i - x_j = -\Phi^{-1}(1 - \hat{p}_{ij}), \tag{A.7}$$

It can be noticed that, if all experts express the same judgment, the model is no more viable: $\hat{p}_{ij}$ values of 1.00 and 0.00 would correspond to $-\Phi^{-1}(1-\hat{p}_{ij})$ values of $\pm\infty$. A simplified approach for tackling this problem is to associate values of $\hat{p}_{ij} \geq 0.977$ with $-\Phi^{-1}(1 - 0.977) = 1.995$ and values of $\hat{p}_{ij} \leq 0.023$ with $-\Phi^{-1}(1 - 0.023) = -1.995$. More sophisticated solutions to deal with this issue have been proposed (Edwards, 1957).

Extending the reasoning to all possible paired comparisons for which $m_{ij} \geq 1$ (i.e., for at least one expert, there is a usable paired-comparison relationship), the relevant $\hat{p}_{ij}$ values can be determined, and the following system of equations can be constructed:

$$\begin{cases} \vdots \\ x_i - x_j + \Phi^{-1}\left(1 - \hat{p}_{ij}\right) = 0 \qquad \forall i, j : m_{ij} \geq 1 \\ \vdots \end{cases}. \tag{A.8}$$

Since, the rank of the system is lower than the number (n) of unknowns of the problem (i.e., x_i $\forall i$) – and the system itself would be indeterminate – the following conventional condition was introduced by Thurstone (1927):

$$\sum_i x_i = 0. \tag{A.9}$$

Eqs. A.8 and A.9 are then aggregated into a new system, which is *over-determined* (i.e., it has rank n while the total number of equations (q) is higher than n) and *linear* with respect to the unknowns:

$$\begin{cases} \left[\begin{array}{l} \vdots \\ x_i - x_j + \Phi^{-1}\left(1 - \hat{p}_{ij}\right) = 0 \qquad \forall i, j : m_{ij} \geq 1 \\ \vdots \end{array} \right. \\ \sum_i x_i = 0 \end{cases}. \tag{A.10}$$

This system can be expressed in matrix form as:

$$\begin{cases} \vdots \\ \sum_{k=1}^{n} \left(a_{hk} \cdot x_k\right) - b_h = 0 \quad \forall h \in [0, q] \\ \vdots \end{cases} \qquad \Rightarrow \qquad \boldsymbol{A} \cdot \boldsymbol{X} - \boldsymbol{B} = \boldsymbol{0}, \tag{A.11}$$

$\boldsymbol{X} = [\ldots, x_i, \ldots]^T \in R^{n \times 1}$ being the column vector containing the unknowns of the problem, a_{hk} being a generic element of matrix $\boldsymbol{A} \in R^{q \times n}$, and b_h being a generic element of vector $\boldsymbol{B} \in R^{n \times 1}$. For details on the construction of $\boldsymbol{A}$ and $\boldsymbol{B}$, see (Gulliksen, 1956).

The next subsection illustrates the proposed solution to the problem of interest.

Initial scaling (3rd level title)

In general, the system in Eq. A.11 will not necessarily be complete, as the number of equations (q) could be lower than $C_2^n + 1$ (i.e., for any paired comparison with $m_{ij} = 0$, no equation can be formulated). In a recent article, Franceschini and Maisano (2019) proposed to solve this "potentially incomplete" system through the *Generalized Least Squares* (GLS) method (Karya and Kurata; Ross, 2014). From a technical point of view, the GLS method allows to obtain a solution that minimizes the weighted sum of the squared residuals of the equations in Eq. A.11, i.e.:

$$\sum_{h=1}^{q} w_h \cdot \left[\sum_{k=1}^{n} (a_{hk} \cdot x_k) - b_h \right]^2 , \tag{A.12}$$

in which weights (w_h) take into account the uncertainty in the $\hat{p}_{ij}$ values. It can be demonstrated that, for a generic equation related to a generic paired comparison (f_i, f_j):

$$w_h = \left[\frac{\partial \Phi^{-1}(1 - \hat{p}_{ij})}{\partial \hat{p}_{ij}} \right]^2 \Big/ \sigma_{p_{ij}}^2 . \tag{A.13}$$

Next, weights are aggregated into a (squared) matrix $\boldsymbol{W} \in R^{(q-1)\times(q-1)}$, which encapsulates the uncertainty related to the equations of the system. A practical way to define $\boldsymbol{W}$ is to apply the *Multivariate Law of Propagation of Uncertainty* (MLPU) to the system in Eq. A.11, referring to the input variables affected by uncertainty (Kariya and Kurata, 2004); these variables – which, focussing on the problem of interest, are essentially the $\hat{p}_{ij}$ values, $\forall i, j : m_{ij} \geq 1$ – can be collected in a column vector $\boldsymbol{\xi} = [\ldots, \hat{p}_{ij}, \ldots]^T$ $\in R^{(q-1)\times 1}$. Precisely, $\boldsymbol{W}$ can be determined propagating the uncertainty of the elements in $\boldsymbol{\xi}$ to the equations of the system:

$$\boldsymbol{W} = \left[\boldsymbol{J}_{\xi} \cdot \Sigma_{\xi} \cdot \boldsymbol{J}_{\xi}^T \right]^{-1} , \tag{A.14}$$

where $\boldsymbol{J}_{\xi}$ is the Jacobian matrix containing the partial derivatives of the first members of Eq. A.11, with respect to the elements in $\boldsymbol{\xi}$, and Σ_{ξ} is the covariance matrix of $\boldsymbol{\xi}$.

By applying the GLS method to the system in Eq. A.11, a final estimate of $\boldsymbol{X}$ can be obtained as (Kariya and Kurata, 2004):

$$\hat{\boldsymbol{X}} = \left(\boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{A} \right)^{-1} \cdot \boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{B} . \tag{A.15}$$

The uncertainty of the solution can be estimated through a covariance matrix $\boldsymbol{\Sigma_X}$, which can be obtained by propagating the uncertainty of input data (i.e., $\hat{p}_{ij}$ values), through the following relationship:

$$\Sigma_{\boldsymbol{X}} = \left(\boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{A} \right)^{-1} \tag{A.16}$$

On the other hand, the partial derivatives in the Jacobian matrix $\boldsymbol{J}_{\xi} \in R^{(q-1)\times(q-1)}$ can be determined in a closed form, by approximating terms $\Phi^{-1}(1 - \hat{p}_{ij})$ (see Eq. A.10) through the following formula (Aludaat and Alodat, 2008):

$$\Phi^{-1}(1 - \hat{p}_{ij}) \approx k \sqrt{\frac{-ln[1 - (1 - 2 \cdot \hat{p}_{ij})^2]}{\sqrt{\pi / 8}}} \qquad \begin{cases} 0 \leq \hat{p}_{ij} \leq 0.5 \rightarrow k = 1 \\ 0.5 < \hat{p}_{ij} \leq 1 \rightarrow k = -1 \end{cases}, \tag{A.17}$$

from which:

$$\begin{aligned} &\frac{\partial \left[\Phi^{-1}(1 - \hat{p}_{ij}) \right]}{\partial \hat{p}_{ij}} \approx \left| \frac{\sqrt{2} \cdot (2 \cdot \hat{p}_{ij} - 1)}{\sqrt{-2 \cdot \sqrt{2 \cdot \pi} \cdot ln\left(-4 \cdot \hat{p}_{ij}^2 + 4 \cdot \hat{p}_{ij}\right) \cdot \hat{p}_{ij} \cdot (1 - \hat{p}_{ij})}} \right| && \text{for } \hat{p}_{ij} \neq 0.5 \\ &\frac{\partial \left[\Phi^{-1}(1 - \hat{p}_{ij}) \right]}{\partial \hat{p}_{ij}} \approx 2.506628 && \text{for } \hat{p}_{ij} = 0.5 \end{aligned} . \tag{A.18}$$

The matrix $\boldsymbol{\Sigma}_\xi \in R^{(q-1)\times(q-1)}$ diagonally contains the variances related to the input variables, i.e., $\hat{p}_{ij}$ terms. Let us now make a brief digression to derive the expression of these variances. Since f_{ij} is determined considering a sample of m_{ij} paired comparisons, it will be distributed binomially; $\hat{p}_{ij}$ is the best estimator of p_{ij}, according to the information available. In formal terms:

$$f_{ij} \sim B\left[\mu_{f_{ij}}, \sigma^2_{f_{ij}}\right] \sim B\left[m_{ij}\cdot\hat{p}_{ij}, m_{ij}\cdot\hat{p}_{ij}\cdot\left(1-\hat{p}_{ij}\right)\right]. \tag{A.19}$$

In the hypothesis that $m_{ij}\cdot\hat{p}_{ij} \geq 5$ when $0 \leq \hat{p}_{ij} \leq 0.5$, or $m_{ij}\cdot\left(1-\hat{p}_{ij}\right) \geq 5$ when $0.5 < \hat{p}_{ij} \leq 1$, the following approximations can be reasonably introduced (Ross, 2014):

$$\begin{aligned} f_{ij} &\sim N\left[\mu_{f_{ij}}, \sigma^2_{f_{ij}}\right] \sim N\left[m_{ij}\cdot\hat{p}_{ij}, m_{ij}\cdot\hat{p}_{ij}\cdot\left(1-\hat{p}_{ij}\right)\right] \\ p_{ij} &\sim N\left[\mu_{p_{ij}}, \sigma^2_{p_{ij}}\right] \sim N\left[\hat{p}_{ij}, \frac{\hat{p}_{ij}\cdot\left(1-\hat{p}_{ij}\right)}{m_{ij}}\right] \end{aligned}. \tag{A.20}$$

It can be noticed that, even when all experts express their usable judgments for all the possible paired comparisons (i.e., $m_{ij} = m \ \forall i, j$), the variance of p_{ij} may change from one paired comparison to one other, as it also depends on the relevant $\hat{p}_{ij}$ value.

The variances of the $\hat{p}_{ij}$ values will therefore be:

$$\sigma^2_{p_{ij}} = \frac{\hat{p}_{ij}\cdot\left(1-\hat{p}_{ij}\right)}{m_{ij}}. \tag{A.21}$$

The relevant covariances can be neglected, upon the reasonable assumption that the estimates of different p_{ij} values are (statistically) independent from each other. Next, it is possible to determine the matrix $\boldsymbol{W}$ (Eq. A.14) and, subsequently, $\hat{\boldsymbol{X}}$ (Eq. A.15) with the relevant uncertainty (Eq. A.16). This solution is defined on an *interval* scale (x), i.e., objects are defined on a scale with meaningful distance but arbitrary zero point (Thurstone, 1927; Roberts, 1979; Franceschini et al., 2019). The following subsection introduces a transformation that allows to "promote" this scaling to a more powerful one.

Transformation of the initial scaling into the final one (3rd level title)

Through the following transformation, the resulting scaling (x) is transformed into a new one (y), which is defined in the conventional range [0, 10]:

$$\hat{y}_i = \hat{y}_i\left(\hat{\boldsymbol{X}}\right) = 10\cdot\frac{\hat{x}_i - \hat{x}_Z}{\hat{x}_M - \hat{x}_Z} \quad \forall i, \tag{A.22}$$

where: $\hat{x}_Z$ and $\hat{x}_M$ are the scale values of f_Z and f_M, resulting from the GLS application; $\hat{x}_i$ is the scale value of a generic i-th object (regular or dummy), resulting from the GLS application; $\hat{y}_i$ is the scale value of a generic i-th object in the new scale y. This transformation can also be expressed in vector form as:

$$\hat{\boldsymbol{Y}} = \hat{\boldsymbol{Y}}\left(\hat{\boldsymbol{X}}\right) = \left[\ldots, \hat{y}_i\left(\hat{\boldsymbol{X}}\right), \ldots\right]^T = \left[\ldots, 10\cdot\frac{\hat{x}_i - \hat{x}_Z}{\hat{x}_M - \hat{x}_Z}, \ldots\right]^T, \tag{A.23}$$

being $\boldsymbol{Y}$ a column vector whose components result from a system of n decoupled equations. Since scale y "inherits" the *interval* property from scale x and has a conventional zero point that corresponds to the absence of the attribute (i.e., $\hat{y}_Z$), it can be reasonably considered as a *ratio* scale, without any conceptually prohibited "promotion". We note that the two dummy objects, f_Z and f_M, are used to "anchor" the x scale to the y scale (Paruolo et al., 2013).

Combining Eqs. A.23 and A.15, the final (ratio) scaling $\boldsymbol{Y}$ can be also expressed as:

$$\hat{\boldsymbol{Y}} = \hat{\boldsymbol{Y}}\left[\hat{\boldsymbol{X}}\right] = \hat{\boldsymbol{Y}}\left[\left(\boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{A}\right)^{-1} \cdot \boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{B}\right]. \tag{A.24}$$

Next, the uncertainty related to the elements in $\hat{\boldsymbol{Y}} = [\ldots, \hat{y}_i, \ldots]^T \in R^{n\times 1}$ can be determined by applying the *delta method* to Eq. A.24 (JCGM100:2008 2008). It is thus obtained:

$$\Sigma_{\boldsymbol{Y}} = \boldsymbol{J}_{\boldsymbol{Y}(\hat{\boldsymbol{x}})} \cdot \Sigma_{\boldsymbol{X}} \cdot \boldsymbol{J}^T_{\boldsymbol{Y}(\hat{\boldsymbol{x}})}, \tag{A.25}$$

where $\boldsymbol{J}_{\boldsymbol{Y}(\hat{\boldsymbol{x}})} \in R^{n\times n}$ is a Jacobian matrix containing the partial derivatives related to the equations of the system in Eq. A.23, with respect to the elements of $\boldsymbol{X}$. In the hypothesis that the n (regular and dummy) objects are ordered as (f_Z, f_M, f_1, f_2, f_3, …) and therefore $\hat{\boldsymbol{X}} = [\hat{x}_Z, \hat{x}_M, \hat{x}_1, \hat{x}_2, \hat{x}_3, \ldots]^T$ and $\hat{\boldsymbol{Y}} = [\hat{y}_Z, \hat{y}_M, \hat{y}_1, \hat{y}_2, \hat{y}_3, \ldots]^T$, $\boldsymbol{J}_{\boldsymbol{Y}(\hat{\boldsymbol{X}})}$ would be:

$$\boldsymbol{J}_{\boldsymbol{Y}(\hat{x})} = \begin{bmatrix} \frac{\partial y_Z}{\partial x_Z} & \frac{\partial y_Z}{\partial x_M} & \frac{\partial y_Z}{\partial x_1} & \cdots & \frac{\partial y_Z}{\partial x_i} & \cdots \\ \frac{\partial y_M}{\partial x_Z} & \frac{\partial y_M}{\partial x_M} & \frac{\partial y_M}{\partial x_1} & \cdots & \frac{\partial y_M}{\partial x_i} & \cdots \\ \frac{\partial y_1}{\partial x_Z} & \frac{\partial y_1}{\partial x_M} & \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_i} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial y_i}{\partial x_Z} & \frac{\partial y_i}{\partial x_M} & \frac{\partial y_i}{\partial x_1} & \cdots & \frac{\partial y_i}{\partial x_i} & \cdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \ddots \end{bmatrix} = \begin{bmatrix} \frac{10}{\hat{x}_M - \hat{x}_Z} & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \frac{-10}{\hat{x}_M - \hat{x}_Z} & 0 & \cdots & 0 & \cdots \\ -10 \cdot \frac{(\hat{x}_M - \hat{x}_1)}{(\hat{x}_Z - \hat{x}_M)^2} & 10 \cdot \frac{\hat{x}_Z - \hat{x}_1}{(\hat{x}_M - \hat{x}_Z)^2} & \frac{-10}{\hat{x}_M - \hat{x}_Z} & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -10 \cdot \frac{(\hat{x}_M - \hat{x}_i)}{(\hat{x}_Z - \hat{x}_M)^2} & 10 \cdot \frac{\hat{x}_Z - \hat{x}_i}{(\hat{x}_M - \hat{x}_Z)^2} & 0 & \cdots & \frac{-10}{\hat{x}_M - \hat{x}_Z} & \cdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \ddots \end{bmatrix}. \tag{A.26}$$

Combining Eqs. A.25 and A.16, $\Sigma_{\boldsymbol{Y}}$ can be expressed as:

$$\Sigma_{\boldsymbol{Y}} = \boldsymbol{J}_{\boldsymbol{Y}(\hat{\boldsymbol{x}})} \cdot [(\boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{A})^{-1}] \cdot \boldsymbol{J}^T_{\boldsymbol{Y}(\hat{\boldsymbol{x}})}. \tag{A.27}$$

Assuming that the p_{ij} and $\hat{y}_i$ values are approximately normally distributed, a 95% confidence interval related to each $\hat{y}_i$ value can be computed as:

$$\hat{y}_i \pm U_{y_i} = \hat{y}_i \pm 2 \cdot \sigma_{y_i} \quad \forall i. \tag{A.28}$$

U_{y_i} being the so-called *expanded uncertainty* of $\hat{y}_i$ with a coverage factor $k = 2$ and $\sigma_{y_i} = \sqrt{\Sigma_{\boldsymbol{Y},(i,i)}}$ (JCGM 100:2008, 2008).

Weighted additive aggregation model (2nd level title)

The fact that the values of S_i, O_i and D_i, are defined on three separate ratio scales entails that their aggregation through the multiplicative model in Eq. 3 is meaningful (Roberts, 1979; Franceschini et al., 2019). Although we are aware of the presumed advantages of multiplicative models with respect to the additive ones, below we propose an alternative weighted additive model:

$$RPN'_i = w_S \cdot S_i + w_O \cdot O_i + w_D \cdot D_i, \tag{A.29}$$

in which:

w_S, w_O and w_D are the percentage weights assigned to the three risk factors of interest (conventionally $w_S + w_O + w_D = 100\%$) and RPN'_i is the resulting composite indicator; the prime symbol "(′)" was introduced to distinguish this composite indicator from the one in Eq. 3. Additionally, while $RPN_i \in [0, 1000]$, $RPN'_i \in [0, 10]$.

The model in Eq. A.29 can be preferred to the one in Eq. 3, for the following reasons:

- It allows the P-FMEA team leader to choose the (strategy) weights (w_S, w_O and w_D) of the three risk factors of interest. For example, for manufacturing processes that are potentially hazardous to personnel safety, it may be appropriate to raise w_S. On the other hand, for processes with high throughput and high level of automation, it may be appropriate (i) to raise w_O, with the purpose of reducing the so-called *mean time between failures* (MTBF), and/or (ii) to raise w_D, with the purpose of reducing the so-called *mean time to failure* (MTTF) (O'Connor and Kleyner, 2012). Of course, the choice of weights should be made according to the strategic objectives of the process. The scientific literature contains a variety of techniques to drive this operation (Vora et al., 2014; Wang et al., 2014).

- The comparability between S_i, O_i, and D_i is ensured by the fact that these indicators are defined on ratio scales with comparable zero and a conventional range [0, 10].
- Although weights could theoretically be introduced into the multiplicative model in Eq. 3, e.g., by changing it into:

$$RPN_i'' = (S_i)^{w_S} \cdot (O_i)^{w_O} \cdot (D_i)^{w_D} , \tag{A.30}$$

we think that it would be relatively difficult to control their influence on the final result. For example, the use of multiplicative models (weighted or not) could make the *substitution rate* of sub-indicators change unpredictably (Franceschini et al., 2019).

- The model in Eq. A.29 allows to visualize the contributions of the three risk factors of interest. E.g., the chart in Fig. A.1 shows the RPN'_i values and relevant contributions for the case study. In this case, the P-FMEA team leader set the following weight combination: $w_S = 40\%$, $w_O = 30\%$ and $w_D = 30\%$.

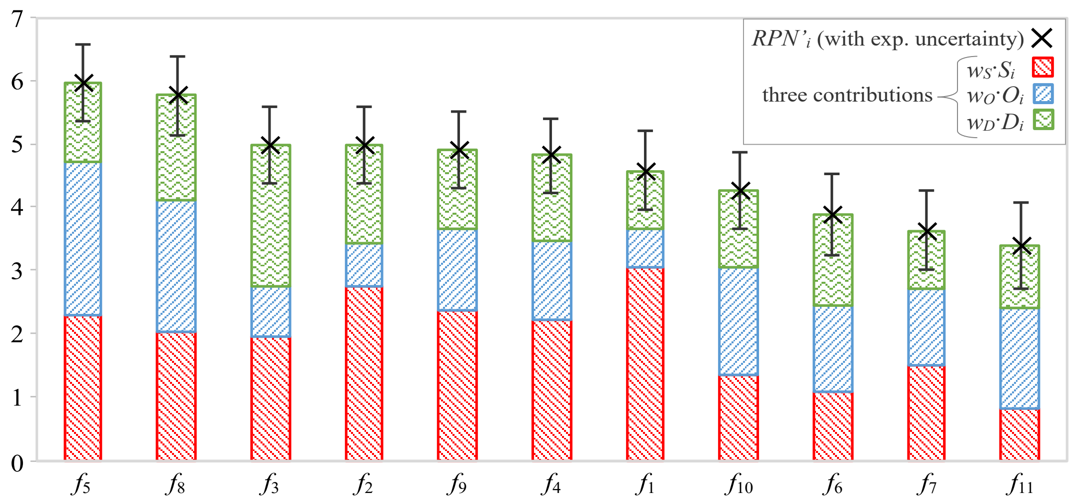

Fig. A.1. Pareto chart of the failure modes based on their RPN'_i values (see Eq. A.29). In addition to expanded-uncertainty bands, this chart allows to visualize the contributions related to S_i, O_i and D_i (numerical data in Table A.1).

Similar to what explained in the section "Uncertainty calculation" for the RPN_i values, the uncertainty related to the RPN'_i values can be determined by applying the *delta method* to Eq. A.29 (JCGM 100:2008, 2008), obtaining:

$$\sigma_{RPN'_i} = \sqrt{w_S^2 \cdot \sigma_{S_i}^2 + w_O^2 \cdot \sigma_{O_i}^2 + w_D^2 \cdot \sigma_{D_i}^2} . \tag{A.31}$$

Weights (w_S, w_O and w_D) are implicitly treated as constants, neglecting the correlations between the S_i, O_i and D_i values.

Assuming that the RPN'_i values are approximately normally distributed, a 95% confidence interval related to each RPN'_i value can be computed as:

$$RPN'_i \pm U_{RPN'_i} = RPN'_i \pm 2 \cdot \sigma_{RPN'_i} \quad \forall i , \tag{A.32}$$

$U_{RPN'_i}$ being the so-called *expanded uncertainty* (JCGM 100:2008, 2008) of RPN'_i with a coverage factor $k = 2$.

Table A.1 contain the results concerned with the case study.

Again, we note that the uncertainty bands of several failure modes are superimposed (see Fig. A.1). Despite the structural differences between the model in Eq. A.29 and that in Eq. 3, the corresponding results are not so dissimilar: the most critical failure modes are f_5 and f_8, while the least critical ones are f_7 and f_{11}. Unlike the model in Eq. 3, we note that the dispersion of the RPN'_i values is rather homogeneous (i.e., the standard deviations are comparable), indicating a certain *homoschedasticity* (Ross, 2014).

Returning to the model in Eq. A.29, we emphasise that the choice of weights (w_S, w_O and w_D) is arbitrary. In such a scenario, it may be appropriate to evaluate the robustness of the results with respect to (small) variations in these weights, through a *sensitivity* *analysis*. For example, the possible variations in the case-study results can be analyzed for three different weight combinations, as shown in Fig. A.1: (1) $w_S = 40\%$, $w_O = 30\%$ and $w_D = 30\%$, (2) $w_S = 33.3\%$, $w_O = 33.3\%$ and $w_D = 33.3\%$, and (3) $w_S = 50\%$, $w_O = 20\%$ and $w_D = 30\%$.

Since the RPN_i' values of the failure modes do not seem to change considerably as the weights vary, the solution provided by the (weighted) additive model in Eq. A.29 appears robust for this case study.

Table A.1. Results of the application of the weighted additive model of Eq. A.29 to the case study.

	S_i values		O_i values		D_i values		Additive contributions			RPN_i' values		
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.	$w_S \cdot S_i$	$w_O \cdot O_i$	$w_D \cdot D_i$	Mean	St.dev.	$U_{RPN_i'}$
f_1	7.69	0.51	2.03	0.60	3.01	0.55	2.32	2.42	1.24	5.98	0.31	0.62
f_2	6.91	0.51	2.23	0.58	5.19	0.51	2.03	2.08	1.66	5.77	0.31	0.62
f_3	4.88	0.52	2.69	0.55	7.46	0.55	1.95	0.81	2.24	5.00	0.31	0.62
f_4	5.60	0.53	4.14	0.51	4.48	0.52	2.76	0.67	1.56	4.99	0.31	0.62
f_5	5.80	0.52	8.06	0.56	4.14	0.54	2.38	1.28	1.27	4.93	0.31	0.62
f_6	2.71	0.61	4.57	0.52	4.82	0.51	2.24	1.24	1.34	4.83	0.30	0.60
f_7	3.76	0.54	4.05	0.54	3.05	0.56	3.08	0.61	0.90	4.59	0.32	0.64
f_8	5.08	0.54	6.93	0.56	5.54	0.52	1.34	1.72	1.21	4.27	0.31	0.62
f_9	5.96	0.52	4.26	0.54	4.23	0.51	1.08	1.37	1.44	3.90	0.33	0.66
f_{10}	3.36	0.55	5.74	0.52	4.03	0.53	1.51	1.21	0.91	3.63	0.32	0.64
f_{11}	2.09	0.66	5.27	0.52	3.29	0.55	0.84	1.58	0.99	3.41	0.35	0.70

In case, weights are conventionally set to $w_S = 40\%$, $w_O = 30\%$, and $w_D = 30\%$.

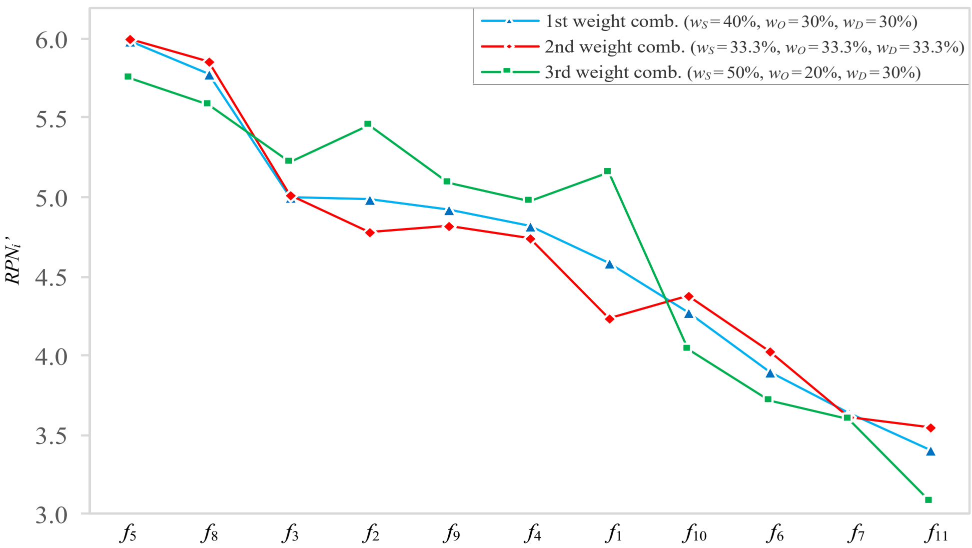

Fig. A.2. Sensitivity analysis of possible variations in results for three different weight combinations. Failure modes are sorted in descending order with respect to the corresponding RPN_i' values, for the first weight combination.