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Probabilistic Framework to Evaluate the Resilience of Engineering Systems Using Bayesian and Dynamic Bayesian Networks

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Abstract

Resilience indicators are a convenient tool to assess the resilience of engineering systems. They are often used in preliminary designs or in the assessment of complex systems. This paper introduces a novel approach to assess the time-dependent resilience of engineering systems using resilience indicators. A Bayesian network (BN) approach is employed to handle the relationships among the indicators. BN is known for its capability of handling causal dependencies between different variables in probabilistic terms. However, the use of BN is limited to static systems that are in a state of equilibrium. Being at equilibrium is often not the case because most engineering systems are dynamic in nature as their performance fluctuates with time, especially after disturbing events (e.g. natural disasters). Therefore, the temporal dimension is tackled in this work using the Dynamic Bayesian Network (DBN). DBN extends the classical BN by adding the time dimension. It permits the interaction among variables at different time steps. It can be used to track the evolution of a system's performance given an evidence recorded at a previous time step. This allows predicting the resilience state of a system given its initial condition. A mathematical probabilistic framework based on the DBN is developed to model the resilience of dynamic engineering systems. Two illustrative examples are presented in the paper to demonstrate the applicability of the introduced framework. One example evaluates the resilience of Brazil. The other one evaluates the resilience of a transportation system.

Keywords: Bayesian Network; Dynamic Bayesian Network; Resilience analysis; critical infrastructure; Resilience indicators, Recovery

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1. Introduction

Research on disaster resilience has recently been fostered due to the noticeable increase in the number of natural and human-caused disasters. Resilience has been defined differently depending on the field of study (Doorn et al. 2018; Hosseini et al. 2016b; Manyena 2006). In engineering, resilience is the ability to withstand a disturbance caused by an external agent and recover quickly if damage occurs (Cimellaro et al. 2010b; Cimellaro et al. 2016a; Cimellaro et al. 2014b). Current scientific contributions are aimed at understanding disaster resilience and finding new ways to measure it, quantitatively or qualitatively. Several solutions exist in the literature to measure resilience at the community level (Cutter et al. 2014; Kammouh and Cimellaro 2018b; Kammouh et al. 2018c) and at the infrastructure level (Gardoni 2018; Liu et al. 2017; Sharma et al. 2018); however, research on resilience quantification is still in the early stage of development and no universal method exists so far. Among others, resilience indicators are an effective tool to compute the resilience of engineering systems because they allow modeling complex systems easily and effectively (Cimellaro et al. 2016b; Cimellaro et al. 2015; Cutter et al. 2008). Several authors have proposed indicator-based methodologies to evaluate the resilience of engineering systems; some are deterministic (Henry and Ramirez-Marquez 2012; Kammouh et al. 2017b; Kammouh et al. 2019b) and the others are probabilistic (Cockburn and Tesfamariam 2012; De Iuliis et al. 2019b; Kammouh et al. 2018b; Kammouh et al. 2018d; Shahriar et al. 2012). Nevertheless, developing a standardized indicator-based methodology to quantify resilience is still challenging. The main challenge is the presence of uncertainty in the resilience model and its inputs. The interdependency and the weighting factors distribution among the variables are also other issues that cannot be handled in a deterministic way. Therefore, probabilistic methods are usually preferred over deterministic methods. Probabilistic models are more powerful to model uncertainties and interdependencies and they are more appropriate to represent reality. In fact, deterministic models are considered a particular case of probabilistic models (Pourret et al. 2008).

While probabilistic models solve the gap of modeling capability that deterministic models suffer from, not all probabilistic approaches are suitable to model the behavior of engineering systems, especially in cases where past data is not readily available (Cimellaro 2016). One way to properly model the behavior of a system in a probabilistic manner is through the use of Fuzzy analysis and Bayesian Networks (BNs). Fuzzy theory represents a method for solving problems which are related to uncertainty and vagueness. It has been implemented in several research works to study the resilience of infrastructure systems (De Iuliis et al. 2019a; Kammouh et al. 2018e). Fuzzy theory is usually implemented in cases with no data, which is replaced with expert judgements. The output of fuzzy-based methods are heavily subjective given that they are based on opinions rather than data. On the other hand, BN is a Directed Acyclic Graph where the nodes represent variables of interest and the links between them indicate causal dependencies. BNs are widely used for knowledge representation and reasoning under uncertainty, especially in the context of partial information. They are effective when different types of variables and knowledge from various sources need to be integrated within a single framework. In addition, BN provides probabilistic relationships among the variables, which allows modeling the interdependencies among them. In the literature, there is a limited number of research contributions that use the BN in resilience analysis. Johansen and Tien (2018) proposed a probabilistic

methodology based on the BN approach to model the interdependencies between critical infrastructure systems. Their research aims at understanding the effect of interdependencies on the fragility of the overall system. Cai et al. (2017) developed a universal resilience metric for infrastructure systems. A BN approach was employed to calculate the resilience metric value. The proposed resilience metric can be used to design and/or optimize different types of engineering systems against various hazards, such as earthquakes, floods, cyber-attacks (Kammouh and Cimellaro 2018a), etc. Moreover, Hosseini and Barker (2016) introduced a resilience quantification methodology using BN. In their research, the resilience is defined using three parameters: absorptive capacity, adaptive capacity, and restorative capacities. A case study on an inland waterway port, an essential component in the intermodal transportation network, was used to demonstrate the methodology. Hosseini et al. (2016a) employed a BN to quantify the supply chain system resilience of sulfuric acid manufacturer in Iran. Their work consists of a formulation for supplier selection, that accounts for operational (e.g., customer demand) and disruption (e.g., natural disaster) risks and their effect on resilient suppliers. (Hossain et al. 2019b) applied Bayesian Network to rank ports infrastructure assets. Their study proposes a formulation for supplier selection, accounting for operational (e.g., customer demand) and disruption (e.g., natural disaster) risks and their effect on resilient suppliers. (Hossain et al. 2019b) introduced a BN approach to rank port infrastructure assets. (Eldosouky et al. 2017) introduced a novel analytical resilience index to measure the effect of each critical infrastructure's physical component on its probability of failure using Bayesian network. The proposed framework was applied to a case study of hydropower dams and their interdependence to the power grid. Finally, (Hossain et al. 2019a) addressed a range of possible risks to the electrical power system and its interdependent networks (EIN) using the BN.

Rather than the static approaches, Tabandeh et al. (2018) developed an indicator-based probabilistic formulation to model the societal impact and estimate the impact considering the immediate consequences and the recovery condition. The methodology uses DBN to integrate the predictive model of the indicators. BNs have also been employed by researchers in fields other than disaster resilience (Cockburn and Tesfamariam 2012; Ismail et al. 2011; Kabir et al. 2016; Kabir et al. 2015; Kammouh et al. 2019a; Siraj et al. 2015). For example, Cockburn and Tesfamariam (2012) used BNs to estimate the risk of several cities located in Canada, Kabir et al. (2015) evaluated the risk of water mains failure using a BN model, while Siraj et al. (2015) employed BNs in the seismic risk assessment of high voltage transformers.

Resilience can be an outcome (static) or a process (dynamic) (Cutter et al. 2008). While most of the abovementioned researches focused on analyzing engineering resilience from a static point of view, there is a significant gap in assessing the dynamic nature of resilience through quantitative approaches. In fact, BN does not capture the behavior of dynamic systems and the interdependencies among the system's components and variables. BN is a snapshot of the system, which implies that the restoration process, which is inherently time-dependent, cannot be modeled. Moreover, a feedback loop is not allowed in a BN model; thus, it cannot be used to model a cyclic relationship. Although there are some research works that use the DBN as an inference tool to express resilience in a dynamic manner, the resilience models adopted in those researches and the transitional model from one step to another in the time space was not clearly defined.

96 This paper first proposes a static framework to model systems of static nature (e.g., assessing a system's
97 performance at a specific instance of time). It employs the BN as a tool to quantify the system's resilience. The
98 framework is demonstrated using a case study in which the resilience of a country, namely Brazil, is assessed.
99 In general, BN is good tool to assess a physical-causal model; however, learning and updating a BN requires an
100 extensive computational load. Updating a BN is necessary for resilience modeling, especially in monitoring the
101 possibility of disruptive events. Thus, this paper also presents a dynamic framework to quantitatively assess the
102 resilience of systems of dynamic nature (i.e., critical infrastructures, buildings, communities, etc.) The
103 framework can be used to assess the resilience of multiple systems at once and it adopts the DBN as an inference
104 tool. In fact, a DBN model can be obtained by expert knowledge, from a database using a combination of
105 machine-learning techniques, or both. These properties make the DBN formalism very useful in the disaster
106 resilience domain as this domain has an abundance of both expert knowledge and databases records. Moreover,
107 a DBN allows performing a transient analysis of the system after the occurrence of disruption until the system
108 was recovered from its disruptive states. The transient analysis can be rather useful to model the restoration
109 process of the damaged system. The proposed resilience framework is presented in the form of a mathematical
110 formulation that integrates the probability distribution of all variables' states. An illustrative example of a
111 transportation network is used to demonstrate the proposed methodology. Results show the ability of the
112 framework to dynamically model complex systems even in cases where data are scarce.

113 The rest of the paper is organized as follows: Section 2 is dedicated to review the basics of the BN and DBN.
114 Section 3 introduces a framework to assess the resilience of static systems where the BN is used as the inference
115 method. In Section 4, an illustrative example is presented to show the applicability of the static resilience
116 framework. Section 5 discusses the general framework for modeling and quantifying the resilience of dynamic
117 systems using the DBN approach. A transportation network is presented in Section 6 to illustrate the
118 effectiveness of the proposed framework.. In Section 7, further considerations and applications are discussed.
119 Finally, conclusions are given in Section 8 together with the proposed future work.

120 ## 2. Bayesian and Dynamic Bayesian Networks

121 ### 2.1 Bayesian Network (BN)

122 The Bayesian Network, also known as Bayesian Belief Network, is a graphical model that allows the design of
123 stochastic relationships among a group of variables (De Iuliis et al. under review). Applications of BN can be
124 found in a variety of fields, from social to economic and biological disciplines (Ismail et al. 2011; Schultz and
125 Smith 2016). BN permits the usage of different types of knowledge, both quantitative and qualitative, and can
126 cope with missing data considering the uncertainty embedded in the system (Balbi et al. 2014). To construct a
127 BN, several hypotheses have to be made. Each hypothesis is decomposed into a set of random variables. Each
128 variable can take values within a finite set of states (also known as beliefs), mutually exclusive and collectively
129 exhaustive (MECE) (Grover 2013). The dependency of one variable on another is represented in the network
130 as a directed edge (or link). The relationships between the variables in a BN are expressed in terms of family
131 relationships. The link starts from the so-called father node and points at the son node, which is the impacted

variable. The set of edges and nodes builds a directed acyclic graph. The network itself is normally learned from data or specified by experts who not only provide the main hypotheses (and consequently the variables to be considered in the model), but also the dependencies between the variables. The foundation and the inference process of BN is set in the Bayes' Theorem. Given a state b for a variable B and a number k of MECE states a_j, $j = 1, \cdots, k$ for a variable A, the updated probability is computed as:

$$P(a_j \mid b) = \frac{P(b \mid a_j)P(a_j)}{P(b)} \quad (1)$$

where $P(a_j|b)$ is one's belief for hypothesis a_j upon observing evidence b, $P(b|a_j)$ is the likelihood that b is observed if a_j is true, $P(a_j)$ is the probability that the hypothesis holds true, and $P(b)$ is the probability that the evidence takes place. $P(a_j|b)$ is known as *posterior* probability and $P(a_j)$ is called prior probability (Laskey 1995). The dependencies of one variable (the son node) on another (the father node) are usually quantified using a Conditional Probability Table (CPT), where the likelihood of the son node to assume a certain state under a certain father node state is assigned (Grover 2013). In the case of a variable with no parents, the probabilities are reduced to the unconditional probability. The quantitative part of the BN starts by assigning conditional probability distributions (CPD) to the nodes. Each node in a BN has a CPT that determines the CPD of the random variable. The CPTs provide information on the probability of a node given its parents (Murphy and Russell 2002).

Once the BN is constructed, we pinpoint that the outcome is highly dependent on the assigned probabilities (Ismail et al. 2011). To test the robustness of the model and the dependency of the outcome on each father node, a sensitivity analysis is usually performed. This allows identifying the most important and impactful variables, leading to consequent emphatic attention in the collection of data for the concerned variables (Laskey 1995). For more details, several examples of BN applications can be found in the literature (Cockburn and Tesfamariam 2012; Ismail et al. 2011; Kabir et al. 2016; Kabir et al. 2015; Siraj et al. 2015).

2.2 Dynamic Bayesian Network (DBN)

BNs are used when the analyzed system is in a static state. This is often not the case in a dynamic, continuously changing world. This raises the need for a tool that is capable of accounting for system changes, such as the Dynamic Bayesian Network. DBN is a Bayesian network extended with additional mechanisms that are capable of modeling influences over time (Murphy and Russell 2002). It extends the classical BN by adding the time dimension. It is suitable for describing dynamic systems where the performance fluctuates (e.g. before and after a disaster). Like the BN, the DBN is a directed acyclic graphical model used for statistical processes. A DBN consists of multiple BNs (often referred to as time-slices or time steps), each with its own variables. The variables within a single and/or successive time-slices are connected using links. A DBN can be defined as (B_1, $B\rightarrow$), where B_1 is a BN that specifies the initial distribution of the variable states $P(Z_1)$ (Murphy and Russell 2002), where $Z_t = (U_t, X_t, Y_t)$ is the input, hidden, and output variables of the model at time step t, while $B\rightarrow$ is called a "two-slice temporal Bayesian network" (2TBN), which defines the transition model $P(Z_t|Z_{t-1})$, as in

Equation (2). The nodes in the first slice of the 2TBN network do not have parameters associated with them, while CPTs are required for the nodes in the second slice.

$$p(Z_t \mid Z_{t-1}) = \prod_{i=1}^{N} p(Z_t^i \mid Pa(Z_t^i)) \quad (2)$$

where Z_t^i is the i^{th} node at time t and could be a component of X_t, Y_t, or U_t. $Pa(Z_t^i)$ are the parents of Z_t^i, which can be in the same or the previous time-slice.

The process in a DBN is stationary and the structure repeats after the second time-slice, so the variables for the slices $t=2, 3, .., T$ remain unchanged. This allows expressing the system using only two slices (i.e., the first and the second time-slices). Therefore, an unbounded sequence length could be modeled using a finite number of parameters. The probability distribution for a sequence of time-slices can be obtained by unrolling the 2TBN network, as follows:

$$p(Z_{1:T}) = \prod_{t=1}^{T} \prod_{i=1}^{N} p(Z_t^i \mid Pa(Z_t^i)) \quad (3)$$

The DBN is often seen as a generalization of other temporal reasoning developments, such as the hidden Markov model (HMM) and the Kalman filter model (KFM) (Hulst 2006). These models, which can be expressed in a compact form, are popular for their fast learning and fast inference techniques. In fact, DBNs generalize HMMs by expressing the state space in not only a single discrete random variable but also in a factored form.

Temporal plate and contemporal nodes

The temporal plate is the area within the DBN model that includes temporal information (i.e., information that changes from a time step to another). The temporal plate includes the variables that evolve over time. These variables are the part of the DBN that can be unrolled. However, nodes that have a constant value at every time step are considered a waste of memory and computational power if copied in each time step. Therefore, it is wise to introduce these nodes outside the temporal plates. The collection of these nodes is called the contemporal space, and the nodes are called contemporal nodes (Murphy and Russell 2002).

Anchor and terminal nodes

One extension of the original DBN formalism was established in (Hulst 2006), where the author introduced nodes that are only connected to the first and last time slices of the DBN. Such variables do not affect the intermediate time slices like the contemporal nodes.

- Anchor node (A): a node that is outside the temporal plate but has at least one child node inside the temporal plate in the first time slice of the unrolled DBN.
- Terminal node (T): a node that is outside the temporal plate and has at least one parent inside the temporal plate in the last time slice of the unrolled DBN.

Figure 1(a) presents a single time slice of a DBN (a snapshot of the system) where all variables appear to be static. Figure 1(b) shows a general DBN where the variables that are in the temporal plate (the dotted rectangle)

are those that are repeated when the DBN is unrolled, and the variables that are outside the temporal plate are the static (contemporal, anchor, or terminal) nodes. Figure 1(c) shows the unrolled DBN where the variables that are inside the temporal plate are connected with one another through temporal arcs and appear in every time-step, while the other variables appear only once since their value is constant. The transition model of the DBN can be represented as follows:

$$p(Z_t|Z_{t-1},Z_{t-2},...,Z_{t-k},C) = \prod_{i=1}^{N} p(Z_t^i \mid Pa(Z_t^i), C^i) \quad (4)$$

where Pa(Z_t^i) is the parents of Z_t^i inside the temporal plate, C^i for $i = 1,...,N$ are the contemporal variables that are a parent of Z_t^i. The joint distribution of a DBN sequence of length T including the additional variables (A) and (T) is given as follows:

$$p(A,C,Z_{1:T},T) = p(C).p(A|C)\prod_{i=1}^{N} p(Z_t^i \mid Pa(Z_1^i), A^i, C^i)\prod_{t=2}^{T}\prod_{i=1}^{N} p(Z_t^i \mid Pa(Z_t^i), C^i)\prod_{i=1}^{N} p(T^i \mid Z_T^i, C^i) \quad (5)$$

where A^i is the anchor variables that are a parent of Z_1^i, T^i is the terminal variables that are a child of Z_T^i.

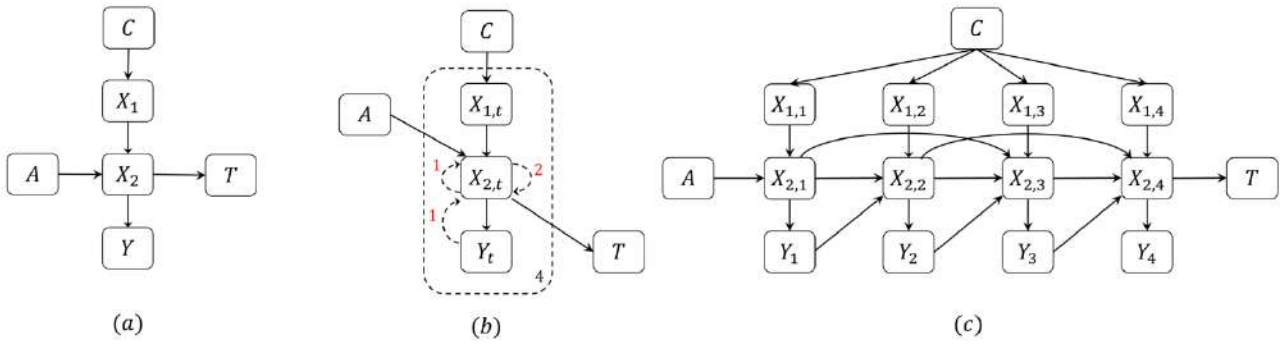


Figure 1 (a) The initial network of a DBN, (b) the 2TBN or a second order DBN, (c) the unrolled DBN model for T= 4 slices.

3. Quantifying Resilience Using Bayesian Network: A Univariate Static Approach

3.1 Static resilience model

In this work, we adopt a model based on the resilience definition provided by Bruneau and Reinhorn (2007) and Bruneau et al. (2003), who describe the resilience of a system using the following three indicators (hereafter we call them the three resilience pillars): reduced failure probability (*reduced vulnerability*); reduced consequences from failure (*robustness*); and reduced time to recovery (*recoverability*) (Figure 2):

- Reduced vulnerability: the reduced likelihood of damage & failure to critical infrastructure systems and components (P_1);

- Robustness: the damage level, in terms of injuries, lives lost, physical damage, and negative economic and social impacts (P_2);
- Recoverability: the time required to restore a specific system or a set of systems to normal or pre-disaster level of functionality (P_3).

Figure 2 The three resilience pillars

A system with a low probability of failure, high robustness, and high recoverability capacity is considered resilient. The three resilience pillars can be typically described using a set of indicators representing the analyzed system. The choice of indicators can be made by experts in the relevant field. Figure 3 shows a general resilience functionality curve where the three resilience pillars are allocated to three time-spans:

- The pre-disaster time span, defined by the system's probability of failure;
- The disaster time span, determined by the robustness level of the system;
- Post-disaster time span, defined by the recovery capacity of the system.

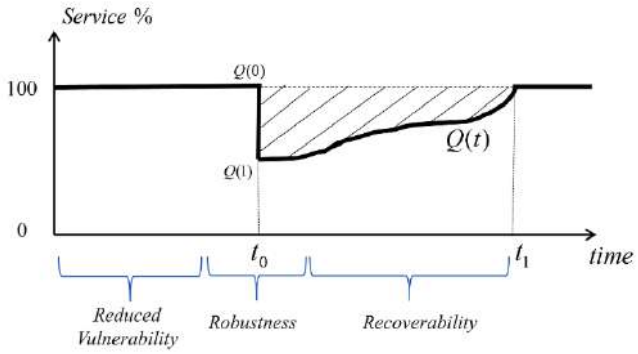


Figure 3 A general resilience function of a system

3.2 Network structure and elements connectivity

Assume that we have a system composed of N indicators (X_1, X_2, …, X_N). The indicators are connected to the three resilience pillars according to their relevance. Such connections can be obtained from past experience or expert knowledge. One indicator can contribute to multiple pillars, as shown in the Bayesian network in Figure 4, where indicator X_4 is connected to R_1 and R_2 while X_7 is connected to R_2 and R_3. The final output (*resilience index*) represents a combination of all factors that contribute towards the resilience pillars.

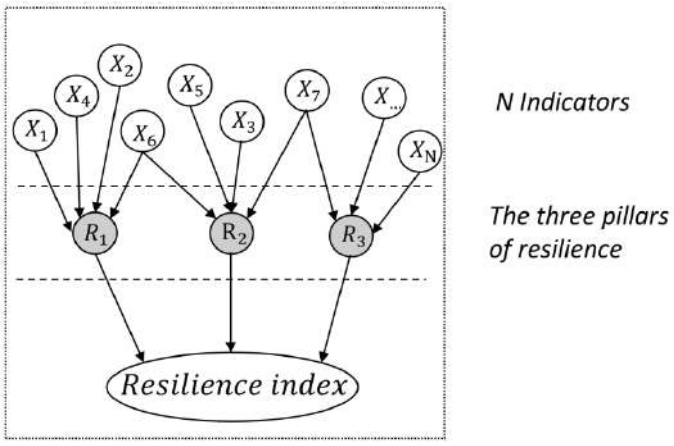


Figure 4 Bayesian network to compute the resilience index of a static system

3.3 Unconditional and Conditional Probability Tables

Once the connections between the son nodes (i.e. resilience pillars) and the father nodes (i.e. indicators) are completed, the CPTs of the resilience pillars given the indicators' states must be defined. The indicators states (or the unconditional probabilities of the basic nodes) are defined using experts knowledge or available data. Only three levels (states) are assigned to each resilience pillar node (High (H), Medium (M), and Low (L)) in order to maintain a low complexity of the network, while five states are assigned to the indicators (High, Good, Medium, Low, and vulnerable), coded as 4, 3, 2, 1, 0 respectively. To obtain a numerical value for the son node, the sum of the numerical values of the father nodes is computed and then divided for the sum of their maximum values, building a global relative value x for the son node ranging between 0 and 1 (Equation 6). Equation**Error! Reference source not found.** 6 is indeed a special case of the min-max normalization where the minimum value is zero.

$$x = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} \max_i} = \frac{\sum_{i=1}^{n} y_i}{n \times \max} \tag{6}$$

where x is the global relative value for the analyzed son node, i is the father node index, n is the number of father nodes under a son node, y_i is the value of the father node i, max_i is the maximum value a father node i can take (fixed for all nodes).

To illustrate this, consider three father nodes [A; B; C], each with a three-level scale High, Medium, and Low, converted to 2, 1, and 0 respectively. Assuming a combination of [Medium, Medium, High] respectively for the three father nodes [A; B; C], which is equivalent to [1; 1; 2], the value x of the son node is computed as follows:

$$x = \frac{1+1+2}{2+2+2} = 0.667 \tag{7}$$

For each combination of father nodes values, the distribution among the three levels (High, Medium and Low) of the son node S is calculated as x^2, $2x(1-x)$, and $(1-x)^2$ respectively. This distribution ensures the

normalization of the distribution and a suitable continuous parametrization, being a binomial distribution were the probability of success is x (Lewis 2011). In fuzzy theory, those equations are called membership functions. A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. Transforming the x value into a degree of membership is called fuzzification. One can choose from several types of membership functions, for instance triangular membership functions (for x [0, 0.5] $\rightarrow$ the membership functions are (-2x+1, 2x , 0) and for x [0.5, 1] $\rightarrow$ the membership functions are (0, -2x+2, 2x-1)). The summation of values at any point should always be equal to one regardless of the membership functions used. In this study, we opted for a set of continuous membership functions. A portion of the CPT of the son node S is presented in Table 1 where the total number of combinations is 27 (3^3)**Error! Reference source not found.**.

Table 1 A CPT of a Son node given the states of the father nodes

Father nodes			Global value	States distribution of the Son Node S		
P1	P2	P3	$x = \frac{\sum_{i=1}^{n} y_i}{n \times \max}$	High x^2	Medium $2x(1-x)$	Low $(1-x)^2$
2	2	2	1.00	1.0000	0.0000	0.0000
2	2	1	0.83	0.6944	0.2778	0.0278
2	2	0	0.67	0.4444	0.4444	0.1111
2	1	2	0.83	0.6944	0.2778	0.0278
2	1	1	0.67	0.4444	0.4444	0.1111
2	1	0	0.50	0.2500	0.5000	0.2500
2	0	2	0.67	0.4444	0.4444	0.1111
2	0	1	0.50	0.2500	0.5000	0.2500
2	0	0	0.33	0.1111	0.4444	0.4444
1	2	2	0.83	0.6944	0.2778	0.0278
…	…	…	…	…	…	…

For the final output (resilience node in Figure 4), five states are defined: High, Good, Moderate, Low, and Vulnerable. In this case, five degrees are preferred for a more accurate understanding of the output level. The same procedure described in the previous paragraphs applies here with the resilience node being the son node and the resilience pillars being the father nodes.

3.4 Joint probability distribution

Each node in a Bayesian network is characterized by a probability distribution. All probabilities together form the joint probability distribution (JPD) of the BN. The JPD of a BN can be written as follows:

$$P(Z) = \prod_{i=1}^{N} P\left[Z^i \mid Pa(Z^i) \right] \tag{8}$$

where Z is the set of all variables, $P(Z)$ is the joint probability of the variables, $Pa(Z^i)$ is the set of variables that are parents of Z^i, $P(X_i|\ Pa(X_i))$ is the local probability distribution, N is the number of variables. Considering the system in Figure 4, the JPD can be calculated using Eq. (9).

$$\begin{aligned} &P(X,R,Re) = P(X_1)\cdot P(X_2)\cdot P(X_3)\cdot P(X_4)\cdot P(X_5)\cdot P(X_6)\cdot P(X_7)\cdot \ldots \cdot P(X_N)\cdot \\ &P(R_1 \mid X_1, X_2, X_4, X_6)\cdot P(R_2 \mid X_3, X_5, X_6, X_7)\cdot P(R_3 \mid X_7, \ldots, X_N)\cdot \\ &P(Re \mid R_1, R_2, R_3) \end{aligned} \tag{9}$$

4. Example 1: resilience evaluation of the state of Brazil using BN

4.1 Model definition: Hyogo Framework for Action

This section illustrates the static resilience framework introduced in Section 3. Given the increase in the number of natural and man-made disasters, the United Nations (UNISDR) have formulated a structured approach to help communities cope with unexpected disruptions. The conceived framework, firstly presented in the 2005 UNISDR report (ISDR 2005; UNISDR 2008), is known as the Hyogo Framework for Action (HFA). It is now considered a global blueprint for minimizing risk associated with natural hazards through the implementation of national laws for risk management and control(Reinhorn et al. 2009). The HFA was originally conceptualized in Kobe, Japan with the goal of encouraging countries to implement resilient measures in their respective laws. The lifespan for the implementation was from 2005 to 2015. After that, each of the participating countries was required to submit a report (a detailed questionnaire) on their own progress. A score was then given by the UN to each of the submitted reports based on the progress each country had made (Kammouh et al. 2018c; UNISDR 2011). The progress recorded by every country is computed on the basis of a five-point scale for each indicator, where 'one point' indicates weak progress while 'five points' implies a great endeavor and commitment in that specific area. The scores of the 22 indicators for 37 countries assessed by the United Nations are reported in Appendix.

The objective of the HFA is the significant reduction in losses after disasters(Cimellaro et al. 2014a). Following the resilience model introduced in Section 3.1, the HFA indicators are unfolded under the three resilience pillars:

a) Reduced Vulnerability: includes consideration of disaster risk that is aimed at preventing and mitigating disaster as well as reducing vulnerability;
b) Robustness: Strengthening of institutions and mechanisms at all level aiming at increasing resilience (P_2);
c) Adaptive and Recovery capacity: Structural embedding of risk reduction methods for emergency preparation, response, and recovery (P_3).

To increase the level of detail and to convert the strategic goals into operationalizable activities, the UNISDR introduces five priorities:

1. Ensure disaster risk reduction;
2. Identify, assess and monitor disaster risks and enhance early warning;
3. Use available physical and non-physical resources to build a culture of safety and resilience;
4. Reduce the underlying risk factors;
5. Strengthen disaster preparedness for effective response.

Each of the five priorities is further disaggregated into four to six indicators, summing up to a total of twenty-two Table 2). The indicators refer to the implementation of activities, mechanisms, or policies with the aim of risk reduction, preparation, and recovery.

4.2 Network structure and elements connectivity

To build the network, linking the indicators with the resilience pillars is necessary. Despite not being introduced in the UNISDR reports (ISDR 2005; UNISDR 2008), the assignment of the indicators to one of the resilience pillars is performed as follows:

- If the indicator clearly refers to a regulatory requirement or action with proactive intent of risk reduction, it is assigned to P_1;
- If the indicator clearly refers to the implementation of institutional mechanisms or the building of resources for the proactive establishment of resilience capabilities, it is assigned to P_2;
- If the indicator clearly refers to the implementation of practices, mechanisms, and programs for emergency response and recovery, it is assigned to P_3. Note that it is possible for an indicator to affect more than one resilience pillar.

Table 2 shows a list of the indicators of the HFA grouped by priority. The BN is built following the procedure described in Section 3.2 (see Figure 5). It can be seen that P_1 is influenced by seven indicators (Q_1, Q_2, Q_{13}, Q_{14}, Q_{15}, Q_{16}, and Q_{19}), P_2 is influenced by eleven indicators (Q_3, Q_4, Q_5, Q_6, Q_8, Q_9, Q_{11}, Q_{12}, Q_{19}, Q_{21}, and Q_{22}), while strategic P_3 is impacted by eight indicators (Q_7, Q_8, Q_{10}, Q_{11}, Q_{17}, Q_{18}, Q_{20}, and Q_{21}). Only four indicators present an overlap between different resilience pillars: Q_8, Q_{11}, Q_{21} (between P_2 and P_3) and Q_{19} (between P_1 and P_2).

Table 2 List of indicators of the Hyogo Framework for Action grouped by priority (UNISDR 2011).

Priority	*Indicator*	*Resilience pillar*
(1) Ensure that disaster risk reduction (DRR) is a national and a local priority with a strong institutional basis for implementation	Q1- National policy and legal framework for disaster risk reduction exists with decentralized responsibilities and capacities at all levels.	P1
	Q2- Dedicated and adequate resources are available to implement disaster risk reduction plans and activities at all administrative levels	P1
	Q3- Community Participation and decentralization is ensured through the delegation of authority and resources to local levels	P2
	Q4- A national multi sectoral platform for disaster risk reduction is functioning.	P2
(2) Identify, assess and monitor disaster risks and enhance early warning	Q5- National and local risk assessments based on hazard data and vulnerability information are available and include risk assessments for key sectors.	P2
	Q6- Systems are in place to monitor, archive and disseminate data on key hazards and vulnerabilities	P2
	Q7- Early warning systems are in place for all major hazards, with outreach to communities.	P3

	Q8- National and local risk assessments take account of regional / trans boundary risks, with a view to regional cooperation on risk reduction.	P2-P3
(3) Use knowledge, innovation, and education to build a culture of safety and resilience at all levels	Q9- Relevant information on disasters is available and accessible at all levels, to all stakeholders (through networks, development of information sharing systems etc.)	P2
	Q10- School curricula, education material and relevant trainings include disaster risk reduction and recovery concepts and practices.	P3
	Q11- Research methods and tools for multi-risk assessments and cost benefit analysis are developed and strengthened.	P2-P3
	Q12- Countrywide public awareness strategy exists to stimulate a culture of disaster resilience, with outreach to urban and rural communities.	P2
(4) Reduce the underlying risk factors	Q13- Disaster risk reduction is an integral objective of environment related policies and plans, including for land use natural resource management and adaptation to climate change.	P1
	Q14- Social development policies and plans are being implemented to reduce the vulnerability of populations most at risk.	P1
	Q15- Economic and productive sectorial policies and plans have been implemented to reduce the vulnerability of economic activities	P1
	Q16- Planning and management of human settlements incorporate disaster risk reduction components, including enforcement of building codes.	P1
	Q17- Disaster risk reduction measures are integrated into post disaster recovery and rehabilitation processes	P3
	Q18- Procedures are in place to assess the disaster risk impacts of major development projects, especially infrastructure.	P3
(5) Strengthen disaster preparedness for effective response at all levels	Q19- Strong policy, technical and institutional capacities and mechanisms for disaster risk management, with a disaster risk reduction perspective are in place.	P1-P2
	Q20- Disaster preparedness plans and contingency plans are in place at all administrative levels, and regular training drills and rehearsals are held to test and develop disaster response programs.	P3
	Q21- Financial reserves and contingency mechanisms are in place to support effective response and recovery when required.	P2-P3
	Q22- Procedures are in place to exchange relevant information during hazard events and disasters, and to undertake post-event reviews	P2

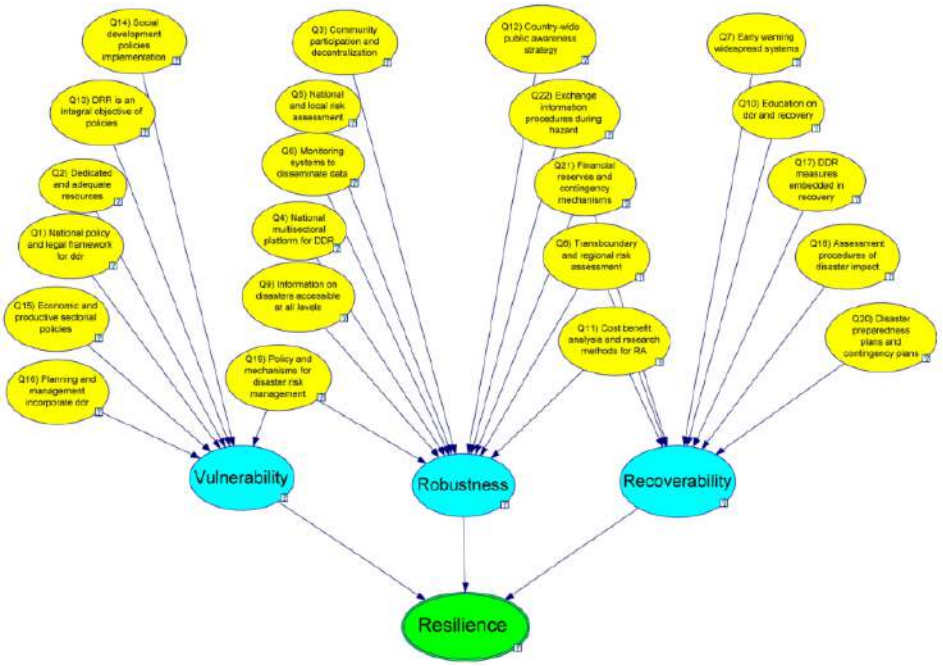


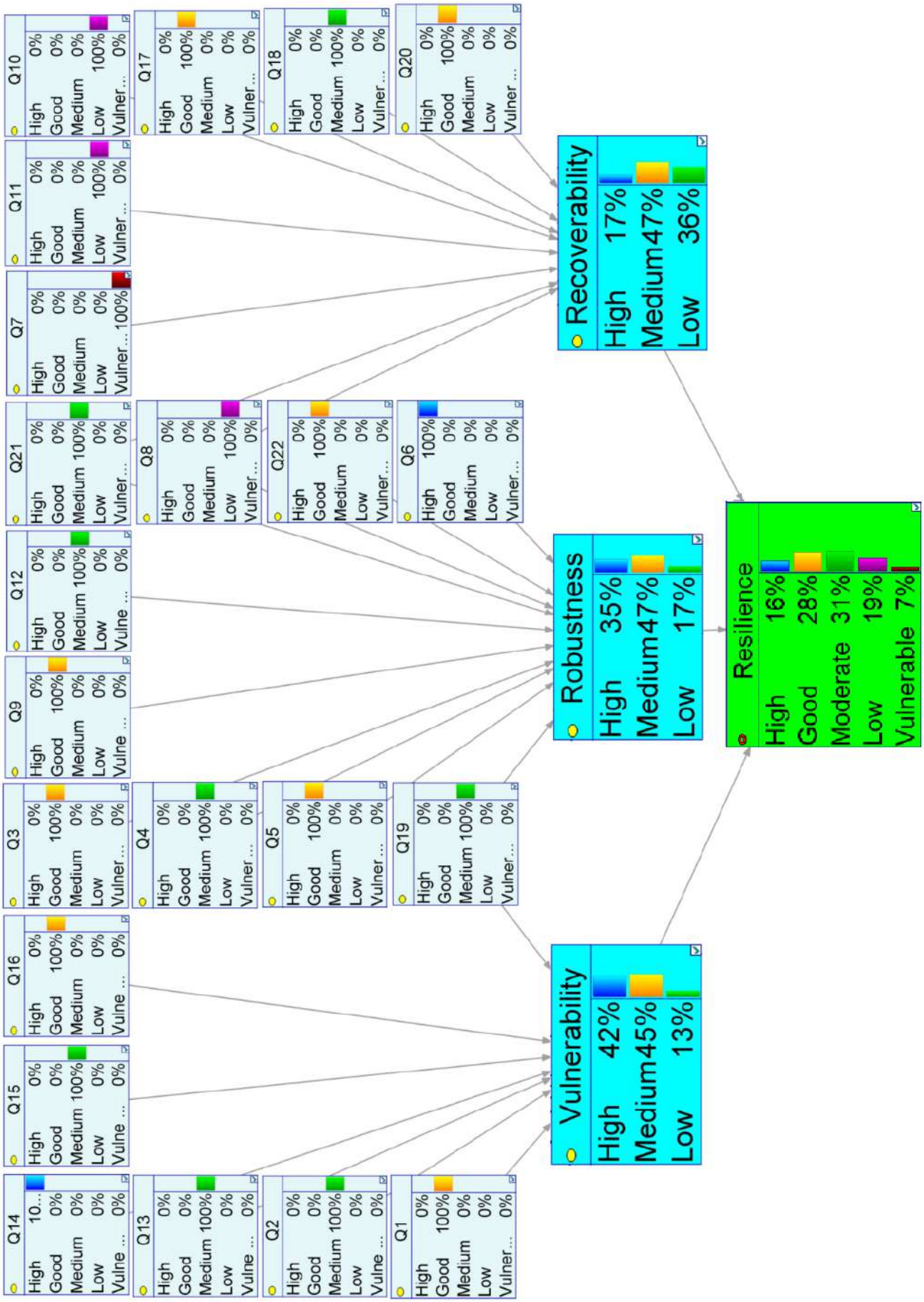
Figure 5 Bayesian Network of the Hyogo framework indicators

4.3 Probability tables and inference

4.4 Results

Figure 6 shows a Bayesian network applied to the data of the country "Brazil" (row 26 in Appendix). The analysis is done using GeNIe modeler, a graphical user interface that allows for interactive model building and learning based on the Bayes' inference theory (BayesFusion 2016). The top level of the network presents the main activities to be performed at the national and local levels, the intermediate level includes the three resilience pillars, and the bottom level node is the output of the network (i.e., resilience). As can be noticed in Figure 6, the three resilience pillars have a probability distribution for their different states despite that the indicators are deterministic. This is caused by the CPTs as well as the characteristics of the Bayesian inference adopted in the study. The final output of resilience presents a range of uncertainty (16% High, 28% Good, 31% Moderate, 19% Low, and 7% Vulnerable) (Note: the sum is 101 instead of 100 because the tool used in the analysis rounds the values to the nearest whole number). In the analyzed scenario, the resilience state of the country Brazil is most likely to be "Moderate" given that this state has obtained the highest probability.

The Bayesian network can also be employed in a backward analysis. A deterministic resilience state can be set (for instance "Good") and the output would be the levels of the indicators required to achieve the assumed resilience state. This is rather useful in case of system design or system improvement.



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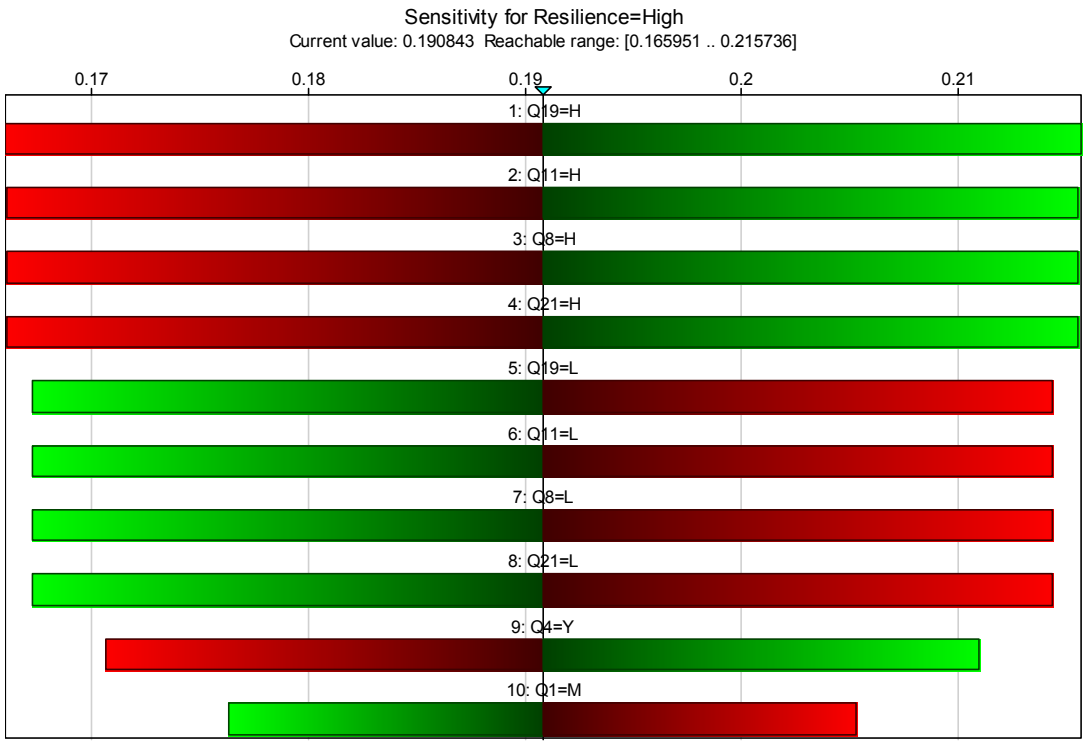
363 Figure 6- BN analysis and resilience results of the country “Brazil”

4.5 Sensitivity analysis

Sensitivity analysis can help validate the probability parameters of a Bayesian network. It is done by investigating the effect of small changes in the values (probabilities) of the input parameters on the output. Identifying highly sensitive parameters allows for a directed allocation of effort in order to obtain accurate results of a Bayesian network model.

In this example, an algorithm proposed by (Kjærulff and van der Gaag 2000) is implemented. Given a set of target nodes, a complete set of the derivatives of the posterior probability distributions over the target nodes over each of the numerical parameters of the Bayesian network are efficiently calculated. These derivatives indicate the importance of precision of network numerical parameters for calculating the posterior probabilities of the targets. A large derivative of a parameter p leads to a large variation in the posteriors of the targets given a variation in the parameter p. If the derivative is small, then even large changes in the parameter make little difference in the posteriors.

Each state of the target node is treated individually when performing a sensitivity analysis. Hence, the sensitivity analysis shows the most sensitive parameters for a selected state of the target node. Figure 7-9 show the sensitivity analysis done for the resilience target node states High, Moderate, and Low. The bar shows the range of changes in the target state as the parameter changes in its range. Only the 10 most influential parameters are plotted. The color of the bar shows the direction of the change in the target state, red expresses negative and green expresses positive change. We can clearly see that each parameter state can have a different sensitivity on the target node. For instance, as we can see in Figure 7, the parameter Q19 with a state “High” is the most sensitive parameter to the target node Resilience with state High. This is followed by Q11(High) and Q8(High). In Figure 8, in which the sensitivity analysis for Resilience with state “Moderate” is done, the most sensitive parameter is Q11(High). This is followed by Q8(High) and Q21(High). Figure 9 presents a different order of sensitive parameters where the analysis is for Resilience with state “Vulnerable”. It is worth to note that these sensitivity results are affected by the conditional probabilities given by the user.



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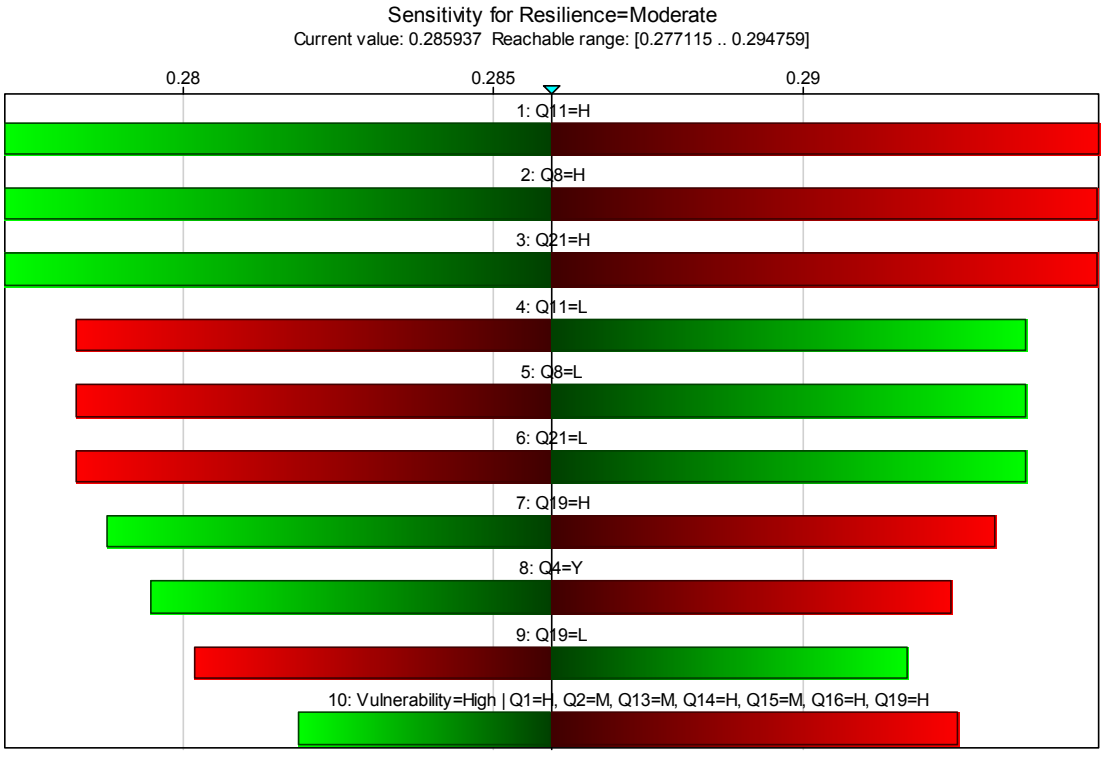
389 Figure 7 Sensitivity analysis for *Resilience* with a state *High*



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391 Figure 8 Sensitivity analysis for *Resilience* with a state *Moderate*

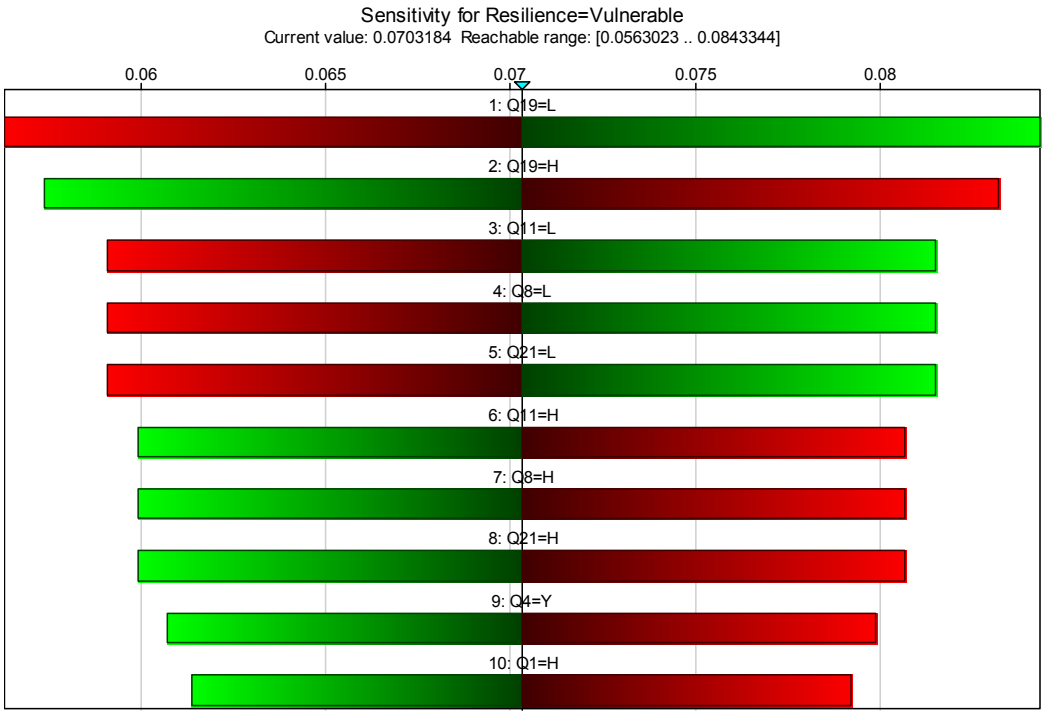


Figure 9 Sensitivity analysis for *Resilience* with a state *Vulnerable*

5. Time-dependent resilience analysis using Dynamic Bayesian Networks

In general, the resilience of a system tends to be a process rather than a state; thus, accounting for the performance variation of a system can be important. Ordinary Bayesian Networks are unable to account for the time dimension in the analysis as they are limited to static systems. In this section, we propose a new methodology to assess the resilience of engineering systems in a dynamic manner.

5.1 Dynamic resilience model

The resilience model used in the dynamic resilience analysis is based on the resilience definition by Bruneau and Reinhorn (2007) who describe the resilience of a system using four components, also called the four R's of resilience (4R's):

- *Robustness* (R_1): refers to the ability of a system to stand a certain level of stress preserving its functionality;
- *Redundancy* (R_2): indicates the alternative resources in the recovery stage when the primary ones are inadequate;
- *Rapidity* (R_3): the capacity to contain losses and avoid future disruption. It represents the slope of the functionality curve during the recovery phase;
- *Resourcefulness* (R_4): considers the human factor and the capacity to move needed resources.

This model is more detailed and more suitable to study dynamic events than the one described in Section 3.1; therefore, this model will be used hereafter. As shown in Figure 10, the first two resilience components (R_1 and

R_2) define the damage level the system may encounter if exposed to a certain hazard. Robust and redundant systems would most likely experience less damage and function almost normally after the disaster. On the other hand, once damage occurs, the system's recovery starts. The recovery process is defined by the recovery capacity and resources availability, such as human resources. Thus, the other two components (R_3 and R_4) interfere during the recovery stage as they are the main drivers of the system's recovery.

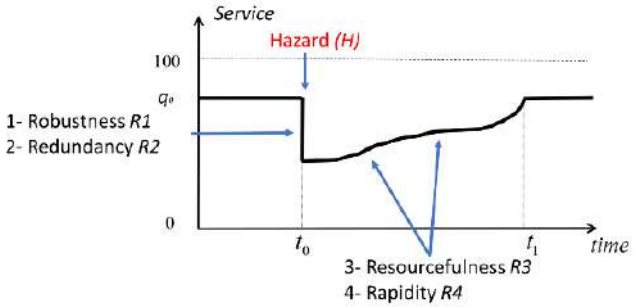


Figure 10 The four resilience components (4R's) and their interaction with the resilience curve

5.2 Network structure and elements connectivity

DBN is a series of Bayesian networks with changing conditions. The elements connectivity within a single time step of a DBN is treated similarly to what introduced before (see Section 3.2 and Figure 4). One main characteristic of DBN is that elements are connected through different time-steps. For example, element A_t can be linked to element B_{t+1} using a temporal link if element B_{t+1} has a dependency on A_t, where t is the time step. The connections between elements at different time steps is done using expert knowledge or from past data. Figure 11 shows a DBN where the individual networks at the different time steps are connected with one another. In our methodology, an element in a BN at time-step t can only affect itself at time-step $t+1$ (i.e., A_t affects A_{t+1} and B_t affects B_{t+1}).

Regarding the four resilience components (4R's), they are incorporated in the network at different time-steps. In Figure 11, the first step ($t=1$) corresponds to the initial state of the system (i.e., before hazard occurrence). At this stage, none of the 4R's is involved as the aim is to assess the initial performance of the system. The second step ($t=2$) is dedicated to assessing the damage that would incur if a hazard of a certain magnitude occurs. The level of damage, or the drop in the functionality, can be determined by acquiring information about the hazard (H) and the system's characteristics (i.e., R_1 and R_2). The combination of the parameters H, R_1, and R_2 can provide valuable information on how a system with a predefined initial state would behave. Thus, the two resilience components R_1 and R_2 are connected to the DBN at the second time-step (t=2).

Once the drop in the serviceability is determined, the recovery needs to be evaluated. Since recovery is not an instantaneous action, several DBN time steps are needed here. Therefore, the recovery period is divided into a finite number of time-steps. Information about the rapidity and the resourcefulness (R_3 and R_4) of the system is integrated at all recovery time-steps as they will define how the variables (i.e., the indicators) will evolve from one step to another. Therefore, the same Bayesian network is copied from time-step $t=3$ until time step $t=T$.

The result of each BN is a performance point. The collection of the performance points creates a resilience function that shows the changes in the system's performance, starting from a stable state (the first uniform part of the function in Figure 11) and ending with a stable state, when the system is fully recovered (the second uniform part of the function). Once obtained, the resilience function can be used to obtain a resilience index. One method uses the area above the resilience curve and links it to the notion "loss of Resilience" (Bruneau and Reinhorn 2007; Cimellaro et al. 2010a) while other methods consider other metrics to quantify the resilience (Sharma et al. 2018).

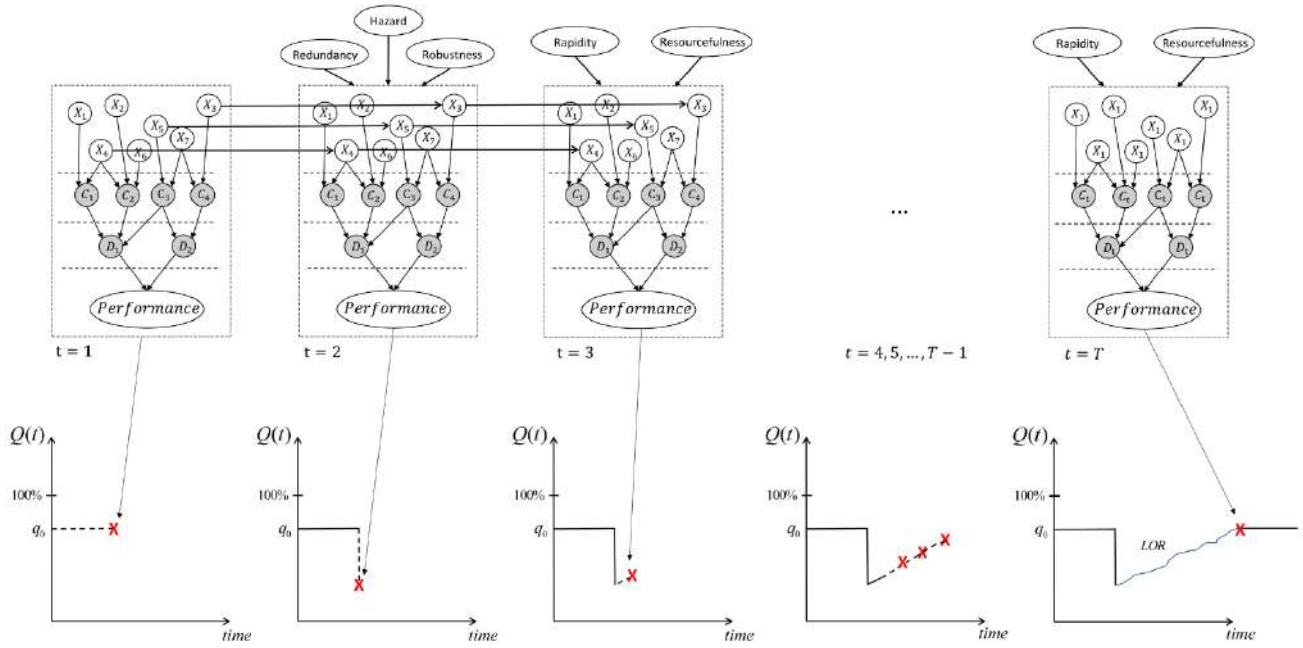


Figure 11 Dynamic Bayesian network of an engineering system considering external factors such as the resilience characteristics (4R's) and the Hazard

5.3 Joint probability distribution

The proposed dynamic resilience analysis using the DBN approach can be mathematically written in probabilistic terms, as follows:

$$P(C, Z_{1:T}, H, R_1, R_2, R_3, R_4) = \left[P(C).\prod_{i=1}^{N} P(Z_{t=1}^{i} \mid Pa(Z_{t=1}^{i}), C^{i}) \right].$$
$$\left[P(H).P(R_1).P(R_2)\prod_{i=1}^{N} P(Z_{t=2}^{i} \mid Pa(Z_{t=2}^{i}), C^{i}, H^{i}, R_1^{i}, R_2^{i}) \right]. \quad (10)$$
$$\left[P(R_3).P(R_4)\prod_{t=3}^{T}\prod_{i=1}^{N} P(Z_{t}^{i} \mid Pa(Z_{t}^{i}), C^{i}, R_3^{i}, R_4^{i}) \right]$$

where C is the set of all static variables (contemporal variables), Z is the set of all dynamic variables (temporal variables), P(C) is the joint probability of the static variables, $Pa(Z^i)$ is the set of variables that are children of Z^i, H is the hazard variable, R_1 is the Redundancy variable, R_2 is the Redundancy variable, R_3 is the Rapidity variable, R_4 is the Resourcefulness variable, N is the number of dynamic indicators, T is the total number of time steps. The first term on the right-hand side of Equation (10) refers to the joint probability of the variables

at the first time-step, the second term refers to the joint probability of the variables at the second time-step, while the third part of the equation considers the remaining time steps.

6. Example 2: resilience evaluation of a transportation network using DBN

6.1 Model definition: Modeling the physical aspect of a transportation network

To illustrate the dynamic methodology introduced above, an illustrative example of a typical transportation system is used. The resilience of engineering systems can be systematically described using a layered diagram. Figure 12 shows a scheme of a general engineering system, being resilience the top level. The resilience node is defined using a set of dimensions. Each dimension is divided into components, and the components are further divided into indicators. The lower level of the diagram is the "Measures" layer, which provides descriptions on how the indicators can be numerically evaluated. Having different layers allows for a detailed description of the system. A similar approach of modeling a port infrastructure system is introduced in (Balbi et al. 2018).

For the sake of this study, a general indicator-based model to describe transportation systems is proposed. The model consists of Seven dimensions divided into 21 Components. The components are further divided into 78 indicators, which are allocated with measures to provide practical information on the computation of each indicator. The indicators included in the model have been collected from exclusively renowned literary publications and then allocated to the proper components. The components themselves have been proposed ensuring a good coverage of the different aspects of the transportation infrastructure. Much effort has been done to reduce the overlap among indicators by removing the duplicated ones. For this, expert opinions have been used. This has led to a condensed list of indicators. The authors have also proposed some indicators when needed to ensure the exhaustiveness of the model.

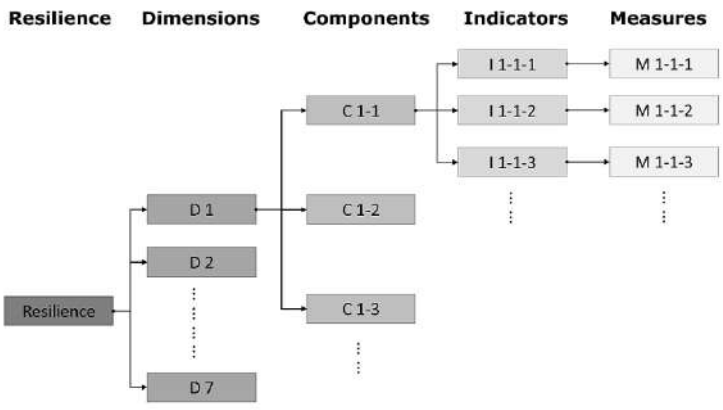


Figure 12 An indicator layered-model to systematically describe engineering systems

Table 3 presents the seven dimensions of the proposed model: (1) Physical infrastructure, (2) User's behavior, (3) Resources, (4) Plan, (5) Organization and management, (6) Social-economic characteristics, and (7) Environment and climate. To keep it simple, only the first dimension (Physical infrastructure) is used in this study, and therefore only the first dimension is expanded with the list of components, indicators, and measures (Table 3). The last two columns in the table represent the importance factor (I) and the Nature (Nat) of the indicators, respectively. The importance factor provides a tool to weight the variable. Several methods for

defining the importance factors exist in the literature. For example, Kammouh et al. (2019b) proposed a matrix-based methodology to compute the weight of variables based on the level of interdependency with other variables. That is, if many variables depend on a certain variable, the latter is assigned a high importance factor. Other methods suggest a subjective assignment of the weighting factors by an expert in the related field. This process is simpler but can produce inaccurate results. The Nature of the indicator (Nat) divides the indicators according to their type "static" or "dynamic".

Figure 13 shows a graphical representation of a static and dynamic indicators. For static indicators, the functionality remains constant with time given that they are not affected by hazards. Dynamic indicators, on the other hand, are affected by hazards, and consequently their functionality changes with time. Dynamic indicators are defined using a set of variables (q_0, q_1, T_r, q_f) where q_0 is the normalized serviceability before the event, q_1 is the residual functionality after the disaster, q_r is the functionality after recovery, T_r is the restoration time or the time needed to finish the recovery process.

Each indicator is normalized with respect to a fixed quantity, the target value (TV). The target value is an essential quantity that provides the baseline to measure the resilience of a system. The system's existing functionality at any instance of time is compared to the target value to know how much functionality deficiency is experienced by the system.

Table 3 Variables of the proposed transportation network model with corresponding importance factors (I) and nature (Nat)

Dimension/ component/ indicator	Measure (0<=value<=1)	Reference	I	Nat
1- Physical infrastructure			**3**	
1-1- Links/ Connectors			3	
-Accessibility	Number of links/passageways per destination ÷ TV	(Ip and Wang 2011)	3	D
-Road density	Number of alternative links between an origin and destination ÷ TV	(Jenelius 2009)	3	D
-Road width	Average width of road ÷ TV	(Jenelius 2009)	2	S
-Lanes of road	Number of lanes available ÷ TV	(Litman 2006)	2	D
-Link (road, track, etc.) condition	% links with full functionality during the event		3	D
1-2- Vehicles			2	
-Mode of transport	Number of multi-mode choices per destination ÷ TV	(Ip and Wang 2011)	3	D
-Service level	Average speed of vehicles in normal condition ÷ TV	(Sarkis et al. 2018)	1	S
-Characteristics of vehicles	Degree of preference of specific vehicles (regarding performance, comfort level, etc.) ÷ TV		1	S
1-3- Other Facilities/ Structures			3	
-Quality of facilities	1-(% deficiency of facilities in past events ÷ TV)	(Tamvakis and Xenidis 2012)	3	S
-Critical components	Number of roundabout/emergency lanes ÷ TV	(Kammouh et al. 2017a; Kammouh et al. 2018a)	2	S

-Maintenance of facilities	Number of maintenances during an interval of period ÷ TV	(Tamvakis and Xenidis 2012)	3	S
-Essential infrastructure robustness	% infrastructures that remained operational during emergencies in past events	(UNISDR 2012)	2	S
-Traffic load capacity	Number of excessive capacity (emergency lanes, tracks, airlines, etc.) ÷ TV	(Cox et al. 2011)	3	D
-Urban form	Number of city centers per 100,000 people ÷ TV	(Mishra et al. 2012)	3	S
-Size of network (connectivity)	Number of connectivity of intersection ÷ TV	(Zhang et al. 2011)	2	D
-Size of network (betweenness)	1-(Number of betweenness of intersections ÷ TV)	(Zhang et al. 2011)	2	D
1-4- Accessories			1	
-Tool kit inside vehicles	1 (Presence of tool kits, like extinguisher, escape hammer, etc.); 0 (otherwise)		2	S
-Path environment	Number of safety elements (isolation strips, traffic lights, etc.) per km ÷ TV	(Soltani-Sobh et al. 2016)	2	S
1-5- Serviceability			2	
-Characteristics of traffic lines	Frequency and capacity of each line ÷ TV	(Dorbritz 2011)	3	D
-Travel time reliability	number of punctual service assisted by control system ÷ total number of service	(Leu et al. 2010)	2	S
2- User's behavior				
3- Resources				
4- Plan				
5- Organization and management				
6- Social-economic characteristics				
7- Environment and climate				

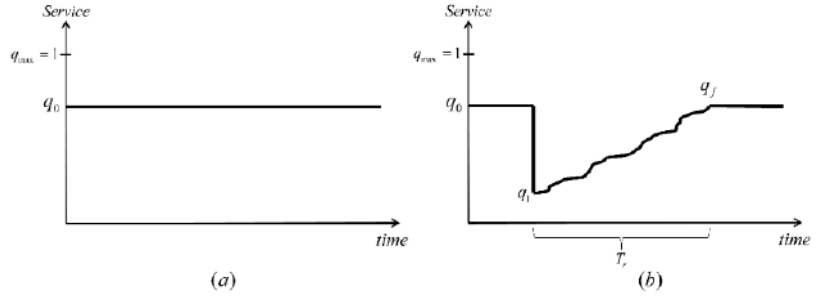


Figure 13 a) Event-non-sensitive indicator (static) b) event-sensitive indicator (dynamic)

6.2 Network structure and elements connectivity

Figure 14 presents the network structure and elements connectivity using the software GeNIe (BayesFusion 2016). The network has been built following Section 5.2. A color code is used to distinguish the variables in the network. Variables that are outside the box are static variables. They are assigned unconditional probability tables (UPTs) that do not change throughout the analysis. Variables inside the green box are dynamic variables. The dynamic indicators (i.e., variables inside the green box and colored in yellow) are assigned UPTs for the first time-step and CPTs for the remaining time steps. The CPTs are used to define the functionality of the indicator at time ($t+1$) given its functionality at time (t) and given external variables (i.e., damage and recovery

variables). The damage variables H, R_1, and R_2 are used to determine the amount of damage the indicators are exposed to following the hazard. Therefore, the damage variables interfere only at the second time-step (see Figure 11) and their effect is reflected in the CPTs of the dynamic indicators at time slice 2. On the other hand, the recovery variables R_3 and R_4 feed the dynamic indicators from time slice 3 until the last time-slice (see Figure 11). The effect of these variables is reflected in the CPTs of the dynamic indicators for all time slices starting from time slice 3. For the first time-slice, the system is assessed for its initial condition. That is, the effect of the damage and recovery variables is not considered, and so the dynamic indicators have no father nodes for this time slice. The tool used for the analysis allows determining at what step each variable interferes.

Other variables inside the green box are the variables colored in Orange (components) and Blue (dimensions). such variables are dynamic, and their value is defined using CPTs that consider the values of their father nodes. The father nodes of the components are the indicators while the father nodes of the dimensions are the components. The damage and recovery variables do not affect the components or the dimensions directly. Their effect is transmitted through the indicators to the lower levels of the network. The connectivity between the indicators and the components or between the components and the dimensions can be defined using expert knowledge and experience.

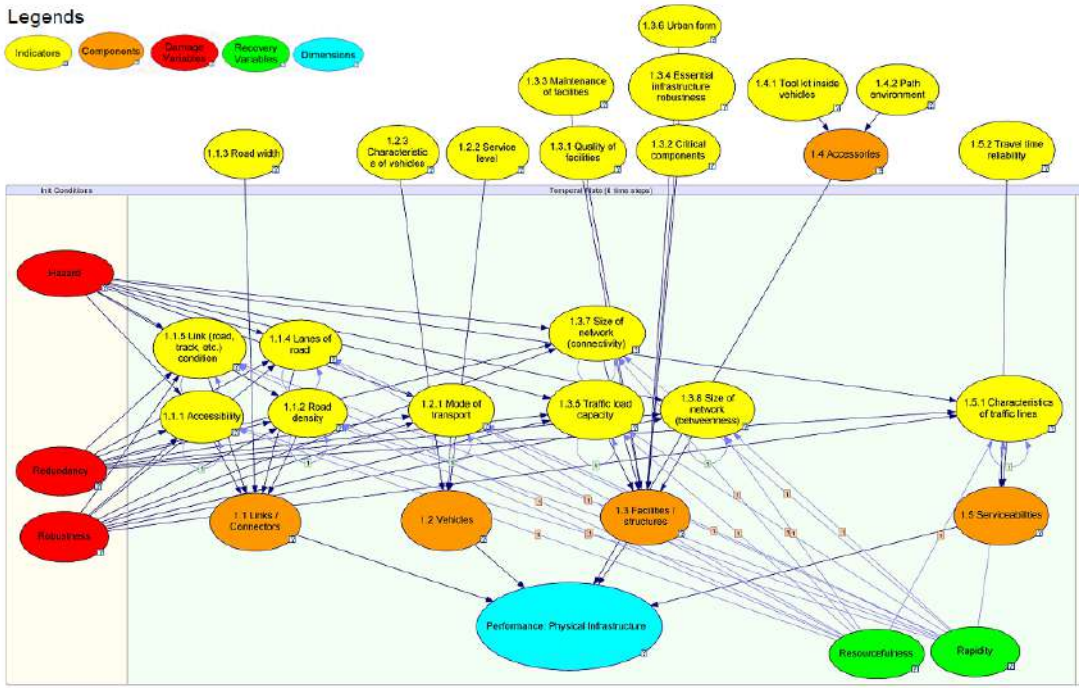


Figure 14 DBN connectivity of the transportation network model

6.3 Probability tables and inference

In the dynamic analysis, CPTs are assigned to variables that have father nodes in the same or different time-slice. For example, "components" are assigned CPTs that consider their father nodes (i.e., indicators), while each dynamic indicator (i.e., indicator that has a temporal link) is assigned a CPT that considers the indicator itself at a previous time-slice as well as the damage and recovery variables, depending on the time step. The same procedure used in Section 3.3 can be used to conclude all CPTs and UPTs of the model's variables.

6.4 Results

Five scenarios have been implemented for comparative reasons. Table 4 summarizes the inputs of the damage and recovery variables for the different scenarios. For the sake of simplicity, each variable is assigned a three-level scale (High, Medium, and Low). For all scenarios, the states of the static indicators are assigned a uniform probability distribution. This is usually done when little or no information about the variables is available. However, when data is available, different probability distribution among the three states can be set. The result of the analysis is the performance level of the system. Since the analysis is dynamic, the result is a curve showing the variation of the performance in time. Four time-steps (or time-slices) are assigned to the analysis as a time interval. In the following, each scenario is tackled separately then a comparison between the scenarios highlighting the effect of the different variables on the performance level of the system is performed.

Table 4 Values for the different input variables

Input	Scenario 1 (Figure 15)	Scenario 2 (Figure 16)	Scenario 3 (Figure 17)	Scenario 4 (Figure 18)	Scenario 5 (Figure 19)
Hazard (H)	High	High	Low	Low	High
Redundancy (R_1)	Low	Low	High	High	Low
Robustness (R_2)	Low	Low	High	High	Low
Resourcefulness (R_3)	High	Low	Low	High	Medium
Rapidity (R_4)	High	Low	Low	High	Medium

Note: the red color implies negative impact on the performance, the green color implies a positive impact, and the orange color implies medium impact.

Scenario 1

Figure 15 shows the result of the first scenario. The states of the damage and recovery variables are set according to Table 4: the damage variables are set to negative impact (i.e., H is set to "High" while R_1 and R_2 are set to "Low") while the recovery variables are set to positive impact (i.e., R_3 and R_4 are set to "High"). To discuss the analysis results, we will focus on the node "Performance" (i.e., node in Blue color). The result is presented as a probability variation for each of the three states of the variable. From Figure 15, we can see that the probability for the node "Performance" being "High" starts very low then it increases rapidly to reach a stable state. This result is expected since our initial input for the damage is set to *negative impact*, which caused the probability for the system's performance of being "High" to be low in the beginning. On the other hand, the recovery variables have been set to *positive impact*, and this caused the probability of the system's performance of being "High" to increase rapidly over time. The probability does not reach 1 because of the uncertainties introduced in the static indicators, which have been transmitted throughout the network. As a complementary, the probability of being "Low" starts relatively high then it reduces over the time with the same rate.

Scenario 2

In the second scenario (Figure 16), the damage variables are kept as before (high damage or negative impact) but the recovery variables have been changed from high to low. The effect of setting the recovery variables to low is reflected in the performance node. We can see that the initial probabilities are exactly like the first

scenario, as the damage variables are the same, but the probabilities do not evolve similarly with time. The probability of being "High" starts low and remains low for all time steps, unlike in the first scenario where there was a noticeable increase in this probability. This is due to the recovery variables which have been set to low, where low stands for limited or no recovery activities.

Scenario 3

As for the third scenario, the damage variables have been switched from negative impact to positive impact. This is done by setting H to "Low" and both R_1 and R_2 to "High" (Figure 17). On the other hand, the recovery variables are kept "Low". The Performance node appears to start with a high probability of being "High" and remains constant with time. This is because, as the inputs suggest, the damage is low and there is no recovery. No recovery is observed for two reasons: a) there is no damage margin to recover, and b) the recovery variables are set to low.

Scenario 4

In the fourth scenario, the damage variables are kept as in the third scenario (i.e., positive impact) while the recovery variables are switched back to "High". The result is shown in Figure 18 where the probability of being "High" starts relatively high and then slightly increases before it becomes stable. The only difference between the result in this scenario and the previous scenario is the slight increase in the performance. This slight increase in the probability is due to the high recovery capacity of the system. However, the high recovery capacity of the system was not needed in this case as there was not a damage margin to recover.

Scenario 5

Last scenario is similar to the first scenario with the only difference that the recovery variables are set to medium instead of high (Figure 19). As a result, the increase in the probability of being "High" of the performance node in this scenario is less than that in scenario 1. We can see a steady increase in the probability until it reaches a stable state at the end of the curve.

In all the cases, we can see that the state "Medium" of the performance node has a certain probability. As mentioned before, this is due to the uncertainty introduced in the static indicators which are propagated in the network. Moreover, the dynamic variables inside the box (i.e., dynamic indicators and dynamic components) are impacted by external variables such as the static indicators and the damage/recovery variables. The dynamic indicators, in particular, are affected by the indicators themselves at previous time steps due to the presence of temporal links (arrow going from the indicator to itself).

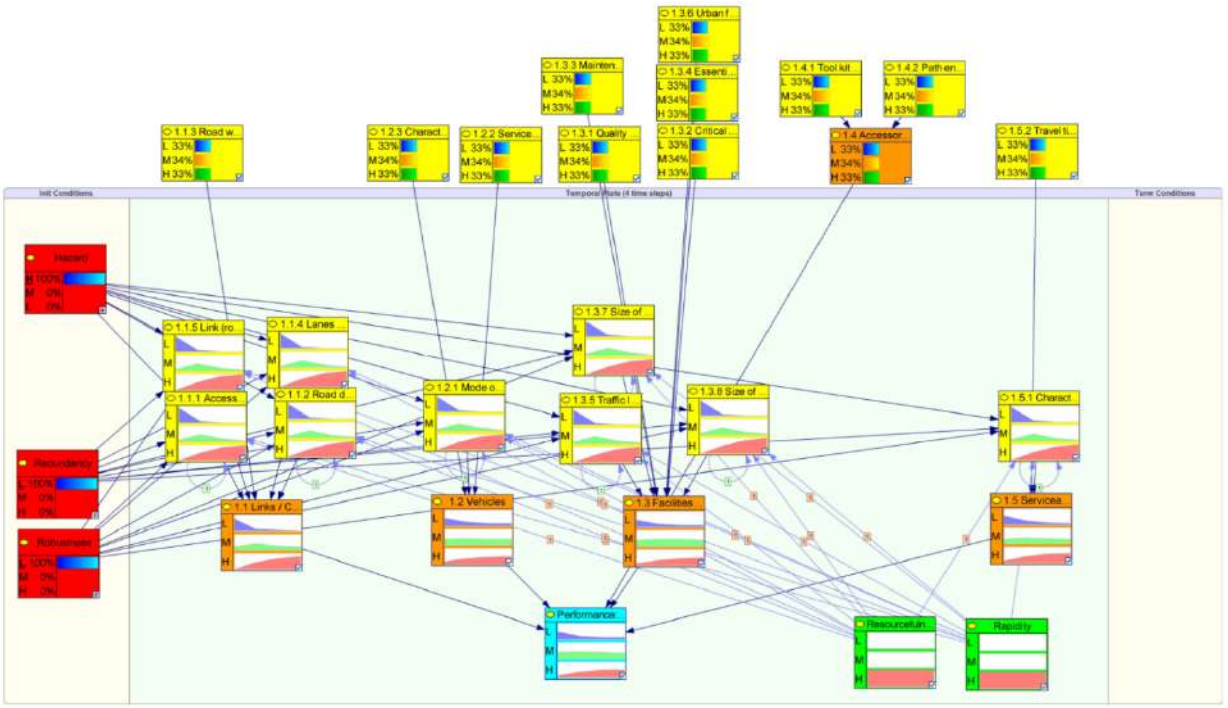


Figure 15 System's performance results for the first scenario of the simulation (high damage, high recoverability)

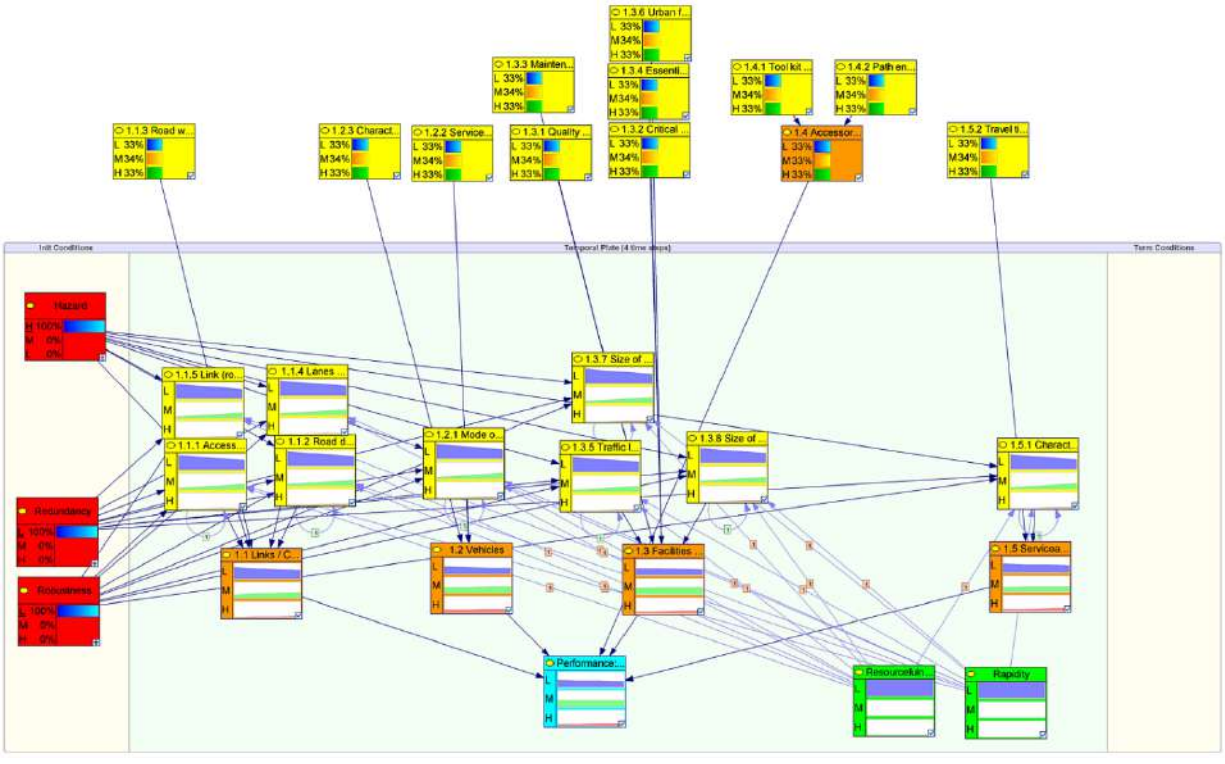
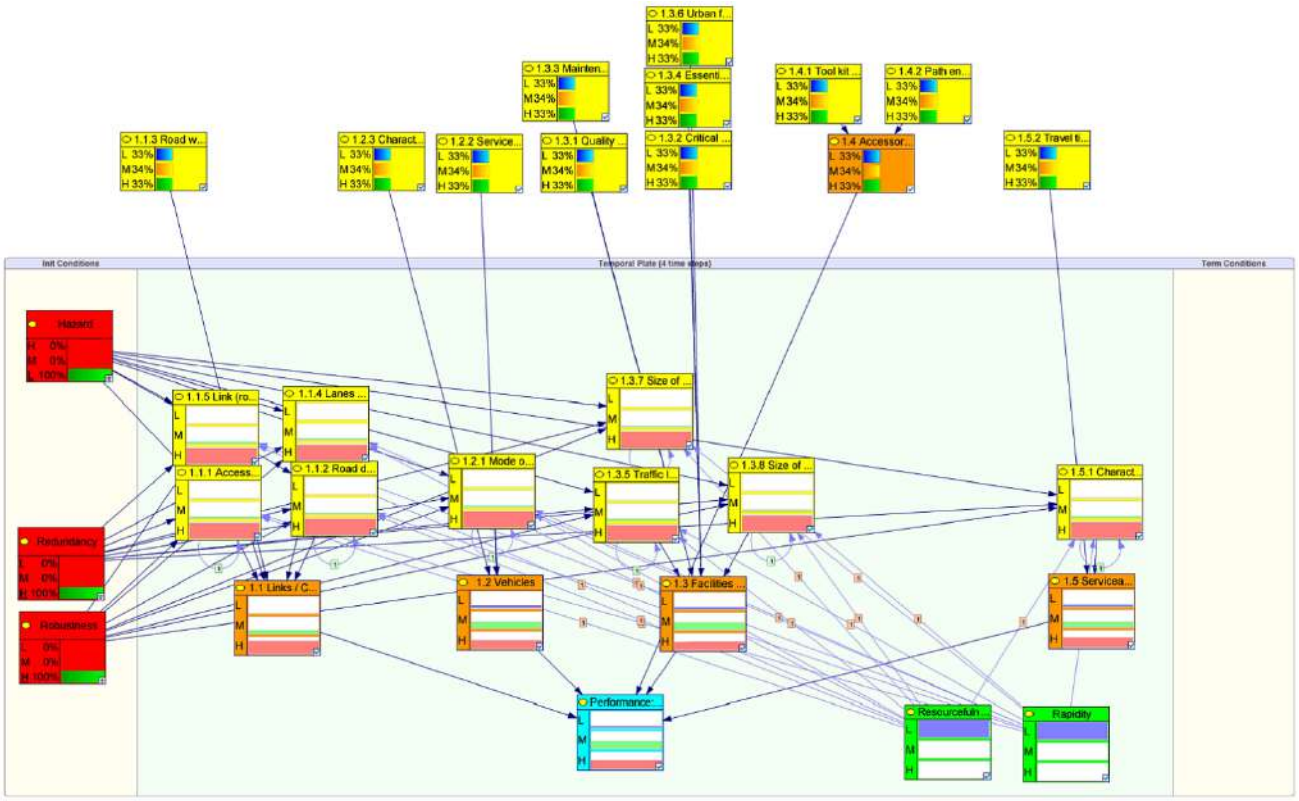


Figure 16 System's performance results for the second scenario of the simulation (high damage, low recoverability)



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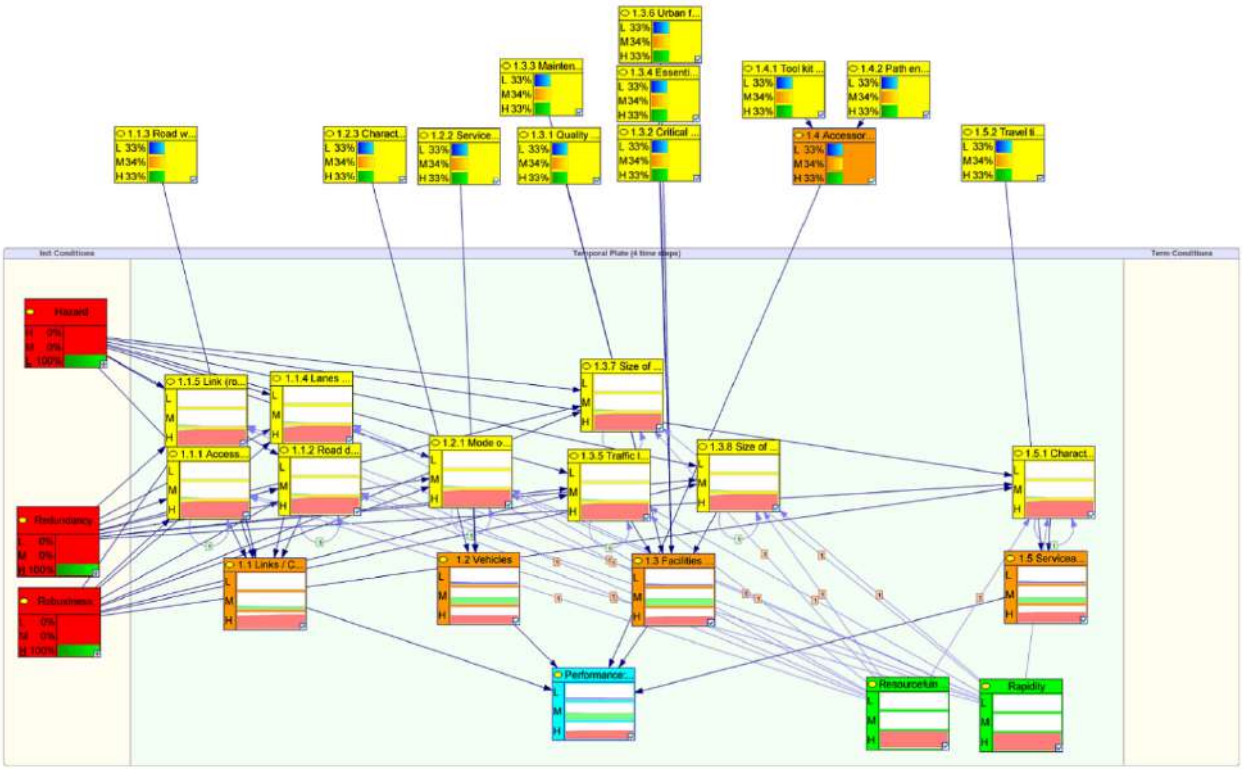
609 Figure 17 System's performance results for the third scenario of the simulation (low damage, low
610 recoverability)



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612 Figure 18 System's performance results for the fourth scenario of the simulation (low damage, high
613 recoverability)

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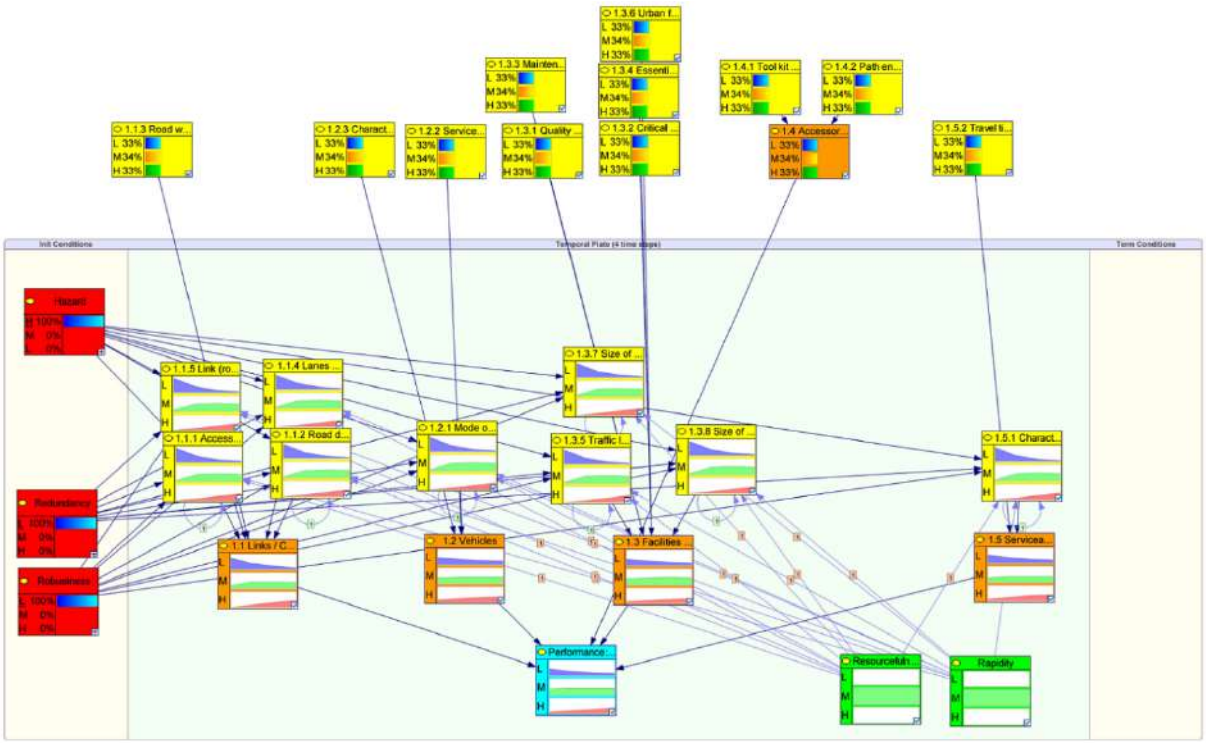

Figure 19 System's performance results for the fifth scenario of the simulation (high damage, medium recoverability)

7. Further considerations and applications of DBN

In our formulation in Equation (10), we assume that only one hazard event can occur. The work can be extended to include a sequence of multiple hazards (e.g. foreshock and aftershock). In this case, the damage variables (H, R_1, and R_2) would appear in other time slices. This is shown in Figure 20 where the resilience function has two drops in functionality instead of one drop due to the presence of two hazards. In such a case, the joint probability formation introduced before in Equation (10) should be rewritten to account for the damage variables at other time slices. There would also be some instances where both damage and recovery variables interfere together.

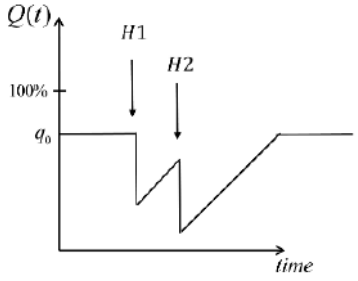

Figure 20 Resilience function with multiple hazards

Moreover, the damage and recovery variables have been expressed using a single variable. However, each of the variables can be described in a separate network that consists of several variables. This allows considering more details that would not be possible to be included if only one variable is considered. Equation (11) presents the damage variables as joint probabilities of other variables.

$$\begin{cases} P(H) = P(H^1, H^2, \cdots, H^k) \\ P(R_1) = P(R_1^1, R_1^2, \cdots, R_1^m) \\ P(R_2) = P(R_2^1, R_2^2, \cdots, R_2^m) \end{cases} \tag{11}$$

As mentioned in Section 2.2, there is also the possibility of introducing special nodes to the first slice or last slice of the DBN when needed (Figure 21). This can be done by introducing the nodes A (anchor) and T (terminal). In this case, Equation (10) must be adjusted accordingly to include the additional variables.

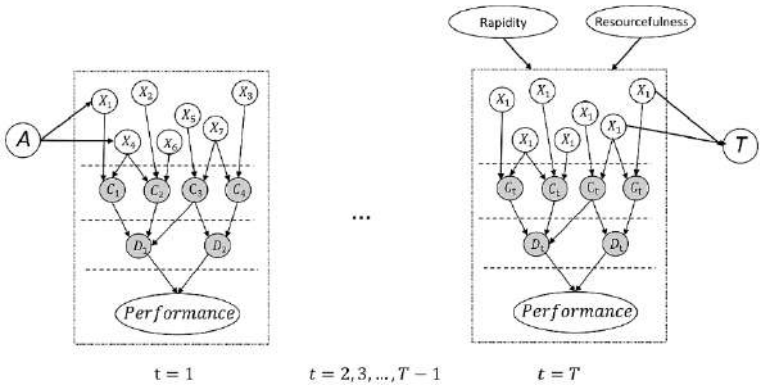

Figure 21 Bayesian network with additional variables in the first and last slices

The use of DBN to model and analyze engineering systems has been very limited in literature. DBN adds to the conventional BN the ability to consider time as a parameter in the analysis. All engineering systems are becoming strongly interdependent and this results in increased dynamic behavior of the systems. The DBN solves the challenge of dynamic modeling of engineering systems while it preserves all features of BN. While in this study we employ DBN to model and analyze the resilience of transportation infrastructure, the proposed model can be used to study the resilience of other infrastructure types as well as other engineering (and non-engineering) phenomenon that requires consideration of time. To give some examples of future applications, the proposed DBN formulation can be used to model, analyze, and improve the intervention activities of infrastructure in terms of scheduling and cost. This type of analysis cannot be performed with a static tool like BN. This, however, entails a detailed model of infrastructure intervention strategy. Another example can be structural degradation modeling and analysis. Degradation is a time-dependent process that cannot be studied at an instance of time, and this provides a good medium for DBN to be utilized. It should be noted that the use of DBN in such applications should be limited to cases where more complex tools cannot be used either due to modeling complexity of data availability.

8. Conclusions

Unlike the static resilience analysis which assumes a constant state of a system and measures the resilience by a static quantity, the dynamic resilience analysis additionally models the evolvement of the system with time. This paper introduced a probabilistic resilience assessment and prediction framework using the Bayesian and Dynamic Bayesian Networks (BN and DBNs). The framework employed resilience indicators for its implementation to make it more usable by decision makers in the industry. The methodology can handle both

static and dynamic engineering systems using quantitative and/or qualitative data. The uncertainty in the inputs and in the variables relationships is accounted for and propagated throughout the model; hence, the output is probabilistic in nature. Two illustrative examples were presented in the paper. The first is a static system that uses the indicators of Hyogo Framework for Action (HFA) to assess the resilience of a country, while the second is a transportation network modelled as a dynamic system. The examples demonstrate the applicability of the framework for both static and dynamic systems.

In the static analysis, the indicators are the main determinant of the resilience output. Highly uncertain state of the indicator (i.e., uniform probability distribution among the indicator's states) would result in a high standard deviation in the probability distribution of the resilience's states. For the dynamic analysis, results show a nonlinear behavior of resilience as a function of time. The recovery variables play a significant role in the resilience assessment, where the resilience function shows an increasing trend whose slope depends on the recovery capacity of the system. The damage variables also contribute to the overall resilience output as they are the primary determinant of the system's functionality drop following the disaster event. A large functionality drop would result in a longer recovery time under the same recovery characteristics of a system. In both static and dynamic analyses, the uncertainty is introduced in the indicators' initial conditions. This is rather useful when deterministic numbers are not available to initiate the analysis.

The quantitative resilience analysis tools that can be readily available to system designers to model and quantify engineering resilience are still underdeveloped. This paper aims at motivating the resilience community to agree on the proposed universal resilience framework. The presented framework provides a tool for decision makers to systematically learn about the state of their systems given a specific event. It allows them to improve the systems' performance using the backward analysis feature of BN. This is done by setting a desirable state of the resilience and getting the variables inputs that lead to the predefined resilience state.

There is a number of limitations in the proposed BN and DBN approaches. First, the need to include subjectivity during the different phases of model development and analysis. This is unavoidable because the main feature of BN is to substitute missing data with expert judgment, which is subjective. This can be partially addressed by using multiple experts. Another limitation is the increased complexity of the model as the analyzed system increases in size and detail. When the system is complex, the mission of classifying the variables and connecting them to one another becomes sophisticated and involves more subjectivity. In addition, BN is a directed acyclic graph, which means that if a variable depends on another variable the reverse is not true. This limits the possibility of modeling some real life situations where two variables can be dependent on each other. This can be artificially solved by introducing the same variable twice in the network, one time as a dependent variable and another time as a leading variable. However, this can also cause some consequences on the final output and on the dynamics of the system. Finally, DBN presents an additional limitation, which is the complexity in connecting variables at different temporal states. It can be challenging to identify which variables can affect other variables at another time step.

Future work will be oriented towards building detailed networks for the damage and recovery variables as this would allow expressing the system in more details. In addition, a procedure to evaluate the interdependency among the variables as well as their weighting factors will be further addressed. Particularly, weighting the indicators can be crucial in determining the conditional probabilities of the father nodes. Future studies can address this aspect using the Analytic hierarchy process (AHP), which has been extensively used in similar problems for organizing and analyzing complex decisions.

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Appendix

The scores of the resilience indicators of 37 countries as assessed by the United Nations

Indicators / Countries	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	Q22
1-Fiji	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
2-Costa Rica	5	4	4	5	3	4	5	4	4	4	5	4	4	4	3	5	5	5	5	5	5	5
3-Singapore	5	5	5	2	5	5	5	5	5	5	5	5	2	5	5	4	1	1	5	5	4	5
4-Japan	5	4	4	5	4	4	4	4	5	4	3	5	4	4	4	4	4	5	5	4	4	4
5-UAE	5	4	5	4	4	4	3	3	3	4	4	5	5	5	5	5	5	5	4	4	3	4
6-Austria	4	5	5	3	4	4	5	5	4	4	3	4	4	4	4	4	4	5	5	4	4	4
7-UK	4	4	5	4	4	4	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	5
8-Greece	4	4	4	4	4	4	5	4	4	4	4	4	4	4	4	5	4	4	4	4	4	4
9-Australia	4	4	4	4	4	4	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
10-Italy	2	4	4	4	4	4	5	4	5	4	4	4	4	3	3	3	5	4	5	5	4	4
11-Cameroon	4	4	4	5	4	4	4	4	4	4	4	4	4	3	4	4	4	4	4	3	4	4
12-New Zealand	4	4	4	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
13-Germany	5	4	4	4	4	4	4	4	4	3	4	4	4	5	3	4	3	3	4	4	4	4
14-Nigeria	4	4	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
15-Canada	4	4	5	4	3	4	4	4	4	3	3	4	3	3	4	4	5	4	5	5	3	3
16-France	4	4	4	4	3	4	5	4	4	3	3	3	4	4	3	4	4	3	4	3	5	5
17-Ecuador	4	4	4	4	3	4	3	4	4	4	4	4	4	4	4	4	4	3	4	4	3	4
18-Ethiopia	4	3	4	4	4	4	4	4	4	4	4	4	4	4	3	3	3	4	4	4	4	4
19-USA	4	4	4	3	4	4	4	4	3	4	4	4	3	3	3	4	4	4	4	4	4	4
20-Chile	4	3	3	4	4	3	4	3	4	4	2	4	3	4	4	4	4	4	4	4	5	4
21-Ghana	4	2	2	4	4	4	4	4	5	1	4	4	4	4	3	3	4	4	4	4	4	4
22-Argentina	3	3	4	4	4	3	4	4	3	3	3	4	3	3	3	4	4	4	4	4	2	4
23-South Africa	4	4	4	4	3	3	3	3	3	3	3	3	4	3	3	3	4	4	3	4	4	4
24-Cook Island	4	3	4	4	3	4	4	4	3	3	3	4	4	3	3	3	4	3	4	3	3	3
25-Pakistan	4	4	4	4	3	3	3	4	3	3	3	3	3	3	3	3	4	3	3	4	4	3
26-Brazil	4	3	4	3	4	5	1	2	4	2	2	3	3	5	3	4	4	3	3	4	3	4
27-Egypt	4	2	4	4	4	3	3	3	3	3	2	4	4	3	4	3	3	3	4	4	3	3
28-Iran	4	3	4	4	3	3	2	2	3	4	3	3	3	3	3	4	3	3	4	3	4	3
29-Qatar	3	4	3	3	4	3	3	3	3	2	3	3	4	3	3	3	3	3	4	3	3	3
30-Samua	4	3	3	4	4	2	3	4	3	3	3	4	4	3	3	2	2	1	4	3	3	3
31-Thailand	4	2	4	4	2	2	4	3	3	3	2	4	3	4	2	3	2	3	4	4	4	2
32-Madagascar	4	3	4	4	4	2	2	2	4	5	4	2	2	1	2	2	4	2	4	4	2	4
33-Mexico	4	3	3	4	2	3	4	3	3	2	3	2	3	3	3	2	3	3	3	2	4	3
34-Morocco	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1	3	3
35-Palestine	3	2	3	4	3	2	4	4	4	3	2	4	3	2	1	2	2	2	3	3	1	2
36-Monaco	3	2	1	3	3	1	3	3	4	4	1	2	3	1	1	1	1	1	3	4	1	1
37-Armenia	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

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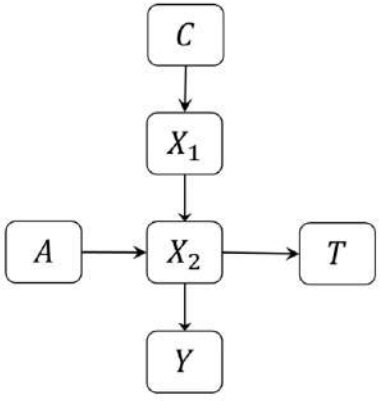
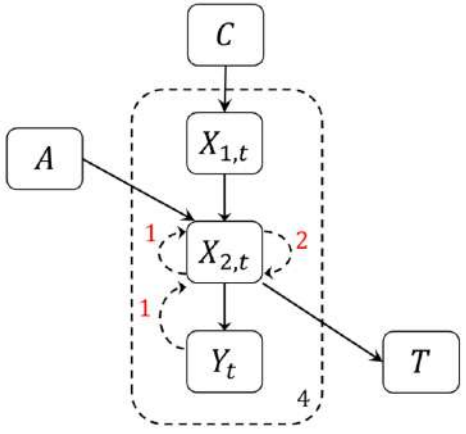
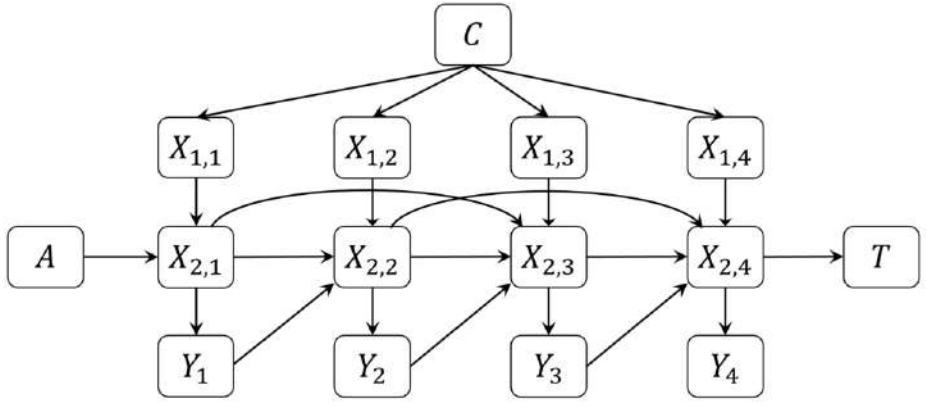
872


C
X1
A
X2
T
Y

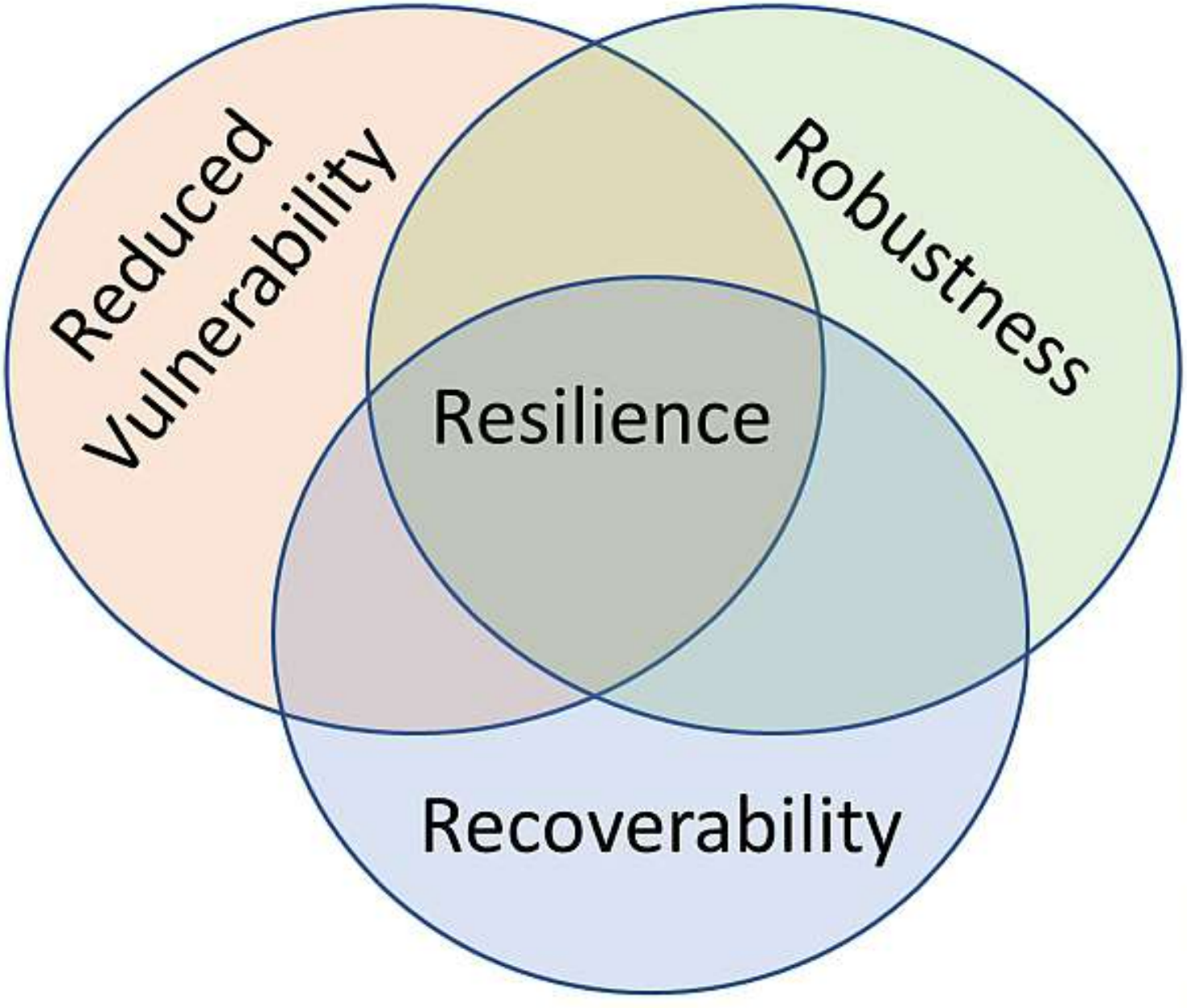
(a)

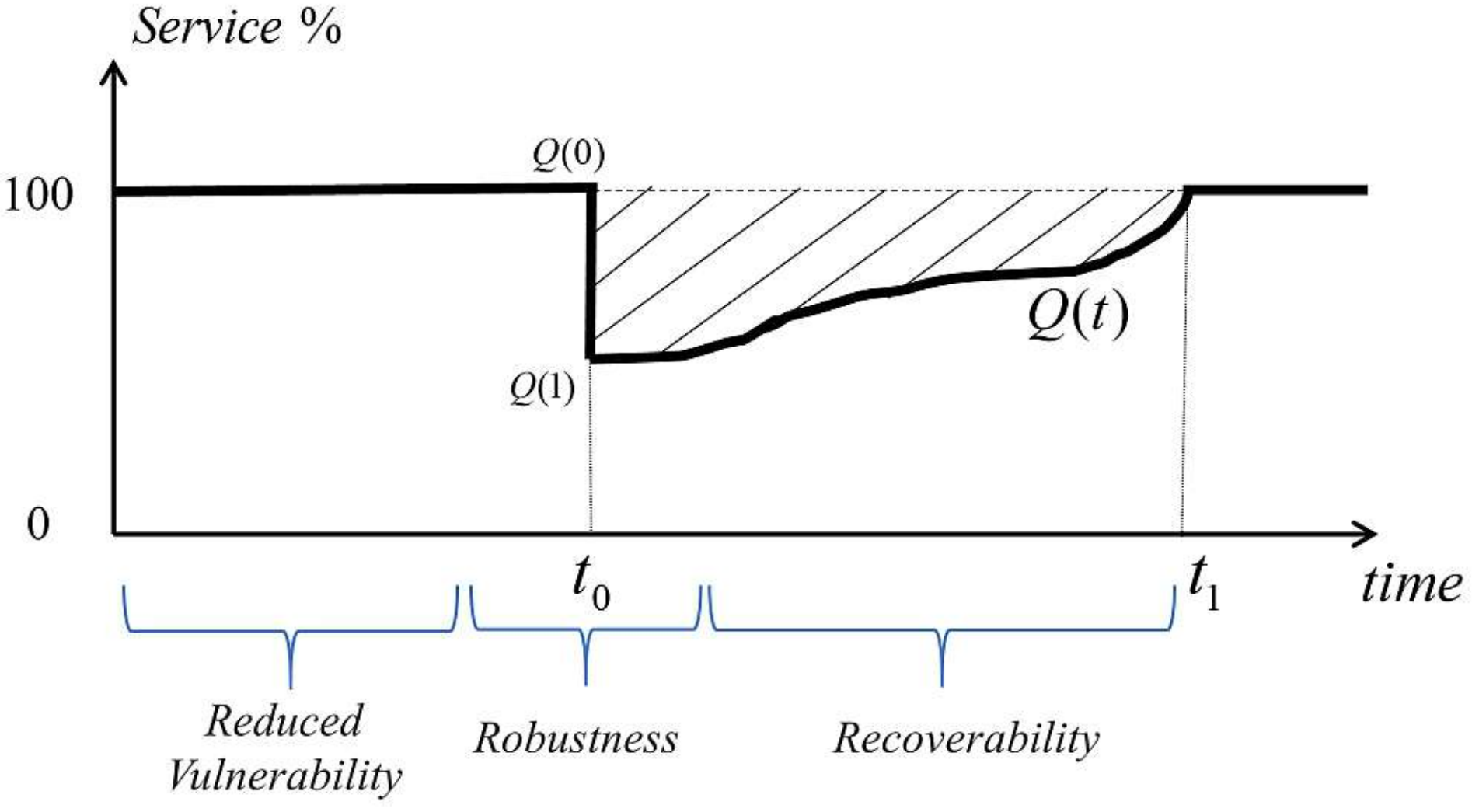

C
A
X1,t
1
X2,t
2
1
Yt
T
4

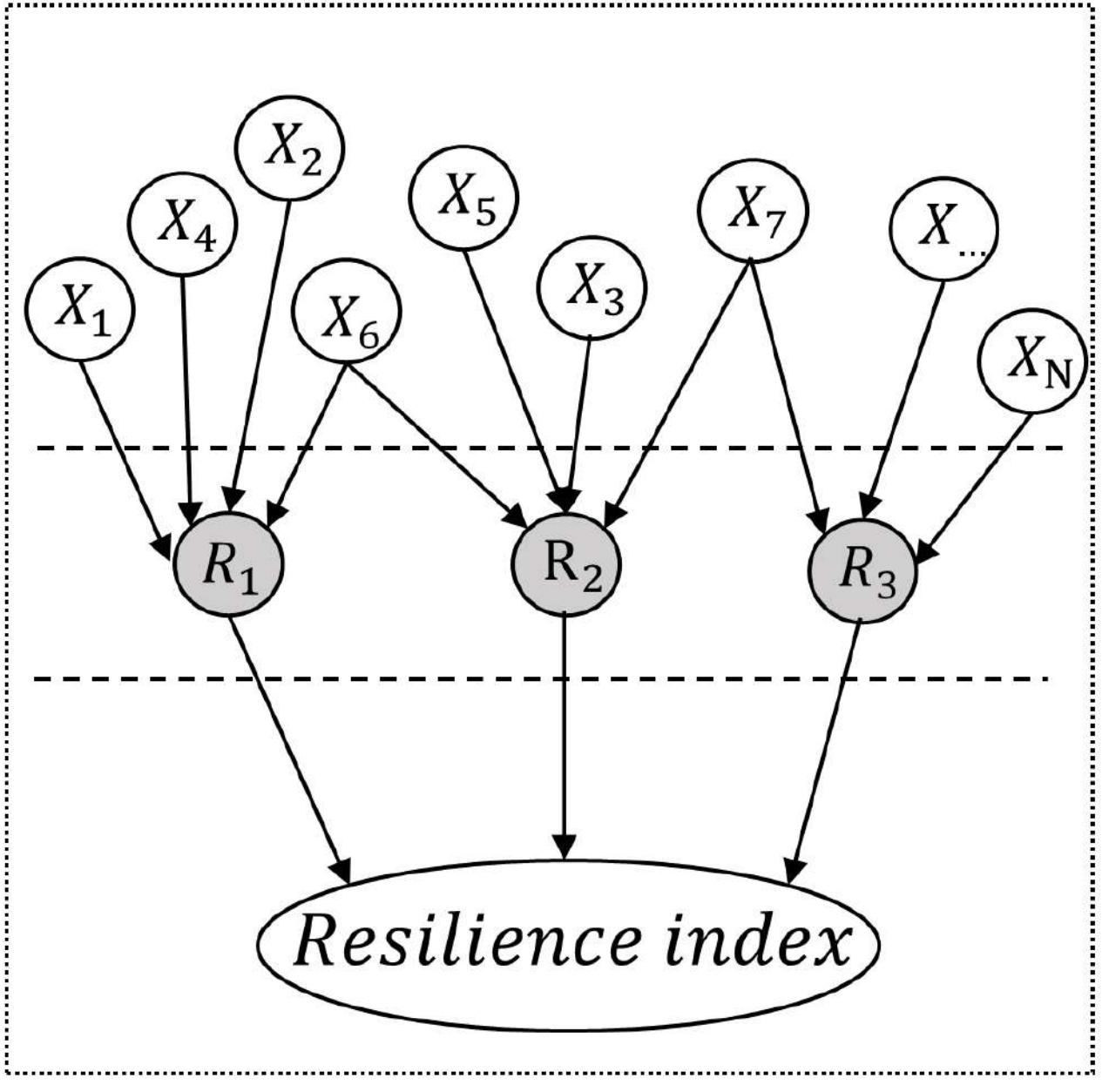
(b)


C
X1,1
X1,2
X1,3
X1,4
A
X2,1
X2,2
X2,3
X2,4
T
Y1
Y2
Y3
Y4

(c)

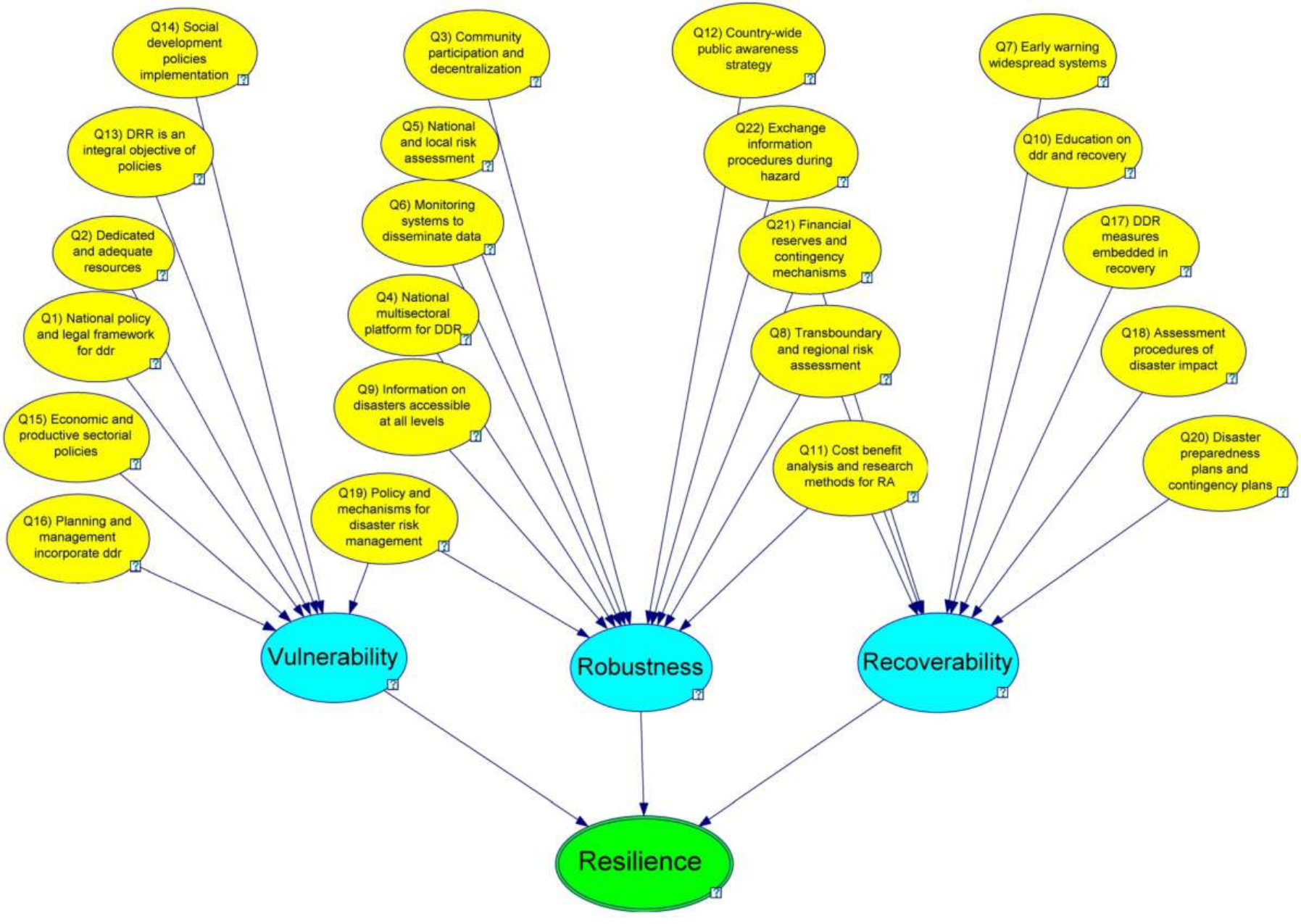

Reduced Vulnerability
Robustness
Resilience
Recoverability

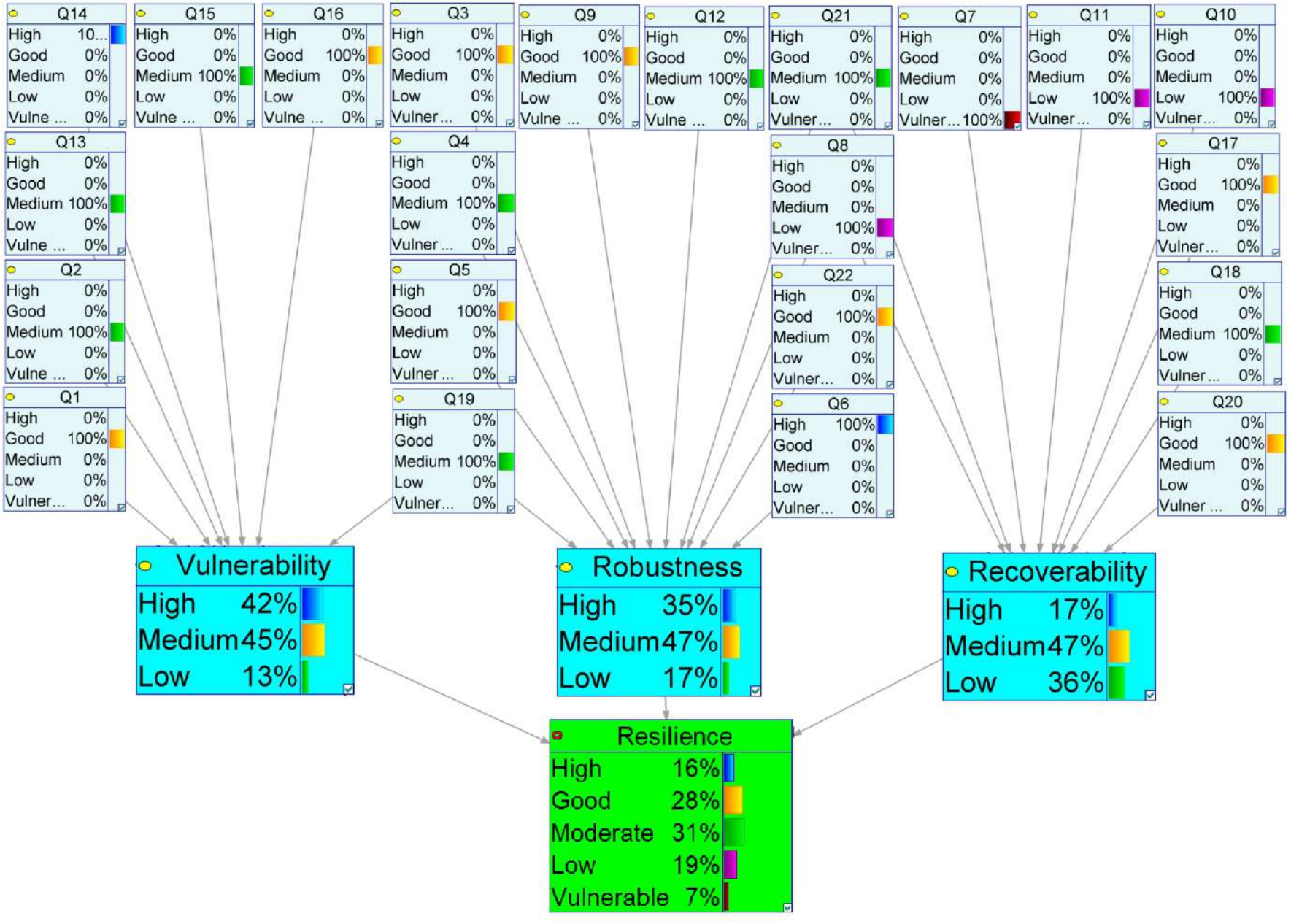

Service %
100
0
Q(0)
Q(1)
Q(t)
t0
t1
time
Reduced Vulnerability
Robustness
Recoverability



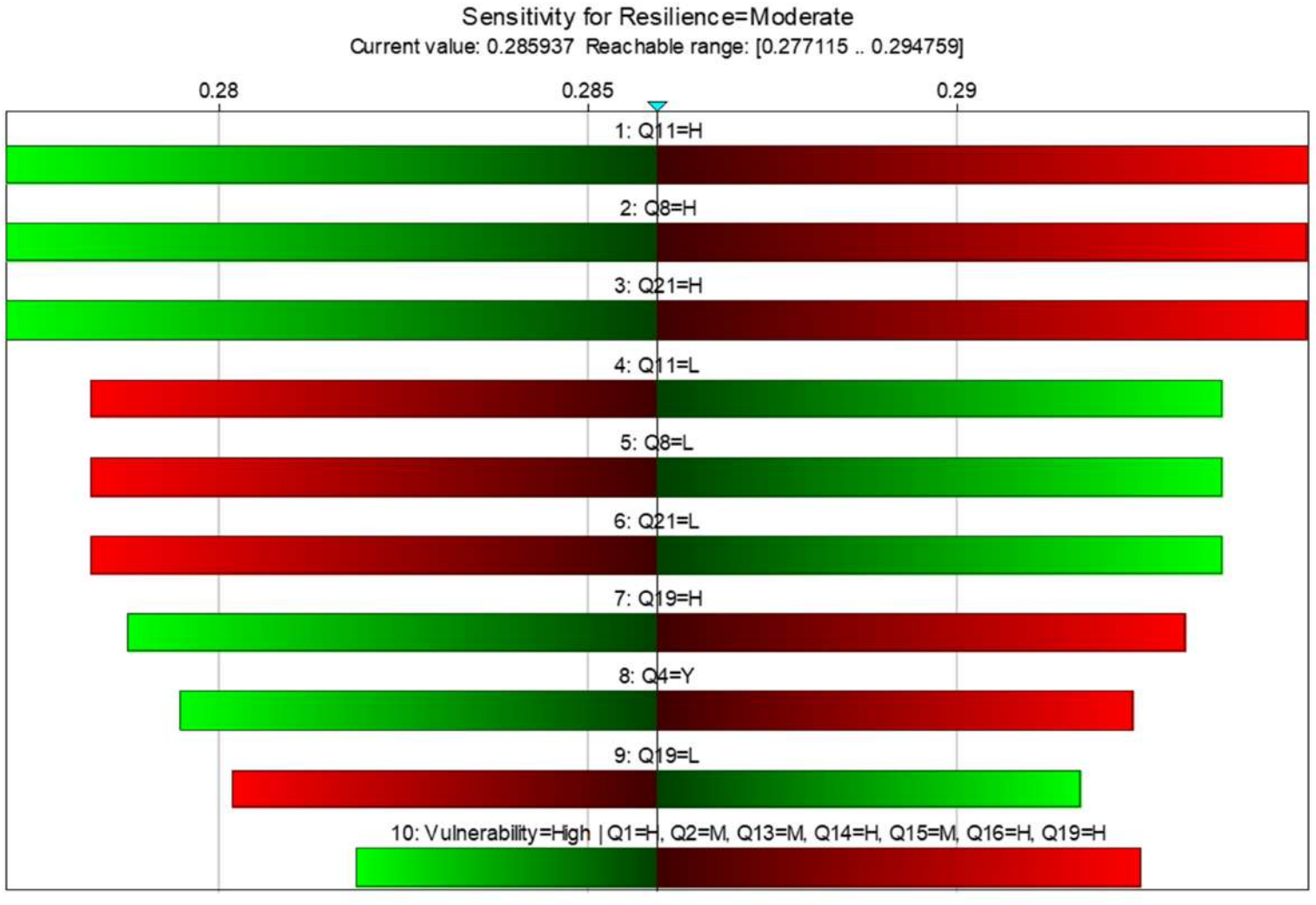
N Indicators

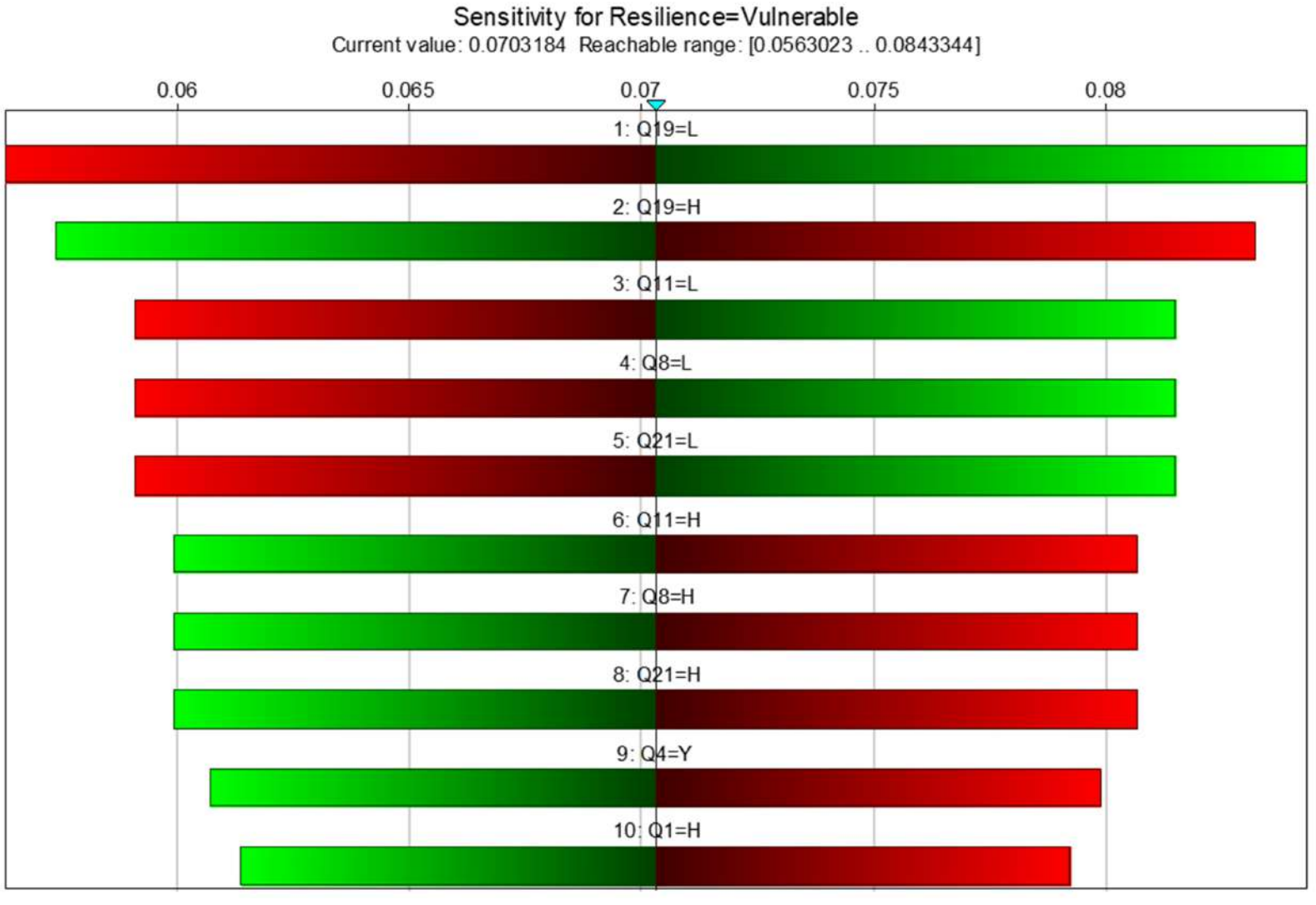
The three pillars of resilience

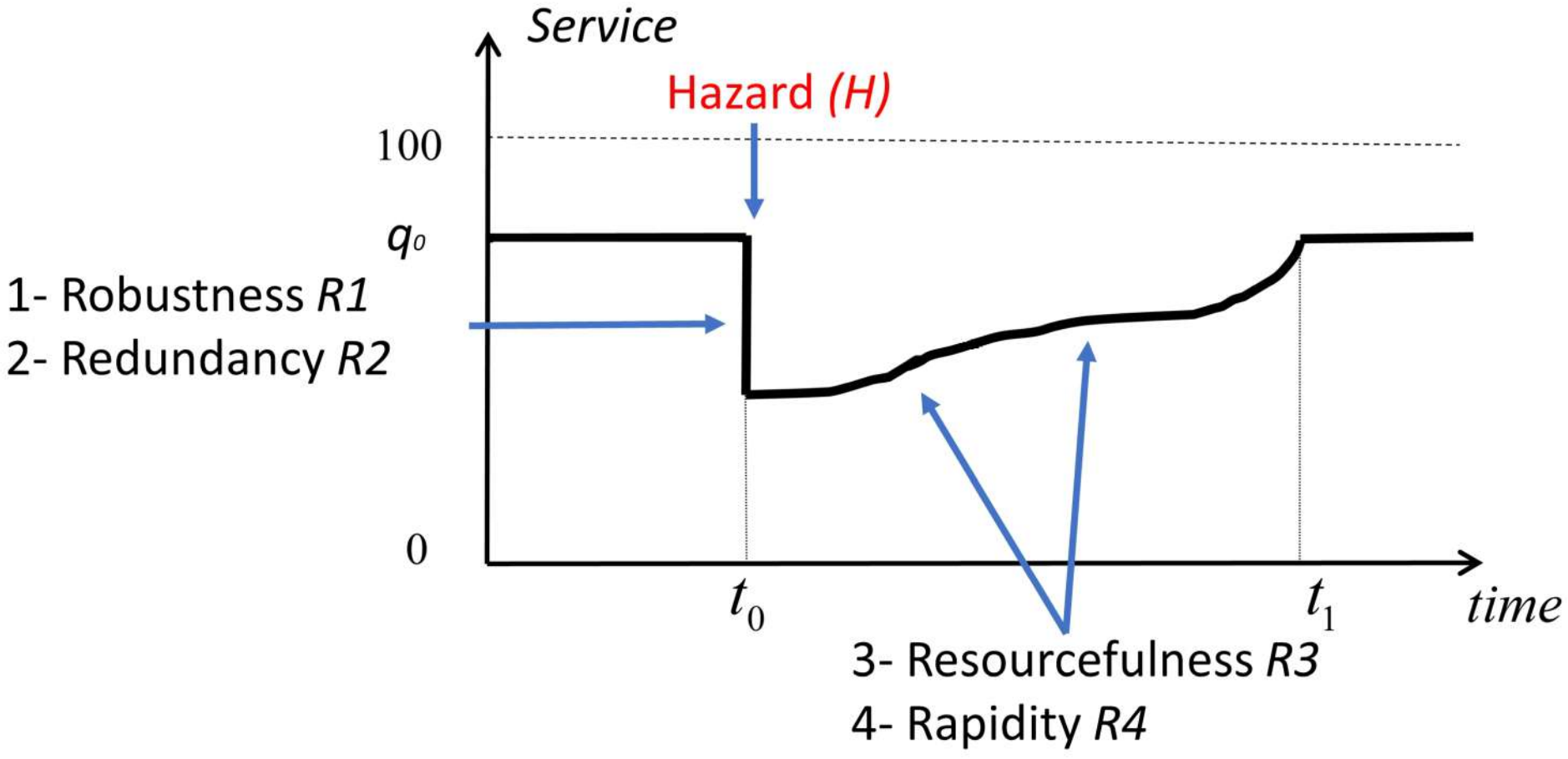

Q14) Social development policies implementation
Q13) DRR is an integral objective of policies
Q2) Dedicated and adequate resources
Q1) National policy and legal framework for ddr
Q15) Economic and productive sectorial policies
Q16) Planning and management incorporate ddr
Q3) Community participation and decentralization
Q5) National and local risk assessment
Q6) Monitoring systems to disseminate data
Q4) National multisectoral platform for DDR
Q9) Information on disasters accessible at all levels
Q19) Policy and mechanisms for disaster risk management
Q12) Country-wide public awareness strategy
Q22) Exchange information procedures during hazard
Q21) Financial reserves and contingency mechanisms
Q8) Transboundary and regional risk assessment
Q11) Cost benefit analysis and research methods for RA
Q7) Early warning widespread systems
Q10) Education on ddr and recovery
Q17) DDR measures embedded in recovery
Q18) Assessment procedures of disaster impact
Q20) Disaster preparedness plans and contingency plans
Vulnerability
Robustness
Recoverability
Resilience

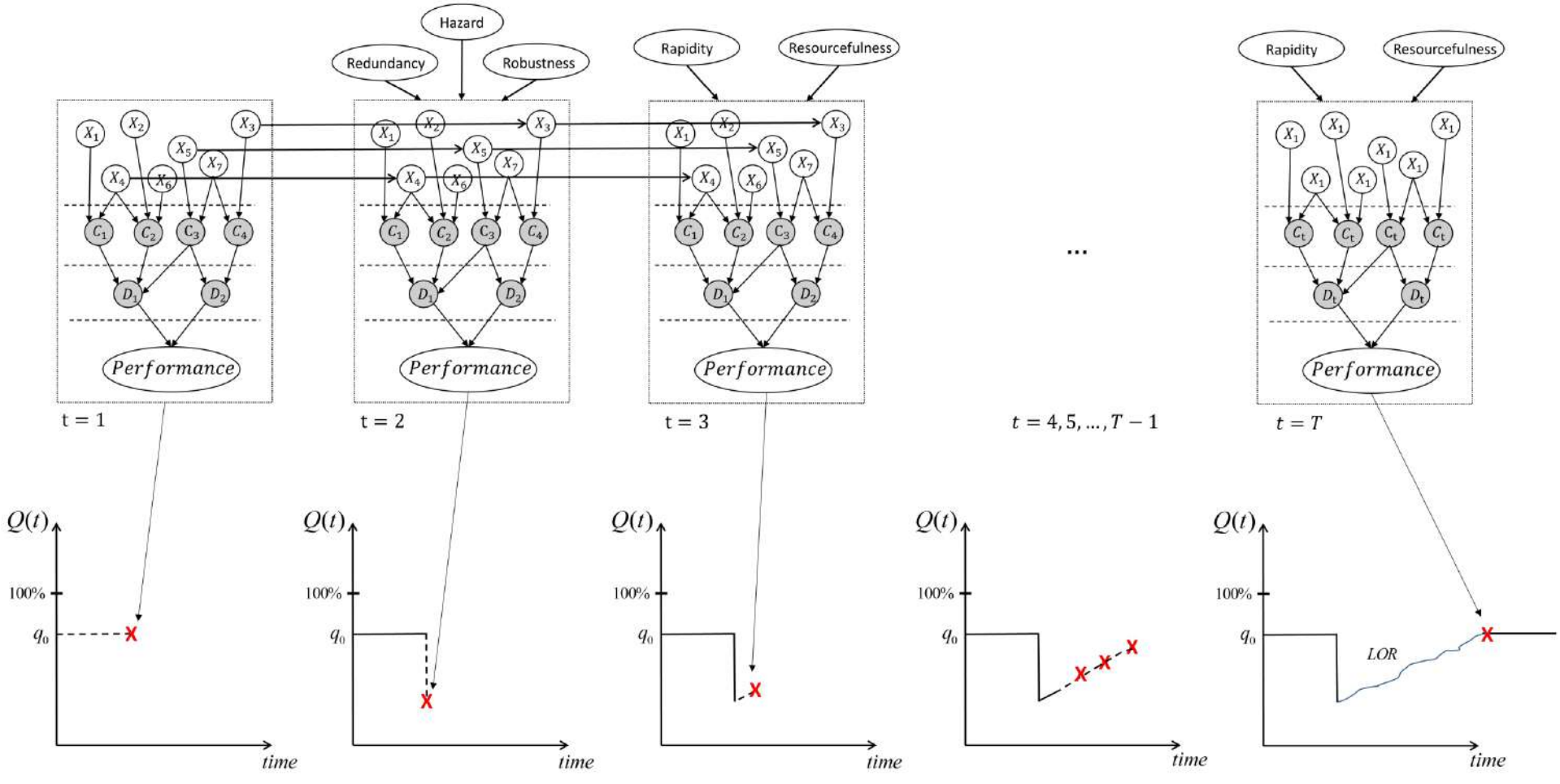

Q14
High 10...
Good 0%
Medium 0%
Low 0%
Vulne ... 0%
Q15
High 0%
Good 0%
Medium 100%
Low 0%
Vulne ... 0%
Q16
High 0%
Good 100%
Medium 0%
Low 0%
Vulne ... 0%
Q3
High 0%
Good 100%
Medium 0%
Low 0%
Vulner... 0%
Q9
High 0%
Good 100%
Medium 0%
Low 0%
Vulne ... 0%
Q12
High 0%
Good 0%
Medium 100%
Low 0%
Vulne ... 0%
Q21
High 0%
Good 0%
Medium 100%
Low 0%
Vulner... 0%
Q7
High 0%
Good 0%
Medium 0%
Low 0%
Vulner... 100%
Q11
High 0%
Good 0%
Medium 0%
Low 100%
Vulner... 0%
Q10
High 0%
Good 0%
Medium 0%
Low 100%
Vulner... 0%
Q13
High 0%
Good 0%
Medium 100%
Low 0%
Vulne ... 0%
Q4
High 0%
Good 0%
Medium 100%
Low 0%
Vulner... 0%
Q8
High 0%
Good 0%
Medium 0%
Low 100%
Vulner... 0%
Q17
High 0%
Good 100%
Medium 0%
Low 0%
Vulner... 0%
Q2
High 0%
Good 0%
Medium 100%
Low 0%
Vulne ... 0%
Q5
High 0%
Good 100%
Medium 0%
Low 0%
Vulner... 0%
Q22
High 0%
Good 100%
Medium 0%
Low 0%
Vulner... 0%
Q18
High 0%
Good 0%
Medium 100%
Low 0%
Vulner... 0%
Q1
High 0%
Good 100%
Medium 0%
Low 0%
Vulner... 0%
Q19
High 0%
Good 0%
Medium 100%
Low 0%
Vulner... 0%
Q6
High 100%
Good 0%
Medium 0%
Low 0%
Vulner... 0%
Q20
High 0%
Good 100%
Medium 0%
Low 0%
Vulner ... 0%
Vulnerability
High 42%
Medium 45%
Low 13%
Robustness
High 35%
Medium 47%
Low 17%
Recoverability
High 17%
Medium 47%
Low 36%
Resilience
High 16%
Good 28%
Moderate 31%
Low 19%
Vulnerable 7%

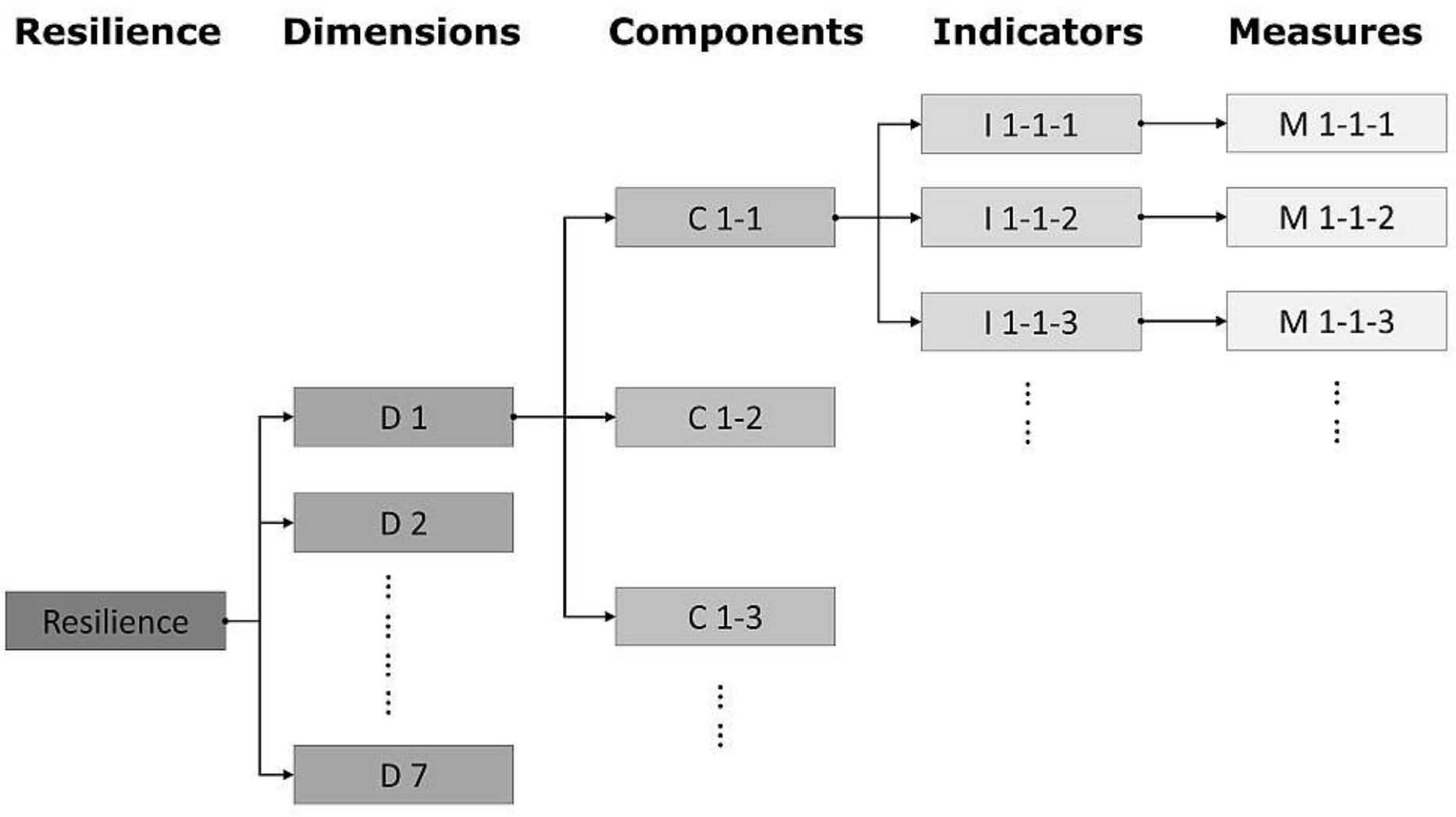
Sensitivity for Resilience=High
Current value: 0.190843 Reachable range: [0.165951 .. 0.215736]
0.17
0.18
0.19
0.2
0.21
1: Q19=H
2: Q11=H
3: Q8=H
4: Q21=H
5: Q19=L
6: Q11=L
7: Q8=L
8: Q21=L
9: Q4=Y
10: Q1=M

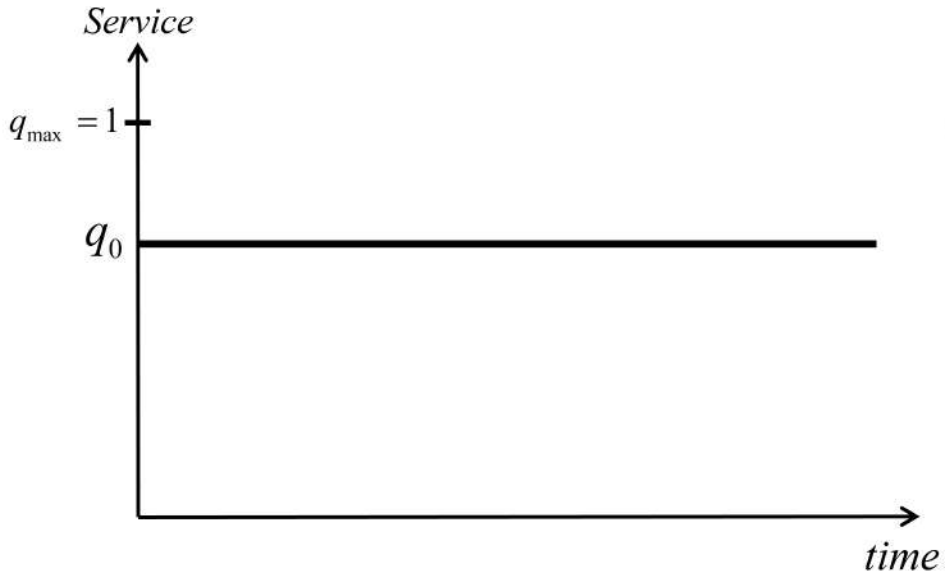

Sensitivity for Resilience=Moderate
Current value: 0.285937 Reachable range: [0.277115 .. 0.294759]
0.28
0.285
0.29
1: Q11=H
2: Q8=H
3: Q21=H
4: Q11=L
5: Q8=L
6: Q21=L
7: Q19=H
8: Q4=Y
9: Q19=L
10: Vulnerability=High | Q1=H, Q2=M, Q13=M, Q14=H, Q15=M, Q16=H, Q19=H


Sensitivity for Resilience=Vulnerable
Current value: 0.0703184 Reachable range: [0.0563023 .. 0.0843344]
0.06
0.065
0.07
0.075
0.08
1: Q19=L
2: Q19=H
3: Q11=L
4: Q8=L
5: Q21=L
6: Q11=H
7: Q8=H
8: Q21=H
9: Q4=Y
10: Q1=H

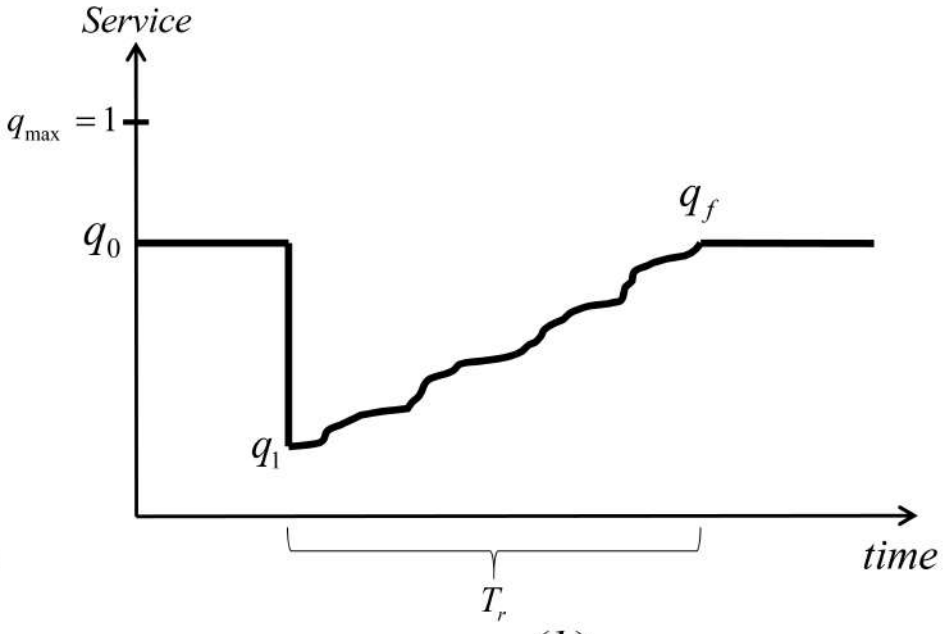

Service
Hazard (H)
100
q_0
0
t_0
t_1
time
1- Robustness R1
2- Redundancy R2
3- Resourcefulness R3
4- Rapidity R4


Hazard
Redundancy
Robustness
Rapidity
Resourcefulness
Rapidity
Resourcefulness
Performance
Performance
Performance
Performance
t = 1
t = 2
t = 3
t = 4, 5, … , T − 1
t = T
Q(t)
100%
q_0
time
LOR

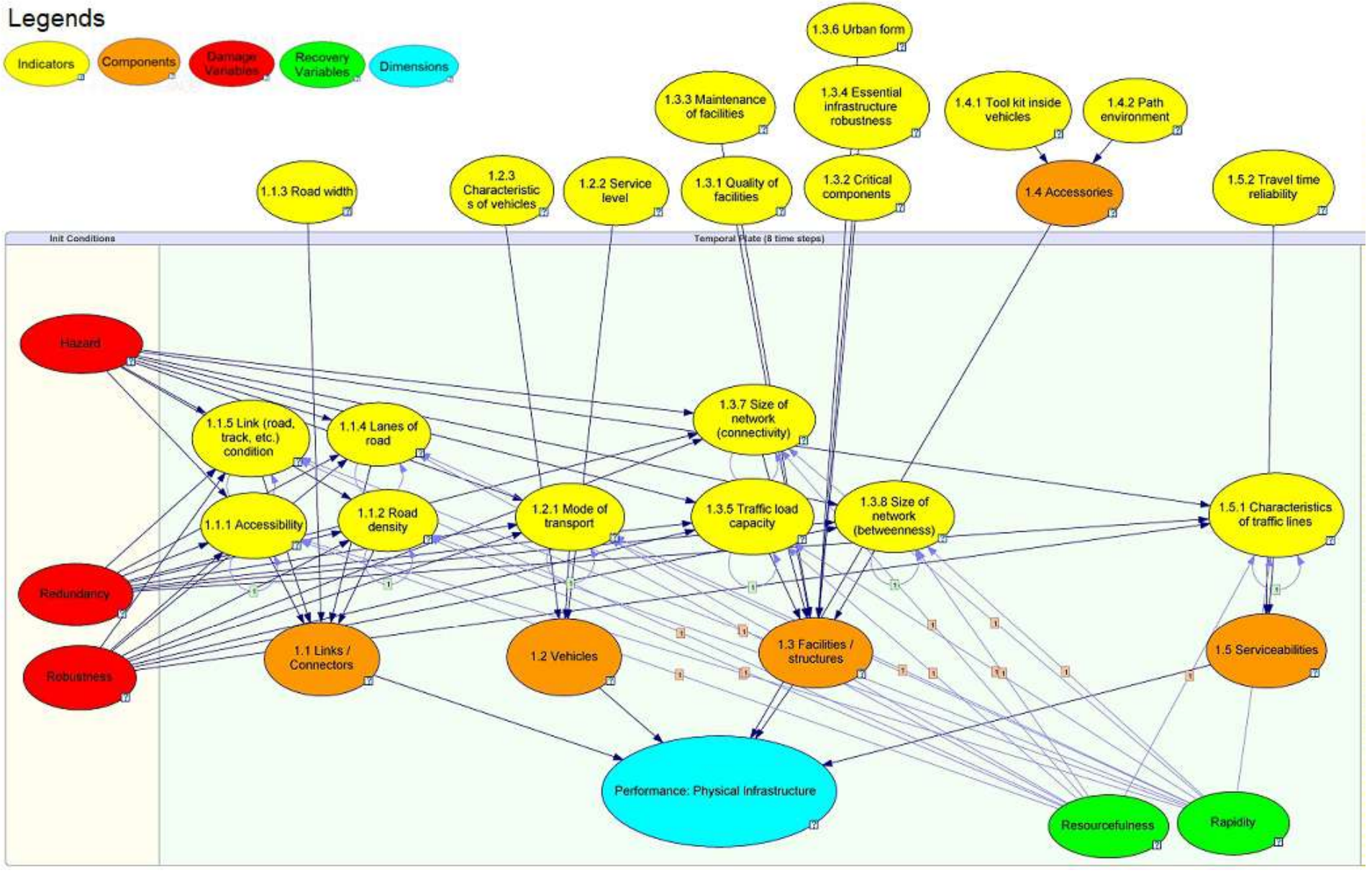

Resilience
Dimensions
Components
Indicators
Measures
Resilience
D 1
D 2
D 7
C 1-1
C 1-2
C 1-3
I 1-1-1
I 1-1-2
I 1-1-3
M 1-1-1
M 1-1-2
M 1-1-3

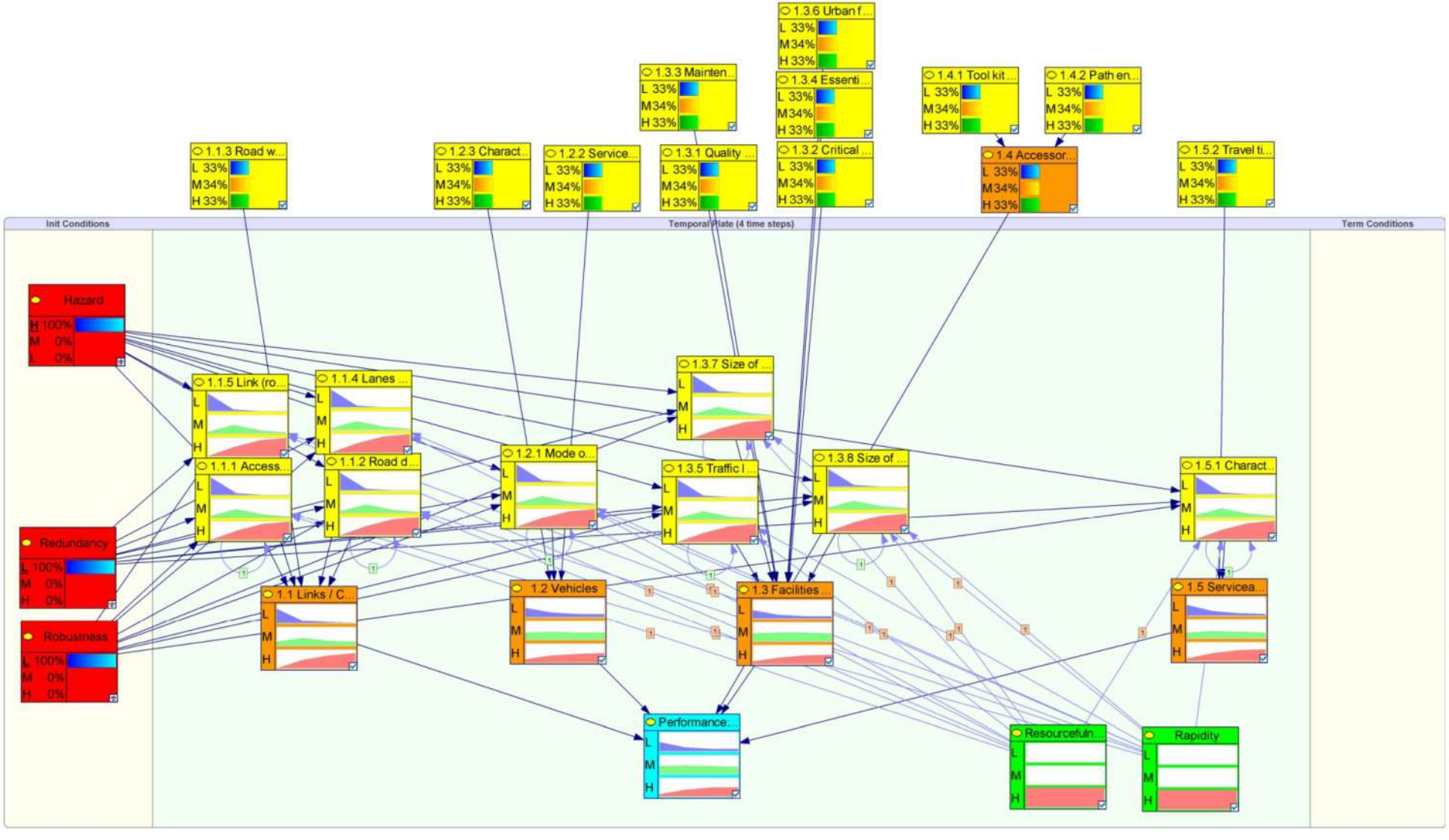


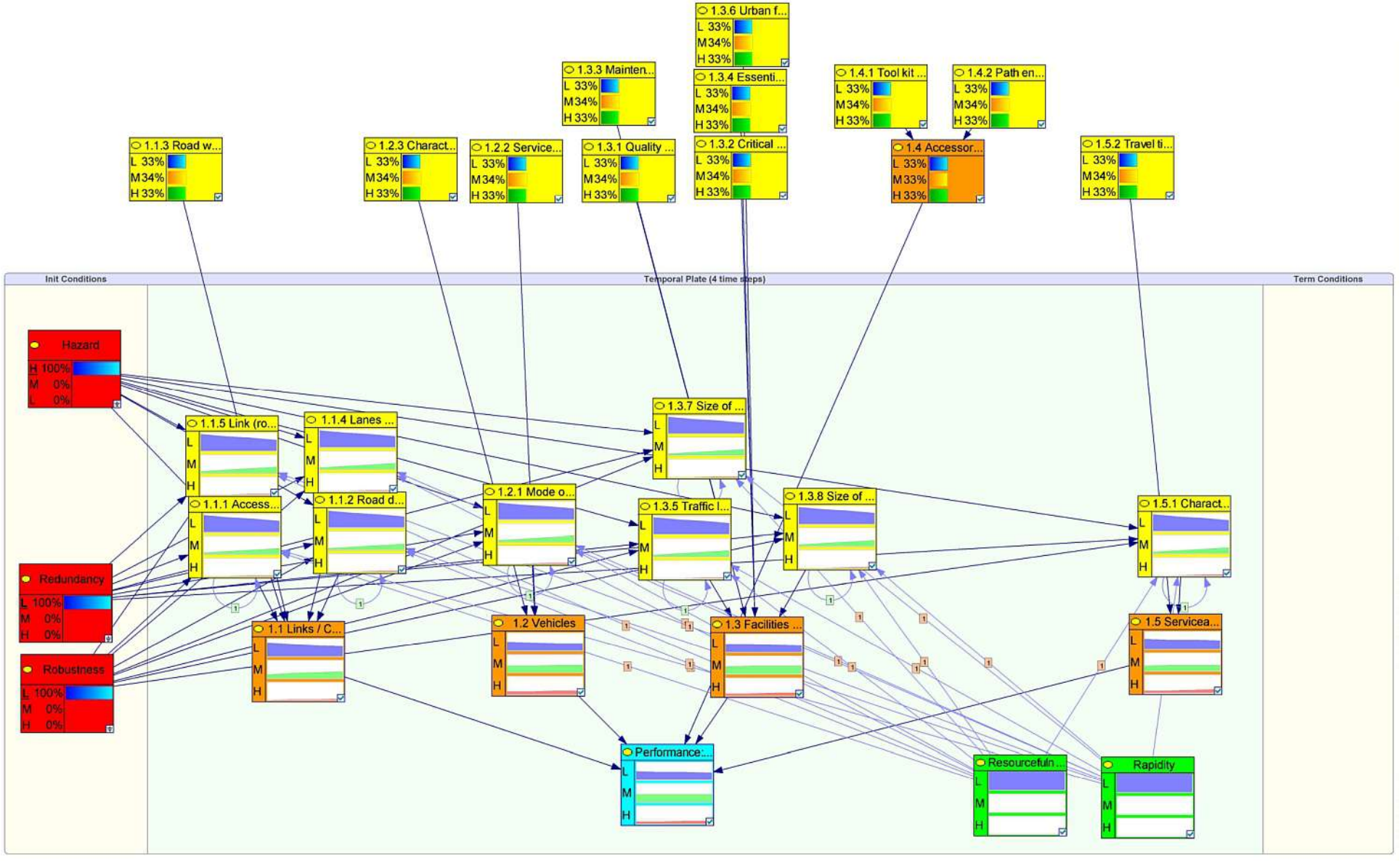
(a)

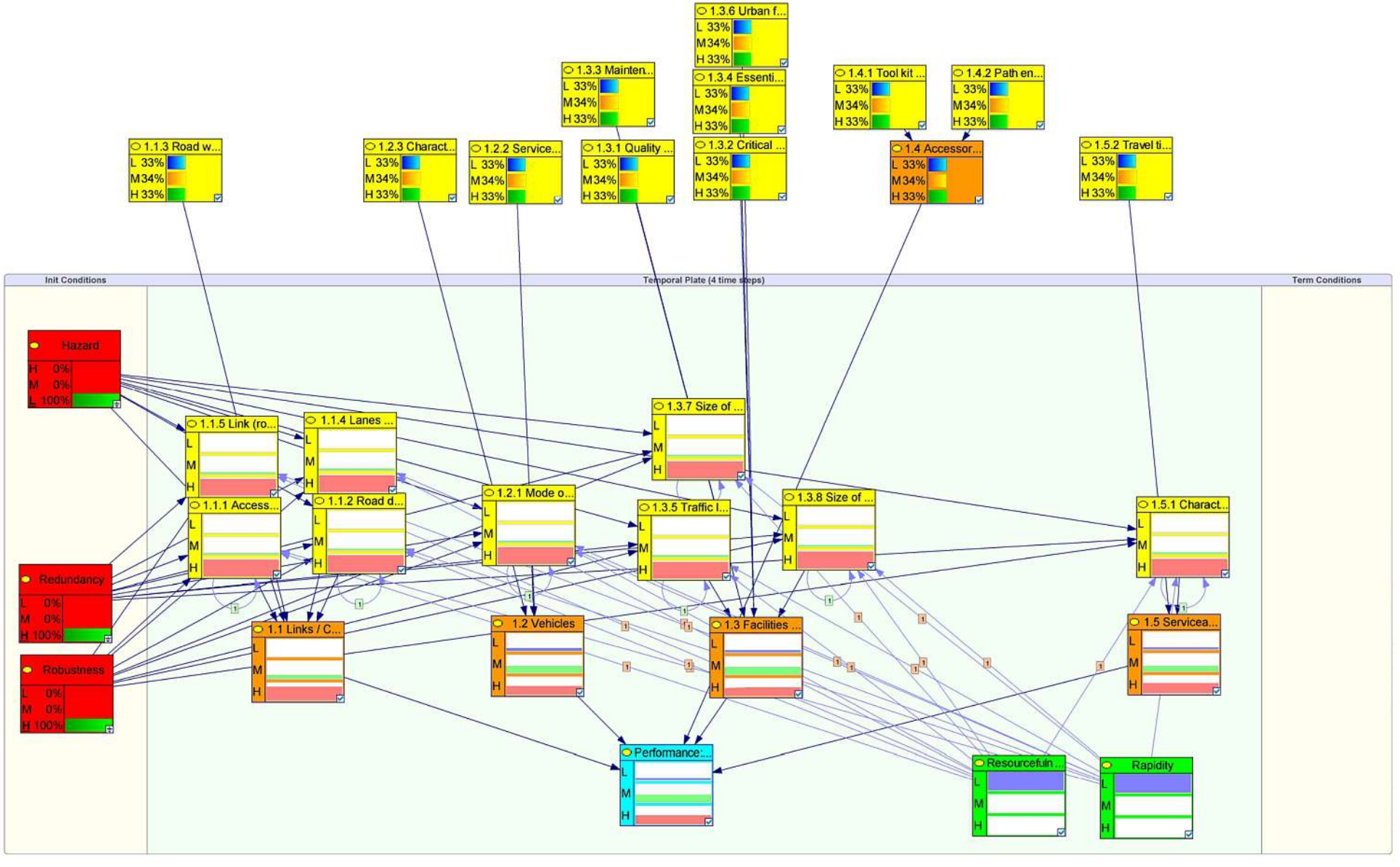


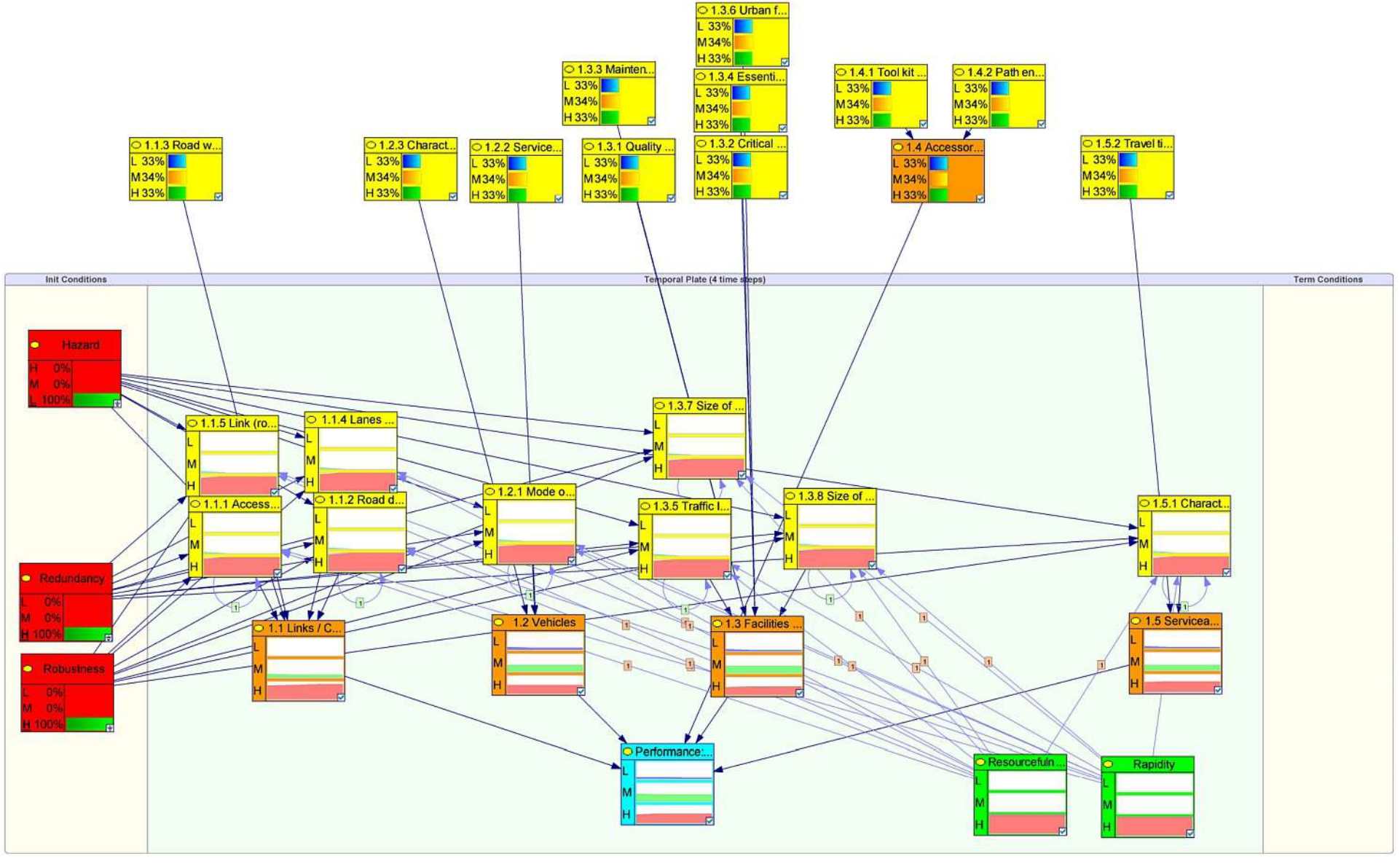
(b)


Legends
Indicators
Components
Damage Variables
Recovery Variables
Dimensions
1.3.6 Urban form
1.3.3 Maintenance of facilities
1.3.4 Essential infrastructure robustness
1.4.1 Tool kit inside vehicles
1.4.2 Path environment
1.1.3 Road width
1.2.3 Characteristics of vehicles
1.2.2 Service level
1.3.1 Quality of facilities
1.3.2 Critical components
1.4 Accessories
1.5.2 Travel time reliability
Init Conditions
Temporal Plate (8 time steps)
Hazard
1.1.5 Link (road, track, etc.) condition
1.1.4 Lanes of road
1.3.7 Size of network (connectivity)
1.1.1 Accessibility
1.1.2 Road density
1.2.1 Mode of transport
1.3.5 Traffic load capacity
1.3.8 Size of network (betweenness)
1.5.1 Characteristics of traffic lines
Redundancy
Robustness
1.1 Links / Connectors
1.2 Vehicles
1.3 Facilities / structures
1.5 Serviceabilities
Performance: Physical Infrastructure
Resourcefulness
Rapidity


Init Conditions
Temporal Plate (4 time steps)
Term Conditions
1.3.6 Urban f...
L 33%
M 34%
H 33%
1.3.3 Mainten...
1.3.4 Essenti...
1.4.1 Tool kit ...
1.4.2 Path en...
1.1.3 Road w...
1.2.3 Charact...
1.2.2 Service...
1.3.1 Quality ...
1.3.2 Critical ...
1.4 Accessor...
1.5.2 Travel ti...
Hazard
H 100%
M 0%
L 0%
1.1.5 Link (ro...
1.1.4 Lanes ...
1.3.7 Size of ...
1.1.1 Access...
1.1.2 Road d...
1.2.1 Mode o...
1.3.5 Traffic I...
1.3.8 Size of ...
1.5.1 Charact...
Redundancy
L 100%
M 0%
H 0%
1.1 Links / C...
1.2 Vehicles
1.3 Facilities ...
1.5 Servicea...
Robustness
L 100%
M 0%
H 0%
Performance...
Resourcefuln...
Rapidity


1.3.6 Urban f...
L 33%
M 34%
H 33%
1.3.3 Mainten...
1.3.4 Essenti...
1.4.1 Tool kit ...
1.4.2 Path en...
1.1.3 Road w...
1.2.3 Charact...
1.2.2 Service...
1.3.1 Quality ...
1.3.2 Critical ...
1.4 Accessor...
1.5.2 Travel ti...
Init Conditions
Temporal Plate (4 time steps)
Term Conditions
Hazard
H 100%
M 0%
L 0%
1.1.5 Link (ro...
1.1.4 Lanes ...
1.3.7 Size of ...
1.1.1 Access...
1.1.2 Road d...
1.2.1 Mode o...
1.3.5 Traffic l...
1.3.8 Size of ...
1.5.1 Charact...
Redundancy
L 100%
M 0%
H 0%
1.1 Links / C...
1.2 Vehicles
1.3 Facilities ...
1.5 Servicea...
Robustness
L 100%
M 0%
H 0%
Performance:...
Resourcefuln...
Rapidity


1.3.6 Urban f...
L 33%
M 34%
H 33%
1.3.3 Mainten...
1.3.4 Essenti...
1.4.1 Tool kit ...
1.4.2 Path en...
1.1.3 Road w...
1.2.3 Charact...
1.2.2 Service...
1.3.1 Quality ...
1.3.2 Critical ...
1.4 Accessor...
1.5.2 Travel ti...
Init Conditions
Temporal Plate (4 time steps)
Term Conditions
Hazard
H 0%
M 0%
L 100%
1.1.5 Link (ro...
1.1.4 Lanes ...
1.3.7 Size of ...
1.1.1 Access...
1.1.2 Road d...
1.2.1 Mode o...
1.3.5 Traffic I...
1.3.8 Size of ...
1.5.1 Charact...
Redundancy
L 0%
M 0%
H 100%
1.1 Links / C...
1.2 Vehicles
1.3 Facilities ...
1.5 Servicea...
Robustness
L 0%
M 0%
H 100%
Performance:...
Resourcefuln...
Rapidity
L
M
H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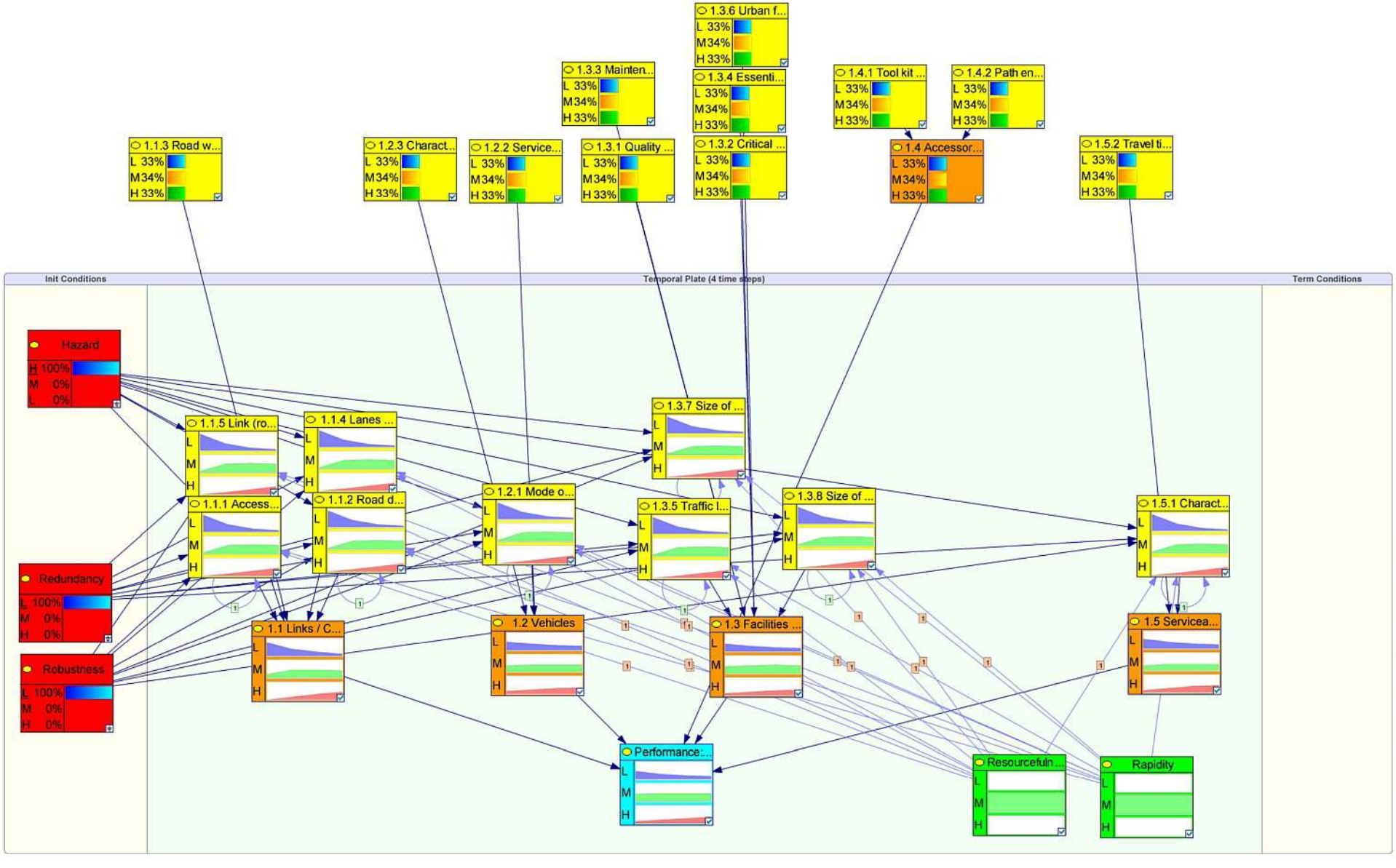
Robustness
L 0%
M 0%
H 100%
Performance:...
Resourcefuln ...
Rapidity


1.3.6 Urban f...
L 33%
M 34%
H 33%
1.3.3 Mainten...
1.3.4 Essenti...
1.4.1 Tool kit ...
1.4.2 Path en...
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Temporal Plate (4 time steps)
Term Conditions
Hazard
H 100%
M 0%
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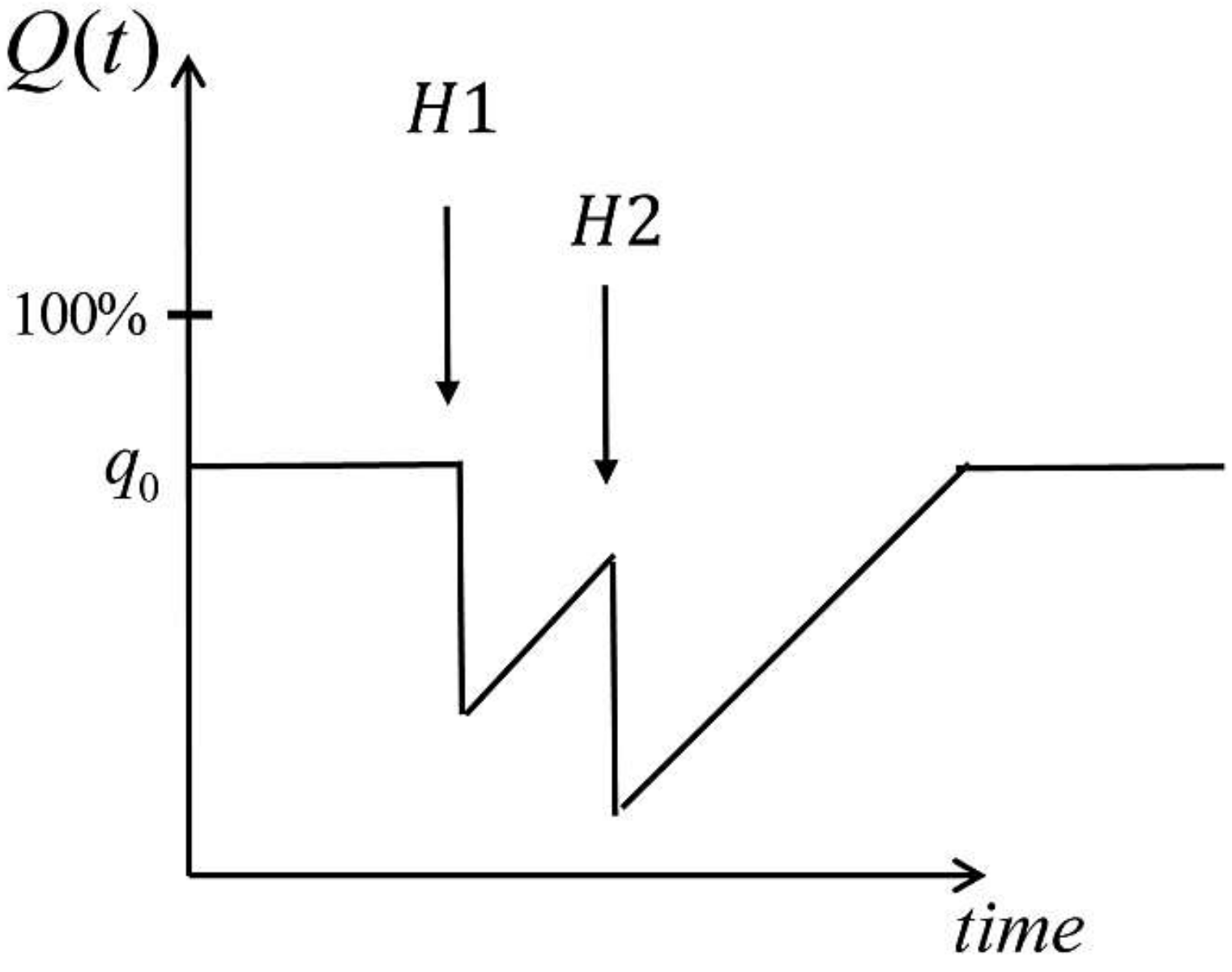
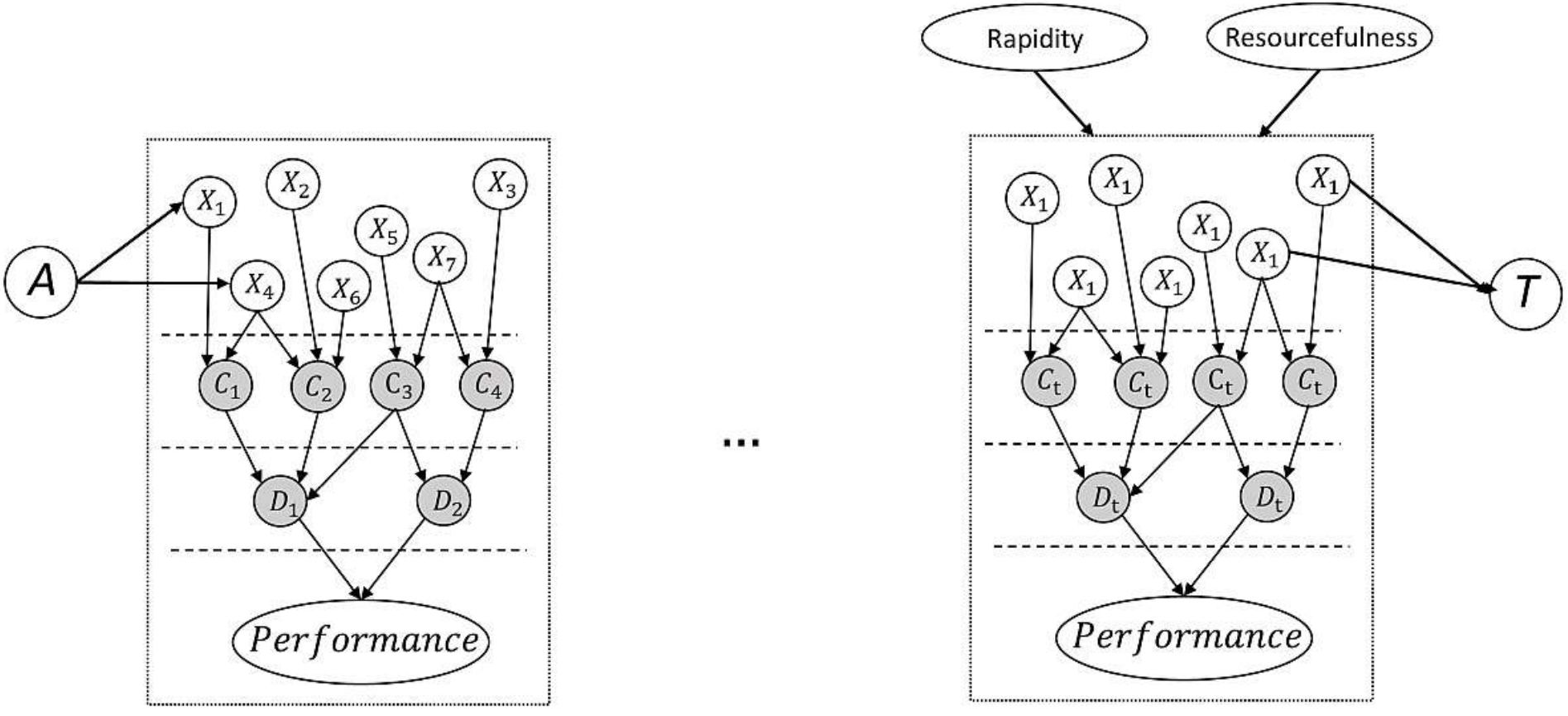
Redundancy
L 100%
M 0%
H 0%
1.1 Links / C...
1.2 Vehicles
1.3 Facilities ...
1.5 Servicea...
Robustness
L 100%
M 0%
H 0%
Performance:...
Resourcefuln...
Rapidity
L
M
H


Q(t)
H1
H2
100%
q0
time



$t = 1$ $t = 2, 3, \ldots, T - 1$ $t = T$