

Time-Dependent Probability of Exceeding a Target Level of Recovery

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# Time-Dependent Probability of Exceeding a Target Level of Recovery in Resilience Analysis

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## Abstract

The resilience of a system is generally defined in terms of its ability to withstand external perturbation(s), adapt, and rapidly recover. This paper introduces a probabilistic formulation to predict the recovery process of a system given past recovery data, and estimate the probability of reaching or exceeding a target value of functionality at any time. A Bayesian inference is used to capture the changes over time of model parameters as recovery data become available during the work progress. The proposed formulation is general and can be applied to continuous recovery processes such as those of economic or natural systems, as well as to discrete recovery processes typical of engineering systems. As an illustration of the proposed formulation, two examples are provided. The paper models the recovery of a reinforced concrete bridge following seismic damage, as well as the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori.

*Keywords:* Decision Support, Recovery, Resilience, Resilience Metrics, Probability, Reliability Analysis.

## 27        **Introduction**

28        Civil infrastructure enables the conveyance of goods, services, and resources to communities (Corotis  
29        2009; Ellingwood et al. 2016; Gardoni et al. 2016). Past disasters continue to show the vulnerability of  
30        civil infrastructure to natural and anthropogenic hazards and highlight the significance of risk mitigation  
31        and management (Murphy and Gardoni 2006; Gardoni et al. 2016). Buildings, bridges, and other  
32        structures and infrastructure may experience extreme natural events, such as floods, earthquakes,  
33        hurricanes, and anthropogenic hazards, such as accidents and terrorist attacks, which may lead to  
34        significant damage making infrastructure networks inoperative (Gardoni and LaFave 2016). Past  
35        disasters stressed the importance of being prepared and to be able to recover in a short period (e.g.,  
36        Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014).

37                The concept of resilience has gained relevance in the last fifteen years as a desirable feature for  
38        communities (Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014; Ellingwood et al.  
39        2016; Guidotti et al. 2016, 2017; Sharma et al. 2018; Gardoni 2018). The relatively recent interest in  
40        resilience has resulted in several definitions of the concept of resilience and several approaches to  
41        measuring resilience across several application domains. In general, resilience is defined as the ability  
42        of systems to recover after a disturbance to the pre-disturbance state or a new (improved) state (e.g.,  
43        Bruneau et al. 2003; Cimellaro et al. 2010a; Bocchini et al. 2012). The U.S. Presidential Policy Directive  
44        21 (PPD 21) defines resilience as *the ability to prepare for and adapt to changing conditions and*  
45        *withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover*  
46        *from deliberate attacks, accidents, or naturally occurring threats or incidents.* A review of the current  
47        state of the research in community resilience can be found in Koliou et al. (2018). Going beyond the  
48        engineering domain, Doorn et al. (2018) explored how philosophical and social science considerations  
49        can be incorporated into a multidisciplinary definition of resilience to account for social justice. The

50 choice of a defined recovery curve plays a key role in resilience analysis in terms of quantifying the  
51 resilience of a system. A recovery curve describes the behavior of a system as a function of time  
52 following the impact of a hazard as the system recovers to achieve a desired state (of functionality or of  
53 reliability.) In absence of disrupting shocks during the recovery phase, the recovery curve is, in general,  
54 a non-decreasing and time-dependent function. Different studies have attempted to model and define the  
55 recovery curve of engineering systems subject to a hazard (e.g., Cimellaro et al. 2010b; Decò et al. 2013;  
56 Titi et al. 2015). Recovery curves are usually assumed based on qualitative attributes, such as the  
57 preparedness of the society, that influence the recovery process. As such, they i) are not based on the  
58 actual physics of the recovery process, ii) do not account for the underlying uncertainties, and iii) are not  
59 able to incorporate additional information as it becomes available (such as ongoing progress of the work  
60 or increased resource availability, which affect the recovery models and reduce the uncertainty involved.)  
61 As a result, models of recovery typically only provide crude approximations and not accounting for the  
62 underplaying uncertainties makes it not possible to estimate the probability of reaching or exceeding a  
63 target percentile of interest of the ultimate desired state (e.g., a target value of functionality or reliability).  
64 To overcome these limitations, Sharma et al. (2018) proposed a mathematical formulation for resilience  
65 analysis that models the recovery curves based on the actual work plan of activities involved in the  
66 recovery process.

67 Once a recovery curve is defined, there is a need to define a metric or a set of metrics of recovery  
68 that distinctively characterize the recovery curve. A typical resilience metric has been defined as the  
69 integral of the recovery curve over a specified interval of time (Bruneau and Reihorn 2007; Cimellaro  
70 et al. 2010a; Bonstrom and Corotis 2016). However, such metrics do not uniquely and fully characterize  
71 a recovery curve. Sharma et al. (2018) defined a set of resilience metrics in analogy with the moments  
72 of a random variable to quantify the resilience of a system. The Sharma et al.'s metrics i) are intuitive

73 because of their analogy with the moments of a random variable, and ii) define a complete set of partial  
74 descriptors that uniquely and fully characterize a recovery curve.

75 This paper contributes to the literature in resilience analysis. In particular, this paper proposes a  
76 probabilistic formulation to predict a recovery process of a system, and then estimate the probability of  
77 reaching or exceeding a target value of functionality (or reliability) of the system at any given time as  
78 the system recovers. The proposed formulation uses Sharma et al.'s resilience metrics obtained from  
79 historical recovery data to predict possible recovery processes along with their likelihood, as well as to  
80 estimate the probability of reaching or exceeding a desired level of recovery by a desired time. The  
81 proposed formulation can be applied to systems in different fields, i.e., economical, natural, and  
82 engineering systems. The proposed formulation first defines the joint probability density function (PDF)  
83 of resilience metrics that captures the underlying uncertainties. Then, parametrized recovery curves are  
84 introduced to model the time-varying recovery process, and the joint PDF of model parameters is  
85 obtained as a function of the joint PDF of the resilience metrics. The joint PDF of the model parameters  
86 defines the variability in the possible recovery curves, which is used to estimate the probability of  
87 reaching or exceeding a target value of functionality by conducting a reliability analysis (Ditlevsen and  
88 Madsen 1996; Gardoni 2017). A Bayesian inference is also proposed to include possible information  
89 from the field while the work for the recovery is in progress. We use field data to update the predicted  
90 recovery curve such that the recovery curve is updated to reflect the advancement of the actual recovery  
91 in the field. Thanks to the Bayesian updating, the uncertainties in the recovery process diminish while  
92 more data become available. The main benefits of the proposed formulation are that the estimates of the  
93 recovery curve can be simply defined as a function of resilience metrics and the modeling can take  
94 advantage of data collected both before and during the recovery process. We illustrate the proposed  
95 formulation considering the recovery of a typical reinforced concrete (RC) bridge following a seismic

96 damage, and the population relocation after the occurrence of a seismic event when no data on the  
97 duration of the recovery are available a priori.

98 Following this introduction, Section 2 gives a brief review of mathematical formulations for  
99 resilience analysis. Section 3 presents the proposed probabilistic formulation. Section 4 illustrates the  
100 proposed formulation considering the recovery of an example bridge following a seismic damage.  
101 Finally, Section 5 uses the propose approach considering the population relocation after a seismic event.

## 102 **Review of Mathematical Formulations for Resilience Analysis**

103 The resilience analysis of engineering systems plays a key role in mitigation planning and allocation of  
104 resources in pre- and post-disruption scenarios (Ellingwood et al. 2016). Resilience of a system is, in  
105 general, defined as its ability to maintain or promptly resume a level of functionality or performance after  
106 a disruption. What is promptly enough is usually defined based on the owner's, customers' or, more  
107 generally, societal needs. A performance measure (e.g., the system functionality), typically indicated as  
108  $Q(t)$ , can be used to describe the system state as a function of time  $t$  (Cimellaro et al. 2010a; Bocchini  
109 et al. 2012; Bonstrom and Corotis 2016; Sharma et al. 2018). An external shock, such as a natural or  
110 anthropogenic event (a shock), might reduce  $Q(t)$  instantaneously. Such reduction is typically a function  
111 of the intensity of the shock, the system design specifications (which define the system robustness at  
112  $t = 0$ , e.g., Bai et al. 2009), and the system state immediately before the shock (which reflect the  
113 deterioration of a system over time and also define the system robustness at time  $t$ , e.g., Kumar and  
114 Gardoni 2014; Kumar et al. 2015; Jia and Gardoni 2018a,b). After a shock, the recovery process starts  
115 to retrieve the system functionality to a desired level, which may be below, same or better than the pre-  
116 disruption value (Ayyub 2014, 2015).

117 Resilience, independently from the field of application, consists of four properties (Bruneau et al.  
118 2003; Tierney and Bruneau 2007): 1) robustness as the ability to withstand a given level of stress or  
119 demand without suffering degradation or loss of function or, if a degradation occurs, the residual level  
120 of  $Q(t)$ ; 2) resourcefulness, interrelated to the ability to diagnose and prioritize issues and to initiate  
121 solutions by identifying and monitoring all resources; 3) redundancy, defined as the extent to which the  
122 system and other elements satisfy and sustain functional requirements in the event of disturbance; and 4)  
123 rapidity as the ability to recover in a timely manner to limit losses and avoid future disruptions. These  
124 four properties define the resilience of a system and characterize the recovery process.

125 Recovery curves capture the changes in system functionality over time and defined how the system  
126 state improves to achieve a desired value of functionality at the end of the recovery process. Different  
127 studies have attempted to quantify the resilience of a system based on the shape of the recovery curves.  
128 As a first attempt to quantify the resilience of a system, Bruneau et al. (2003) proposed to measure the  
129 resilience as the area underneath the recovery curve. Chang and Shinozuka (2004) assessed the resilience  
130 as the probability that the time needed for the recovery due to a performance loss after a disruption would  
131 be less than a predefined threshold. Garbin (2007) outlined an approach to quantitatively measure the  
132 resilience of a network as the percentage of links damaged and the percentage of nodes damaged versus  
133 a network performance measure. Bruneau and Reinhorn (2007) proposed metrics for measuring  
134 resiliency based on the expected degradation in the quality of an infrastructure by quantifying robustness,  
135 redundancy, resourcefulness, and rapidity to recovery. While these contributions show the importance  
136 of quantifying resilience in an objective and formal way, the metrics they define only provide partial  
137 information about the actual resilience and might not be able to distinguish among different resilience  
138 levels (as noted in Sharma et al. 2018). Uniquely and fully characterizing the resilience of the system

139 requires capturing all of the relevant characteristics of the recovery curve. Consequently, a single metric  
140 cannot represent a curve and capture all of its attributes.

141 Sharma et al. (2018) showed that the existing metrics are not able to uniquely and fully characterize  
142 recovery curves with different shapes and might not be able to capture the difference in the resilience  
143 levels. To address this issue, they developed a complete set of resilience metrics able to fully describe  
144 the recovery process and capture the differences in the shapes of different recovery curves. Sharma et  
145 al.'s resilience metrics are analogous to the partial descriptors commonly adopted in probability and  
146 statistics (e.g., mean, standard deviation and higher moments of a random variable.) The recovery curve  
147  $Q(t)$ , which Sharma et al. (2018) call the *cumulative resilience function* (CRF) in analogy with the  
148 *cumulative distribution function* (CDF) of a random variable, represents the overall recovery process as  
149 a function of time. If the CRF is a continuous and differentiable function of the time, it is possible to  
150 describe the instantaneous rate of recovery as the *resilience density function* (RDF)  $q$  defined as the time  
151 derivative of the CRF (in analogy with the definition of the *probability density function* (PDF) of a  
152 random variable). If the CRF is not continuous and differentiable, it is possible to define a *resilience*  
153 *mass function* (RMF) that describes the instantaneous change of the recovery occurring as a step-wise  
154 function (in analogy with the probability mass function (PMF) of a random variable).

155 Based on these definitions, Sharma et al. (2018) introduced a set of resilience metrics to capture the  
156 specific characteristics of the recovery process in analogy to the moments of random variables. In  
157 analogy to the mean and standard deviation of a random variables, Sharma et al. (2018) defined the *center*  
158 *of resilience*  $\rho$  and the *resilience bandwidth*,  $\chi$  as two fundamental partial descriptions. The definition  
159 of these metrics is general and can be systematically extended to higher order metrics to fully characterize any  
160  $Q(t)$ . The metric  $\rho$  defines where the recovery curve is centered with respect to the time of the initial



161 shock. In addition, Sharma et al. (2018) also introduced the resilience quantile,  $\rho_\omega$ , which is the time instant  
162 corresponding to the  $\omega^{th}$  ( $0 \leq \omega \leq 1$ ) quantile of the CRF. Mathematically, the recovery quantile can be  
163 written as  $\rho_\omega := \min\{t \in [0, T_R] : \omega \leq [Q(t)/Q(T_R)]\}$ , where  $T_R$  is the recovery time (i.e., the time needed  
164 to reach a desired final level of  $Q(t)$ .) The metric  $\chi$  gives the breath of the recovery process, small  
165 values represent a situation in which a significant percentage of the recovery process is completed over  
166 a short period concentrated around  $\rho$ . By contrast, a large value of  $\chi$  captures a recovery process  
167 spread over a prolonged period of time. To further characterize the recovery curve, Sharma et al. (2018)  
168 also introduced the skewness of the recovery,  $\psi$ . If  $\psi = 0$  the recovery progress is symmetric about  $\rho$   
169 (i.e., the recovery process has the same pace before and after  $\rho$ .) If  $\psi < 0$ , the process is slower during  
170 the initial phases (i.e., in the interval  $[0, \rho]$ ) and then it becomes faster over the next period  $(\rho, T_R]$ ,  
171 which is the most typical case for recovery processes that include a lengthy planning phase in the post-  
172 disruption period. If planning is done ahead of the disruptive event as a pre-disruption planning and  
173 preparation, then  $\psi > 0$ . In this case, the recovery progress picks up quickly and the relative most time-  
174 consuming portion is the completing of the repairs/reconstruction (i.e., faster in the interval  $[0, \rho]$ , and  
175 slower in the interval,  $(\rho, T_R]$ ). Finally, to uniquely and fully characterize the recovery curve, Sharma  
176 et al. (2018) also introduced higher order partial descriptors (in analogy with higher order moments or a  
177 random variable). However, in most cases,  $\rho$  and  $\chi$  are sufficient to characterize a recovery process.  
178 Based on Sharma et al. (2018), we can write the center of resilience as

$$\rho := \frac{\int_0^{T_R} \tau q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (1)$$

179 Likewise, we can write the resilience bandwidth as

$$\chi^2 := \frac{\int_0^{T_R} (\tau - \rho)^2 q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (2)$$

180 Finally, as a generalization, the  $n^{\text{th}}$  recovery moment can be written as

$$\rho^{(n)} := \frac{\int_0^{T_R} \tau^n q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (3)$$

181

## 182 **Proposed Probabilistic Formulation**

183 This section explains the proposed probabilistic formulation to develop recovery curves accounting for  
 184 the relevant uncertainties and estimate the probability of reaching or exceeding a target level of  
 185 functionality at any time.

186 Work progress for civil structures and infrastructure typically advances continuously, or near-  
 187 continuously, over time (Klinger and Susong 2006; Gardoni et al. 2007), whereas, the system state  
 188 changes only at completion of a group of activities (Sharma et al. 2018). As a result, the functionality of  
 189 a system typically changes in a step-wise fashion with discrete increments at the completion of each  
 190 group of activities. Besides civil structures and infrastructure, or more generally, engineering systems,

191 the recovery might be a continuous function of time when we deal with the restoration of natural systems,  
192 such as the recovery and resilience of tropical forests (Cole et al. 2014;. van Leeuwen 2008), or the  
193 Gross Domestic Product (GDP) as a monetary measure of the market value of all final goods and  
194 services produced in a period to quantify the economic performance of a whole country or region.  
195 The proposed methodology is general and allows to estimate processes described either by discrete or  
196 continuous recovery curves. The proposed formulation has the following four steps: Step 1: Obtaining  
197 the joint PDF of the Sharma et al.'s resilience metrics, Step 2: Obtaining the joint PDF of the model  
198 parameters of the recovery curve, Step 3: Obtaining point and predictive estimates of the recovery curve  
199 and confidence bounds, Step 4: Estimating the probability of reaching or exceeding a target percentile of  
200 interest of the ultimate desired state, and Step 5: Updating the model parameters as new data become  
201 available.

## 202 **Obtaining the joint PDF of the resilience metrics**

203 The first step of the proposed formulation consists in collecting historical recovery data for the system  
204 of interest and with them obtaining estimates of the statistics (means, standard deviations and  
205 correlation coefficients) and marginal PDFs of Sharma et al.'s resilience metrics (reviewed in Section  
206 2). Based on the obtained statistics and marginal PDFs, we can then construct the joint PDF of the  
207 resilience metrics using a Nataf formulation (Liu and Der Kiureghian 1986). Let  $f_p(\rho)$ ,  $f_x(\chi)$ , up  
208 to  $f_{p^{(n)}}(\rho^{(n)})$  be the marginal PDFs of Sharma et al.'s resilience metrics, and let  $r_{ij}$  be the estimated  
209 correlation coefficients between the  $i^{th}$  and the  $j^{th}$  resilience metric. Following the Nataf  
210 formulation, the joint PDF of the resilience metrics is

$$f_{\mathbf{p}}(\mathbf{p}) = f_p(\rho) f_x(\chi) \dots f_{p^{(n)}}(\rho^{(n)}) \frac{\varphi_n(\mathbf{z}, \mathbf{R}')}{\varphi(z_1) \varphi(z_2) \dots \varphi(z_n)} \quad (4)$$

211 where  $z_i = \Phi^{-1}[F_p(\rho_i)]$ ,  $\varphi(\cdot)$  is the standard normal PDF,  $\varphi_n(\mathbf{z}, \mathbf{R}')$  is the n-dimensional standard  
 212 normal PDF with correlation matrix  $\mathbf{R}'$ . The elements  $r_{ij}'$  in the correlation matrix  $\mathbf{R}'$  are obtained  
 213 based on the correlation coefficients  $r_{ij}$  through the integral

$$\begin{aligned} r_{ij} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{\rho^{(i)} - \mu_i}{\sigma_i} \right) \left( \frac{\rho^{(j)} - \mu_j}{\sigma_j} \right) f_{\rho^{(i)}}(\rho^{(i)}) f_{\rho^{(j)}}(\rho^{(j)}) \\ &\quad \times \frac{\varphi_2(z_i, z_j, \rho_{ij}')}{\varphi(z_i) \varphi(z_j)} d\rho^{(i)} d\rho^{(j)} \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{\rho^{(i)} - \mu_i}{\sigma_i} \right) \left( \frac{\rho^{(j)} - \mu_j}{\sigma_j} \right) \varphi_2(z_i, z_j, r_{ij}') dz_i dz_j \end{aligned} \quad (5)$$

214 In case historical recovery data for the system of interest are not available, one can choose a  
 215 distribution that either reflect some degree of judgement and experience, or a distribution with  
 216 minimal information (i.e., a noninformative distribution as usually done in Bayesian inference), to  
 217 reflect the fact that little or no information is available a priori. In addition, the Bayesian inference  
 218 discussed later in Section 3.5, can be used to update the state of knowledge every time new knowledge  
 219 becomes available (i.e., recovery data are collected as the recovery unfolds) (Box and Tiao 1992)

## 220 **Obtaining the joint PDF of the model parameters of the recovery curve**

221 The second step consists in introducing parametrized recovery curves to describe the recovery process  
 222 over time. In general, the functional form of the selected parametrized recovery curve may affect the  
 223 time-varying recovery process of a general performance measure. However, one can choose the  
 224 parametrized recovery curve based on engineering judgement and experience of the problem. In  
 225 addition, one can use flexible functional forms, such that the recovery curve can be updated as the actual

226 recovery progresses and data become available. Example of parametrized recovery curves can be found  
 227 in Gardoni et al. (2007) and Ayyub (2015). A parametrized CRF describes the time-varying recovery  
 228 process of a general performance measure in the following form:

$$T[Q(\Theta, \tau)] = Q(\theta, \tau) + \sigma\varepsilon \quad (6)$$

229 where,  $T[\cdot]$  is a transformation function,  $\Theta = (\theta, \sigma)$ ;  $\theta = (\theta_1, \theta_2, \dots)$  is a vector of unknown model  
 230 parameters associated with  $Q$ , that needs to be estimated; and  $\sigma\varepsilon$  is an additive model error term of  $Q$   
 231 (additivity assumption), in which  $\sigma$  is the standard deviation of the model error, assumed not to depend  
 232 on  $\tau$  (homoskedasticity assumption), and  $\varepsilon$  is a standard normal random variable (normality  
 233 assumption). The additivity, normality and homoskedasticity assumptions typically can be satisfied  
 234 using an appropriate variance stabilizing transformation from the parametrized family of transformations  
 235 introduced by Box and Cox (1964). We then define the joint PDF of the unknown model parameters  
 236  $\Theta$  based on the joint PDF of the resilience metrics (Hogg et al. 2012; Ang and Tang 2006). Let the set  
 237  $(\rho, \chi, \dots, \rho^{(n)})$  have a jointly continuous distribution with PDF  $f_{\mathbf{p}}(\rho, \chi, \dots, \rho^{(n)})$  on a defined support  
 238 set  $C$ . According to the definition of the resilience metrics, the resilience metrics are a function of  
 239 the model parameters  $\theta = (\theta_1, \dots, \theta_n)$  in the support set  $D$ , such that  $\rho = k_1(\theta_1, \dots, \theta_n)$ ,  $\chi = k_2(\theta_1, \dots, \theta_n)$ ,  
 240 up to  $\rho^{(n)} = k_n(\theta_1, \dots, \theta_n)$ , where the generic  $i^{th}$  function  $k_i(\theta_1, \dots, \theta_n)$  represents the expression of the  $i^{th}$   
 241 resilience metric  $\rho^{(i)}$  based on Equations (1)-(3), after introducing a parametrized recovery curve  
 242 according to Equation (6). We first evaluate the  $n \times n$  Jacobian given by

$$J = \begin{bmatrix} \frac{\partial \rho}{\partial \theta_1} & \frac{\partial \rho}{\partial \theta_2} & \cdots & \frac{\partial \rho}{\partial \theta_n} \\ \frac{\partial \chi}{\partial \theta_1} & \frac{\partial \chi}{\partial \theta_2} & \cdots & \frac{\partial \chi}{\partial \theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \rho^{(n)}}{\partial \theta_1} & \frac{\partial \rho^{(n)}}{\partial \theta_2} & \cdots & \frac{\partial \rho^{(n)}}{\partial \theta_n} \end{bmatrix} \quad (7)$$

243 Then, let us consider two subsets of the supports, respectively named  $A$  and  $B$ , where  $B$  denotes  
 244 the mapping of  $A$  under a one-to-one transformation. Due to the conservation of the probability the  
 245 event  $\{(\rho, \chi, \dots, \rho^{(n)}) \in A\}$  is equivalent to the event  $\{(\theta_1, \theta_2, \dots, \theta_n) \in B\}$ . Therefore, we can write

$$\mathbb{P}[(\theta_1, \theta_2, \dots, \theta_n) \in B] = \mathbb{P}[(\rho, \chi, \dots, \rho^{(n)}) \in A] = \int \cdots \int_A f_{\mathbf{P}}(\rho, \chi, \dots, \rho^{(n)}) d\rho d\chi \dots d\rho^{(n)} \quad (8)$$

246 We change variables of integration by writing  $\theta_1 = h_1(\rho, \chi, \dots, \rho^{(n)})$ ,  $\theta_2 = h_2(\rho, \chi, \dots, \rho^{(n)})$ , up to  
 247  $\theta_n = h_n(\rho, \chi, \dots, \rho^{(n)})$  such that

$$\begin{aligned} & \int \cdots \int_A f_{\mathbf{P}}(\rho, \chi, \dots, \rho^{(n)}) d\rho d\chi \dots d\rho^{(n)} = \\ & \int \cdots \int_B f_{\theta} [k_1(\theta_1, \theta_2, \dots, \theta_n), k_2(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| d\theta_1 d\theta_2 \dots d\theta_n \end{aligned} \quad (9)$$

248 Therefore, for every set  $B \subseteq D$ , we can write

$$\begin{aligned} & \mathbb{P}[(\theta_1, \theta_2, \dots, \theta_n) \in B] = \\ & \int \cdots \int_B f_{\theta} [k_1(\theta_1, \theta_2, \dots, \theta_n), k_2(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| d\theta_1 d\theta_2 \dots d\theta_n \end{aligned} \quad (10)$$

249 We conclude that the joint PDF of interest  $f_{\theta}(\theta_1, \theta_2, \dots, \theta_n)$  is

$$f_{\theta}(\theta_1, \theta_2, \dots, \theta_n) = \begin{cases} f_{\theta} [k_1(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| & (\theta_1, \theta_2, \dots, \theta_n) \in D \\ 0 & \text{elsewhere} \end{cases} \quad (11)$$

250 Eq. (11) represents the state of knowledge on the model parameters  $\theta = (\theta_1, \dots, \theta_n)$ . We can now  
251 derive the expected recovery curve and the related uncertainties based on the distribution of the  
252 parameters in Eq. (6).

### 253 **Obtaining point and predictive estimates of the recovery curve and confidence bounds**

254 Different estimates of the recovery curves can be obtained depending on how we treat the model  
255 parameters. Following Gardoni et al. (2002), we can obtain point estimates or predictive estimates.  
256 A point estimate of the recovery curve is obtained using a point estimate of  $\Theta$ , in place of  $\Theta$ . In  
257 general, the mean value of  $\Theta$  or the maximum likelihood estimate (MLE)  $\Theta_{MLE}$  can be used.  
258 However, the point estimate does not incorporate the epistemic (statistical) uncertainties in the model  
259 parameters  $\Theta$ . To incorporate these uncertainties, we need to consider  $\Theta$  as random variable. The  
260 predictive estimate of the recovery curve is then the expected value of the recovery curve over the  
261 space of the model parameters, i.e.,

$$\tilde{Q}(\tau) = \int Q(\Theta, \tau) f(\Theta) d\Theta \quad (12)$$

262 This estimate incorporates the epistemic uncertainties in the model parameters  $\Theta$ . In addition, we can  
263 construct probability bounds on the recovery curve using the PDF of the model parameters, as  
264 illustratively shown in Figure 1.

### 265 **Estimating the probability of reaching or exceeding a target percentile of interest of the ultimate** 266 **desired state**

267 Once we obtained  $Q(\Theta, \tau)$ , we can estimate the probability of reaching or exceeding a target value of  
268  $Q$  by reliability analysis (Ditlevsen and Madsen 1996; Gardoni 2017). We can write a limit-state  
269 function  $g(\Theta, \tau)$  as

$$g(\Theta, \tau) = Q(\Theta, \tau) - Q_T \quad (13)$$

270 where  $Q_T$  is a level of performance we desire to reach or exceed, expressed as a percentile of the ultimate  
 271 desired state,  $Q_\infty$ . Mathematically, we can write the probability that the recovery process is above  $Q_T$   
 272 at a time  $\tau$ ,  $H(\Theta, \tau)$ , as

$$H(\Theta, \tau) = 1 - \mathbb{P}[g(\Theta, \tau) \leq 0 | \tau] \quad (14)$$

273 Figure 2 shows a conceptual representation of  $Q(\Theta, \tau)$  and the corresponding  $H(\Theta, \tau)$  over time.  
 274 Following Gardoni et al. (2002), we can construct a point estimate of  $H(\tau)$ , a predictive estimate as well  
 275 as confidence bounds as previously proposed for the recovery curve. Hence, we define the point estimate  
 276 of the probability that the recovery process is above  $Q_T$  at a time  $\tau$  using a point estimate of  $\hat{\Theta}$ , in place  
 277 of  $\Theta$ , whereas the predictive estimate  $\tilde{H}(\tau)$  is defined taking the expected value of the quantity of  
 278 interest over the space of the model parameters, in the same way as previously showed for the recovery  
 279 curve. Furthermore, we obtain confidence bounds on the estimate in Eq. (14). We can define the  
 280 reliability index as

$$\beta(\Theta, \tau) = \Phi^{-1}[H(\Theta, \tau)] \quad (15)$$

281 where  $\Phi^{-1}(\cdot)$  indicates the inverse of the standard normal CDF. Following Gardoni et al. (2002), the  
 282 variance of  $\beta(\Theta, \tau)$  can be estimates as

$$\sigma_\beta^2(\tau) \approx \nabla_\Theta \beta(\tau) \Sigma_{\Theta\Theta} \nabla_\Theta \beta(\tau)^T \quad (16)$$



283 where  $\nabla_{\Theta}\beta(\tau)$  is the gradient of  $\beta(\Theta, \tau)$  evaluated at the mean value and  $\Sigma_{\Theta\Theta}$  is the estimated covariance  
 284 matrix. The gradient vector  $\nabla_{\Theta}\beta(\tau)$  is obtained by performing a FORM (First-Order Reliability  
 285 Method) analysis (Ditlevsen and Madsen 1996). Therefore, we obtain

$$\{\Phi[-\tilde{\beta}(\tau) - \sigma_{\beta}(\tau)], \Phi[-\tilde{\beta}(\tau) + \sigma_{\beta}(\tau)]\} \quad (17)$$

286 as one standard deviation bounds, where  $\tilde{\beta}(\tau) = \Phi^{-1}[\tilde{H}(\tau)]$ . The bounds represent approximately 15%  
 287 and 85% probability levels.

### 288 **Updating the model parameters as new data become available**

289 Finally, Bayesian inference can be used to update the model parameters  $\Theta$  combining existing  
 290 information with new information as it might become available during the actual recovery process  
 291 (Gardoni et al. 2007). Steps 3 and 4 can then be repeated to obtain updated recovery curves and updated  
 292 probabilities of reaching or exceeding a desired level  $Q_T$ . Mathematically, we can write the posterior  
 293 distribution  $f''(\Theta)$  that includes the updated status of knowledge about  $\Theta$  as (Box and Tiao 1992)

$$f''(\Theta | \mathbf{Q}) = \kappa L(\Theta | \mathbf{Q}) f'(\Theta) \quad (18)$$

294 where  $L(\Theta | \mathbf{Q})$  is the likelihood function that contains the objective information on  $\Theta$  in a set of  
 295 observations,  $f'(\Theta)$  is the prior distribution, reflecting the state of knowledge about  $\Theta$  prior to obtaining  
 296 the observations  $\mathbf{Q} = (Q_1, \dots, Q_m)$ , and  $\kappa = \left[ \int L(\Theta | \mathbf{Q}) f'(\Theta) d\Theta \right]^{-1}$  is a normalizing factor.

297 The prior distribution includes the status of knowledge based on previous experiences,  
 298 engineering judgments, and/or past data. The likelihood function is proportional to the conditional  
 299 probability of observing the recorded data  $\mathbf{Q} = (Q_1, \dots, Q_m)$  for given values of the parameters  $\Theta$ . In

300 general, the likelihood function permits to include lower, upper and equality data (see Gardoni et al.  
 301 2002). A lower bound datum is defined as an observation of  $Q$  that is larger than a certain value  $Q_i$  at  
 302 time  $\tau$ ; an upper bound datum is defined as an observation that is smaller than a certain value  $Q_i$  at time  
 303  $\tau$ ; an equality datum is defined as the value of  $Q$  recorded at time  $\tau$ . Following Gardoni et al. (2002),  
 304 the likelihood function can be written as

$$\begin{aligned}
 L(\boldsymbol{\theta}, \sigma) &\propto \prod_{\text{equality data}} \mathbb{P}[\sigma \varepsilon_i = Q_i - Q(\boldsymbol{\theta}, \tau)] \\
 &\times \prod_{\text{lower bound data}} \mathbb{P}[\sigma \varepsilon_i > Q_i - Q(\boldsymbol{\theta}, \tau)] \\
 &\times \prod_{\text{upper bound data}} \mathbb{P}[\sigma \varepsilon_i < Q_i - Q(\boldsymbol{\theta}, \tau)]
 \end{aligned} \tag{19}$$

305 Based on the normality assumption, we can then write

$$\begin{aligned}
 L(\boldsymbol{\theta}, \sigma) &\propto \prod_{\text{equality data}} \left\{ \frac{1}{\sigma} \varphi \left[ \frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\} \\
 &\times \prod_{\text{lower bound data}} \left\{ \Phi \left[ -\frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\} \\
 &\times \prod_{\text{upper bound data}} \left\{ \Phi \left[ \frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\}
 \end{aligned} \tag{20}$$

306 where  $\varphi(\cdot)$  is the standard normal PDF, and  $\Phi(\cdot)$  is the standard normal CDF.

307 Eqs. (15)-(20) can be used every time additional information is available to update the model  
 308 parameters. For instance, when a set of samples  $\mathbf{Q}_1$  is available, we can write

$$f''(\boldsymbol{\Theta} | \mathbf{Q}_1) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_1) f'(\boldsymbol{\Theta}) \tag{21}$$

309 Then, let us suppose another set of samples  $\mathbf{Q}_2$  is available and this is independent from the previous  
 310 one, we can update the posterior PDF evaluated in Eq. (21) such that

$$f'''(\boldsymbol{\Theta} | \mathbf{Q}_1, \mathbf{Q}_2) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_1)L(\boldsymbol{\Theta} | \mathbf{Q}_2)f''(\boldsymbol{\Theta}) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_2)f''(\boldsymbol{\Theta} | \mathbf{Q}_1) \quad (22)$$

311 Generally, if  $n$  independent set of observations are available, we can write

$$f^{(k+1)}(\boldsymbol{\Theta} | \mathbf{Q}_1, \dots, \mathbf{Q}_k) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_k)f^{(k)}(\boldsymbol{\Theta} | \mathbf{Q}_1, \dots, \mathbf{Q}_{k-1}) \quad (23)$$

$$k = 2, \dots, n$$

312 Field measurements can often be inexact and include measurement errors (Gardoni et al. 2002;  
 313 Murphy et al. 2011). Following Gardoni et al. (2002), measurement errors can be incorporated in the  
 314 updating process. To incorporate the measurements errors in the updating process, we assume that  
 315  $Q_i = \hat{Q}_i + e_{Q_i}$  is the true value of the  $i^{\text{th}}$  observation, where  $\hat{Q}_i$  represents the measured value and  $e_{Q_i}$  is  
 316 the measurement error. We also assume that  $e_{Q_i}$  has zero mean, which reflects that the measurements  
 317 have been corrected from any systematic errors, and variance  $s_i^2$ , which represents the uncertainties  
 318 inherent in the measurements. For the equality data we have  $\hat{Q}_i + e_{Q_i} = Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$ , for the lower bound  
 319 data we have  $\hat{Q}_i + e_{Q_i} < Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$ , and for the upper bound data we have  $\hat{Q}_i + e_{Q_i} > Q(\boldsymbol{\theta}, \tau) + \sigma\varepsilon_i$ .  
 320 Therefore, the conditions for the three type of data can be, respectively, written as  $\sigma\varepsilon_i - e_{Q_i} = \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ ,  
 321  $\sigma\varepsilon_i - e_{Q_i} > \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ , and  $\sigma\varepsilon_i - e_{Q_i} < \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ . The left-hand sides of these expressions are a  
 322 normal random variable with zero mean and variance  $\hat{\sigma}(\boldsymbol{\theta}, \sigma) = \sigma^2 + s_i^2$ . Hence, in presence of  
 323 measurement errors, the likelihood function is

$$\begin{aligned}
L(\boldsymbol{\theta}, \sigma) \propto & \prod_{\substack{\text{equality} \\ \text{data}}} \left\{ \frac{1}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \varphi \left[ \frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{lower bound} \\ \text{data}}} \left\{ \Phi \left[ -\frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{upper bound} \\ \text{data}}} \left\{ \Phi \left[ \frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\}
\end{aligned} \tag{24}$$

### 324 **Example 1: Recovery curves for an example bridge**

325 This section presents the proposed formulation considering the recovery process of a typical RC bridge  
326 subject to seismic excitations. The first example demonstrates the application of the formulation in a  
327 realistic case related to civil structures in support of risk and resilience analysis.

328 We previously discussed that for civil structures the work progress is a continuous, or near-  
329 continuous, function, whereas a discrete function describes the performance indicators (e.g.,  
330 functionality) with jumps when a group of activities is completed. This section illustrates the proposed  
331 formulation applied to a RC bridge. Figure 3 shows the configuration of the considered (single column,  
332 single bent) testbed bridge from Kumar and Gardoni (2014a) and Jia et al. (2017). Following the  
333 proposed formulation, we obtain the estimates of the first two resilience metrics to describe the recovery  
334 process of the selected engineering system. Figure 4 shows the pair  $(\rho, \chi)$  used in this example, and  
335 their correlation. Based on the data in Figure 4, we assume that both  $\rho$  and  $\chi$  follow a lognormal  
336 distribution, whose parameters are listed in Table 1. Then, based on the estimated coefficient of  
337 correlation and the marginal PDFs we can construct the joint PDF of the resilience metrics as described  
338 in Section 3.1. Next, we introduce a parametrized recovery curve to describe the changes of a selected  
339 performance measure over time. The performance indicator considered in this example is the reliability  
340 index  $\beta$ . Moreover, in this example we assume that there is only one recovery step that restores the

341 reliability of the bridge, as described in Sharma et al. (2018). Consequently, we consider the recovery  
342 curve in the following form:

$$Q_1(\Theta, \tau) = \begin{cases} \theta_1 & \tau < \theta_2 \\ Q_\infty & \tau \geq \theta_2 \end{cases} \quad (25)$$

343 where  $\theta_1$  is the residual reliability index after the occurrence of the hazard, and before the completion of  
344 the recovery; and  $\theta_2$  is the time at which the reliability index reaches the ultimate desired value  $Q_\infty$ . To  
345 model the reliability, we consider the reliability-based resilience metrics coming from previous analyses.  
346 Thus, we do not need to model the occurrence of earthquake mainshock-aftershocks sequence and their  
347 impact on structural properties because the resilience metrics capture all these information. Based on the  
348 definition of the resilience metrics in Eqs. (1) and (2), the parameters  $\theta_1$  and  $\theta_2$  can be written as a  
349 function of the resilience metrics. Specifically, Figure 5 shows the RMF of the adopted parametrized  
350 curve. Following the proposed methodology, we compute the joint PDF of the model parameters and  
351 the corresponding expected recovery process in terms of the reliability index  $\beta$ . We observe that the  
352 number of resilience metrics needed to adopt the formulation is at least equal to the number of the model  
353 parameters of the selected parametrized recovery curve. Therefore, considering the possibility of having  
354 a drop in the functionality, during the recovery process due to aftershocks, would require implying higher  
355 resilience metrics. Next, we estimate the probability of reaching or exceeding a target value of  
356 functionality at any time setting, for instance,  $Q_T = 3.5$ .

357 **Initial estimate of the recovery curve and corresponding probability of exceeding the target**  
358 **value of functionality**

359 As previously discussed, in this example we assume that there is only one recovery step that restores the  
360 reliability of the bridge. Nevertheless, we can also estimate the behavior of the system toward the desired  
361 value of the functionality at the end of the recovery in terms of the mean value of the different probable

362 recovery curves. Figure 6 shows the expected changes of the instantaneous reliability index over time.  
363 Adopting the reliability-based definitions for the damage state proposed in Sharma et al. (2018), the  
364 initial damage level is moderate. Figure 7 shows the probability of exceeding the target value of  $Q_T$ .  
365 The figure also presents the confidence band due to the statistical uncertainty in  $\Theta$ . Based on the  
366 expected initial value of the reliability index, we can observe that the probability of exceeding the target  
367 value of functionality,  $Q_T = 3.5$ , at time  $\tau = 25$  days, is equal to 0.5. The observed result matches the  
368 results provided in Sharma et al. (2018), where the expected value of the time to recover is approximately  
369 26 days when the initial damage level is moderate.

370 **Updated estimate of the recovery curve and corresponding probability of exceeding the**  
371 **target value of functionality**

372 We assume that after the occurrence of the hazard we collect data on the state of damage for the first 10  
373 days, and then we update the model parameters. In the presented example, we assume that inspection  
374 data are collected after the occurrence of the hazard. Specifically, we assume that a qualitative  
375 description of the damage state indicates moderate damage following the definition in ATC-38 (ATC  
376 2000) and Bai et al. (2009) (i.e., “Repairable structural damage has occurred. The existing elements can  
377 be repaired in place, without substantial demolition or replacement of elements”). Then, the qualitative  
378 definition of the damage state is mapped into a reliability-based definition in terms of the corresponding  
379 reliability index  $\beta$  (i.e.,  $1.5 \leq \beta \leq 2.5$ ) following in Sharma et al. (2018). As a result, we obtain the new  
380 expected changes in the reliability index and the corresponding time-varying probability of exceeding  
381 the same target value of functionality, as shown in Figure 8 and 9. Figure 8 shows the expected changes  
382 of the reliability index over time, after updating the model parameters based on the observed data. First,  
383 we can observe that the recovery process follows the observed data in terms of its mean; then, the

384 Bayesian inference also reduces the relevant uncertainties. The probability of exceeding the target value  
385 of functionality reflects both the effects of the Bayesian inference.

### 386 **Example 2: Population relocation after a seismic event**

387 The second example shows the application of the formulation in a scenario where historical recovery  
388 data are not available. In this example, we consider the population relocation of the city of Seaside, OR,  
389 after the occurrence of an earthquake originated from the Cascadia Subduction Zone. We consider a  
390 seismic event of magnitude  $M_w = 7.0$ , located 25 km southwest of the city. Since no data are available,  
391 we consider a noninformative PDF of the first resilience metric  $\rho$  in the form  $f_p(\rho) = 1/\rho$ ,  $\rho > 0$ , which  
392 reflects the fact that little is known a priori. We consider a parametrized S-shape recovery curve proposed  
393 in Gardoni et al. (2007) in the following form:

$$Q_2(\Theta, \tau) = 1 - Q_R + (Q_\infty - Q_R) \left( \frac{\tau}{\theta_1} \right)^2 \left[ 3 - 2 \left( \frac{\tau}{\theta_1} \right) \right] \quad \tau \leq \theta_1 \quad (26)$$

394 where  $Q_R$  represents the percentage of population dislocation at time  $t_{0^+}$  (i.e., after the occurrence of the  
395 seismic event),  $Q_\infty$  represents the percentage of the population that relocates at the end of the recovery,  
396 and  $\theta_1$  is the time at which the recover ends.

397 Ground Motion Prediction Equations (Boore and Atkinson, 2008) are used to obtain maps of the  
398 seismic intensity measure at the residential building location. Next, we perform a building damage  
399 analysis using different fragility functions (e.g., HAZUS-MH (FEMA 2015), and Steelman et al. 2007).  
400 Then, we estimate the initial percentage of population dislocation due to structural damage using a  
401 logistic regression model (Lin 2009). For the purpose of this example, we assume that the entire  
402 population returns to their homes at the end of the recovery (i.e.,  $Q_\infty = 100\%$ .)

403 Based on the definition of the resilience metric in Eq. (1), the parameter  $\theta_1$  can be written as a  
404 function of the resilience metric  $\rho$ . Therefore, we obtain the PDF  $f_{\theta_1}(\theta_1)$  according to Eq. (11), as well  
405 as the corresponding estimate of  $Q_2$  over time (shown in Figure 10(b).) More generally,  $Q_\infty$  could also  
406 be taken as a parameter (i.e.,  $Q_\infty = \theta_2$ ). In this case, we would obtain the joint PDF  $f_{\theta}(\theta_1, \theta_2)$  again using  
407 Eq. (11) given  $\rho$  and  $\chi$ .

408 After the occurrence of the seismic event, recovery activities start to retrieve structures and  
409 infrastructure functionality, thereby we can observe the population returning to their homes. For the  
410 purpose of this example, we assumed that data on the population relocation are available at given time-  
411 steps. The relocation data at different times can be used to obtain the corresponding values of  $Q_2$  (shown  
412 by dots in Figure 10(b).) Using these values of  $Q_2$ , we obtain the new expected value of  $Q_2$  as a function  
413 of time, as shown in Figure 10(b). In particular, we can observe that the uncertainties in the initial  
414 estimate in Figure 10(a) reflect the fact that little is known in terms of the duration of the recovery. In  
415 Figure 10(b) the confidence band is significantly smaller around the mean line indicating that the values  
416 of  $Q_2$  used to update the mean prediction also reduce the prediction uncertainty.

417 Finally, Figure 11 shows the probability that the population dislocation is higher than 25% of the  
418 total population (i.e.,  $Q_T = 0.25$ ) before and after we update the recovery curve, including the confidence  
419 band due to the statistical uncertainty in  $\Theta$ . We can see that the information used to update the model  
420 parameter can also adjust the prediction in terms of the probability of exceeding a target level of  
421 functionality, as well as reduce the prediction uncertainty.



422       **Conclusions**

423       The paper proposed a formulation to (i) predict the recovery curves that define the recovery of  
424       engineering systems subject to a hazard, and (ii) estimate the probability of reaching or exceeding a target  
425       value of a selected performance indicator at any given time. The formulation uses the resilience metrics  
426       defined in Sharma et al. (2018), which quantify the resilience of systems and form a complete set of  
427       partial descriptors that characterize the recovery curve of the system of interest. To evaluate the recovery  
428       process of an engineering system, the paper proposed to use the probability density function (PDF) of  
429       the resilience metrics, defined based on historical data, to obtain the PDF of the model parameters that  
430       define the recovery curve. The proposed formulation incorporates the Bayesian inference to update the  
431       estimates of the unknown parameters when additional information is available. The paper illustrated the  
432       implementation of the proposed formulation by predicting the recovery of a single-bent, single-column  
433       reinforced concrete (RC) bridge subject to seismic damage, and the population relocation after the  
434       occurrence of a seismic event when no data on the duration of the recovery are available a priori. The  
435       proposed formulation is general and suited to applications such as risk analysis and mitigation, and  
436       resilience-based design.

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557 **Figures**

558 **Figure 1.** 90% Confidence bounds on the estimate of the recovery curve

559 **Figure 2.** Conceptual representation of probability of reaching or exceeding a target value of functionality at any time

560 **Figure 3.** The considered RC bridge (adapted from Jia et al. 2017)

561 **Figure 4.** Correlation between the adopted resilience metrics

562 **Figure 5.** RMF of the adopted parametrized recovery curve

563 **Figure 6.** Mean value and corresponding 95% confidence band of the recovery curve

564 **Figure 7.** Probability of exceeding the selected value of  $Q_T = 3.5$

565 **Figure 8.** Mean value and corresponding 95% confidence band of the recovery curve after updating the model parameters

566 **Figure 9.** Probability of exceeding the selected value of  $Q_T = 3.5$  after updating the model parameters

567 **Figure 10.** Mean value and corresponding 95% confidence band of the population dislocation recovery curve (a) before and  
568 (b) after updating the model parameter

569 **Figure 11.** Probability of population dislocation (i.e.,  $Q_T = 25\%$ ) (a) before and (b) after updating the model parameter

570

571 **Tables**

572 **Table 1.** Distribution parameters of the resilience metrics

Resilience metric	$\lambda$	$\xi$
$\rho$	-0.94	0.22
$\chi$	1.14	0.20

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