

Time-Dependent Probability of Exceeding a Target Level of Recovery

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# Time-Dependent Probability of Exceeding a Target Level of Recovery in Resilience Analysis

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## Abstract

The resilience of a system is generally defined in terms of its ability to withstand external perturbation(s), adapt, and rapidly recover. This paper introduces a probabilistic formulation to predict the recovery process of a system given past recovery data, and estimate the probability of reaching or exceeding a target value of functionality at any time. A Bayesian inference is used to capture the changes over time of model parameters as recovery data become available during the work progress. The proposed formulation is general and can be applied to continuous recovery processes such as those of economic or natural systems, as well as to discrete recovery processes typical of engineering systems. As an illustration of the proposed formulation, two examples are provided. The paper models the recovery of a reinforced concrete bridge following seismic damage, as well as the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori.

**Keywords:** Decision Support, Recovery, Resilience, Resilience Metrics, Probability, Reliability Analysis.

## Introduction

Civil infrastructure enables the conveyance of goods, services, and resources to communities (Corotis 2009; Ellingwood et al. 2016; Gardoni et al. 2016). Past disasters continue to show the vulnerability of civil infrastructure to natural and anthropogenic hazards and highlight the significance of risk mitigation and management (Murphy and Gardoni 2006; Gardoni et al. 2016). Buildings, bridges, and other structures and infrastructure may experience extreme natural events, such as floods, earthquakes, hurricanes, and anthropogenic hazards, such as accidents and terrorist attacks, which may lead to significant damage making infrastructure networks inoperative (Gardoni and LaFave 2016). Past disasters stressed the importance of being prepared and to be able to recover in a short period (e.g., Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014).

The concept of resilience has gained relevance in the last fifteen years as a desirable feature for communities (Bruneau et al. 2003; McAllister 2013; Caverzan and Solomos 2014; Ellingwood et al. 2016; Guidotti et al. 2016, 2017; Sharma et al. 2018; Gardoni 2018). The relatively recent interest in resilience has resulted in several definitions of the concept of resilience and several approaches to measuring resilience across several application domains. In general, resilience is defined as the ability of systems to recover after a disturbance to the pre-disturbance state or a new (improved) state (e.g., Bruneau et al. 2003; Cimellaro et al. 2010a; Bocchini et al. 2012). The U.S. Presidential Policy Directive 21 (PPD 21) defines resilience as *the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents*. A review of the current state of the research in community resilience can be found in Koliou et al. (2018). Going beyond the engineering domain, Doorn et al. (2018) explored how philosophical and social science considerations can be incorporated into a multidisciplinary definition of resilience to account for social justice. The

choice of a defined recovery curve plays a key role in resilience analysis in terms of quantifying the resilience of a system. A recovery curve describes the behavior of a system as a function of time following the impact of a hazard as the system recovers to achieve a desired state (of functionality or of reliability.) In absence of disrupting shocks during the recovery phase, the recovery curve is, in general, a non-decreasing and time-dependent function. Different studies have attempted to model and define the recovery curve of engineering systems subject to a hazard (e.g., Cimellaro et al. 2010b; Decò et al. 2013; Titi et al. 2015). Recovery curves are usually assumed based on qualitative attributes, such as the preparedness of the society, that influence the recovery process. As such, they i) are not based on the actual physics of the recovery process, ii) do not account for the underlying uncertainties, and iii) are not able to incorporate additional information as it becomes available (such as ongoing progress of the work or increased resource availability, which affect the recovery models and reduce the uncertainty involved.) As a result, models of recovery typically only provide crude approximations and not accounting for the underplaying uncertainties makes it not possible to estimate the probability of reaching or exceeding a target percentile of interest of the ultimate desired state (e.g., a target value of functionality or reliability). To overcome these limitations, Sharma et al. (2018) proposed a mathematical formulation for resilience analysis that models the recovery curves based on the actual work plan of activities involved in the recovery process.

Once a recovery curve is defined, there is a need to define a metric or a set of metrics of recovery that distinctively characterize the recovery curve. A typical resilience metric has been defined as the integral of the recovery curve over a specified interval of time (Bruneau and Reihorn 2007; Cimellaro et al. 2010a; Bonstrom and Corotis 2016). However, such metrics do not uniquely and fully characterize a recovery curve. Sharma et al. (2018) defined a set of resilience metrics in analogy with the moments of a random variable to quantify the resilience of a system. The Sharma et al.'s metrics i) are intuitive

73 because of their analogy with the moments of a random variable, and ii) define a complete set of partial  
74 descriptors that uniquely and fully characterize a recovery curve.

75 This paper contributes to the literature in resilience analysis. In particular, this paper proposes a  
76 probabilistic formulation to predict a recovery process of a system, and then estimate the probability of  
77 reaching or exceeding a target value of functionality (or reliability) of the system at any given time as  
78 the system recovers. The proposed formulation uses Sharma et al.'s resilience metrics obtained from  
79 historical recovery data to predict possible recovery processes along with their likelihood, as well as to  
80 estimate the probability of reaching or exceeding a desired level of recovery by a desired time. The  
81 proposed formulation can be applied to systems in different fields, i.e., economical, natural, and  
82 engineering systems. The proposed formulation first defines the joint probability density function (PDF)  
83 of resilience metrics that captures the underlying uncertainties. Then, parametrized recovery curves are  
84 introduced to model the time-varying recovery process, and the joint PDF of model parameters is  
85 obtained as a function of the joint PDF of the resilience metrics. The joint PDF of the model parameters  
86 defines the variability in the possible recovery curves, which is used to estimate the probability of  
87 reaching or exceeding a target value of functionality by conducting a reliability analysis (Ditlevsen and  
88 Madsen 1996; Gardoni 2017). A Bayesian inference is also proposed to include possible information  
89 from the field while the work for the recovery is in progress. We use field data to update the predicted  
90 recovery curve such that the recovery curve is updated to reflect the advancement of the actual recovery  
91 in the field. Thanks to the Bayesian updating, the uncertainties in the recovery process diminish while  
92 more data become available. The main benefits of the proposed formulation are that the estimates of the  
93 recovery curve can be simply defined as a function of resilience metrics and the modeling can take  
94 advantage of data collected both before and during the recovery process. We illustrate the proposed  
95 formulation considering the recovery of a typical reinforced concrete (RC) bridge following a seismic

96 damage, and the population relocation after the occurrence of a seismic event when no data on the  
97 duration of the recovery are available a priori.

98 Following this introduction, Section 2 gives a brief review of mathematical formulations for  
99 resilience analysis. Section 3 presents the proposed probabilistic formulation. Section 4 illustrates the  
100 proposed formulation considering the recovery of an example bridge following a seismic damage.  
101 Finally, Section 5 uses the propose approach considering the population relocation after a seismic event.

## 102 **Review of Mathematical Formulations for Resilience Analysis**

103 The resilience analysis of engineering systems plays a key role in mitigation planning and allocation of  
104 resources in pre- and post-disruption scenarios (Ellingwood et al. 2016). Resilience of a system is, in  
105 general, defined as its ability to maintain or promptly resume a level of functionality or performance after  
106 a disruption. What is promptly enough is usually defined based on the owner's, customers' or, more  
107 generally, societal needs. A performance measure (e.g., the system functionality), typically indicated as  
108  $Q(t)$ , can be used to describe the system state as a function of time  $t$  (Cimellaro et al. 2010a; Bocchini  
109 et al. 2012; Bonstrom and Corotis 2016; Sharma et al. 2018). An external shock, such as a natural or  
110 anthropogenic event (a shock), might reduce  $Q(t)$  instantaneously. Such reduction is typically a function  
111 of the intensity of the shock, the system design specifications (which define the system robustness at  
112  $t = 0$ , e.g., Bai et al. 2009), and the system state immediately before the shock (which reflect the  
113 deterioration of a system over time and also define the system robustness at time  $t$ , e.g., Kumar and  
114 Gardoni 2014; Kumar et al. 2015; Jia and Gardoni 2018a,b). After a shock, the recovery process starts  
115 to retrieve the system functionality to a desired level, which may be below, same or better than the pre-  
116 disruption value (Ayyub 2014, 2015).

117 Resilience, independently from the field of application, consists of four properties (Bruneau et al.  
118 2003; Tierney and Bruneau 2007): 1) robustness as the ability to withstand a given level of stress or  
119 demand without suffering degradation or loss of function or, if a degradation occurs, the residual level  
120 of  $Q(t)$ ; 2) resourcefulness, interrelated to the ability to diagnose and prioritize issues and to initiate  
121 solutions by identifying and monitoring all resources; 3) redundancy, defined as the extent to which the  
122 system and other elements satisfy and sustain functional requirements in the event of disturbance; and 4)  
123 rapidity as the ability to recover in a timely manner to limit losses and avoid future disruptions. These  
124 four properties define the resilience of a system and characterize the recovery process.

125 Recovery curves capture the changes in system functionality over time and defined how the system  
126 state improves to achieve a desired value of functionality at the end of the recovery process. Different  
127 studies have attempted to quantify the resilience of a system based on the shape of the recovery curves.  
128 As a first attempt to quantify the resilience of a system, Bruneau et al. (2003) proposed to measure the  
129 resilience as the area underneath the recovery curve. Chang and Shinozuka (2004) assessed the resilience  
130 as the probability that the time needed for the recovery due to a performance loss after a disruption would  
131 be less than a predefined threshold. Garbin (2007) outlined an approach to quantitatively measure the  
132 resilience of a network as the percentage of links damaged and the percentage of nodes damaged versus  
133 a network performance measure. Bruneau and Reinhorn (2007) proposed metrics for measuring  
134 resiliency based on the expected degradation in the quality of an infrastructure by quantifying robustness,  
135 redundancy, resourcefulness, and rapidity to recovery. While these contributions show the importance  
136 of quantifying resilience in an objective and formal way, the metrics they define only provide partial  
137 information about the actual resilience and might not be able to distinguish among different resilience  
138 levels (as noted in Sharma et al. 2018). Uniquely and fully characterizing the resilience of the system

139 requires capturing all of the relevant characteristics of the recovery curve. Consequently, a single metric  
140 cannot represent a curve and capture all of its attributes.

141 Sharma et al. (2018) showed that the existing metrics are not able to uniquely and fully characterize  
142 recovery curves with different shapes and might not be able to capture the difference in the resilience  
143 levels. To address this issue, they developed a complete set of resilience metrics able to fully describe  
144 the recovery process and capture the differences in the shapes of different recovery curves. Sharma et  
145 al.'s resilience metrics are analogous to the partial descriptors commonly adopted in probability and  
146 statistics (e.g., mean, standard deviation and higher moments of a random variable.) The recovery curve  
147  $Q(t)$ , which Sharma et al. (2018) call the *cumulative resilience function* (CRF) in analogy with the  
148 *cumulative distribution function* (CDF) of a random variable, represents the overall recovery process as  
149 a function of time. If the CRF is a continuous and differentiable function of the time, it is possible to  
150 describe the instantaneous rate of recovery as the *resilience density function* (RDF)  $q$  defined as the time  
151 derivative of the CRF (in analogy with the definition of the *probability density function* (PDF) of a  
152 random variable). If the CRF is not continuous and differentiable, it is possible to define a *resilience*  
153 *mass function* (RMF) that describes the instantaneous change of the recovery occurring as a step-wise  
154 function (in analogy with the probability mass function (PMF) of a random variable).

155 Based on these definitions, Sharma et al. (2018) introduced a set of resilience metrics to capture the  
156 specific characteristics of the recovery process in analogy to the moments of random variables. In  
157 analogy to the mean and standard deviation of a random variables, Sharma et al. (2018) defined the *center*  
158 *of resilience*  $\rho$  and the *resilience bandwidth*,  $\chi$  as two fundamental partial descriptions. The definition  
159 of these metrics is general and can be systematically extended to higher order metrics to fully characterize any  
160  $Q(t)$ . The metric  $\rho$  defines where the recovery curve is centered with respect to the time of the initial



161 shock. In addition, Sharma et al. (2018) also introduced the resilience quantile,  $\rho_\omega$ , which is the time instant  
 162 corresponding to the  $\omega^{th}$  ( $0 \leq \omega \leq 1$ ) quantile of the CRF. Mathematically, the recovery quantile can be  
 163 written as  $\rho_\omega := \min\{t \in [0, T_R] : \omega \leq [Q(t)/Q(T_R)]\}$ , where  $T_R$  is the recovery time (i.e., the time needed  
 164 to reach a desired final level of  $Q(t)$ .) The metric  $\chi$  gives the breath of the recovery process, small  
 165 values represent a situation in which a significant percentage of the recovery process is completed over  
 166 a short period concentrated around  $\rho$ . By contrast, a large value of  $\chi$  captures a recovery process  
 167 spread over a prolonged period of time. To further characterize the recovery curve, Sharma et al. (2018)  
 168 also introduced the skewness of the recovery,  $\psi$ . If  $\psi = 0$  the recovery progress is symmetric about  $\rho$   
 169 (i.e., the recovery process has the same pace before and after  $\rho$ .) If  $\psi < 0$ , the process is slower during  
 170 the initial phases (i.e., in the interval  $[0, \rho]$ ) and then it becomes faster over the next period  $(\rho, T_R]$ ,  
 171 which is the most typical case for recovery processes that include a lengthy planning phase in the post-  
 172 disruption period. If planning is done ahead of the disruptive event as a pre-disruption planning and  
 173 preparation, then  $\psi > 0$ . In this case, the recovery progress picks up quickly and the relative most time-  
 174 consuming portion is the completing of the repairs/reconstruction (i.e., faster in the interval  $[0, \rho]$ , and  
 175 slower in the interval,  $(\rho, T_R]$ ). Finally, to uniquely and fully characterize the recovery curve, Sharma  
 176 et al. (2018) also introduced higher order partial descriptors (in analogy with higher order moments or a  
 177 random variable). However, in most cases,  $\rho$  and  $\chi$  are sufficient to characterize a recovery process.  
 178 Based on Sharma et al. (2018), we can write the center of resilience as

$$\rho := \frac{\int_0^{T_R} \tau q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (1)$$

179 Likewise, we can write the resilience bandwidth as

$$\chi^2 := \frac{\int_0^{T_R} (\tau - \rho)^2 q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (2)$$

180 Finally, as a generalization, the  $n^{th}$  recovery moment can be written as

$$\rho^{(n)} := \frac{\int_0^{T_R} \tau^n q(\tau) d\tau}{\int_0^{T_R} q(\tau) d\tau} \quad (3)$$

181

## 182 **Proposed Probabilistic Formulation**

183 This section explains the proposed probabilistic formulation to develop recovery curves accounting for  
 184 the relevant uncertainties and estimate the probability of reaching or exceeding a target level of  
 185 functionality at any time.

186 Work progress for civil structures and infrastructure typically advances continuously, or near-  
 187 continuously, over time (Klinger and Susong 2006; Gardoni et al. 2007), whereas, the system state  
 188 changes only at completion of a group of activities (Sharma et al. 2018). As a result, the functionality of  
 189 a system typically changes in a step-wise fashion with discrete increments at the completion of each  
 190 group of activities. Besides civil structures and infrastructure, or more generally, engineering systems,

191 the recovery might be a continuous function of time when we deal with the restoration of natural systems,  
192 such as the recovery and resilience of tropical forests (Cole et al. 2014;. van Leeuwen 2008), or the  
193 Gross Domestic Product (GDP) as a monetary measure of the market value of all final goods and  
194 services produced in a period to quantify the economic performance of a whole country or region.  
195 The proposed methodology is general and allows to estimate processes described either by discrete or  
196 continuous recovery curves. The proposed formulation has the following four steps: Step 1: Obtaining  
197 the joint PDF of the Sharma et al.'s resilience metrics, Step 2: Obtaining the joint PDF of the model  
198 parameters of the recovery curve, Step 3: Obtaining point and predictive estimates of the recovery curve  
199 and confidence bounds, Step 4: Estimating the probability of reaching or exceeding a target percentile of  
200 interest of the ultimate desired state, and Step 5: Updating the model parameters as new data become  
201 available.

## 202 **Obtaining the joint PDF of the resilience metrics**

203 The first step of the proposed formulation consists in collecting historical recovery data for the system  
204 of interest and with them obtaining estimates of the statistics (means, standard deviations and  
205 correlation coefficients) and marginal PDFs of Sharma et al.'s resilience metrics (reviewed in Section  
206 2). Based on the obtained statistics and marginal PDFs, we can then construct the joint PDF of the  
207 resilience metrics using a Nataf formulation (Liu and Der Kiureghian 1986). Let  $f_p(\rho)$ ,  $f_x(\chi)$ , up  
208 to  $f_{p^{(n)}}(\rho^{(n)})$  be the marginal PDFs of Sharma et al.'s resilience metrics, and let  $r_{ij}$  be the estimated  
209 correlation coefficients between the  $i^{th}$  and the  $j^{th}$  resilience metric. Following the Nataf  
210 formulation, the joint PDF of the resilience metrics is

$$f_p(\mathbf{p}) = f_p(\rho) f_x(\chi) \dots f_{p^{(n)}}(\rho^{(n)}) \frac{\varphi_n(\mathbf{z}, \mathbf{R}')}{\varphi(z_1) \varphi(z_2) \dots \varphi(z_n)} \quad (4)$$

211 where  $z_i = \Phi^{-1}[F_{p_i}(\rho_i)]$ ,  $\varphi(\cdot)$  is the standard normal PDF,  $\varphi_n(\mathbf{z}, \mathbf{R}')$  is the n-dimensional standard  
 212 normal PDF with correlation matrix  $\mathbf{R}'$ . The elements  $r_{ij}'$  in the correlation matrix  $\mathbf{R}'$  are obtained  
 213 based on the correlation coefficients  $r_{ij}$  through the integral

$$\begin{aligned} r_{ij} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{\rho^{(i)} - \mu_i}{\sigma_i} \right) \left( \frac{\rho^{(j)} - \mu_j}{\sigma_j} \right) f_{\rho^{(i)}}(\rho^{(i)}) f_{\rho^{(j)}}(\rho^{(j)}) \\ &\quad \times \frac{\varphi_2(z_i, z_j, r_{ij}')}{\varphi(z_i) \varphi(z_j)} d\rho^{(i)} d\rho^{(j)} \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \frac{\rho^{(i)} - \mu_i}{\sigma_i} \right) \left( \frac{\rho^{(j)} - \mu_j}{\sigma_j} \right) \varphi_2(z_i, z_j, r_{ij}') dz_i dz_j \end{aligned} \quad (5)$$

214 In case historical recovery data for the system of interest are not available, one can choose a  
 215 distribution that either reflect some degree of judgement and experience, or a distribution with  
 216 minimal information (i.e., a noninformative distribution as usually done in Bayesian inference), to  
 217 reflect the fact that little or no information is available a priori. In addition, the Bayesian inference  
 218 discussed later in Section 3.5, can be used to update the state of knowledge every time new knowledge  
 219 becomes available (i.e., recovery data are collected as the recovery unfolds) (Box and Tiao 1992)

## 220 **Obtaining the joint PDF of the model parameters of the recovery curve**

221 The second step consists in introducing parametrized recovery curves to describe the recovery process  
 222 over time. In general, the functional form of the selected parametrized recovery curve may affect the  
 223 time-varying recovery process of a general performance measure. However, one can choose the  
 224 parametrized recovery curve based on engineering judgement and experience of the problem. In  
 225 addition, one can use flexible functional forms, such that the recovery curve can be updated as the actual

226 recovery progresses and data become available. Example of parametrized recovery curves can be found  
 227 in Gardoni et al. (2007) and Ayyub (2015). A parametrized CRF describes the time-varying recovery  
 228 process of a general performance measure in the following form:

$$T[Q(\Theta, \tau)] = Q(\theta, \tau) + \sigma \varepsilon \quad (6)$$

229 where,  $T[\cdot]$  is a transformation function,  $\Theta = (\theta, \sigma)$ ;  $\theta = (\theta_1, \theta_2, \dots)$  is a vector of unknown model  
 230 parameters associated with  $Q$ , that needs to be estimated; and  $\sigma \varepsilon$  is an additive model error term of  $Q$   
 231 (additivity assumption), in which  $\sigma$  is the standard deviation of the model error, assumed not to depend  
 232 on  $\tau$  (homoskedasticity assumption), and  $\varepsilon$  is a standard normal random variable (normality  
 233 assumption). The additivity, normality and homoskedasticity assumptions typically can be satisfied  
 234 using an appropriate variance stabilizing transformation from the parametrized family of transformations  
 235 introduced by Box and Cox (1964). We then define the joint PDF of the unknown model parameters  
 236  $\Theta$  based on the joint PDF of the resilience metrics (Hogg et al. 2012; Ang and Tang 2006). Let the set  
 237  $(\rho, \chi, \dots, \rho^{(n)})$  have a jointly continuous distribution with PDF  $f_{\mathbf{p}}(\rho, \chi, \dots, \rho^{(n)})$  on a defined support  
 238 set  $C$ . According to the definition of the resilience metrics, the resilience metrics are a function of  
 239 the model parameters  $\theta = (\theta_1, \dots, \theta_n)$  in the support set  $D$ , such that  $\rho = k_1(\theta_1, \dots, \theta_n)$ ,  $\chi = k_2(\theta_1, \dots, \theta_n)$ ,  
 240 up to  $\rho^{(n)} = k_n(\theta_1, \dots, \theta_n)$ , where the generic  $i^{th}$  function  $k_i(\theta_1, \dots, \theta_n)$  represents the expression of the  $i^{th}$   
 241 resilience metric  $\rho^{(i)}$  based on Equations (1)-(3), after introducing a parametrized recovery curve  
 242 according to Equation (6). We first evaluate the  $n \times n$  Jacobian given by

$$J = \begin{bmatrix} \frac{\partial \rho}{\partial \theta_1} & \frac{\partial \rho}{\partial \theta_2} & \cdots & \frac{\partial \rho}{\partial \theta_n} \\ \frac{\partial \chi}{\partial \theta_1} & \frac{\partial \chi}{\partial \theta_2} & \cdots & \frac{\partial \chi}{\partial \theta_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \rho^{(n)}}{\partial \theta_1} & \frac{\partial \rho^{(n)}}{\partial \theta_2} & \cdots & \frac{\partial \rho^{(n)}}{\partial \theta_n} \end{bmatrix} \quad (7)$$

243 Then, let us consider two subsets of the supports, respectively named  $A$  and  $B$ , where  $B$  denotes  
 244 the mapping of  $A$  under a one-to-one transformation. Due to the conservation of the probability the  
 245 event  $\{(\rho, \chi, \dots, \rho^{(n)}) \in A\}$  is equivalent to the event  $\{(\theta_1, \theta_2, \dots, \theta_n) \in B\}$ . Therefore, we can write

$$\mathbb{P}[(\theta_1, \theta_2, \dots, \theta_n) \in B] = \mathbb{P}[(\rho, \chi, \dots, \rho^{(n)}) \in A] = \int \cdots \int_A f_{\mathbf{p}}(\rho, \chi, \dots, \rho^{(n)}) d\rho d\chi \dots d\rho^{(n)} \quad (8)$$

246 We change variables of integration by writing  $\theta_1 = h_1(\rho, \chi, \dots, \rho^{(n)})$ ,  $\theta_2 = h_2(\rho, \chi, \dots, \rho^{(n)})$ , up to  
 247  $\theta_n = h_n(\rho, \chi, \dots, \rho^{(n)})$  such that

$$\begin{aligned} \int \cdots \int_A f_{\mathbf{p}}(\rho, \chi, \dots, \rho^{(n)}) d\rho d\chi \dots d\rho^{(n)} = \\ \int \cdots \int_B f_{\theta}[k_1(\theta_1, \theta_2, \dots, \theta_n), k_2(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| d\theta_1 d\theta_2 \dots d\theta_n \end{aligned} \quad (9)$$

248 Therefore, for every set  $B \subseteq D$ , we can write

$$\begin{aligned} \mathbb{P}[(\theta_1, \theta_2, \dots, \theta_n) \in B] = \\ \int \cdots \int_B f_{\theta}[k_1(\theta_1, \theta_2, \dots, \theta_n), k_2(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| d\theta_1 d\theta_2 \dots d\theta_n \end{aligned} \quad (10)$$

249 We conclude that the joint PDF of interest  $f_{\theta}(\theta_1, \theta_2, \dots, \theta_n)$  is

$$f_{\theta}(\theta_1, \theta_2, \dots, \theta_n) = \begin{cases} f_{\theta}[k_1(\theta_1, \theta_2, \dots, \theta_n), \dots, k_n(\theta_1, \theta_2, \dots, \theta_n)] |J| & (\theta_1, \theta_2, \dots, \theta_n) \in D \\ 0 & \text{elsewhere} \end{cases} \quad (11)$$

Eq. (11) represents the state of knowledge on the model parameters  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$ . We can now derive the expected recovery curve and the related uncertainties based on the distribution of the parameters in Eq. (6).

### **Obtaining point and predictive estimates of the recovery curve and confidence bounds**

Different estimates of the recovery curves can be obtained depending on how we treat the model parameters. Following Gardoni et al. (2002), we can obtain point estimates or predictive estimates. A point estimate of the recovery curve is obtained using a point estimate of  $\hat{\boldsymbol{\theta}}$ , in place of  $\boldsymbol{\theta}$ . In general, the mean value of  $\boldsymbol{\theta}$  or the maximum likelihood estimate (MLE)  $\boldsymbol{\theta}_{MLE}$  can be used. However, the point estimate does not incorporate the epistemic (statistical) uncertainties in the model parameters  $\boldsymbol{\theta}$ . To incorporate these uncertainties, we need to consider  $\boldsymbol{\theta}$  as random variable. The predictive estimate of the recovery curve is then the expected value of the recovery curve over the space of the model parameters, i.e.,

$$\tilde{Q}(\tau) = \int Q(\boldsymbol{\theta}, \tau) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \quad (12)$$

This estimate incorporates the epistemic uncertainties in the model parameters  $\boldsymbol{\theta}$ . In addition, we can construct probability bounds on the recovery curve using the PDF of the model parameters, as illustratively shown in Figure 1.

### **Estimating the probability of reaching or exceeding a target percentile of interest of the ultimate desired state**

Once we obtained  $Q(\boldsymbol{\theta}, \tau)$ , we can estimate the probability of reaching or exceeding a target value of  $Q$  by reliability analysis (Ditlevsen and Madsen 1996; Gardoni 2017). We can write a limit-state function  $g(\boldsymbol{\theta}, \tau)$  as

$$g(\Theta, \tau) = Q(\Theta, \tau) - Q_T \quad (13)$$

270 where  $Q_T$  is a level of performance we desire to reach or exceed, expressed as a percentile of the ultimate  
 271 desired state,  $Q_\infty$ . Mathematically, we can write the probability that the recovery process is above  $Q_T$   
 272 at a time  $\tau$ ,  $H(\Theta, \tau)$ , as

$$H(\Theta, \tau) = 1 - \mathbb{P}[g(\Theta, \tau) \leq 0 | \tau] \quad (14)$$

273 Figure 2 shows a conceptual representation of  $Q(\Theta, \tau)$  and the corresponding  $H(\Theta, \tau)$  over time.  
 274 Following Gardoni et al. (2002), we can construct a point estimate of  $H(\tau)$ , a predictive estimate as well  
 275 as confidence bounds as previously proposed for the recovery curve. Hence, we define the point estimate  
 276 of the probability that the recovery process is above  $Q_T$  at a time  $\tau$  using a point estimate of  $\hat{\Theta}$ , in place  
 277 of  $\Theta$ , whereas the predictive estimate  $\tilde{H}(\tau)$  is defined taking the expected value of the quantity of  
 278 interest over the space of the model parameters, in the same way as previously showed for the recovery  
 279 curve. Furthermore, we obtain confidence bounds on the estimate in Eq. (14). We can define the  
 280 reliability index as

$$\beta(\Theta, \tau) = \Phi^{-1}[H(\Theta, \tau)] \quad (15)$$

281 where  $\Phi^{-1}(\cdot)$  indicates the inverse of the standard normal CDF. Following Gardoni et al. (2002), the  
 282 variance of  $\beta(\Theta, \tau)$  can be estimates as

$$\sigma_\beta^2(\tau) \approx \nabla_\Theta \beta(\tau) \Sigma_{\Theta\Theta} \nabla_\Theta \beta(\tau)^T \quad (16)$$



283 where  $\nabla_{\Theta}\beta(\tau)$  is the gradient of  $\beta(\Theta, \tau)$  evaluated at the mean value and  $\Sigma_{\Theta\Theta}$  is the estimated covariance  
 284 matrix. The gradient vector  $\nabla_{\Theta}\beta(\tau)$  is obtained by performing a FORM (First-Order Reliability  
 285 Method) analysis (Ditlevsen and Madsen 1996). Therefore, we obtain

$$\left\{ \Phi\left[-\tilde{\beta}(\tau) - \sigma_{\beta}(\tau)\right], \Phi\left[-\tilde{\beta}(\tau) + \sigma_{\beta}(\tau)\right] \right\} \quad (17)$$

286 as one standard deviation bounds, where  $\tilde{\beta}(\tau) = \Phi^{-1}[\tilde{H}(\tau)]$ . The bounds represent approximately 15%  
 287 and 85% probability levels.

### 288 **Updating the model parameters as new data become available**

289 Finally, Bayesian inference can be used to update the model parameters  $\Theta$  combining existing  
 290 information with new information as it might become available during the actual recovery process  
 291 (Gardoni et al. 2007). Steps 3 and 4 can then be repeated to obtain updated recovery curves and updated  
 292 probabilities of reaching or exceeding a desired level  $Q_T$ . Mathematically, we can write the posterior  
 293 distribution  $f''(\Theta)$  that includes the updated status of knowledge about  $\Theta$  as (Box and Tiao 1992)

$$f''(\Theta | \mathbf{Q}) = \kappa L(\Theta | \mathbf{Q}) f'(\Theta) \quad (18)$$

294 where  $L(\Theta | \mathbf{Q})$  is the likelihood function that contains the objective information on  $\Theta$  in a set of  
 295 observations,  $f'(\Theta)$  is the prior distribution, reflecting the state of knowledge about  $\Theta$  prior to obtaining  
 296 the observations  $\mathbf{Q} = (Q_1, \dots, Q_m)$ , and  $\kappa = \left[ \int L(\Theta | \mathbf{Q}) f'(\Theta) d\Theta \right]^{-1}$  is a normalizing factor.

297 The prior distribution includes the status of knowledge based on previous experiences,  
 298 engineering judgments, and/or past data. The likelihood function is proportional to the conditional  
 299 probability of observing the recorded data  $\mathbf{Q} = (Q_1, \dots, Q_m)$  for given values of the parameters  $\Theta$ . In

300 general, the likelihood function permits to include lower, upper and equality data (see Gardoni et al.  
 301 2002). A lower bound datum is defined as an observation of  $Q$  that is larger than a certain value  $Q_i$  at  
 302 time  $\tau$ ; an upper bound datum is defined as an observation that is smaller than a certain value  $Q_i$  at time  
 303  $\tau$ ; an equality datum is defined as the value of  $Q$  recorded at time  $\tau$ . Following Gardoni et al. (2002),  
 304 the likelihood function can be written as

$$\begin{aligned}
 L(\boldsymbol{\theta}, \sigma) &\propto \prod_{\substack{\text{equality} \\ \text{data}}} \mathbb{P}[\sigma \varepsilon_i = Q_i - Q(\boldsymbol{\theta}, \tau)] \\
 &\times \prod_{\substack{\text{lower bound} \\ \text{data}}} \mathbb{P}[\sigma \varepsilon_i > Q_i - Q(\boldsymbol{\theta}, \tau)] \\
 &\times \prod_{\substack{\text{upper bound} \\ \text{data}}} \mathbb{P}[\sigma \varepsilon_i < Q_i - Q(\boldsymbol{\theta}, \tau)]
 \end{aligned} \tag{19}$$

305 Based on the normality assumption, we can then write

$$\begin{aligned}
 L(\boldsymbol{\theta}, \sigma) &\propto \prod_{\substack{\text{equality} \\ \text{data}}} \left\{ \frac{1}{\sigma} \varphi \left[ \frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\} \\
 &\times \prod_{\substack{\text{lower bound} \\ \text{data}}} \left\{ \Phi \left[ -\frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\} \\
 &\times \prod_{\substack{\text{upper bound} \\ \text{data}}} \left\{ \Phi \left[ \frac{Q_i - Q(\boldsymbol{\theta}, \tau)}{\sigma} \right] \right\}
 \end{aligned} \tag{20}$$

306 where  $\varphi(\cdot)$  is the standard normal PDF, and  $\Phi(\cdot)$  is the standard normal CDF.

307 Eqs. (15)-(20) can be used every time additional information is available to update the model  
 308 parameters. For instance, when a set of samples  $\mathbf{Q}_1$  is available, we can write

$$f''(\boldsymbol{\Theta} | \mathbf{Q}_1) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_1) f'(\boldsymbol{\Theta}) \tag{21}$$

309 Then, let us suppose another set of samples  $\mathbf{Q}_2$  is available and this is independent from the previous  
 310 one, we can update the posterior PDF evaluated in Eq. (21) such that

$$f'''(\boldsymbol{\Theta} | \mathbf{Q}_1, \mathbf{Q}_2) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_1) L(\boldsymbol{\Theta} | \mathbf{Q}_2) f''(\boldsymbol{\Theta}) \propto L(\boldsymbol{\Theta} | \mathbf{Q}_2) f''(\boldsymbol{\Theta} | \mathbf{Q}_1) \quad (22)$$

311 Generally, if  $n$  independent set of observations are available, we can write

$$\begin{aligned} f^{(k+1)}(\boldsymbol{\Theta} | \mathbf{Q}_1, \dots, \mathbf{Q}_k) &\propto L(\boldsymbol{\Theta} | \mathbf{Q}_k) f^{(k)}(\boldsymbol{\Theta} | \mathbf{Q}_1, \dots, \mathbf{Q}_{k-1}) \\ k &= 2, \dots, n \end{aligned} \quad (23)$$

312 Field measurements can often be inexact and include measurement errors (Gardoni et al. 2002;  
 313 Murphy et al. 2011). Following Gardoni et al. (2002), measurement errors can be incorporated in the  
 314 updating process. To incorporate the measurements errors in the updating process, we assume that  
 315  $Q_i = \hat{Q}_i + e_{Q_i}$  is the true value of the  $i^{th}$  observation, where  $\hat{Q}_i$  represents the measured value and  $e_{Q_i}$  is  
 316 the measurement error. We also assume that  $e_{Q_i}$  has zero mean, which reflects that the measurements  
 317 have been corrected from any systematic errors, and variance  $s_i^2$ , which represents the uncertainties  
 318 inherent in the measurements. For the equality data we have  $\hat{Q}_i + e_{Q_i} = Q(\boldsymbol{\theta}, \tau) + \sigma \varepsilon_i$ , for the lower bound  
 319 data we have  $\hat{Q}_i + e_{Q_i} < Q(\boldsymbol{\theta}, \tau) + \sigma \varepsilon_i$ , and for the upper bound data we have  $\hat{Q}_i + e_{Q_i} > Q(\boldsymbol{\theta}, \tau) + \sigma \varepsilon_i$ .  
 320 Therefore, the conditions for the three type of data can be, respectively, written as  $\sigma \varepsilon_i - e_{Q_i} = \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ ,  
 321  $\sigma \varepsilon_i - e_{Q_i} > \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ , and  $\sigma \varepsilon_i - e_{Q_i} < \hat{Q}_i - Q(\boldsymbol{\theta}, \tau)$ . The left-hand sides of these expressions are a  
 322 normal random variable with zero mean and variance  $\hat{\sigma}(\boldsymbol{\theta}, \sigma) = \sigma^2 + s_i^2$ . Hence, in presence of  
 323 measurement errors, the likelihood function is

$$\begin{aligned}
L(\boldsymbol{\theta}, \sigma) \propto & \prod_{\substack{\text{equality} \\ \text{data}}} \left\{ \frac{1}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \varphi \left[ \frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{lower bound} \\ \text{data}}} \left\{ \Phi \left[ -\frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\} \\
& \times \prod_{\substack{\text{upper bound} \\ \text{data}}} \left\{ \Phi \left[ \frac{\hat{Q}_i - Q(\boldsymbol{\theta}, \tau)}{\hat{\sigma}(\boldsymbol{\theta}, \sigma)} \right] \right\}
\end{aligned} \tag{24}$$

### Example 1: Recovery curves for an example bridge

This section presents the proposed formulation considering the recovery process of a typical RC bridge subject to seismic excitations. The first example demonstrates the application of the formulation in a realistic case related to civil structures in support of risk and resilience analysis.

We previously discussed that for civil structures the work progress is a continuous, or near-continuous, function, whereas a discrete function describes the performance indicators (e.g., functionality) with jumps when a group of activities is completed. This section illustrates the proposed formulation applied to a RC bridge. Figure 3 shows the configuration of the considered (single column, single bent) testbed bridge from Kumar and Gardoni (2014a) and Jia et al. (2017). Following the proposed formulation, we obtain the estimates of the first two resilience metrics to describe the recovery process of the selected engineering system. Figure 4 shows the pair  $(\rho, \chi)$  used in this example, and their correlation. Based on the data in Figure 4, we assume that both  $\rho$  and  $\chi$  follow a lognormal distribution, whose parameters are listed in Table 1. Then, based on the estimated coefficient of correlation and the marginal PDFs we can construct the joint PDF of the resilience metrics as described in Section 3.1. Next, we introduce a parametrized recovery curve to describe the changes of a selected performance measure over time. The performance indicator considered in this example is the reliability index  $\beta$ . Moreover, in this example we assume that there is only one recovery step that restores the

341 reliability of the bridge, as described in Sharma et al. (2018). Consequently, we consider the recovery  
 342 curve in the following form:

$$Q_1(\Theta, \tau) = \begin{cases} \theta_1 & \tau < \theta_2 \\ Q_\infty & \tau \geq \theta_2 \end{cases} \quad (25)$$

343 where  $\theta_1$  is the residual reliability index after the occurrence of the hazard, and before the completion of  
 344 the recovery; and  $\theta_2$  is the time at which the reliability index reaches the ultimate desired value  $Q_\infty$ . To  
 345 model the reliability, we consider the reliability-based resilience metrics coming from previous analyses.  
 346 Thus, we do not need to model the occurrence of earthquake mainshock-aftershocks sequence and their  
 347 impact on structural properties because the resilience metrics capture all these information. Based on the  
 348 definition of the resilience metrics in Eqs. (1) and (2), the parameters  $\theta_1$  and  $\theta_2$  can be written as a  
 349 function of the resilience metrics. Specifically, Figure 5 shows the RMF of the adopted parametrized  
 350 curve. Following the proposed methodology, we compute the joint PDF of the model parameters and  
 351 the corresponding expected recovery process in terms of the reliability index  $\beta$ . We observe that the  
 352 number of resilience metrics needed to adopt the formulation is at least equal to the number of the model  
 353 parameters of the selected parametrized recovery curve. Therefore, considering the possibility of having  
 354 a drop in the functionality, during the recovery process due to aftershocks, would require implying higher  
 355 resilience metrics. Next, we estimate the probability of reaching or exceeding a target value of  
 356 functionality at any time setting, for instance,  $Q_T = 3.5$ .

#### 357 **Initial estimate of the recovery curve and corresponding probability of exceeding the target** 358 **value of functionality**

359 As previously discussed, in this example we assume that there is only one recovery step that restores the  
 360 reliability of the bridge. Nevertheless, we can also estimate the behavior of the system toward the desired  
 361 value of the functionality at the end of the recovery in terms of the mean value of the different probable

recovery curves. Figure 6 shows the expected changes of the instantaneous reliability index over time. Adopting the reliability-based definitions for the damage state proposed in Sharma et al. (2018), the initial damage level is moderate. Figure 7 shows the probability of exceeding the target value of  $Q_T$ . The figure also presents the confidence band due to the statistical uncertainty in  $\Theta$ . Based on the expected initial value of the reliability index, we can observe that the probability of exceeding the target value of functionality,  $Q_T = 3.5$ , at time  $\tau = 25$  days, is equal to 0.5. The observed result matches the results provided in Sharma et al. (2018), where the expected value of the time to recover is approximately 26 days when the initial damage level is moderate.

#### **Updated estimate of the recovery curve and corresponding probability of exceeding the target value of functionality**

We assume that after the occurrence of the hazard we collect data on the state of damage for the first 10 days, and then we update the model parameters. In the presented example, we assume that inspection data are collected after the occurrence of the hazard. Specifically, we assume that a qualitative description of the damage state indicates moderate damage following the definition in ATC-38 (ATC 2000) and Bai et al. (2009) (i.e., “Repairable structural damage has occurred. The existing elements can be repaired in place, without substantial demolition or replacement of elements”). Then, the qualitative definition of the damage state is mapped into a reliability-based definition in terms of the corresponding reliability index  $\beta$  (i.e.,  $1.5 \leq \beta \leq 2.5$ ) following in Sharma et al. (2018). As a result, we obtain the new expected changes in the reliability index and the corresponding time-varying probability of exceeding the same target value of functionality, as shown in Figure 8 and 9. Figure 8 shows the expected changes of the reliability index over time, after updating the model parameters based on the observed data. First, we can observe that the recovery process follows the observed data in terms of its mean; then, the

Bayesian inference also reduces the relevant uncertainties. The probability of exceeding the target value of functionality reflects both the effects of the Bayesian inference.

### **Example 2: Population relocation after a seismic event**

The second example shows the application of the formulation in a scenario where historical recovery data are not available. In this example, we consider the population relocation of the city of Seaside, OR, after the occurrence of an earthquake originated from the Cascadia Subduction Zone. We consider a seismic event of magnitude  $M_w = 7.0$ , located 25 km southwest of the city. Since no data are available, we consider a noninformative PDF of the first resilience metric  $\rho$  in the form  $f_p(\rho) = 1/\rho$ ,  $\rho > 0$ , which reflects the fact that little is known a priori. We consider a parametrized S-shape recovery curve proposed in Gardoni et al. (2007) in the following form:

$$Q_2(\Theta, \tau) = 1 - Q_R + (Q_\infty - Q_R) \left( \frac{\tau}{\theta_1} \right)^2 \left[ 3 - 2 \left( \frac{\tau}{\theta_1} \right) \right] \quad \tau \leq \theta_1 \quad (26)$$

where  $Q_R$  represents the percentage of population dislocation at time  $t_{0^+}$  (i.e., after the occurrence of the seismic event),  $Q_\infty$  represents the percentage of the population that relocates at the end of the recovery, and  $\theta_1$  is the time at which the recover ends.

Ground Motion Prediction Equations (Boore and Atkinson, 2008) are used to obtain maps of the seismic intensity measure at the residential building location. Next, we perform a building damage analysis using different fragility functions (e.g., HAZUS-MH (FEMA 2015), and Steelman et al. 2007). Then, we estimate the initial percentage of population dislocation due to structural damage using a logistic regression model (Lin 2009). For the purpose of this example, we assume that the entire population returns to their homes at the end of the recovery (i.e.,  $Q_\infty = 100\%$ .)

Based on the definition of the resilience metric in Eq. (1), the parameter  $\theta_1$  can be written as a function of the resilience metric  $\rho$ . Therefore, we obtain the PDF  $f_{\theta_1}(\theta_1)$  according to Eq. (11), as well as the corresponding estimate of  $Q_2$  over time (shown in Figure 10(b).) More generally,  $Q_\infty$  could also be taken as a parameter (i.e.,  $Q_\infty = \theta_2$ ). In this case, we would obtain the joint PDF  $f_\theta(\theta_1, \theta_2)$  again using Eq. (11) given  $\rho$  and  $\chi$ .

After the occurrence of the seismic event, recovery activities start to retrieve structures and infrastructure functionality, thereby we can observe the population returning to their homes. For the purpose of this example, we assumed that data on the population relocation are available at given time-steps. The relocation data at different times can be used to obtain the corresponding values of  $Q_2$  (shown by dots in Figure 10(b).) Using these values of  $Q_2$ , we obtain the new expected value of  $Q_2$  as a function of time, as shown in Figure 10(b). In particular, we can observe that the uncertainties in the initial estimate in Figure 10(a) reflect the fact that little is known in terms of the duration of the recovery. In Figure 10(b) the confidence band is significantly smaller around the mean line indicating that the values of  $Q_2$  used to update the mean prediction also reduce the prediction uncertainty.

Finally, Figure 11 shows the probability that the population dislocation is higher than 25% of the total population (i.e.,  $Q_T = 0.25$ ) before and after we update the recovery curve, including the confidence band due to the statistical uncertainty in  $\Theta$ . We can see that the information used to update the model parameter can also adjust the prediction in terms of the probability of exceeding a target level of functionality, as well as reduce the prediction uncertainty.



## **Conclusions**

The paper proposed a formulation to (i) predict the recovery curves that define the recovery of engineering systems subject to a hazard, and (ii) estimate the probability of reaching or exceeding a target value of a selected performance indicator at any given time. The formulation uses the resilience metrics defined in Sharma et al. (2018), which quantify the resilience of systems and form a complete set of partial descriptors that characterize the recovery curve of the system of interest. To evaluate the recovery process of an engineering system, the paper proposed to use the probability density function (PDF) of the resilience metrics, defined based on historical data, to obtain the PDF of the model parameters that define the recovery curve. The proposed formulation incorporates the Bayesian inference to update the estimates of the unknown parameters when additional information is available. The paper illustrated the implementation of the proposed formulation by predicting the recovery of a single-bent, single-column reinforced concrete (RC) bridge subject to seismic damage, and the population relocation after the occurrence of a seismic event when no data on the duration of the recovery are available a priori. The proposed formulation is general and suited to applications such as risk analysis and mitigation, and resilience-based design.

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557	<b>Figures</b>
558	<b>Figure 1.</b> 90% Confidence bounds on the estimate of the recovery curve
559	<b>Figure 2.</b> Conceptual representation of probability of reaching or exceeding a target value of functionality at any time
560	<b>Figure 3.</b> The considered RC bridge (adapted from Jia et al. 2017)
561	<b>Figure 4.</b> Correlation between the adopted resilience metrics
562	<b>Figure 5.</b> RMF of the adopted parametrized recovery curve
563	<b>Figure 6.</b> Mean value and corresponding 95% confidence band of the recovery curve
564	<b>Figure 7.</b> Probability of exceeding the selected value of $Q_T = 3.5$
565	<b>Figure 8.</b> Mean value and corresponding 95% confidence band of the recovery curve after updating the model parameters
566	<b>Figure 9.</b> Probability of exceeding the selected value of $Q_T = 3.5$ after updating the model parameters
567	<b>Figure 10.</b> Mean value and corresponding 95% confidence band of the population dislocation recovery curve (a) before and
568	(b) after updating the model parameter
569	<b>Figure 11.</b> Probability of population dislocation (i.e., $Q_T = 25\%$ ) (a) before and (b) after updating the model parameter
570	

571     **Tables**

572     **Table 1.** Distribution parameters of the resilience metrics

Resilience metric	$\lambda$	$\xi$
$\rho$	-0.94	0.22
$\chi$	1.14	0.20

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