

New trends towards enhanced structural efficiency and aesthetic potential in tall buildings: The case of diagrids

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New trends towards enhanced structural efficiency and aesthetic potential in tall buildings: The case of diagrids / Scaramozzino, D.; Lacidogna, G.; Carpinteri, A.. - In: APPLIED SCIENCES. - ISSN 2076-3417. - STAMPA. - 10:3917(2020), pp. 1-31. [10.3390/app10113917]

Availability:

This version is available at: 11583/2838332 since: 2020-07-04T13:21:45Z

Publisher:

MDPI

Published

DOI:10.3390/app10113917

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1 Review

2 New trends towards enhanced structural efficiency 3 and aesthetic potential in tall buildings: The case of 4 diagrids

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10 Received: date; Accepted: date; Published: date

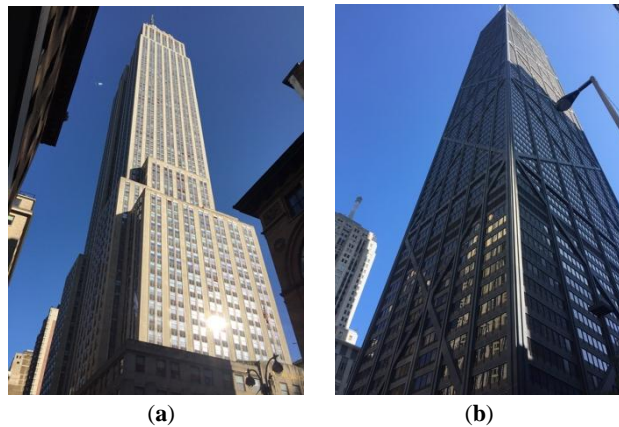
11 **Abstract:** Due to the increasing number of people and activities within the cities, tall buildings are
12 exploited worldwide to address the need for new living and commercial spaces, while limiting the
13 amount of used land. In the last decades, the design of tall buildings has experienced a remarkable
14 improvement thanks to the development of new computational tools and technological solutions.
15 This has led to the realization of innovative structural systems, like diagrids, which allow to reach
16 high structural performances and remarkable architectural effects. In this paper, a thorough and
17 updated review of diagrid structural systems is provided. Simplified methodologies for the
18 preliminary design and structural analysis are reported. Special attention is also paid to the
19 optimization of the structural response based on the geometrical pattern. A discussion on the effect
20 of local deformability, stability and shear-lag phenomenon is carried out. Results from nonlinear
21 and dynamic analyses for the seismic assessment of diagrid systems are reported, and attention is
22 also paid to the recent research on diagrid nodes. Eventually, an overview of twisted, tapered,
23 tilted and freeform diagrid towers is carried out, with a final mention to hexagrids, another recent
24 evolution of tubular systems for tall buildings.

25 **Keywords:** diagrid; preliminary design; structural analysis; stiffness-based methodology;
26 optimization; hexagrid.
27

28 1. Introduction

29 The evolution of tall buildings has experienced remarkable developments in the last century.
30 The first buildings reaching few tens of stories were firstly built in the United States in the late
31 nineteenth century, mostly in the cities of New York and Chicago. At the beginning of the twentieth
32 century, a race for the realization of the tallest skyscrapers took place, which led to the completion of
33 the 102-story tall Empire State Building in 1931 (Figure 1a). Although at that time the height of those
34 buildings was worthy of note, their realization was not achieved by means of significant
35 technological innovations. They usually employed the same steel frames which were adopted for
36 shorter buildings, leading to excessive material usage and quite over-designed solutions [1].
37 Bracings were employed to withstand lateral loads arising from wind pressures and earthquake
38 actions. It was already recognized that lateral actions usually govern the design solutions in tall
39 buildings. In fact, as the building becomes taller, the lateral drifts turn out to be more critical and
40 there is greater demand of suitable structural systems to carry lateral forces. This leads to the
41 dramatic increase of material consumption with the increase in the number of stories, which is
42 usually referred to as the “premium for height” [1,2].

43 Due to aesthetic and constructability considerations, the bracings were usually embedded
 44 within the interior core of the building. Although their shear resistance based on the axial
 45 deformation of the diagonals was beneficial to resist the lateral actions, compared to the mechanism
 46 of the conventional moment resisting frames, their placement within the interior of the building
 47 prevented their effective employment to withstand the overturning moment. Therefore, new
 48 solutions exploiting bracings on the external perimeter of the building were developed. One of the
 49 first examples was the 100-story tall John Hancock Center built in Chicago in 1970 (Figure 1b). The
 50 John Hancock is an example of braced tube, where the mega-diagonals spanning over several stories
 51 are effective to resist the shear and bending moment deriving from lateral actions. The braced tube
 52 was a variation of the typical framed tube, where closely spaced perimeter columns were in charge
 53 of providing the necessary lateral stiffness. The adoption of mega-diagonals on the external surface
 54 offered higher lateral stiffness, while reducing some detrimental phenomena of the framed tube
 55 such as the shear-lag effect. With this new solution, higher number of stories and an overall
 56 enhanced structural performance could be achieved, leading also to important advantages from a
 57 material consumption perspective.
 58



59
60

61 **Figure 1.** Different structural systems for twentieth century tall buildings: (a) Moment resisting
 62 frame: Empire State Building (New York, USA); (b) Braced tube: John Hancock Center (Chicago,
 63 USA). Pictures taken by D. Scaramozzino.

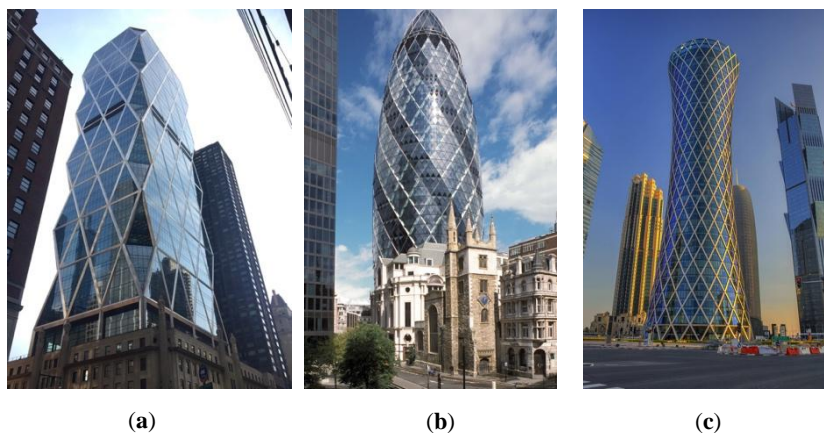
64 Based on the structural behavior of the braced tube, where vertical columns and external
 65 bracings were designed to carry gravity and lateral loads respectively, it was realized that the
 66 external mega-diagonals were able to resist vertical and horizontal loads simultaneously, without
 67 the need of conventional vertical columns. This led to the realization of the diagrid (“diagonal” +
 68 “grid”) structural system.

69 The idea of removing vertical columns and considering only inclined diagonals was not new.
 70 As a matter of fact, the first diagrid structure was realized before the construction of the John
 71 Hancock braced tube, in the 1920s by the Russian architect Vladimir Shukhov, for the realization of a
 72 broadcasting tower in Moscow [3]. The external pattern, made up of a triangular tessellation,
 73 allowed to reduce the wind load while reaching a stable stiff configuration. The first application of
 74 diagrid system in building design occurred in the 1960s, with the completion of the 13-story tall IBM
 75 Building (Pittsburgh, USA). The steel diagrid exoskeleton was integrated with the glazing system
 76 and assisted in the overall stability of the building [3].

77 However, it was not until the early twenty-first century that diagrid systems started to be
 78 thoroughly applied for the design and construction of tall buildings. The first examples are the
 79 Hearst Tower in New York (Figure 2a) and the 30 St. Mary Axe (also known as Swiss Re Tower or
 80 The Gherkin) in London (Figure 2b), both by Sir Norman Foster. These buildings allowed to reach
 81 180 meters and provided the first references for the suitability of diagrid systems in tall building
 82 design. Thanks to the stiff diagrid façades which create a pleasant diamond-like pattern, the Hearst
 83 Tower was realized using 20% less steel than an equivalent conventional moment frame structure

84 [4]. The aerodynamic form of the Swiss Re Tower, obtained through an external free-form diagrid envelope, allowed to reduce the wind action on the building and led to column-free flexible internal
 85 envelope, allowed to reduce the wind action on the building and led to column-free flexible internal
 86 spaces [5]. These two examples already showed the valuable features of diagrids for tall buildings:
 87 enhanced structural performance, saving of material consumption compared to traditional
 88 solutions, and significant aesthetic potential.

89 Many diagrid structures were realized worldwide in the following years, where various forms
 90 and shapes were adopted for the external diagrid façades. Among others, examples worthy of notes
 91 are the Guangzhou Financial Center, the CCTV Tower and the Poly International Plaza in China, the
 92 Tornado Tower (Figure 2c) in Qatar, the Capital Gate in Arab Emirates, and the Bow Tower in
 93 Canada [3]. Nowadays, most of the built diagrid structures are made of steel, mostly due to the
 94 easier and faster construction, simpler joints and less expensive formworks [3]. However, concrete
 95 and composite diagrids are also experiencing an increasing popularity, as they allow to realize even
 96 more complex-shaped diagrid patterns, e.g. the O-14 Building in Dubai [6].
 97



98
99

100 **Figure 2.** Examples of diagrid systems in tall buildings: (a) Hearst Tower (New York, USA), picture
 101 taken by D. Scaramozzino; (b) Swiss Re Tower (London, UK), from <https://larryspeck.com/>; (c)
 102 Tornado Tower (Doha, Qatar), from <http://www.asergeev.com/>.

103 The significant use of diagrid systems in recent tall buildings was mainly due to the following
 104 reasons: (1) high lateral stiffness (thus low lateral deformability), which allowed to reach the lateral
 105 deflection limit target by using lower amount of structural material compared to other conventional
 106 systems; (2) architectural flexibility, allowing a more rational use of the interior space with fewer
 107 columns; (3) modularity, which led to the realization of complex-shaped structures of various forms.
 108 These three points arise from the successful use of the triangular module coupled with the inherent
 109 structural performance of the tubular structure [7,8].

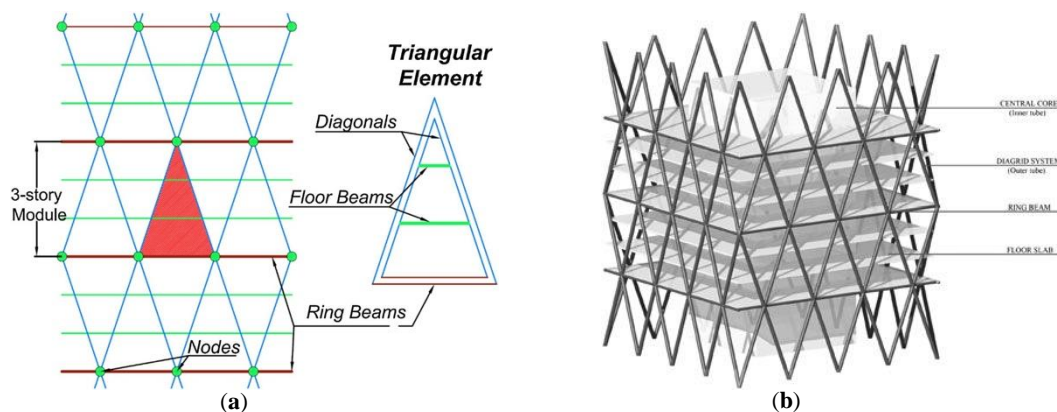
110 The triangular element, which is made up of two inclined diagonals and a ring beam, is the
 111 basic component of the diagrid façade. The diagonals carry the vertical and lateral loads mostly by
 112 axial forces (compression or tension). For this reason, they are usually considered to be pinned at the
 113 panel nodes, as reflected in Figure 3a. Since the inclined diagonals often extend over multiple stories,
 114 the external floor beams of intermediate stories are often supported by the diagonals and
 115 consequently induce slight shear and bending stresses on them. However, in preliminary design
 116 stages, these are usually neglected when compared to the high axial stresses arising from the vertical
 117 and lateral loads on the building. Note that in Figure 3a, a 3-story module is reported as the height of
 118 diagrid module has the same height of the triangular element. In other research works, as will be
 119 shown in further figures in the remaining of the paper, the diagrid module is defined in a way that it
 120 covers two triangular elements.

121 In Figure 3b, the three-dimensional view of the tubular diagrid structure is shown, as reported
 122 in [9]. Usually, the tube-in-tube configuration is found in real diagrid buildings, where an internal
 123 (concrete or steel braced) core is coupled to the external diagrid tube. In preliminary design stages,
 124 the diagrid is usually designed to carry the lateral actions alone, while the internal core is designed

125 only for gravity loads. However, further details about the diagrid-core interaction will be shown in
 126 the remaining of the paper.

127 Hence, it is the combination of the axial resisting mechanism of the triangular element,
 128 characterized by modularity and arrangement flexibility, coupled to the structural efficiency of the
 129 tubular configuration that ultimately led to the success of the diagrids in recent days.

130



131

132

133 **Figure 3.** Fundamental diagrid geometrical features: (a) diagrid module and basic triangular
 134 element, used with permission from Asadi and Adeli [8]; (b) diagrid tubular configuration, used
 135 with permission from Angelucci and Mollaioli [9].

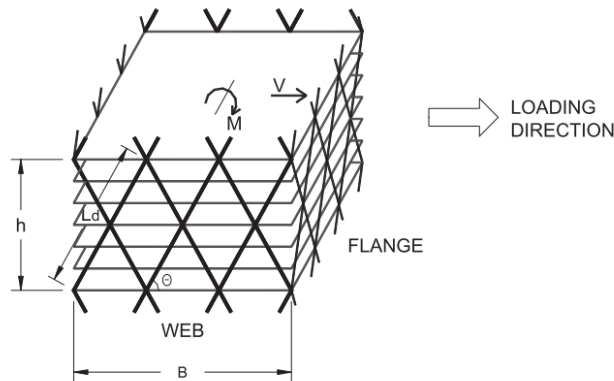
136 In this paper a thorough and up-to-date survey of the research studies on diagrid systems is
 137 reported. In particular, in Section 2 the fundamental stiffness-based approaches for the preliminary
 138 design is described, as firstly proposed by Moon et al. [10] and further developed in the following
 139 years. Moreover, strength-based design methodologies are also discussed and their implication on
 140 the preliminary design is analyzed. Section 3 describes the various methodologies available today,
 141 besides the typical Finite Element Method (FEM), for the structural analysis of diagrid structures in
 142 preliminary design stages, e.g. hand-based calculations, modular and matrix-based methods, etc. In
 143 Section 4, the subject related to the optimization of the diagrid performance based on its geometrical
 144 features is also addressed. This problem has been thoroughly tackled by various researchers in the
 145 last decade with different methodologies and has led to significant results. Section 5 describes the
 146 problem of local structural issues in the design of diagrid tall buildings, e.g. excessive inter-story
 147 drifts and stability of interior columns. The mathematical formulation to identify these problems is
 148 reported, as well as the solutions which have been suggested to tackle them, such as the insertion of
 149 secondary bracing systems (SBSs) as firstly proposed by Montuori et al. [11]. In Section 6, the
 150 shear-lag phenomenon in diagrid tubes is discussed and its influence depending on the diagrid
 151 geometrical parameters is analyzed. Section 7 also discusses the research studies which have dealt
 152 with the nonlinear behavior of diagrid tubes, in order to assess their seismic and robustness
 153 performance. In Section 8, the recent research on diagrid nodes, which represent a crucial
 154 component for the correct behavior of the diagrid, is also reported. Section 9 provides comments
 155 about the new trends regarding unconventional diagrids, which are applied to the realization of
 156 twisted, tilted, tapered and freeform buildings. A further evolution of the grid tubular structure,
 157 which has experienced a significant growth in recent tall building design, is finally presented in
 158 Section 10.

159 2. Simplified approaches for the preliminary design of diagrid tubes

160 The first simplified stiffness-based approach for the preliminary design of diagrid systems has
 161 been proposed by Moon et al. in 2007 [10]. It is based on the evaluation of the shear and bending
 162 stiffness of the diagrid modules, aimed at limiting the lateral deflection of the structure. The building
 163 is treated as a vertical cantilever beam, fixed on the ground and subjected to lateral loads.
 164 Accordingly, the building undergoes horizontal displacements, that depend on the stiffness of the

165 diagrid tubular structure. For sake of the preliminary design, the contribution of the internal cores to
 166 the lateral stiffness of the building is neglected, as they are only designed to carry gravity loads.

167 The elementary diagrid module is depicted in Figure 4. The diagrid module covers an height h
 168 with two triangular elements. The diagonals have a length L_d and their inclination with respect to the
 169 horizontal plane is θ . Depending on the loading direction, each façade can act either as a web or a
 170 flange. V_i and M_i are the shear force and bending moment acting on the level of the i^{th} module. These
 171 are carried by the web and flange diagonals, respectively. Diagonals are assumed to be pinned at
 172 their end, thus carrying only axial force, and remain in the linear elastic regime. In this way, the
 173 cross-sectional areas of the web and flange members are the only factors to obtain in order to
 174 accomplish the preliminary design.
 175



176
 177 **Figure 4.** Scheme of the elementary diagrid module for the definition of the stiffness-based approach
 178 for the diagrid preliminary design. Used with permission from Moon et al. [10].

179 The shear stiffness $K_{T,i}$ and bending stiffness $K_{B,i}$ of the i^{th} diagrid module link the shear force V_i
 180 and bending moment M_i to the module displacement Δu_i and rotation $\Delta \beta_i$, respectively. By applying
 181 compatibility, constitutive and equilibrium equations, $K_{T,i}$ and $K_{B,i}$ are obtained as follows:
 182

$$K_{T,i} = 2N_w \left(\frac{A_{d,w,i} E}{L_d} \right) \cos^2 \theta, \quad (1a)$$

$$K_{B,i} = N_f \left(\frac{B^2 A_{d,f,i} E}{2L_d} \right) \sin^2 \theta, \quad (1b)$$

183 where N_w and N_f is the total number of diagonals in the web and flange façade, respectively, $A_{d,w,i}$
 184 and $A_{d,f,i}$ the cross-sectional area of the web and flange members, E the elastic modulus of the
 185 diagonals and B the web dimension. The displacement Δu_i and rotation $\Delta \beta_i$ are equal to the product
 186 of the module height h and the shear and bending deformation, γ and χ , respectively. Specifying the
 187 desired values of shear and bending deformation, γ^* and χ^* , the member dimensions can be easily
 188 obtained as [10]:
 189
 190

$$A_{d,w,i} = \frac{V_i L_d}{2N_w E h \gamma^* \cos^2 \theta'} \quad (2a)$$

$$A_{d,f,i} = \frac{2M_i L_d}{N_f B^2 E h \chi^* \sin^2 \theta'} \quad (2b)$$

191 Since the horizontal load can act in either direction, the maximum value of the cross-sectional
 192 areas from Eqs. (2a-b) should be assigned to each diagonal which can act as either a web or flange
 193

194 member. The desired values of γ^* and χ^* are specified based on the desired deformation mode of the
 195 building. Assuming that the building sway mechanism is equivalent to the deformation of a
 196 cantilever beam, the lateral deflection at the top of the building $u(H)$ can be written as follows:
 197

$$u(H) = \gamma^*H + \frac{\chi^*H^2}{2}, \quad (3)$$

198 being γ^*H and $\chi^*H^2/2$ the shear and bending contribution, respectively. In order to assess the relative
 199 contribution of bending versus shear deformation, Moon et al. [10] introduces a non-dimensional
 200 parameter s , given by the ratio of the bending to the shear contribution, i.e.:
 201
 202

$$s = \frac{\chi^*H^2}{2\gamma^*H}. \quad (4)$$

203
 204 Combining Eqs. (3-4) and considering that the top lateral displacement is usually specified as a
 205 fraction of the total building height, i.e. $u(H) = H/\alpha$ (α usually being 500), one obtains the following
 206 relations between γ^* , χ^* and s :
 207

$$\gamma^* = \frac{1}{(1+s)\alpha}, \quad (5a)$$

$$\chi^* = \frac{2\gamma^*s}{H} = \frac{2s}{(1+s)\alpha H}. \quad (5b)$$

208
 209 Substituting Eqs. (5a-b) into Eqs. (2a-b), the member sizes can be obtained for the different values of
 210 the parameter s .

211 Adopting different s values leads to different preliminary sizing for the external diagonals.
 212 When s is extremely low, the shear deformation mode of the structure prevails over the bending
 213 mode and this leads to excessive material usage in the flange members to limit the bending
 214 deflection. Conversely, when s is high, the bending deformation prevails and the obtained
 215 cross-sectional areas are mainly governed by the web façades to limit the shear deformability.
 216 Therefore, an optimal value of s is shown to exist, s_{opt} , which balances the need to limit both shear
 217 and bending deformability [10]. In this case, the member sizes at the higher stories are usually
 218 governed by the shear deformation, while the ones at the lower stories are mostly controlled by the
 219 bending deformation. The s_{opt} depends on the building aspect ratio (H/B), and leads to the most
 220 efficient solutions that comply with the target maximum displacement while employing the
 221 minimum amount of material. For diagrid structures taller than 40 stories, with H/B greater than 5
 222 and diagonal angles between 60° and 70° , the empirical equation $s_{opt} = H/B - 3$ is proposed [10].

223 The other fundamental parameter that plays a key role in the preliminary design of diagrids is
 224 the diagonal inclination. Investigating a set of 20- to 60-story tall buildings, Moon et al. show that, for
 225 diagrid structures having aspect ratios of about 7, the optimal angle is between 65° and 75° , whereas
 226 for diagrids having aspect ratios of about 5 the optimal angle is lower than around 10° [10]. This is
 227 due to the competition between shear and bending stiffnesses in governing the deformation mode,
 228 and their dependence on the diagonal angle. Shear rigidity is maximum when the inclination is
 229 about 35° , while bending rigidity achieves its maximum value when the elements are vertical, i.e. $\theta =$
 230 90° . The optimal value to maximize both shear and bending rigidity lies between these two. Since
 231 shear mechanism prevails in shorter buildings and bending prevails in taller ones, it is expected that
 232 as the aspect ratio increases the building behaves more like a bending beam, thus the optimal angle
 233 increases [10]. This consideration has been strongly exploited in the analysis and design of diagrid
 234 systems, by considering various angle-based strategies and patterns to optimize the diagrid
 235 performance. More details about this subject are reported in Section 4.

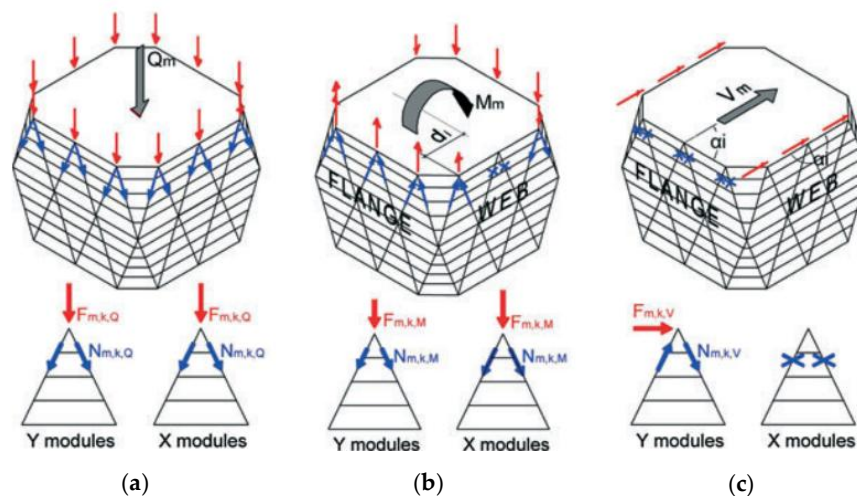
236 The same stiffness-based approach reported in the previous paragraphs is also applied by
 237 Moon to braced tubes in [12]. In this case, the shear force is carried by the external mega-diagonals,
 238 while bending moment is carried by the perimeter vertical columns. Analyses based on 40-
 239 100-story tall braced tubes show that the optimal angle in this case is close to 45° and is less
 240 dependent on the building aspect ratio. This is due to the negligible involvement of external
 241 diagonals in carrying bending moment. For braced tubes with an aspect ratio greater than 6, Moon
 242 suggests a different empirical equation for the optimal s value, i.e. $s_{opt} = H/2B - 1$ [12]. It has to be
 243 noted that, in the same paper, the same analysis has been applied to diagrids with a broader range of
 244 heights than previously analyzed, i.e. from 40 to 100 stories. As a result, the author proposes a new
 245 empirical equation for the s_{opt} for diagrid structures with aspect ratios greater than 6, i.e. $s_{opt} = H/B - 2$.

246 In the cases investigated by Moon [10,12], it is found that the stiffness requirements drive the
 247 preliminary design and the strength criteria are usually fulfilled. Only a few members in the leeward
 248 façade of the building are found to fail when the maximum allowable displacement is increased, i.e.
 249 $\alpha < 500$. However, thanks to the high rigidity of the diagonalized façades, which make the diagrid
 250 structure highly efficient, strength requirements may be of paramount importance and in specific
 251 cases they might even govern the design criteria, as suggested by Montuori et al. in [13]. In this
 252 paper, a simplified strength-based methodology for the preliminary design of diagrid tubes is
 253 provided. Figure 5 shows the adopted scheme for the development of the strength-based approach.
 254 Both gravity and lateral loads are applied to the building.

255 Assuming that the internal core occupies the 25% of the floor area, the diagrid carries the 37.5%
 256 of the gravity load at the level of the m^{th} module, Q_m (Figure 5a). This vertical loading condition
 257 generates a uniform compression state in all the n_k diagonals of the module, $N_{m,k,Q} = 0.375Q_m \sin\theta / 2n_k$.
 258 Lateral loads generate the bending moment M_m and shear force V_m at the module level. The former
 259 induces a uniform compression state in the diagonals of the leeward flange, a uniform tension state
 260 in the windward flange and a linear distribution of tension-compression axial forces in the webs,
 261 depending on the distance d_i of the i^{th} diagonal from the center of the floor (Figure 5b). This leads to
 262 the expression of the axial force $N_{m,k,M} = \pm M_m d_k \sin\theta / 2 \sum d_i$. Conversely, the shear force induces only
 263 tension-compression stresses in the web diagonals, therefore $N_{m,k,V} = \pm V_m \cos\alpha_k \cos\theta / 2 \sum \cos\alpha_i$, being α
 264 the direction of the horizontal force with respect to the orientation of the diagrid module (Figure 5c).

265 Considering all the loading conditions, one obtains the total axial force in the generic diagonal,
 266 as $N_{m,k} = N_{m,k,Q} + N_{m,k,M} + N_{m,k,V}$. This value is finally used to define the member size, based on the
 267 tensile strength and the buckling compressive resistance of the diagonal. In the same paper, the
 268 authors also propose an analytical formulation, based on Euler-Bernoulli and Timoshenko beam
 269 theories, to obtain an alternative optimal s value for the stiffness-based approach, i.e. $s_{opt} =$
 270 $0.19H^2 / \tan\theta L^2$.

271



272
273

274 **Figure 5.** Scheme of the elementary diagrid module for the definition of the strength-based approach
 275 for the diagrid preliminary design, under: (a) gravity loads; (b) overturning moment; (c) shear force.
 276 Used with permission from Montuori et al. [13].

277 The strength- and stiffness-based approaches are simultaneously applied for the preliminary
278 design of a rectangular 100-story tall diagrid tube, considering three different diagonal angles (64°,
279 69° and 79°), under both gravity and wind loads. The results show that, on the broad side of the
280 buildings, strength requirements always prevail at the upper modules, whereas stiffness criteria
281 drive the design of the lower modules. Conversely, on the shorter side, strength prevails over
282 stiffness for the entire height of the building with $\theta = 64^\circ$, stiffness prevails for $\theta = 79^\circ$, while in the
283 case of 69° (which is close to the optimal angle inclination) the stiffness- and strength-based
284 approaches provide almost the same result [13].

285 After carrying out the structural analyses on the designed buildings, it is found that the
286 stiffness-based methodology leads to very efficient structures as regards the top lateral deflection,
287 which is very close to the target value. However, this approach usually leads to unsatisfactory
288 results in terms of inter-story drifts of the upper modules, as well as in terms of member strength
289 demand-to-capacity ratio (DCR). In fact, besides the case of $\theta = 79^\circ$, where only 0.3% of the diagonals
290 fail the strength requirements, 26% and 23% of them exhibit DCR greater than 1 for $\theta = 64^\circ$ and 69°,
291 respectively. On the contrary, adopting the strength-based design, the fraction of elements with DCR
292 greater than 1 is 0%, 0.5% and 0.3%, for $\theta = 64^\circ$, 69° and 79°. However, with this approach,
293 unsatisfactory results are obtained in terms of lateral deformability, especially in the case of 69° and
294 79° [13]. Therefore, stiffness-based approaches might lead to unsatisfactory strength results, while
295 strength criteria might fail stiffness requirements. A compromise should then be found depending
296 on the specific building characteristics. In both cases, large inter-story drifts are usually found at the
297 upper modules. This issue has been thoroughly analyzed by Montuori et al. [11] and tackled by
298 providing special internal systems, like SBSs. More details about this will be provided in Section 5.

299 Further investigation regarding the suitability of stiffness- and strength-based criteria for the
300 preliminary design is subsequently developed to a broader range of diagrid structures [14,15]. In
301 [14], Mele investigates the effect of both approaches on 90-story tall diagrid tubes, with diagonal
302 angles of 60°, 70° and 80°. The results are in line with the previous findings. For smaller diagonal
303 angles, strength usually drives the design, while stiffness-based approach leads to inadequate DCR
304 values. For greater angles, stiffness-based design prevails, while strength criteria lead to excessive
305 lateral deflections. In the range of the optimal angle, both criteria concur to define the member sizes.

306 More recently, the effect of both slenderness and diagonal angle has been taken into account
307 simultaneously for the preliminary design [15]. Diagrids with aspect ratios ranging from 2 to 8 and
308 diagonal angles from 50° to 80° are considered. It is found that, for aspect ratio from 2 to 4, the
309 design is mainly governed by strength requirements, independently on the diagrid angle, and the
310 “premium for height” is mostly linear with the increase of slenderness. Conversely, for aspect ratios
311 greater than 6, the design is mainly driven by stiffness, and the weight increases more than linearly
312 with the slenderness. Aspect ratios around 5 are found to be the threshold, where stiffness- and
313 strength-based designs provide comparable solutions [15].

314 Based on these results, it is concluded that, due to the extreme rigidity of the diagrid tubular
315 system, it is not always possible to know *a priori* whether stiffness- or strength-based criteria should
316 be considered for the preliminary design. Both approaches are necessary and unavoidable, and none
317 of them should be used without the other [13]. The geometrical diagrid parameters, e.g. building
318 aspect ratio and diagonal angle, drive the prevalence of one over the other. In any case, simplified
319 approaches for the preliminary design represent an effective way to quickly define and assess the
320 structural characteristics and performance of the diagrid.

321 3. Methods for the structural analysis of diagrid tall buildings

322 In the academic literature, the most common procedure to deal with the structural analysis of
323 diagrid systems is the Finite Element Method (FEM). However, simplified methodologies have also
324 been developed for a quick evaluation of the overall diagrid structural behavior.

325 Mele et al. [16] have proposed a hand-based method for the evaluation of the axial stress in the
326 diagrid members. The method is based on the analysis of the internal forces arising in the basic
327 triangular element due to gravity and vertical loads, taking also into account the effect of horizontal

328 and vertical curvatures of the diagrid façade. Although it does not allow to calculate directly the
 329 displacements of the structure, this methodology is proven effective in the computation of the axial
 330 forces in the diagonals. It is applied to three real case studies, the Swiss Re Building (London), the
 331 Hearst Headquarters (New York) and West Tower (Guangzhou), and the axial stresses arising from
 332 hand-calculations show a very good correspondence with FEM results. Design considerations on the
 333 optimal diagonal inclination for the investigated cases are also provided.

334 More recently, Liu and Ma have developed a simplified methodology, called the modular
 335 method (MM), to perform the structural analysis of diagrid tubes with arbitrary polygonal shape
 336 [17]. So far, most of the research had been focused on rectangular diagrids, having vertical façades
 337 acting as webs or flanges (Figures 4-5); however, little attention was paid on diagrids with polygonal
 338 shapes.

339 The modular method relies on the modularization of the diagrid and the calculation of the
 340 lateral stiffness of the diagrid modules in order to compute the total lateral deflection. The lateral
 341 displacement u_i of the i^{th} module can be obtained by superimposing the contribution of the shear
 342 displacement $u_{V,i}$ and bending displacement $u_{M,i}$. Based on the evaluation of the shear and bending
 343 rigidity of the i^{th} module, $K_{V,i}$ and $K_{M,i}$, the two contributions can be computed as follows:
 344

$$u_{V,i} = \frac{V_1}{K_{V,1}} + \frac{V_2}{K_{V,2}} + \dots + \frac{V_i}{K_{V,i}}, \quad (6a)$$

$$u_{M,i} = \frac{M_1}{K_{M,1}} h i + \frac{M_2}{K_{M,2}} h(i-1) + \dots + \frac{M_i}{K_{M,i}} h[i-(i-1)]. \quad (6b)$$

345 where V_i and M_i are the shear force and bending moment at the level of the i^{th} module, respectively,
 346 and h the height of the module. The key of the MM is the calculation of the shear and bending
 347 rigidities, $K_{V,i}$ and $K_{M,i}$, and is based on the usual assumptions for diagrid tubes: the diagonals are
 348 only subjected to axial stress and remain in the linear elastic regime, the building floors behave as
 349 rigid bodies without any internal deformation, the intra-module floors are neglected for the
 350 calculations of the modular rigidities.
 351

352 Shear rigidity is defined as the total shear force F_V required for unitary horizontal displacement
 353 of the module Δv (Figure 6a), and bending rigidity is defined as the bending moment M required for
 354 unitary floor rotation $\Delta \beta$ (Figure 6b). Applying independently unitary floor displacements and
 355 rotations, and computing the total shear force and bending moment, leads to the direct evaluation of
 356 $K_{V,i}$ and $K_{M,i}$. The calculation of the shear force and bending moment is based on the geometrical
 357 compatibility equations, the constitutive relations of the diagonals and finally the equilibrium
 358 equations at the level of the floor. This finally allows to obtain the following formulations for K_V and
 359 K_M :
 360

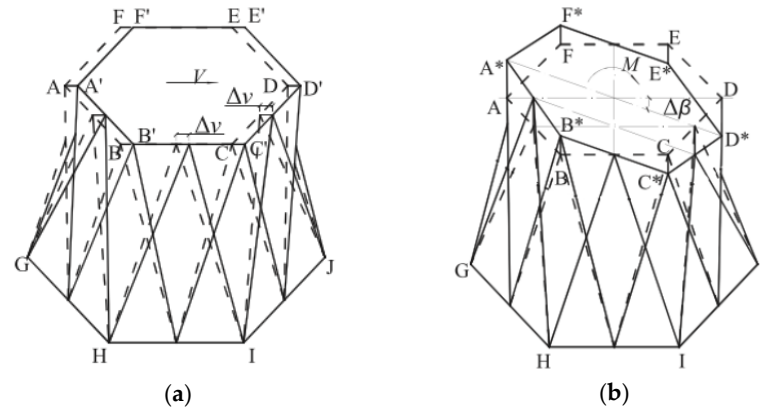
$$K_V = \frac{EA \cos^2 \theta \sin \theta \sin \gamma}{h} \sum_{d=1}^N \cos^2 \alpha_d + \frac{EA \sin^3 \theta \cos^2 \gamma \sin \gamma}{h} \sum_{d=1}^N \sin^2 \alpha_d, \quad (7a)$$

$$K_M = \frac{EA \sin^3 \theta \sin^3 \gamma}{h} \sum_{d=1}^N B_d^2, \quad (7b)$$

361 where E and A are the Young modulus and cross-sectional areas of the diagonals, θ the angle
 362 between the diagonal and the main ring beam in the façade, γ the angle between the ring beam plane
 363 and the façade, N the number of total diagonals in the module, α the angle between the ring beam
 364 and shear direction and B_d is the distance between the diagonal d and the neutral axis in the main
 365 ring beam plane [17]. Note that Eqs. (7a-b) resemble Eqs. (1a-b), but they also include the effect of
 366 not-vertical façades (γ angle) and polygonal planar shapes (α angle). Making use of Eqs. (7a-b) for
 367

368 each module, together with the application of Eqs. (6a-b), one can finally evaluate the lateral
 369 deformation of the diagrid building under horizontal loads.

370 The MM is verified against FEM calculations, analyzing square, hexagonal and octagonal
 371 diagrid tubes with vertical and inclined façades under different horizontal loading conditions. The
 372 variations in terms of top displacements from FEM results are always within 10%, which verifies the
 373 proposed methodology. Based on the evaluation of the shear and bending rigidities, the MM is also
 374 employed to define the analytical framework for the preliminary design of diagrids [17].
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378 **Figure 6.** Scheme for the calculation of (a) shear rigidity; (b) bending rigidity, according to the
 379 modular method (MM). Used with permission from Liu and Ma [17].

380 Subsequently, a new method has been developed for the structural analysis of diagrids by
 381 Lacidogna et al. in 2019 [18], which has been called the matrix-based method (MBM). This
 382 methodology is grounded on matrix calculus and is similar to the FE method, although it drastically
 383 reduces the degrees of freedom (DOFs) of the diagrid structure. It is based on the same assumptions
 384 employed by the previous authors [10,17]: the diagonals carry only axial force and remain in linear
 385 elastic regime, the intra-module floors are neglected and the building deformation obeys the plane
 386 section assumption. The major reduction in the system DOFs, compared to conventional FE models,
 387 is due to the fact that, under the above assumptions, the considered DOFs in the MBM are only the
 388 displacements and rotations of the rigid floors, rather than the nodal displacements and rotations
 389 associated to all the structural elements.

390 The structural problem for the 3D free-form diagrid tubes is formulated through the
 391 generalized Hooke's law as $\{F\} = [K]\{\delta\}$, $\{F\}$ and $\{\delta\}$ being the force and displacement vectors,
 392 respectively, and $[K]$ the stiffness matrix of the diagrid. Considering a number of floors equal to N ,
 393 the total dimension of the structural problem is $6N \times 6N$, being six the number of DOFs per floor, i.e.
 394 three translations and three rotations. The matrix equation can be expanded as follows, where all the
 395 DOFs contributions are highlighted:
 396

$$\begin{Bmatrix} \{F_x\} \\ \{F_y\} \\ \{M_z\} \\ \{M_x\} \\ \{M_y\} \\ \{F_z\} \end{Bmatrix} = \begin{bmatrix} [K_{F_x\delta_x}] & [K_{F_x\delta_y}] & [K_{F_x\varphi_z}] & [K_{F_x\varphi_x}] & [K_{F_x\varphi_y}] & [K_{F_x\delta_z}] \\ [K_{F_y\delta_x}] & [K_{F_y\delta_y}] & [K_{F_y\varphi_z}] & [K_{F_y\varphi_x}] & [K_{F_y\varphi_y}] & [K_{F_y\delta_z}] \\ [K_{M_z\delta_x}] & [K_{M_z\delta_y}] & [K_{M_z\varphi_z}] & [K_{M_z\varphi_x}] & [K_{M_z\varphi_y}] & [K_{M_z\delta_z}] \\ [K_{M_x\delta_x}] & [K_{M_x\delta_y}] & [K_{M_x\varphi_z}] & [K_{M_x\varphi_x}] & [K_{M_x\varphi_y}] & [K_{M_x\delta_z}] \\ [K_{M_y\delta_x}] & [K_{M_y\delta_y}] & [K_{M_y\varphi_z}] & [K_{M_y\varphi_x}] & [K_{M_y\varphi_y}] & [K_{M_y\delta_z}] \\ [K_{F_z\delta_x}] & [K_{F_z\delta_y}] & [K_{F_z\varphi_z}] & [K_{F_z\varphi_x}] & [K_{F_z\varphi_y}] & [K_{F_z\delta_z}] \end{bmatrix} \begin{Bmatrix} \{\delta_x\} \\ \{\delta_y\} \\ \{\varphi_z\} \\ \{\varphi_x\} \\ \{\varphi_y\} \\ \{\delta_z\} \end{Bmatrix} \quad (8)$$

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In Eq. (8) $\{F_x\}$, $\{F_y\}$, $\{F_z\}$ represent the forces acting at the floor level on the horizontal (X, Y) and
 vertical (Z) direction, respectively, and $\{\delta_x\}$, $\{\delta_y\}$, $\{\delta_z\}$ are the corresponding displacements. $\{M_x\}$ and

400 $\{M_y\}$ contain the out-of-plane moments acting along the X and Y directions, respectively, while $\{\phi_x\}$
 401 and $\{\phi_y\}$ are the corresponding out-of-plane rotations. Finally, $\{M_z\}$ and $\{\phi_z\}$ denote the torque
 402 moments and rotations acting around the vertical axis. Based on the expansion of the force and
 403 displacement vectors reported in Eq. (8), the $6N \times 6N$ stiffness matrix is partitioned accordingly,
 404 where each $N \times N$ submatrix relates a force-moment vector to the generic displacement-rotation
 405 vector. The procedure for the calculation of the stiffness matrices is grounded on the displacements
 406 method, and is similar to the scheme adopted by Liu and Ma in the MM [17]. The stiffness
 407 coefficients are obtained by applying unitary displacements-rotations at the floor levels and
 408 evaluating the total reacting forces-moments according to compatibility, constitutive and
 409 equilibrium equations.

410 The MBM is more general than the MM, since it does not consider only shear and bending
 411 rigidities (contained in the matrices $[K_{F_x, \delta_x}]$, $[K_{F_y, \delta_y}]$, $[K_{M_x, \phi_x}]$ and $[K_{M_x, \phi_x}]$), but also the vertical and
 412 torsional ones, i.e. $[K_{F_z, \delta_z}]$ and $[K_{M_z, \phi_z}]$. Besides these 6 sub-matrices that lie on the diagonal of the
 413 stiffness matrix, the MBM also evaluates other 30 mixed submatrices, although only 15 of them need
 414 to be computed due to the symmetry properties of $[K]$. For regular-form diagrids most of these
 415 out-of-diagonal matrices are zero; nevertheless, the evaluation of the matrices $[K_{F_x, \phi_x}]$, $[K_{F_y, \phi_y}]$, $[K_{M_x, \delta_x}]$
 416 and $[K_{M_x, \delta_y}]$ is extremely important, as they contain the information about the coupling between
 417 shear and bending stiffnesses, therefore concurring to the correct definition of the lateral deflection.

418 After the complete calculation of the stiffness coefficients of the 21 non-identical submatrices in
 419 Eq. (8), the application of forces and moments at the floor levels leads to the evaluation of the
 420 corresponding displacements and rotations, through the inversion of Eq. (8). Eventually, known the
 421 deformation of the structure, the compatibility and constitutive equations are applied once again, in
 422 order to find the axial forces in the diagonals. The MBM is applied to perform the structural analysis
 423 of the double-curvature Swiss Re Tower and comparisons with FEM calculations show the
 424 consistency of the suggested method for the evaluation of both lateral and vertical displacements, as
 425 well as torsional rotations [18].

426 The MBM has not only been developed for the structural analysis of diagrids, but also to
 427 investigate the interaction between a diagrid tube and other resisting elements embedded within the
 428 building. To do so, the MBM has been built within the framework of the General Algorithm (GA), a
 429 matrix-based analytical methodology proposed in 1985 for the preliminary analysis of tall buildings
 430 [19]. The GA was firstly developed for the analysis of 3D civil buildings with moment resisting
 431 frames and closed-section shear walls. Further on, open-section shear walls were also taken into
 432 account [20], which observe the Vlasov's theory of deformation and exhibit the warping effects
 433 typical of thin-walled structures [21]. In recent years, the GA has also allowed to study the
 434 interaction between structures of different heights [22], secondary effects in tall buildings [23],
 435 unconventionally-designed systems such as tapered and twisted towers [24]. It has also been applied
 436 to investigate the dynamic behavior of tall buildings [25], as well as real case studies in Northern
 437 Italy [26,27].

438 The framework of GA takes into account only 3 DOFs per floor, namely the two horizontal
 439 displacements and one torsional rotation. To make the MBM suitable for insertion into the GA,
 440 Lacidogna et al. [18] carry out a static condensation procedure, where the contributions of vertical
 441 displacements and out-of-plane rotations are condensed. Specifically, Eq. (8) is re-written in the
 442 following form:

443

$$\begin{Bmatrix} \{F_H\} \\ \{F_V\} \end{Bmatrix} = \begin{bmatrix} [K_{HH}] & [K_{HV}] \\ [K_{VH}] & [K_{VV}] \end{bmatrix} \begin{Bmatrix} \{\delta_H\} \\ \{\delta_V\} \end{Bmatrix} \quad (9)$$

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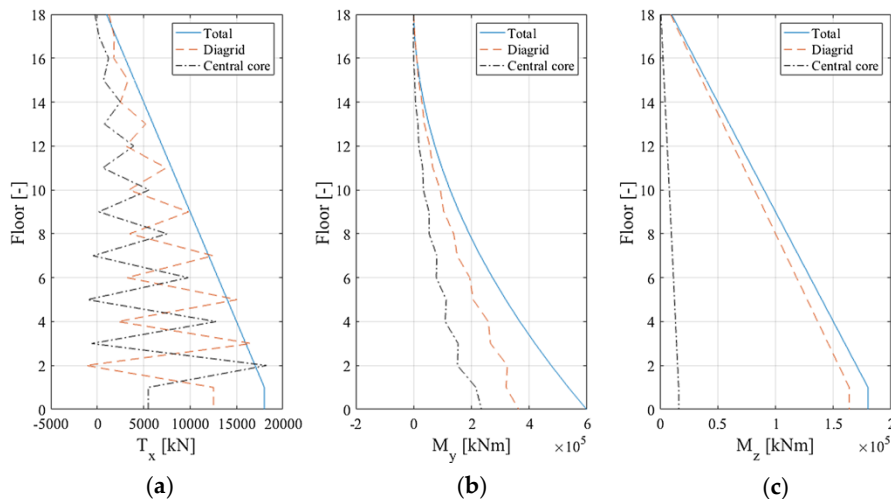
445 where $\{\delta_H\}$ ($\{F_H\}$) include the contributions of horizontal displacements (forces) and torsional
 446 rotations (moments), and $\{\delta_V\}$ ($\{F_V\}$) include the contribution of vertical displacements (forces) and
 447 out-of-plane rotations (moments). The condensation procedure yields the following relation, where
 448 only the horizontal DOFs are considered through the $3N \times 3N$ condensed stiffness matrix $[K_{HH}]^*$:

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$$\{F_H\} = ([K_{HH}] - [K_{HV}][K_{VV}]^{-1}[K_{VH}])\{\delta_H\} = [K_{HH}]^*\{\delta_H\}. \quad (10)$$

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The MBM has been used within the GA framework to investigate the stiffness interaction between an external steel diagrid and an internal concrete core. In particular, a square 18-story tall building is considered in [18] and the coupled behavior is analyzed under lateral forces and torque moments. Although the torsional behavior is obviously governed by the external diagrid tube, the distribution of shear forces at the various floor levels is not trivial and gives rise to an oscillating trend along the height of the building, due to the shear-bending coupling of the two structural systems (Figure 7).



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Figure 7. External steel diagrid tube coupled with a central concrete core. Distribution of: (a) shear forces; (b) bending moments; (c) torque moments, according to the MBM and GA. Used with permission from Lacidogna et al. [18].

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In a more recent paper, Lacidogna et al. [28] investigate the effect of the diagonal inclination on the diagrid-core coupled system. As already pointed out by Moon et al. [10], when the diagrid angle is in the optimal range, the diagrid lateral stiffness prevails over the internal core's one. The influence of the type of internal core, i.e. closed- or open-section shear wall, is also investigated in [28]. Although the diagrid-core coupling mechanism is almost the same in terms of the lateral deformability, a remarkable difference between the two types of internal cores (open- and closed-section shear wall) is observed in terms of torsional behavior. In the case of internal open-section shear wall and steep diagrid angles, a clear inflection point in the torsional deformation curve is obtained due to the warping effect of the internal shear wall [28].

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Although simplified, the methods presented in this section for the structural analysis of diagrid systems, integrated with the stiffness- and strength-based methodologies for the preliminary design shown in Section 1, can provide a valid and efficient alternative to FE calculation in the preliminary stages. As a matter of fact, they allow the quick investigation of the overall structural behavior, while capturing the fundamental parameters governing the diagrid performance. It has to be noted that, due to the increasing power of nowadays computing technologies, simplifying the structural model, i.e. reducing the DOFs of the system, is not often an imperative need. Current FE software in modern computers are able to perform the structural analysis of buildings with very large number of DOFs, and the trend nowadays is to consider even more detailed models for the sake of general analysis. However, although using detailed models is necessary in the ultimate design stages, it can make lose sight of the overall structural behavior during the preliminary stages. As a matter of fact, in these stages, the correct comprehension of the overall structural behavior has important implications on the definition of the optimal resisting elements. These choices in the preliminary phases are in turn known to have a strong influence on the cost and efficiency of the final solution, especially in tall

487 buildings. For this reason, using simplified methodologies for the preliminary design can help the
488 designer to acquire awareness on the overall structural behavior, that the application of very
489 detailed FE models might not reveal at first sight.

490 4. Optimization of the diagrid performance based on the geometrical features

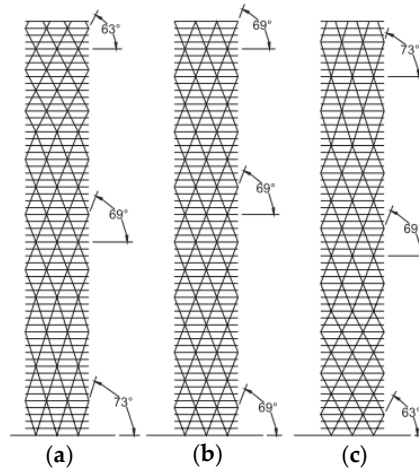
491 Besides the great stiffness of the diagonalized façades and the capability to realize
492 complex-shaped systems, one of the main successful aspects of diagrids is the possibility to reach
493 high structural performance thanks to the optimization of the geometrical features. In the last
494 decade, various researchers have thoroughly investigated the structural behavior of diagrids upon
495 changing the external diagonal pattern, in order to reach the optimal solutions.

496 Moon et al. [10] show for the first time that there exists a diagonal angle capable of satisfying the
497 stiffness requirements with the minimum amount of employed material. The optimal angle results
498 from the need to limit both shear and bending deformation, and it is found to increase as the aspect
499 ratio of the building increases. As already remarked in Section 2, for 60-story tall diagrids having an
500 aspect ratio of about 7, the optimal angle is in the range 65° – 75° , while it decreases of about 10° for
501 aspect ratios close to 5 [10]. Approximately the same results are found in [12] for a set of 40- to
502 100-story tall diagrids.

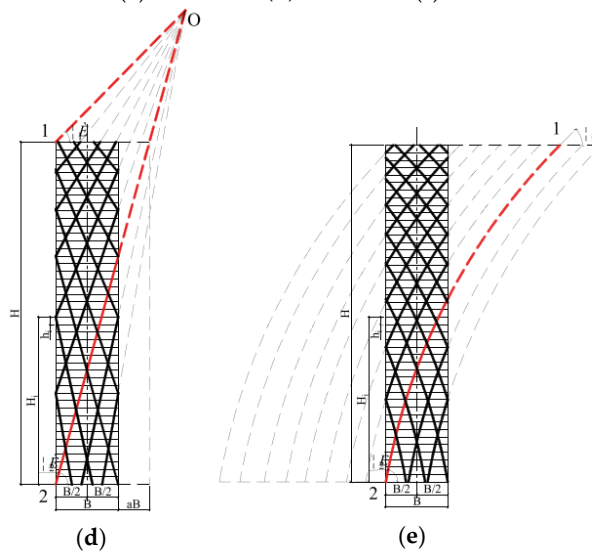
503 Under lateral actions, shear forces and bending moments have different trends along the
504 building elevation. For example, if we consider a uniform horizontal load, the shear force is zero at
505 the top of the building and increases linearly towards the base, while the bending moment increases
506 quadratically. This means that the need to resist shear and bending actions is different in different
507 parts of the structure. Shear force prevails in the upper portion, while bending moment drives the
508 design of the lower part. Based on this consideration, Moon investigates diagrid buildings with
509 different patterns of diagonal angles [12]. Figure 8a shows a varying-angle diagrid with steeper
510 angles at the base, Figure 8b a uniform-angle diagrid, and Figure 8c a varying-angle diagrid with
511 steeper angles at the top. Steeper diagonals are more suitable for carrying bending moment, while
512 shallower diagonals are more appropriate to carry shear force. Therefore, the solution in Figure 8a
513 should enhance the structural performance of the diagrid. Conversely, the solution with steeper
514 diagonals at the top behaves against structural logics and is only considered for sake of
515 completeness, as it is not supposed to provide any beneficial effect.

516 Based on the results, it is found that, for shorter buildings with aspect ratio lower than 7, the
517 uniform-angle configuration provides the most efficient design in terms of material consumption.
518 Shorter buildings behave like shear beams and, while the steeper diagonals at the base enhance the
519 bending stiffness, the negative effect of the reduced shear rigidity causes the varying-angle diagrids
520 to lose efficiency. Contrariwise, for taller buildings with aspect ratio greater than 7, the bending
521 behavior prevails. The reduced shear rigidity at the base due to the steeper diagonals is balanced by
522 the significant increase in bending stiffness. Therefore, in this case, the varying-angle configuration
523 is found to provide the most efficient solution [12]. The same results are found in another paper by
524 Moon [30], where the author takes also into account the “speed” of variation of the diagonal angles
525 along the height of the building, with smooth or more radical changes.
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530 **Figure 8.** Different diagonal angle patterns diagrids: (a) varying-angle with steeper diagonals at the
 531 base; (b) uniform-angle; (c) varying-angle with steeper angle at the top, used with permission from
 532 Moon [12]; (d) varying-angle with straight diagonals; (e) varying-angle with curved diagonals, used
 533 with permission from Zhao and Zhang [29].

534 In the solutions provided by Moon with variable angles, the diagonals do not remain straight in
 535 their length over the full height of the building, because of their changing direction at the interface of
 536 two diagrid modules with different angles. To overcome this, Zhang et al. [31] propose a different
 537 strategy for the generation of varying-angle diagrid tubes. As shown in Figure 8d, a graphic
 538 approach is suggested to generate a varying-angle pattern with straight diagonals that extend over
 539 the full height of the building. This pattern is governed by two fundamental parameter, the top angle
 540 θ_1 and the bottom angle θ_2 . The stiffness- and strength-based design criteria are applied to a set of
 541 30- to 75-storey tall varying-angle diagrids with straight diagonals, with aspect ratios ranging from
 542 3.6 to 9. Several $\theta_1 - \theta_2$ combinations are considered to investigate the optimal solutions under
 543 gravity and wind loads. Based on the results, the following empirical formulas are suggested for the
 544 optimal values of θ_1 and θ_2 , depending on the building aspect ratio H/B :
 545

$$\theta_{1,opt} = \begin{cases} \theta_{2,opt}, & H/B \leq 3.5 \\ \frac{1}{\left(1 + \ln \frac{H/B}{3.5}\right)^2} \left(\theta_{2,opt} - \arcsin \frac{1}{\sqrt{3}} \right) + \arcsin \frac{1}{\sqrt{3}}, & H/B > 3.5 \end{cases}, \quad (11a)$$

$$\theta_{2,opt} = \arctan \frac{H/B}{1 + 0.475 \sqrt{\frac{H/B}{4.75}}}. \quad (11b)$$

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547 As H/B increases, the optimal bottom angle $\theta_{2,opt}$ increases, while the optimal top angle $\theta_{1,opt}$
 548 decreases. A critical value of the aspect ratio, $(H/B)_{crit}$ is found, which defines the interface between
 549 the efficiency of uniform- versus varying-angle diagrids, meaning that below $(H/B)_{crit}$ uniform-angle
 550 diagrids are more efficient, while above this value varying-angle structures provide the most
 551 economical solutions. In this paper, $(H/B)_{crit}$ is found to be 4.5-5, smaller than the value of 7
 552 previously suggested by Moon [12,30]. This is mainly due to the different definition of the diagonal
 553 pattern. For aspect ratios less than $(H/B)_{crit}$, the bottom angle rather than the top angle drives the
 554 design. Conversely, for greater aspect ratios, the top angle becomes one of the determining factors
 555 [31].

556 In a following paper, Zhao and Zhang [29] propose an additional diagrid configuration, where
 557 the varying-angle solution is obtained with curved diagonals (Figure 8e). In the same paper, they
 558 also consider seismic loads for the evaluation of the optimal diagrid pattern. It is found that, for
 559 varying-angle straight diagonals, the optimal bottom angle $\theta_{2,opt}$ is not affected by the load type, thus
 560 Eq. (11b) holds also for seismic loads. Whereas, the optimal top angle $\theta_{1,opt}$ is always close to the
 561 lower limit for seismic loads, i.e. $\theta_{1,opt} = \arcsin 1/\sqrt{3}$, thus correcting Eq. (11a). In the case of diagrids
 562 with curved diagonals, they propose the following equations for the optimal angles, which are valid
 563 for both wind and seismic loads:
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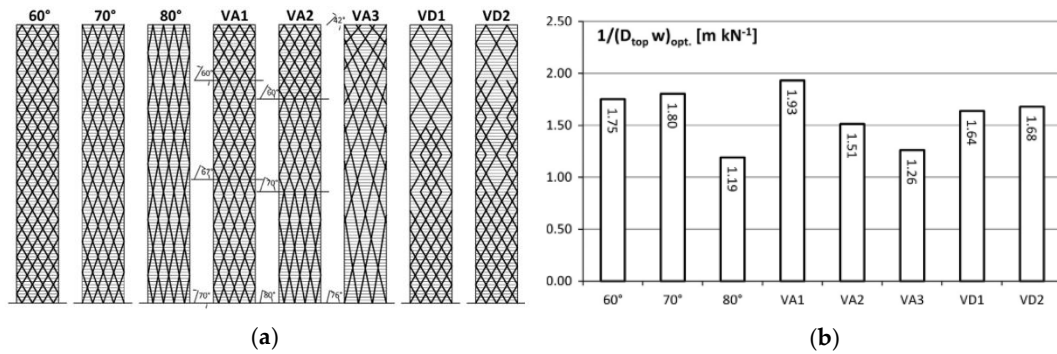
$$\theta_{1,opt} = 0.8 \left(\frac{H/B}{8} \right)^{\frac{1}{8}} \theta_{2,opt}, \quad (12a)$$

$$\theta_{2,opt} = \arctan(H/B), \quad (12b)$$

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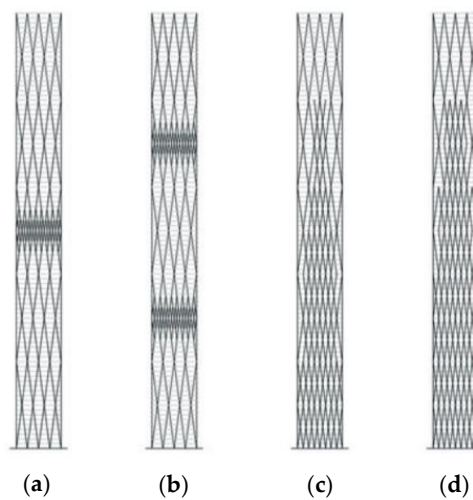
566 H/B being in the range 3.6 – 9. With these values, the optimal top angle $\theta_{1,opt}$ lies in the range 50°–70°,
 567 greater than the top angles in diagrids with straight diagonals (35°–45°). Thus, the smaller difference
 568 between $\theta_{1,opt}$ and $\theta_{2,opt}$ in this case results in a small curvature of the diagonals [29].

569 Further developments in the external diagrid patterns are carried out by Montuori et al. [32]. In
 570 addition to the consideration of uniform- and varying-angle (VA) solutions, the authors also propose
 571 diagrid patterns with variable density (VD) in the diagonal layout (Figure 9a). FE calculations are
 572 performed on a 90-storey tall building with aspect ratio of 6.62, under gravity and wind loads, and
 573 the structural responses are analyzed in terms of top lateral deflection, inter-story drifts and
 574 diagonals DCR. For each solution, an efficiency parameter is proposed as $1/D_{top}w$, D_{top} and w being
 575 the top lateral displacement and the employed steel weight per total floor area. The lower the lateral
 576 displacement and the amount of steel, the greater the efficiency of the investigated solution. The
 577 obtained efficiency parameters are shown in Figure 9b for all the considered solutions. From the
 578 results, it is found that the 80° and VA3 solutions result the less efficient for the investigated case,
 579 whereas VA1 is the most efficient one. Uniform-angle solutions with 60° and 70°, as well as the
 580 variable patterns VA2, VD1 and VD2, show similar efficiency values [32].
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Figure 9. (a) Different geometrical patterns from Montuori et al. [32]: uniform-angle patterns (60°, 70°, 80°), varying-angle patterns according to Moon approach (VA1, VA2) [12,30], varying-angle pattern according to Zhang approach (VA3) [31], variable-density patterns (VD1, VD2). (b) Efficiency parameters for the investigated solutions. Used with permission from Montuori et al. [32].



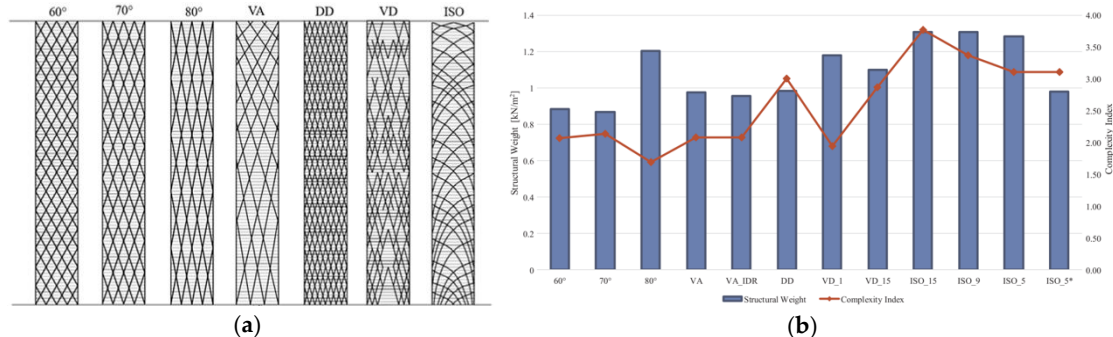
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Figure 10. Variable-density (VD) patterns proposed by Angelucci and Mollaioli [9]: (a-b) concentrated outrigger-like VD pattern; (c-d) distributed VD pattern. Used with permission from Angelucci and Mollaioli [9].

594 Additional analyses regarding different pattern configurations can be found in the work of
595 Angelucci and Mollaioli [9]. After exploring the effectiveness of stiffness-based approaches for a
596 351-meter tall diagrid with optimal (69°) and non-optimal (82°) diagonal angle in order to evaluate
597 whether common approaches lead to optimized member sizes, the authors propose additional
598 variable-density (VD) patterns of the diagonal arrangement (Figure 10). Two VD strategies are
599 suggested for the non-optimal (82°) diagrid tube to meet the stiffness requirements: a localized
600 pattern, resembling one-outtrigger-like (Figure 10a) or two-outtrigger-like (Figure 10b) schemes; a
601 more uniform VD pattern, which provides distributed additional stiffness over the building
602 elevation (Figures 10c-d). The outcomes from FE calculations show that the local density increments
603 (Figures 10a-b) are not efficient strategies to meet stiffness and strength requirements. Conversely,
604 the solutions involving a more uniform VD pattern (Figures 10c-d), where the diagonal
605 concentration rarefies towards the top of the building, turn out to be appropriate solutions to limit
606 the lateral displacements, while obtaining notable material savings [9].

607 The previous work of Montuori et al. [32] has been subsequently developed by Tomei et al. [33],
608 who propose additional diagonal patterns for the 90-story tall diagrid building (Figure 11a). Besides
609 considering the usual uniform- and varying-angle patterns, the authors also suggested a
610 double-density pattern (DD), where the diagonal layout is doubled and mirrored over the diagrid
611 façade, a variable-density pattern (VD), generated starting from the DD pattern with further
612 topology optimization, and a diagrid-like pattern (ISO), where the diagonals follow the principal
613 stress lines obtained from the equivalent building cantilever. Stiffness- and strength-based

614 preliminary designs are carried out, together with optimization procedures based on Genetic
 615 Algorithms through the use of commercial software. The optimization procedure aims at
 616 minimizing the unit structural weight of the building, while complying to the stiffness and strength
 617 requirements. This is achieved by formulating an objective function (OF) to be minimized and
 618 specifying the constraints of the optimization problem, as thoroughly described in [15,33].
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 622 **Figure 11.** (a) Geometrical patterns for the 90-story tall diagrid building considered by Tomei et al.
 623 [33]: uniform-angle patterns (60°, 70°, 80°), varying-angle pattern according to Zhang approach (VA)
 624 [31], double-density pattern (DD), variable-density pattern (VD), stress lines pattern (ISO). (b) Unit
 625 structural weight (blue bars) and complexity index (red curve) for the investigated diagrid patterns.
 626 VA_IDR, VD_1, VD_15, ISO_15, ISO_9, ISO_5, ISO_5* refer to additional subsets of the
 627 corresponding patterns, as reported in [33]. Used with permission from Tomei et al. [33].
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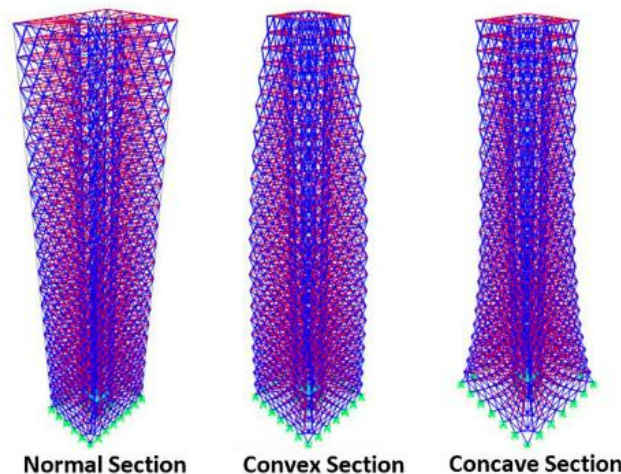
629 The results of the analysis are analyzed in terms of unit structural weight, diagonal
 630 cross-section distribution along the elevation, deformed configuration, lateral displacements,
 631 inter-story drifts, diagonal DCR, highlighting the most efficient solutions from the structural
 632 viewpoint. The authors also propose a complexity index, which accounts for the “constructability”
 633 of the diagrid structure. This is defined taking into account five main metrics, i.e. the total number of
 634 nodes, the number of different cross-sections, the number of diagonal splices necessary for
 635 transportation purposes, the total number of diagonals and the number of different diagonal
 636 lengths. The results of the complexity index, together with the obtained structural weight, are shown
 637 in Figure 11b for each geometrical pattern. Graphs like the one reported in Figure 11b can be
 638 extremely useful for evaluating both the structural efficiency and constructability of the investigated
 639 diagrid solutions [33].

640 The analyses shown in the previous paragraph of this Section, for the assessment of diagrid
 641 performance, take into account only square and rectangular buildings. To consider also different
 642 plan shapes, Mirniazmandan et al. [34] recently investigate the simultaneous effect of diagonal
 643 inclination and planar shape on the top lateral displacement and diagrid weight. Sixty-four
 644 parametric models of a 180-meter tall building, with various cross-sectional shape, are generated by
 645 randomly increasing the number of sides at both the base and top plans. Five diagonal angles are
 646 also considered, in the 33°–81° range. By means of Genetic Algorithms coupled with FE structural
 647 analyses, the authors find out that the diagonal angle of 63° provides the least amount of top lateral
 648 deflection, while reducing the employed structural material. Furthermore, it is found that increasing
 649 the sides of the base and top plans leads to the most efficient solutions in terms of lateral
 650 displacements, although the increase of structural performance is not as evident as when changing
 651 the diagonal inclination.

652 More recently, a similar analysis has been carried out by Lacidogna et al. [35], to investigate the
 653 influence of both the diagonal inclination and plan shape on the structural behavior of diagrid tubes.
 654 In this study, the structural response for square, hexagonal, octagonal and circular diagrids is
 655 evaluated not only in terms of lateral deflection, but also in terms of torsional rotations. The analysis
 656 is carried out by means of the previously developed matrix-based method (MBM) [18], and shows
 657 that the diagonal angle is the main parameter governing the structural response rather than the
 658 specific plan shape. As already found out previously by Moon et al. [10], the optimal angle to

659 minimize the lateral displacement increases with the aspect ratio of the building, as it results from
 660 the need of limiting both shear and bending deformability. Conversely, the optimal angle to
 661 minimize torsional rotations is always found to be the shallowest one, close to 35°, and it does not
 662 depend on the building aspect ratio. This is mainly due to the different mechanisms which drive the
 663 lateral and torsional deformability of the diagrid. As already reported, the former is affected by both
 664 the shear and bending stiffness of the diagrid modules, whereas only the shear rigidity concurs in
 665 the definition of the torsional behavior. Due to the fact that shear rigidity is maximum for shallower
 666 diagonal angles, these are the most effective to resist torque moments [28,35]. Therefore, when
 667 torque actions need to be taken into account, this aspect must be considered in the definition of the
 668 optimal grid pattern.

669 Finally, all the analyses presented so far have mainly dealt with tubular structures with vertical
 670 façades. In a very recent paper, Ardekani et al. [36] investigate the influence of the plan shape,
 671 together with the convexity and concavity of the diagrid surface (Figure 12). Based on FE
 672 calculations on a set of 40-story tall buildings, the outcomes show that, compared to rectangular
 673 diagrids, other polygonal forms might lead to beneficial material savings, while meeting the stiffness
 674 requirements. Furthermore, with respect to the normal models, the buildings with convex and
 675 concave façades achieve better results in terms of structural performance.
 676



677 **Figure 12.** Diagrid structures with vertical, convex and concave façades. Used with permission of
 678 Taylor & Francis Ltd (www.tandfonline.com), from Ardekani et al. [36].
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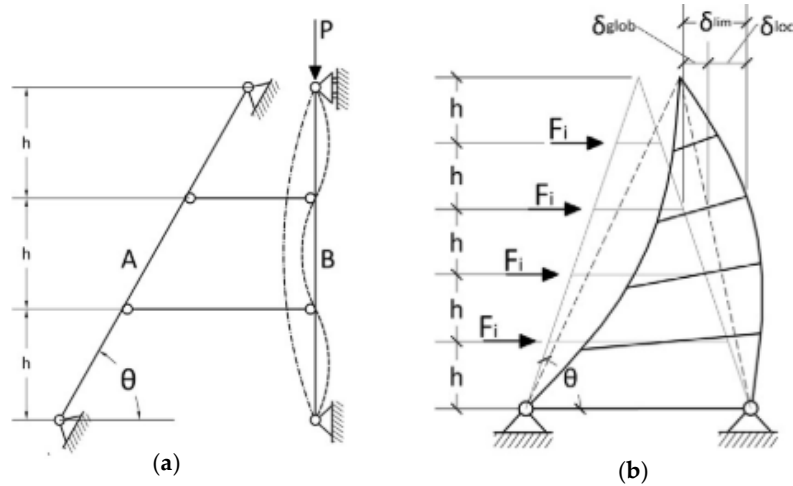
681 As can be easily recognized from the studies reported in this Section, one of the main aspects
 682 that has caused the notable diffusion of diagrids in recent years is related to the versatility of its
 683 external diagonal layout. A rational and optimized diagonal pattern allows to achieve remarkable
 684 structural performance, together with beneficial material savings. The application of expeditious FE
 685 calculations, as well as simplified methodologies such as the ones reported in the previous Sections,
 686 together with optimization techniques, can help engineers and designers to reach high-performance
 687 structures in the preliminary stages of the tall building design.

688 5. Strategies for tackling excessive inter-story drifts and stability phenomena in diagrids

689 In the previous Section, we have seen that the external diagonal layout can be properly
 690 modified to meet the necessary stiffness and strength requirements. Accordingly, the external
 691 mega-bracings can extend over multiple stories. As pointed out by Montuori et al. [11], this can give
 692 rise to two local structural issues which need to be carefully addressed by the designer: (a) the
 693 instability of interior columns and (b) excessive inter-story drifts. Both of them are mainly due to the
 694 lack of flexural resistance of the diagonals. This section investigates these local issues, as reported in
 695 the fundamental work of Montuori et al. [11].

696 The first local issue is shown in Figure 13a, where a 3-story diagrid module is represented.
 697 Element A and B represents the external diagonal and the interior column, respectively. The column

698 usually extends over the full height of the building and it is subjected to high compression forces due
 699 to gravity loads, which might induce Eulerian buckling. The column's resistance to lateral buckling
 700 mode relies on the external diagrid structure, which fully braces the interior column only at the
 701 panel points. Within the module height, the multi-story buckling mode is only prevented by the
 702 flexural resistance of the diagonals. If this is not enough, the multi-story sway mechanism takes
 703 place, which can occur at lower buckling loads than the one-story mode.
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 707 **Figure 13.** Local issues in the design of diagrid tall buildings: (a) stability of interior columns; (b)
 708 excessive inter-story drift of intra-module floors. Used with permission from Montuori et al. [11].
 709

710 Based on a simplified analytical formulation, Montuori et al. [11] propose a simple equation in
 711 order to check whether the flexural resistance of the diagonals is sufficient to the purpose,
 712 specifically:
 713

$$n_{dg} I_{dg} > (k^2 - 1) \sin \theta n_{col} I_{col}, \quad (13)$$

714
 715 n_{dg} and I_{dg} being the number of diagonals along the perimeter and their inertia moment, respectively,
 716 k the number of intra-module stories, θ the diagonal inclination, n_{col} and I_{col} the number and inertia
 717 moment of the gravity columns. If Eq. (13) is not satisfied, the internal columns buckle in a
 718 multi-story sway mode (Figure 13a). In this case, either the columns are designed to sustain greater
 719 buckling loads or a secondary system is necessary.

720 The second local issue is related to the excessive inter-story drifts of intra-module floors. As in
 721 the case of the column stability, the lateral displacements of these floors rely on the flexural
 722 resistance of the mega-diagonals. Based on the scheme reported in Figure 13b, a simple expression is
 723 obtained by Montuori et al. to assess the need of additional systems for the limitation of inter-story
 724 drifts [11]:
 725

$$\frac{(\sum_{i=1}^{k-1} F_i) \sin \theta L^2}{24EI_{dg}n_{dg}} < \frac{500 - \alpha}{500\alpha}, \quad (14)$$

726
 727 F_i being the horizontal force applied at the i^{th} intra-module floor, L the total span of the diagonal, E
 728 the elastic modulus, α the limiting factor for the inter-story drift (usually $\alpha = 300$), and k , I_{dg} and n_{dg}
 729 with the same meaning reported above. If Eq. (14) is not satisfied, either the inertia moment of the
 730 diagonals I_{dg} is increased or, again, an additional structural system is needed.

731 Since the diagonals in diagrid systems are usually designed to carry only axial load, their
 732 flexural resistance is often not enough to prevent the multi-story sway mode of interior gravity
 733 columns, as well as the excessive inter-story drifts of the intra-module floors. For this reason,

734 Montuori et al. [11] propose the adoption of a secondary bracing system (SBS), realized with limited
735 modifications to the simple frames of the interior service core. Either rigid connections at
736 beam-column joints or triangulation of the structural framework are proposed, thus obtaining a
737 moment resisting frame (MRF) or concentric braced frame (CBF) respectively. The design of the SBS
738 is carried out to address the lack of stability of the interior columns, the excessive inter-story drifts,
739 or both.

740 Under the authors' assumptions, the first issue gives rise to a force in the bracing system F_{br}
741 equal to $0.004P_{cr,col,NS}$, $P_{cr,col,NS}$ being the buckling load of the fully braced column. This force can be
742 directly employed to design the members of the SBS, e.g. the diagonals of the CBF, for the
743 stabilization of the internal gravity columns. Similarly, the second local issue is tackled with the
744 design of an SBS able to provide a required lateral stiffness $\beta_{req,d}$ equal to $250\alpha(\Sigma F_i)/(500h-\alpha h)$ [11].

745 In the paper, the authors analyze a 90-story tall diagrid building, with diagonal angles equal to
746 60° , 70° and 80° , to test the efficacy of the proposed formulation for the SBS. Application of Eq. (13)
747 shows that the 70° and 80° buildings have almost all the diagonal members with inertia less than the
748 minimum required for the stability of internal columns, whereas in the 60° case only the upper
749 diagrid modules are able to provide enough resistance against the multi-sway mode. This is mainly
750 due to the lower number of intra-module floors in the 60° solution. Thus, a SBS is found to be
751 necessary to stabilize interior columns. Similarly, the application of Eq. (14) reveals that, in all cases,
752 SBSs are needed to limit the excessive inter-story drifts at the upper modules.

753 For this reason, SBSs are designed consisting of four CBFs, to both stabilize interior columns
754 and comply with the imposed drift limitation ($\alpha = 300$). In Figure 14, the results are shown for the
755 three building solutions in terms of lateral deflections (Figures 14a,c,e) and inter-story drift ratios
756 (Figures 14b,d,f) under wind forces. Resulting in a total 3% increase of the total structural weight
757 due to the insertion of the SBS, the additional structural system is proven effective in limiting the
758 inter-story drifts. As can be seen in Figures 14b,d,f, the inter-story drift ratios before the insertion of
759 the SBS are much greater than the maximum allowable value, especially at the upper modules, and
760 they increase as the diagonal angle increases, due to the greater number of intra-module floors. From
761 Figures 14a,c,e it is also evident that the insertion of the SBS does not affect the global stiffness of the
762 building, since the top lateral deflection remains the same.

763 The efficacy of the SBSs has also been assessed in the investigation of real diagrid buildings, i.e.
764 the Hearst Tower (New York) and The Bow (Calgary) [14]. Since the majority of diagrids are not
765 stand-alone systems but present central cores that provide local floor-to-floor restraints to the
766 diagonal members, avoiding their flexural engagement, the adoption of SBS-like structures can
767 preserve the axial-dominated behavior in the diagrid structure, thus better exploiting the
768 extraordinary efficiency of the external tube mechanism [14].

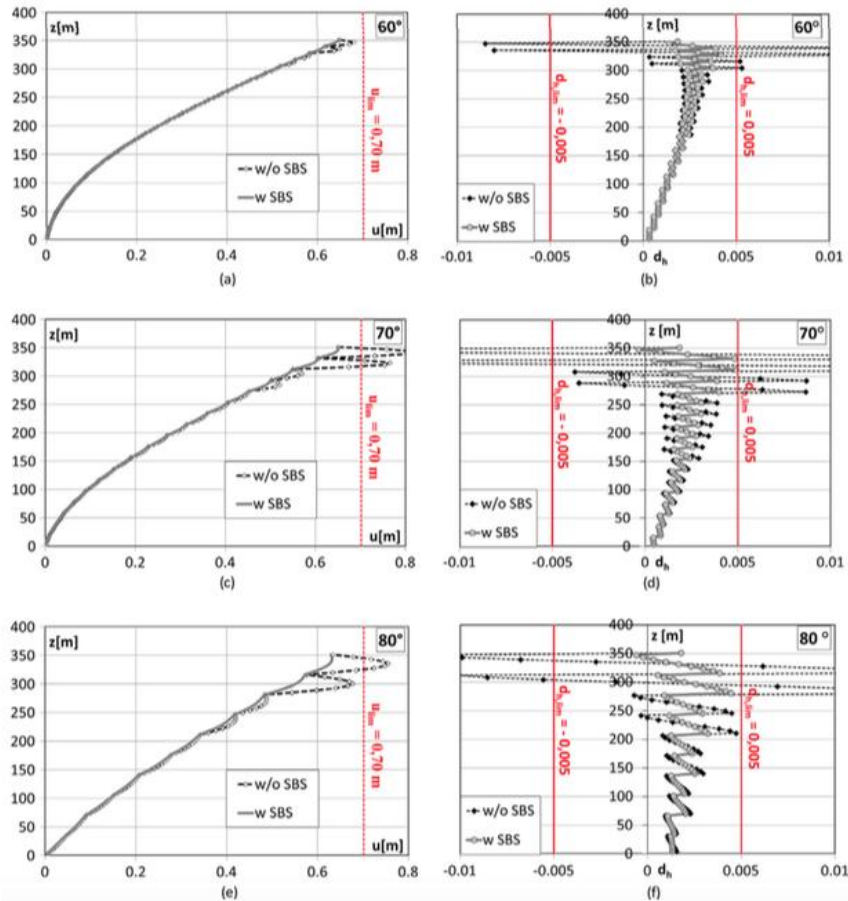


Figure 14. Efficacy of SBS in limiting inter-story drifts: (a-b) 60°; (c-d) 70°; (e-f) 80° diagonal pattern. (a,c,e) lateral deflections; (b,d,f) inter-story drift ratios. Used with permission from Montuori et al. [11].

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The stability requirements for diagrid tubes are also investigated by Rahimian in [37]. SBSs are introduced in this paper, with suggestions regarding their design for column lateral bracing, similarly to the considerations of Montuori et al. [11]. However, in this analysis, the SBS aims also at stabilizing the diagrid itself, against the lateral buckling of the diagrid modules. The buckling deformation mode of the diagrid modules arises from the vertical loads acting on the diagrid nodes, both at the panel level and at the level of intermediate floors. The required stiffness of the SBS is function of the diagrid geometry and compression force in diagrid members. The SBS methodology proposed by the author is applied to the case of the Hearts Tower (New York), where the efficacy of the designed SBS is discussed. Besides the typical lateral deformations due to wind and seismic actions, limiting the lateral diagrid displacements due to buckling sway mechanisms under gravity loads is essential for an efficient structural behavior and design.

785 6. Shear-lag effect in diagrid tubes

786 One of the most important problems in external tubes composed of beams and conventional
787 vertical columns is the shear-lag effect, which undermines their efficiency in high-rise buildings. As
788 shown in Figure 15, for a framed tube subjected to later loads the actual axial force distribution in the
789 vertical columns does not follow the Euler-Bernoulli distribution, i.e. linear and constant trend in the
790 web and flange respectively [1]. Conversely, due to the nature of the framed tube with
791 closely-spaced columns, both distributions are non-linear and result in higher stresses in the corner
792 columns, compared to the ones in the middle of the flange. This phenomenon is known as the
793 shear-lag effect. Shear-lag coefficients can be defined based on the non-linearity of the stress
794 distribution in the web and flange façades. In the design of a framed tube, the limitation of the
795 shear-lag effect often drives the design of the structural elements.

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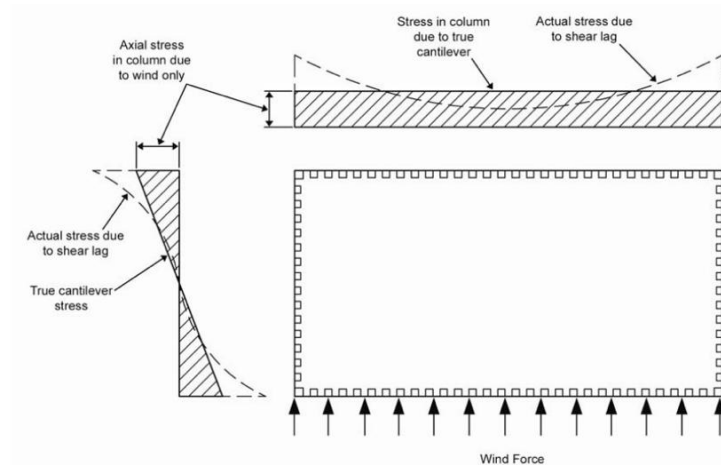


Figure 15. Shear-lag effect in framed tubes. Used with permission of Taylor & Francis Ltd (www.tandfonline.com), from Ali and Moon [1].

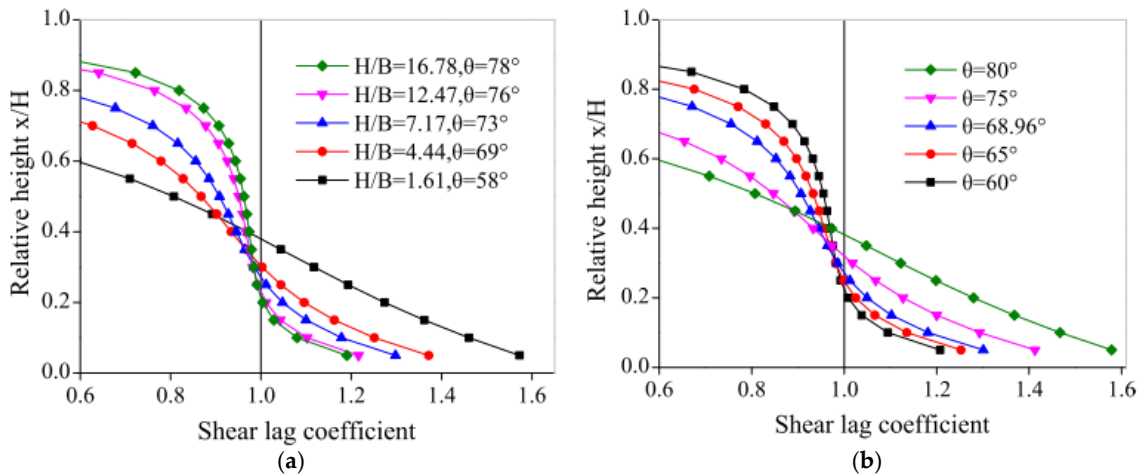
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801 Diagrids are known to be stiffer than traditional framed tubes. However, being external tubular
802 systems, they might experience the shear-lag phenomenon as well. For the first time, Leonard
803 investigates the influence of the shear-lag in a 60-story tall square diagrid building, with comparison
804 with the conventional framed tube solution [38]. It is found that the diagrid performs better both in
805 terms of lateral deflection and shear-lag effect, compared to the framed tube. However, the shear-lag
806 effect strongly depends on the external diagonal pattern. Steeper diagonal angles can increase the
807 severity of the shear-lag effect, whereas the number of the diagonal bays on the building perimeter
808 does not have a significant influence. Interestingly, it is also found that no direct correlation between
809 the shear-lag and lateral deflection appears in diagrids. Sometimes, the solutions with higher
810 shear-lag coefficients provide quite small lateral deflections [38]. This mainly derives from the
811 different mechanical behavior of the diagrid with respect to the conventional framed tube: the
812 former exploits the axial deformation of the external bracings, while the latter is dependent on the
813 flexural and shear deformation of vertical columns and horizontal beams. As a consequence of the
814 different mechanism, the shear-lag is less severe in diagrids than in conventional framed tubes.

815 The shear-lag effect is also analyzed for hybrid structures, where frame and diagrid tubes act
816 together in different parts of the building [39]. In this study, it is still observed that the shear-lag
817 effect in conventional framed tubes is much more significant than in diagrid systems. However, in
818 hybrid diagrid-frame tubes, the shear-lag coefficients depend on the specific geometrical
819 combination of the two systems over the height of the building and might not be negligible.

820 The previous studies investigate the shear-lag effects by means of FE calculations [38,39]. In a
821 more recent work, Shi and Zhang [40] propose an analytical formulation for a quick evaluation of
822 the shear-lag effect in diagrid tubes. To this purpose, the diagrid tube is equated to an elastic
823 orthotropic membrane, where the material properties are derived based on the stiffness equivalence.
824 Simple equations allow to compute the internal stresses in the equivalent tube under horizontal
825 loads. From the non-linearity in the distributions of web and flange stresses, a measure of the
826 severity of the shear-lag effect is provided. The suggested methodology is applied to a 52-story tall
827 rectangular diagrid and validated against FE calculations. Different diagrid tubes, with different
828 aspect ratios and diagonal angles, are also investigated. It is found that, for shorter buildings the
829 shear-lag effect obviously increases; moreover, increasing the steepness of the external diagonals
830 leads to greater shear-lag coefficients. Figure 16 shows the influence of aspect ratio (Figure 16a) and
831 diagonal inclination (Figure 16b) on the maximum shear-lag coefficient along the height of the
832 structure, as reported by Shi and Zhang [40]. The greater the distance of the shear-lag coefficient
833 from 1, the greater the influence of the shear-lag effect. Therefore, in shorter diagrid buildings and/or
834 with steeper diagonal inclinations, the shear-lag effect should be carefully taken into account in the
835 design stages.

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Figure 16. Shear-lag coefficient in diagrids depending on the diagonal angle and building aspect ratio: (a) effect of aspect ratio under optimal angle; (b) effect of diagonal angle under fixed aspect ratio. Used with permission from Shi and Zhang [40].

842 7. Non-linear analyses and seismic performance of diagrid structures

843 Most of the studies that have been reported in Sections 2-6 deal with the investigation of the
844 structural behavior of diagrid systems under static loads and within the linear elastic regime.
845 Although linear static analyses can provide extremely important information for the preliminary
846 design stages, the non-linear response of diagrid tubes is of paramount importance for the
847 evaluation of their performance. In the same way, analyses under dynamic loading conditions can
848 reveal significant information, especially regarding the seismic assessment as well as the resistance
849 against progressive collapse.

850 One of the first works dealing with the seismic performance of diagrid tubes has been carried
851 out by Kim and Lee in 2012 [41]. In the study, the authors analyze a set of 36-story tall square
852 buildings, with diagonal angles ranging from 50.2° to 79.5° . Non-linear static analyses, i.e. push-over
853 analyses, are carried out by applying lateral loads proportional to multi-mode story-wise
854 distribution pattern. The non-linearity in the behavior of the structural members is also considered,
855 according to FEMA-356 suggestions [42]. The outcomes show a quite brittle response for the diagrid
856 structure, if compared to the traditional framed tube which shows more ductile behavior. Increasing
857 the diagonal angle leads to lower ultimate shear forces carried by the diagrid before the final
858 collapse. Non-linear dynamic analyses are also performed, where the equations of motion of the
859 structure subjected to seven different earthquakes are numerically solved. The outcomes reveal that
860 greater diagonal angles are usually found to induce greater lateral displacements. It is also found
861 that both strength and ductility of the diagrid are increased when the diagonal members are
862 replaced by buckling-restrained braces [41].

863 In a following paper, Kim and Kong [43] make use of non-linear static and dynamic analyses to
864 investigate the resisting capacity of axisymmetric rotor-type diagrid buildings against progressive
865 collapse. Based on arbitrary column removal scenarios, the robustness of 33-story tall diagrids, with
866 cylindrical, concave, convex and gourd shapes, is evaluated. The outcomes show satisfactory
867 resisting capability against progressive collapse when one or two diagonal members are removed
868 from the first level, regardless of the geometrical shape. However, concave-type buildings exhibit
869 lower collapse resistance when two pairs of bracings are removed from the first story. In the study, a
870 thorough investigation of the collapse strength and formation of plastic hinges is also carried out,
871 depending on the diagonal inclination and location of member removal [43].

872 The ultimate capacity of diagrids in the damaged state, when certain diagonals are removed
873 from the nominal structure, is also investigated by means of FE non-linear analyses by Milana et al.
874 [44]. The results show that the ultimate resistance of diagrids upon damaging is quite satisfactory,
875 although it depends on the specific location of the bracing removal. In the same study, push-over

876 analyses are carried out on a set of 40-story tall buildings, and their performance is evaluated in
877 terms of strength, stiffness, ductility and sustainability aspects.

878 Although not common, experimental tests on prototype models can be also carried out to
879 investigate the dynamic properties of diagrid tubes. For example, Liu et al. [45] conduce shaking
880 table tests on a Plexiglas model of the Guangzhou West Tower, and compare the resulting dynamic
881 features (mode shapes, vibrational frequencies, acceleration magnification coefficients, etc.) to FE
882 time-history calculations. A crucial aspect in conducting such tests relies on the correct definition of
883 the geometrical, inertia, stiffness and damping parameters of the prototype, which should reflect the
884 real parameters of the tall building based on similarity laws. This procedure can also be a rational
885 way to validate FE models [45].

886 The seismic assessment of diagrid towers has been further investigated in more recent papers
887 [46–51]. In [46], Sadeghi and Rofooei quantify the seismic performance factors (SPFs) of steel diagrid
888 buildings, i.e. the response modification coefficient (R -factor), the overstrength factor (Ω_0) and the
889 displacement modification factor (C_d), based on FEMA P695 methodology [52]. FE push-over
890 analyses and incremental dynamic analyses (IDAs) are carried out. It is found that diagrids exhibit a
891 quite brittle behavior, as observed from the push-over curves, and the ductility increases as the
892 diagonal angle increases. R -factors also depend on the diagonal inclination, varying between 1.5 and
893 3, for diagonal angles ranging from 45° to 71.5° . The restraining end-conditions of the diagonals (pin
894 or rigid) do not have a significant influence on the stiffness of the structure; however, the pin-ended
895 solutions better tolerate larger displacements, improving the building ultimate seismic performance.

896 In a series of following studies, Asadi et al. [47–49] perform a comprehensive investigation of
897 the non-linear performance of low- to mid-rise steel diagrid structures, using static, time-history
898 dynamic and incremental dynamic analyses. Special attention is paid to corner columns, due to the
899 shear-lag effect, as well as to the diagonal inclination on the evaluation of the seismic assessment and
900 loss estimation of very short (4- and 8-story tall) diagrid buildings [47]. Mid-rise buildings, in the 8-
901 to 30-story range, are also investigated and their non-linear behavior is analyzed and compared to
902 traditional solutions, such as moment resisting frames and concentrically braced frames, in terms of
903 weight, story drift, lateral stiffness, fundamental period and evolution of plastic hinge formation
904 [48]. A set of 4- to 30-story tall diagrid buildings is further investigated for the evaluation of the SPFs,
905 and the authors recommend specific values of the SPFs for diagrid frames lying in different story
906 ranges [49].

907 Very recently, Heshmati et al. [50] investigate the influence of the interior cores on the seismic
908 performance of diagrid tubes. By the application of push-over analysis, it is found that the interior
909 tube can indeed help as a backup load-resisting system after the yielding of the perimeter diagrid
910 structure, procrastinating the insurgence of damage and providing an enhanced safety margin.
911 Non-linear time-history analyses also reveal that most of the buildings perform well under severe
912 earthquakes, dissipating large amount of the input energy and leading to quite uniform plastic
913 hinges distribution [50]. The seismic reliability of diagrids is also recently investigated by Mohsenian
914 et al. [51], who develop an efficient performance-based design strategy, based on a new multi-level
915 response modification factor.

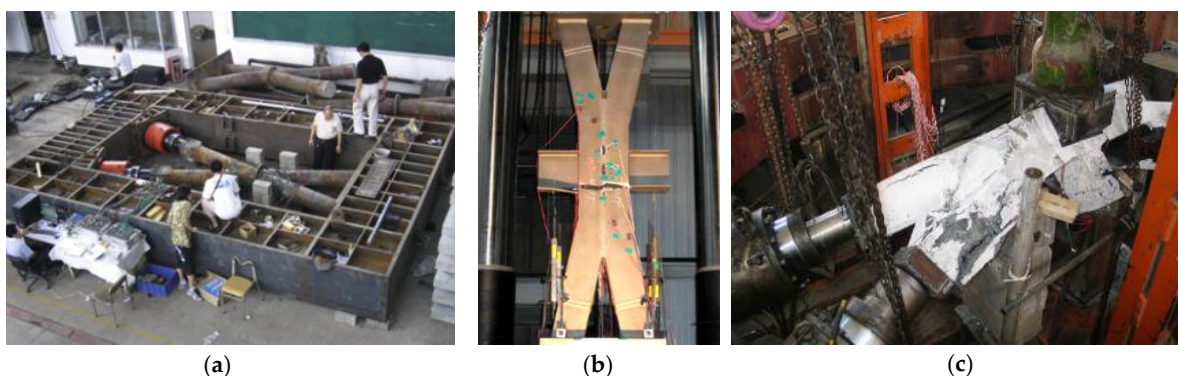
916 All these studies show that the non-linear and dynamic behavior of diagrids should be carefully
917 taken into account right after the preliminary design stage, as it strongly depends on the diagrid
918 features (diagonal angle, building height, etc.) that are often defined to satisfy the static
919 requirements as shown in Section 4. As briefly remarked from the papers cited above, the analyses
920 for the seismic assessment of structural systems can be very diverse. Various types of analyses can be
921 carried out, such as linear modal analysis, non-linear static analysis (i.e. push-over analysis),
922 time-history analysis, etc. These methodologies rely on the accurate modeling of the
923 three-dimensional building and the non-linear behavior of the structural members needs to be
924 properly taken into account for the correct comprehension of plastic hinge formation, local collapses,
925 force redistribution, etc. The different modeling and design approaches can therefore lead to slightly
926 different outcomes, that need to be related to the specific analysis and the adopted design
927 approaches and modeling procedures.

928 8. Research on diagrid nodes

929 A crucial element in the assessment of diagrid structures lies in the correct design and
 930 realization of the nodes, where a connection usually consists of four intersecting diagonal columns
 931 and several beams (Figure 3a). These elements are pivotal as they are in charge of transferring the
 932 high axial stresses between the diagonal members. The failing of one single node induces a
 933 redistribution of the load path and might compromise the overall stability and resistance of the
 934 diagrid, especially under cyclic and dynamic loading conditions. As a matter of fact, due to the
 935 seismic concept “stronger connection, weaker component”, special attention needs to be conveyed to
 936 the mechanical behavior of diagrid joints [53].

937 The joints are mainly divided in three types depending on the employed material, namely steel,
 938 reinforced concrete and concrete-filled steel tube (CFST) joints. Each of them is characterized by
 939 different mechanical behaviors, particularly under cyclic loading conditions and with reference to
 940 the hysteretic energy dissipation. Huang et al. [54] investigate the bearing capacity of CFST joints
 941 (Figure 17a), where the influence of connection detail, intersecting angle between the diagonals and
 942 loading type is analyzed on the bearing performance of the node. Based on the experimental results,
 943 it is found that the diagonal angle plays a key role in the definition of the joint failure mode, whereas
 944 the loading type (symmetric or asymmetric) has little influence. The authors also propose a simple
 945 equation for the calculation of joint bearing capacity, which is verified against the experimental
 946 outcomes. Kim et al. [55] perform an experimental campaign to analyze the cyclic behavior of the
 947 steel nodes from the Lotte Super Tower in Seoul (Figure 17b). Open- and box-section joints are
 948 realized and their cyclic performance is investigated in terms of stiffness and strength. Attention is
 949 also paid by the researchers to different welding methods. Subsequently, Jung et al. [56] study
 950 web-continuous steel connections for diagrid nodes under cyclic loads. Different welding methods
 951 and design details are taken into account, and they are not found to provide significant influence on
 952 the joint initial stiffness and yielding strength. Conversely, they can significantly modify the joint
 953 failure modes as well as the energy dissipation characteristics. Spatial concrete nodes are also
 954 studied by Zhou et al. [57], who investigate their failure mode and bearing capacity, focusing
 955 particular attention on the influence of transverse stirrups amount on the connection performance. It
 956 is shown that the volume ratio of transverse stirrups affects the bearing capacity of the joint, by
 957 effectively confining the concrete core under high compressive loads (Figure 17c).

958 These studies are fundamental for a thorough evaluation of the mechanical behavior of diagrid
 959 nodes, which in turn strongly affects the overall structural response of the diagrid system.
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 962
 963 **Figure 17.** Research on diagrid nodes: (a) CFST joint, used with permission from Huang et al. [54]; (b)
 964 steel joint, used with permission from Kim et al. [55]; (c) concrete spatial joints, used with permission
 965 from Zhou et al. [57].

966 9. Twisted, tilted, tapered, freeform diagrids

967 Besides the notable structural efficiency of diagrid tubes in resisting lateral forces, one of the
 968 key factors which has led to their successful exploitation is the capability to realize complex-shaped
 969 structures. As a matter of fact, due to the versatility and modularity of the elementary triangular

970 unit, diagrid systems can be effectively employed to build unconventional towers, such as twisted,
971 titled, tapered and even freeform structures.

972 These unconventional shapes are deeply investigated by Moon in [58–60]. 60-story tall twisted
973 diagrid towers are analyzed under lateral loads, with various twisting rates, namely 0, 1, 2 and 3
974 degrees per floor [58,59]. It is found that, as the rate of twist increases, the diagrid lateral stiffness
975 decreases and the top lateral deflection increases. This is mainly due to the fact that, the reference
976 un-twisted structure being designed with the optimal diagonal angle, increasing the twisting leads
977 to higher deviation of the diagonal angle from its optimal value. This in turn causes the lower
978 efficiency of the twisted tower compared to the un-twisted structure. The same analysis is conducted
979 for 80- and 100-story tall diagrid buildings and the same conclusions are drawn in [59]. The
980 performance of twisted diagrids is also investigated in terms of progressive collapse resistance and
981 seismic performance by Kwon and Kim [61]. Based on arbitrary column removal scenarios on a set
982 of 36-story tall twisted diagrid buildings, it is shown that the resistance against progressive collapse
983 is decreased as the twisting angle increases, whereas the twisting angle is beneficial for improving
984 the failure probability under seismic events.

985 Tilted towers are also investigated in [58,59], under both gravity and lateral loads. For 60-story
986 tall diagrid buildings with various tilting angles (ranging from 0° to 13°), it is found that the top
987 lateral displacement due to wind loads is not significantly affected by the tilting angle. Conversely,
988 lateral displacements due to the eccentricity of gravity loads in tilted towers are found to be
989 remarkably significant and these can become even greater than the lateral displacements due to
990 horizontal actions for great tilting angles. This aspect obviously needs to be taken into account
991 carefully for the realization of tilted diagrid structures.

992 Moon [59,60] also investigates the efficiency of tapered buildings compared to traditional
993 vertical structures. Such an effectiveness arises from the more rational employment of the structural
994 material in tapered tall buildings since this is more abundant in the lower part of the structure,
995 where the gravity, shear and bending actions are more important. As pointed out by the author,
996 attention should be paid when generating the tapered diagrid frame, as the inclination of the
997 external façades obviously affects the inclination of the diagonal members, which is known to be a
998 crucial parameter for the diagrid behavior. The analysis is carried out on a set of 60-, 80- and
999 100-story tall square diagrids under wind loads, with taper angles of 0, 1, 2 and 3 degrees. From the
1000 outcomes, it is shown that, as the taper angle increases, the top lateral deflection decreases, thus
1001 enhancing the lateral stiffness of the diagrid. This result is more significant as the building aspect
1002 ratio increases [59,60].

1003 Finally, diagrid structures with irregular shapes along the building elevation, namely freeform
1004 diagrids, are also analyzed. In particular, in [59] freeform geometries are generated using “sine”
1005 curves of various amplitudes and frequencies. Lateral loads are applied to the freeform buildings
1006 and the outcomes show that the top lateral deflection increases as the freeform shape deviates more
1007 from the original rectangular box form.

1008 From the previous considerations, it is clear that twisted, tilted, tapered and freeform diagrid
1009 systems offer a great variety of architectural solutions to the design of unconventional tall buildings.
1010 However, their structural performance needs to be carefully evaluated from the early design stages,
1011 in order to lead to feasible and sustainable solutions.

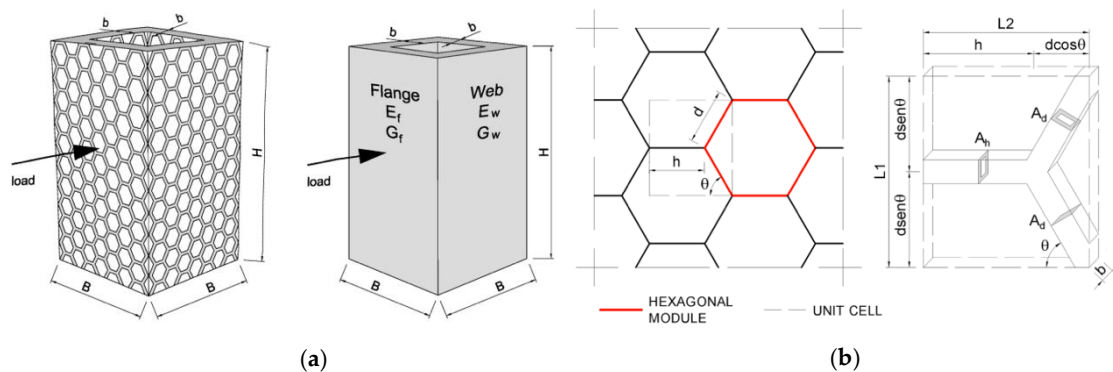
1012 **10. New evolutions of grid structures: hexagrids and beyond**

1013 A further development of grid tubular structures for the realization of tall buildings has been
1014 inspired by natural materials, such as the honeycomb pattern (beehive). This is the case of hexagrids,
1015 where six-node hexagonal elements are placed all over the building perimeter to resist gravity and
1016 lateral loads. Examples are the Sinosteel International Plaza in Tianjin and the Al Bahr Towers in
1017 Abu Dhabi. Hexagrids can mainly be divided in two types, according to the orientation of the
1018 hexagonal cell: horizontal hexagrids, where the hexagon is composed of four diagonal members and
1019 two horizontal beams, and vertical hexagrids, where the four diagonals are coupled with two

1020 vertical columns. The different orientation of the basic hexagonal unit obviously leads to differences
 1021 in the structural performance, which needs to be investigated in the preliminary design.

1022 Although the concept of hexagrids is very similar to diagrids, both exploiting an external
 1023 tubular structure to withstand external actions, their structural behavior is somewhat different. As a
 1024 matter of fact, diagrids resist gravity, shear and bending actions mainly by the axial stress of the
 1025 diagonal members. Conversely, besides the axial forces in the hexagonal members, the resisting
 1026 mechanism of hexagrids also involves the bending deformation of the diagonals and of the
 1027 horizontal/vertical elements.

1028 Based on the seminal work from de Meijer [62], Montuori et al. [63] investigate the mechanical
 1029 properties of hexagrid structures and their applicability in tall buildings. A general homogenization
 1030 approach is applied, where the hexagrid tube is converted into an equivalent orthotropic solid
 1031 membrane (Figure 18a). The hexagonal module and the unit cell are defined (Figure 18b) and
 1032 representative volume elements (RVEs) are identified based on the loading conditions. A
 1033 stiffness-based approach is then followed to calculate the equivalent elastic properties of the solid
 1034 tube, based on the grid mechanical and geometrical properties. Note that in Figure 18 the horizontal
 1035 hexagrid is shown, with diagonal members and horizontal beams. The same scheme has also been
 1036 adopted to investigate hexagrids with vertical elements [63].
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Figure 18. (a) Analogy between the hexagrid tube and an orthotropic solid membrane; (b) hexagonal module and unit cell. Used with permission from Montuori et al. [63].

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 1044 Based on the stiffness equivalence, both the equivalent elastic axial modulus E^* and shear
 1045 modulus G^* are evaluated, and subsequently employed to calculate the displacements of the
 1046 building equivalent cantilever. The effect of rigid floor diaphragms is also investigated, as it is found
 1047 to have a strong impact on the evaluation of E^* through the modification of the RVE. The application
 1048 of the simplified methodology is carried out on a 90-story tall hexagrid building, by changing the
 1049 module height and the inclination of the diagonal members. A comparative analysis is also
 1050 performed with similar diagrid structures [63]. From the results, it is found that the optimal angle of
 1051 the diagonal members is close to 60° for horizontal hexagrids, whereas it is lower for vertical
 1052 hexagrids, lying in the range 40° – 50° . Compared to diagrid structures, the hexagrids are usually less
 1053 stiff, being more bending-dominated, and consequently less structurally efficient. However, they
 1054 can provide new architectural solutions with notable aesthetic effects.

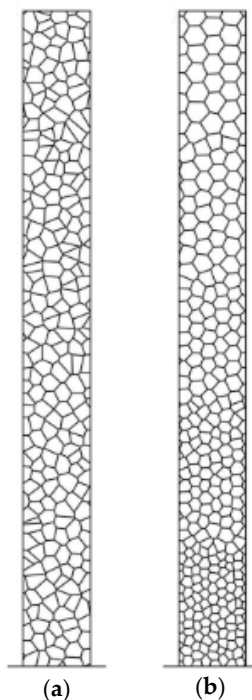
1055 The structural performance of hexagrid systems is also investigated by Lee and Kim [64] on
 1056 60-story tall square buildings. Different patterns of horizontal and vertical hexagrids, coupled with a
 1057 central core, are analyzed under gravity and load actions. It is found that the vertical hexagrids are
 1058 usually stiffer than the horizontal ones under lateral actions. This is mainly due to the axial
 1059 contribution of the vertical elements in the flange façades. The gravity loads are equally distributed
 1060 between the central core and the perimetral hexagrid, regardless the specific hexagonal pattern,
 1061 whereas the lateral loads are absorbed by the hexagrids for the 50–80%, with differences depending
 1062 on the arrangement of the hexagon module [64].

1063 Hexagrids are also investigated under dynamic loading conditions such as earthquakes, as well
 1064 as under arbitrary member removals to assess their resistance against progressive failure. In [65],

1065 Mashhadiali and Kheyroddin show that the shear mode deformation in hexagrids is usually greater
1066 than that occurring in diagrids, and hexagrids exhibit greater ductility under dynamic loadings. In a
1067 following paper, 28- and 48-story tall buildings with diagrid and hexagrid solutions are investigated
1068 in terms of resistance against progressive collapse, upon removal of corner elements [66]. The
1069 outcomes of non-linear static and dynamic analyses show that, although the specific geometrical
1070 configurations play an important role, hexagrids seem to be less vulnerable to progressive failure
1071 than diagrids, as they show greater potential for force redistribution.

1072 Hexagrids are not the only further development of grid tubular systems in tall buildings.
1073 Taranath et al. [67] investigate the efficiency of hexagrids compared to another grid system, the
1074 pentagrid. The latter is based on the arrangement of various pentagons on the surface of the
1075 building, where all the elements are designed to share a similar amount of stress. From the outcomes
1076 of the structural analysis, the authors find that the pentagrid is more structurally efficient than the
1077 hexagrid, although the cost of constructability of the pentagrid might be superior [67]. Other grid
1078 evolutions also count octagrids and Voronoi-like grid systems.

1079 Voronoi tessellation has been exploited in recent works as a new solution for grid tubular
1080 systems [68–70]. Angelucci and Mollaioli focus their attention on the evaluation of the mechanical
1081 characteristics of irregular Voronoi-like patterns for tall buildings [69]. Starting from a regular
1082 hexagrid solution, irregularity in the pattern is applied through random parametric generation, to
1083 realize more irregular building models (Figure 19a). The effect of varying-density pattern in the
1084 irregular Voronoi-like grid is also taken into account by the researchers (Figure 19b). Static and
1085 dynamic analyses are carried out on square 351-meter tall buildings. The outcomes reveal that cell
1086 irregularities do not affect the lateral stiffness significantly, and that the gradually rarefication of the
1087 pattern is a suitable strategy to optimize the lateral response [69].
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Figure 19. Irregular Voronoi-like grid pattern for tall buildings: (a) uniform density; (b) gradually rarefying density. Used with permission from Angelucci and Mollaioli [69].

1094 The mathematical and numerical framework for the stiffness homogenization procedure of
1095 Voronoi-like grid tubes is thoroughly presented in [70], where the authors define the concept of the
1096 testing volume element (TVE), which replaces the RVE used in regular hexagrid structures. Based on
1097 numerical analyses, the polynomial expressions for the correction factors of the mechanical
1098 properties of the homogenized tube are proposed, when dealing with irregular Voronoi-like grids.

1099 FE calculations on various 351-meter buildings are also carried out to investigate the efficiency of
1100 different Voronoi-like patterns [70].

1101 Conclusions

1102 In this paper, a fairly complete and up-to-date review of diagrid structural systems has been
1103 provided. The fundamental characteristics of diagrid tubes, which rely on the axial-dominated
1104 mechanism of the external mega-bracing, have been shown in the beginning of the paper, together
1105 with an overview of the structural solutions which brought to the realization and success of diagrids
1106 in recent years.

1107 The simplified approaches for the preliminary design, based on the seminal works of Moon et
1108 al. [10] and Montuori et al. [13], have been reported and their application thoroughly analyzed. Some
1109 of the recent simplified methodologies for the structural analysis of diagrids, which do not rely on
1110 the common FE calculations, have also been described. The great variety of works regarding the
1111 optimization of the diagrid performance based on the geometrical characteristics has been discussed
1112 and their implications analyzed. Local structural issues, such as excessive inter-story drifts and
1113 stability problems of the interior gravity columns, have also been addressed. A discussion regarding
1114 the shear-lag effect in diagrid rectangular tubes has also been provided, based on the current
1115 research literature. Space has also been given to the non-linear and dynamic analyses which have
1116 been performed in the last decade to assess the seismic performance of diagrid systems, as well as
1117 their resistance against progressive collapse. A quick overview of the research about diagrid nodes
1118 has also been carried out, as well as the analysis on unconventional shapes for diagrids, such as
1119 twisted, tilted, tapered and freeform towers. Finally, some final remarks have been provided
1120 regarding the latest evolution of the tubular grid structures, e.g. hexagrids, pentagrid and irregular
1121 patterns based on Voronoi tessellation.

1122 Throughout this review, we have seen that diagrids are efficient systems for tall buildings.
1123 Their efficiency mainly relies on the mechanism of the tubular system, coupled with the
1124 axial-dominated behavior of the basic triangular element. Because of the modularity characteristics
1125 and versatility of the reticulated surface, complex-shaped buildings can be realized with remarkable
1126 aesthetic potential. We have also seen that the power of diagrids also relies in the capability to
1127 further optimize their structural performance based on the geometrical features. This is a crucial
1128 point for sustainability purposes. With the need to limit material resources, while complying to
1129 safety and serviceability requirements, diagrid (and in general grid-based) tubes are the major
1130 candidates for the realization of the efficient, attractive and sustainable tall buildings of the future.
1131 Further researches dealing with all these aspects, following multi-criteria approaches [71] and
1132 involving different professional and academic figures and competences, will certainly lead diagrid
1133 structures to be more exploited worldwide in tall building design and construction.

1134 **Author Contributions:** Conceptualization, D.S. and G.L.; methodology, D.S.; formal analysis, D.S.;
1135 writing—original draft preparation, D.S.; writing—review and editing, D.S., G.L., A.C.; visualization, D.S.;
1136 supervision, G.L., A.C.; project administration, G.L., A.C. All authors have read and agreed to the published
1137 version of the manuscript.

1138 **Conflicts of Interest:** The authors declare no conflict of interest.

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