EVALUATION OF AIDING AND OPPOSING LOAD EFFICIENCY FOR AN ACTUATOR PLANETARY DRIVE

Pier Carlo Berri   Matteo D. L. Dalla Vedova Paolo Maggiore   Andrea Manuello Bertetto

Dept. of Mechanical and Aerospace Engineering (DIMEAS), Politecnico di Torino, Turin, ITA

ABSTRACT

Planetary gears offer compact and light solutions to achieve the high gear ratios required by robotics and actuation applications. This characteristic is usually combined to high torque density, high stiffness, and a good mechanical efficiency. In this work, we consider a particular layout of a compound planetary gearing, aiming to achieve very high gear ratios while reducing the amount of recirculating power that has a negative impact on the overall efficiency of the transmission. The efficiency of the reducer is computed in opposing and aiding load conditions, and compared to alternative solutions offering the same gear ratio and compactness, showing a better performance than commonly employed planetary drives. Additionally, the results are compared to a simplified model for mechanical transmission efficiency.

Keywords: Planetary gears, Efficiency, Aiding and Opposing load, Back-Drivable transmission

1 INTRODUCTION

Planetary gears are widely used in the efficient transmission of power. They are renowned for their high efficiency rating. These types of gears are compact and have a higher torque to weight ratio, than the ordinary one. With good performances and efficiency, the planetary gears use is becoming more and more prevalent in many fields as the aerospace, high efficiency plants, automotive field, naval realisations, civil and military projects [1-5]. Compared to other gears, the planetary gears compact design offers a high torque transmission rating and requires minimal installation space.

This interesting mechanism is based on a principle that has been known since ancient times, here are some interesting and fascinating examples that date back to the classical era in Mediterranean region and in the ancient Chinese East.

Around the sixth century BC, in Greece, the idea of epicyclic motion with rotation and revolution motion was known. Based on this theory, Claudio Ptolemy was able, in his Almagest, to predict the orbits of the planets.

Contact author: P. C. Berri[1], A. Manuello Bertetto[1]

[1] Dept. of Mechanical and Aerospace engineering, Politecnico di Torino, c.so Duca degli Abruzzi 24, 10129 Turin (IT)
E-mail: pier.berri@polito.it, andrea.manuello@polito.it

Almagest was written in the second century AD. and, for more than a thousand years, was the basis of astronomical knowledge in Europe and the Islamic world [6]. The Antikythera Mechanism, dated in the first century BC, was a complex planetary mechanism, capable of predicting data, motions of the planets, the calendars of Olympic games. It was able to approximate the moon's elliptical path through the heavens, in addition it corrects for the nine-year precession of that path [7 - 9]. In China was invented and realised a chariot, by the Yellow Emperor (Huang Di), named South Pointing Chariot. This mechanical-geared two wheeled vehicle was equipped by a planetary differential gear, in order to guide the troops out of enemy's smoke screen. A statue indicating the correct direction was linked to the planetary carrier of the differential gear [10]. The planetary gears have retained their value over the centuries and have been reinvented in modern times. The high power to mass ratio and the reduced encumbrance makes planetary gears the preferred fit in the design of drive solutions. Surely planetary gears can achieve a given speed ratio with a light mass and a reduced size, then would be required with an ordinary gear having similar characteristics and performances. In any case, many power losses arise in a gear train. Among these losses are frictional losses in teeth sliding, lubrication shaking losses, bearing losses, seals and guide bearing friction. In an ordinary single gear train, the power loss due teeth friction is usually about some unit percent of the input power. Sometimes, on the contrary, the power loss in planetary gears can be high, bringing the transmission efficiency low.
Therefore, a method to estimate the efficiency of planetary gears trains is one of the most important steps in the design of this type of transmission. It should not only give a numerical estimate of the efficiency, but also its analytical expression. This is relevant because the efficiency analytical expression of a planetary transmission depends on the sense of the power flow transmitted and on the speed ratio.
In [11] Macmillan discusses the fundamental relations among torques, velocities and power flow, derived from first principles, for a general differential mechanism. Studies on the efficiency of single D.o.F. planetary gear trains are those of Jose M. del Castillo [12], where two procedures are presented for obtaining the analytical expression for the efficiency of planetary gear trains.
Li Jianying and Hu Qigchun, in [13], considering a specific example, analyse the power distribution of complex and planetary gears and the overall efficiencies are computed taking into account power loss. Berri et al, in [14], discuses accurate yet simple models for the efficiency of mechanical transmissions, these models, which can be used within system level simulations, are intended to run in real-time for prognostic and monitoring tasks. In [15] Dezman and Gams propose a modification that decreases the stiffness varying motor torque in a motor-transmission system, thus increasing mechanisms energy efficiency, is proposed. The main design parameters are optimized with respect to the highest torque/deflection workspace, while staying below the peak torque constraints of motors gearboxes.
In this paper, we apply a model for the estimation of mechanical efficiency similar to that proposed in [12] to a particular compound planetary gearing [16 - 18]. The performance of this layout is compared with three alternative solutions offering similar characteristics in terms of gear ratio and torque density: the Harmonic Drive [19] and two carrier-driven planetary gearings. Additionally, the results of this approach are compared with those obtained with the simplified model proposed in [14].

2 METHODOLOGY

We employ the methodology proposed by [12], adapting it to a slightly different planetary gearing layout. The strategy finds a relationship between the efficiency of a planetary gearbox and the efficiency in the reference frame of the planet carrier.

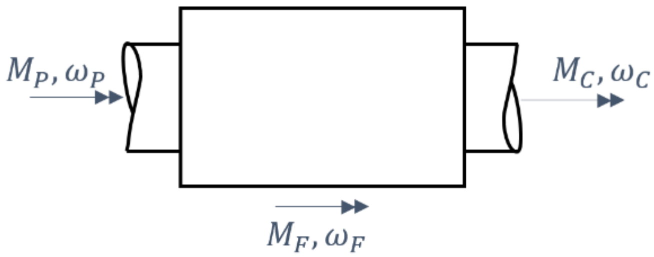


Figure 1 Definition of torques and angular speeds for a generic gearbox.

Considering the generic planetary gearbox of Figure 1, including a planet carrier P, a fixed case F and a moving shaft C, we can write the equilibrium equation:

$$M_P + M_C + M_F = 0 \tag{1}$$

where M_P is the torque applied to the planet carrier, M_C is the torque applied on the output shaft, and M_F is the torque applied to the case of the transmission. The power budget in the reference frame of the planet carrier can be written as:

$$-\omega_P M_F + (\omega_C - \omega_P) M_C = P_W \tag{2}$$

where ω_P is the angular speed of the planet carrier, ω_C is the angular speed of the moving shaft C, and the waste power P_W is the energy loss due to internal friction forces within the gearbox. Equation (2) can be rewritten as:

$$P_{in} + P_{out} = P_W \tag{3}$$

where $P_{in} > 0$ is an input power contribution and $P_{out} < 0$ is an output power contribution. The correspondence between the power contributions of Equations (2) and (3) depend on the particular layout of the gearbox and on its efficiency. The efficiency ρ of the transmission in the reference frame of the planet carrier is:

$$\rho = \frac{|P_{out}|}{|P_{in}|} \tag{4}$$

Typical values of ρ range from 99.2% to 99.5% for each external meshing, and in the range from 99.5% to 99.8% for each internal meshing, depending on a number of factors, namely the gear modulus, surface finish, manufacturing tolerances, preload, friction coefficient, and backlash. The power budget in the fixed reference frame is:

$$\eta M_P \omega_P + M_C \omega_C = 0 \tag{5}$$

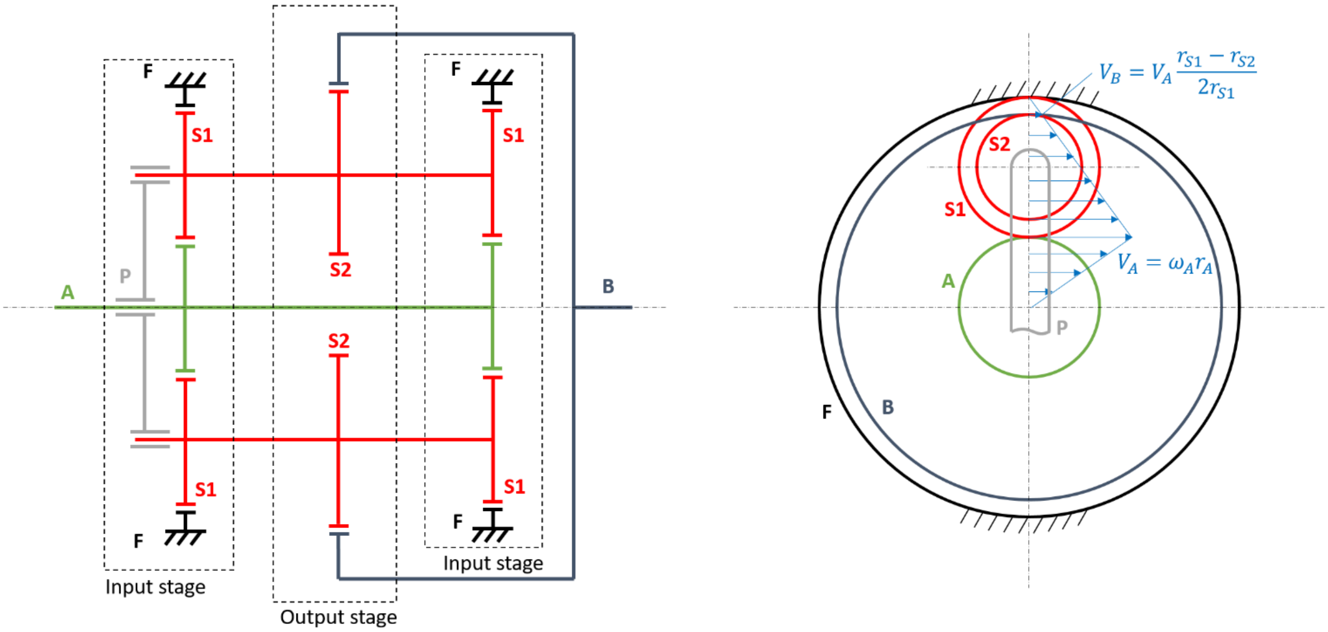
if P is the input shaft, or:

$$M_P \omega_P + \eta M_C \omega_C = 0 \tag{6}$$

if C is the input shaft. Writing Equation (4) in terms of torques and angular speeds, and substituting into Equation (5) or (6), η can be expressed as a function of ρ and the gear ratio k in the reference frame of the planet carrier.

3 APPLICATION TO THE PROPOSED REDUCER

We refer to the compound planetary gearbox with layout shown in Figure 2 (a). This particular architecture allows to achieve a very high gear ratio within a compact form factor, similarly to Harmonic Drives. The input shaft (A) carries a sun gear and engages with satellite gears (S1). The satellites engage with a fixed ring gear (F), and are fixed on a common shaft with the second-stage satellite gears (S2). The S2 gears mesh with a second ring gear (B), which is the output of the reducer. The first stage gearing, including the wheels A, S1 and F, is mirrored on the other side of the output ring gear B, in order to balance the forces on the satellites. Additionally, the use of herringbone gears and rolling rings in parallel with the gears allows to support axial and radial loads on the satellites, as well as on the input and output shafts [20].



(a) (b)

Figure 2 (a) Layout of the gearbox and (b) velocity distribution.

This way, the planet carrier (P) is a virtual component (i.e. a physical planet carrier is not needed, since it does not bear any loads), and the system behaves as a Combined Gear Bearing [20].
Figure 2 (b) shows the velocity distribution on the gears. The peripheral speed of the sun gear A is:

$$V_A = \omega_A r_A \tag{7}$$

while the peripheral velocity of the output ring gear B is:

$$V_B = V_A \frac{r_{S1} - r_{S2}}{2r_{S1}} = \omega_B r_B \tag{8}$$

This yields the expression for the gear ratio:

$$i = \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} \frac{2r_{S1}}{r_{S1} - r_{S2}} \tag{9}$$

The modulus of the input stage (gears A, S1 and F) may not be the same as that of the output stage (gears S2 and B). Using two different moduli allows for a much larger flexibility in the selection of the gear ratio. If using the same modulus, the gear ratio *i* can be written as a function of the numbers of teeth *z* directly from Equation (9):

$$i = \frac{z_B}{z_A} \frac{2z_{S1}}{z_{S1} - z_{S2}} \tag{10}$$

Otherwise, the expression of *i* can be derived with Willis' equation [16], yielding:

$$i = \frac{\frac{z_F}{z_A} + 1}{1 - \frac{z_F z_{S2}}{z_B z_{S1}}} \tag{11}$$

From Equations (9) to (11) appears clearly that very high gear ratios can be achieved if the wheels are chosen so that $z_F z_{S2} \approx z_B z_{S1}$.
The design of a compound planetary gearbox considered in this work is then applicable to actuation and robotics applications, similarly to Harmonic Drives. As highlighted in [16], the compound planetary gearbox has some advantages with respect to a Harmonic Drive of the same gear ratio and dimensions, in terms of reduced inertia and increased stiffness, so it may be preferable in applications requiring fast dynamics and precise positioning, while dealing with large, time-variable loads.

3.1 EFFICIENCY IN OPPOSING LOAD CONDITION

To estimate the efficiency of the reducer, we adapted the method described in Section 2, considering that the compound planetary gearbox layout of Figure 2 includes an additional rotating shaft A. Considering the equilibrium to rotation of the whole gearbox, it is possible to write:

$$M_A + M_B + M_F = 0 \tag{12}$$

Being $M_P = 0$ since the planet carrier is virtual. The power balance of the gearbox in the reference frame of the planet carrier can be expressed as:

$$-M_F\omega_P + M_A(\omega_A - \omega_P) + M_B(\omega_B - \omega_P) = P_W \tag{13}$$

where ω_A, ω_B, ω_P are the angular rates of the input and output shafts and the virtual planet carrier respectively, and P_W is the waste power. To identify the input and output power contributions, two cases must be considered, depending on the number of teeth of the wheels:

- case 1: if $r_{S1} < r_{S2}$, Equation (9) yields $i < 0$;
- case 2: if $r_{S1} > r_{S2}$, Equation (9) yields $i > 0$.

The two cases are analysed in the following sections.

3.1.1 Case 1: *i* < 0

In this case, the satellites S2 are bigger than S1, and the output ring gear B is bigger than the fixed ring gear F. As a result, the output and input shafts rotate in opposite directions. Considering that the first stage, including the gears A, S1, F and the carrier P, is equivalent to a standard planetary gearing, we can write:

$$|\omega_A| > |\omega_P| \tag{14}$$

$$\omega_A\omega_P > 0 \tag{15}$$

Additionally, for a practical configuration, the output B is much slower than both A and P:

$$|\omega_A| > |\omega_P| > |\omega_B| \tag{16}$$

while the assumption of $i < 0$ yields:

$$\omega_A\omega_B < 0 \tag{17}$$

Additionally, since A is the input shaft and B is the output shaft, $\omega_A M_A > 0$ and $\omega_B M_B < 0$. Eventually, being the transmission ratio high, $M_A \ll M_B$; then, Equation (12) yields $M_B M_F < 0$ and $M_B M_A > 0$. Within the considerations listed above, it is possible to identify input and output power contributions in Equation (13):

- $-M_F\,\omega_P$ is an input power contribution;
- $M_A\,(\omega_A - \omega_P)$ is an input power contribution;
- $M_B\,(\omega_B - \omega_P)$ is an output power contribution.

Then, we can write the definition of the efficiency in the reference frame of the planet carrier (Equation (4)) in terms of torques and angular velocities:

$$\rho = \frac{|P_{out}|}{|P_{in}|} = \frac{-M_B(\omega_B - \omega_P)}{M_A(\omega_A - \omega_P) - M_F\omega_P} \tag{18}$$

Equation (18) can be rearranged by introducing the gear ratios k_A and k_B in the reference frame of the planet carrier, yielding the following:

$$k_A = \frac{\omega_A - \omega_P}{\omega_P} = \frac{z_F}{z_A} \tag{19}$$

$$k_B = \frac{\omega_B - \omega_P}{\omega_P} = -\frac{z_F}{z_{S1}}\frac{z_{S2}}{z_B} \tag{20}$$

$$\frac{1}{\rho} = -\frac{M_A(\omega_A - \omega_P)}{M_B(\omega_B - \omega_P)} + \frac{M_F\omega_P}{M_B(\omega_B - \omega_P)} = -\frac{M_A}{M_B}\frac{k_A}{k_B} + \frac{M_F}{M_B}\frac{1}{k_B} \tag{21}$$

Note that the overall gear ratio i can be expressed in terms of k_A and k_B as:

$$i = \frac{\omega_A}{\omega_B} = \frac{1 + k_A}{1 + k_B} \tag{22}$$

The efficiency η in the fixed reference frame is defined as:

$$\eta = -\frac{M_B}{iM_A} \tag{23}$$

which yields:

$$\frac{M_A}{M_B} = -\frac{1}{i\eta} \tag{24}$$

Substituting Equation (24) into Equation (21) and solving for M_F / M_B, we obtain:

$$\frac{M_F}{M_B} = \frac{k_B}{\rho} - \frac{1}{i\eta} k_A \tag{25}$$

Then, it is possible to rearrange Equation (12) by dividing both terms by M_B:

$$\frac{M_A}{M_B} + 1 + \frac{M_F}{M_B} = 0 \tag{26}$$

and substituting Equations (24) and (25):

$$-\frac{1}{i\eta} + 1 + \frac{k_B}{\rho} - \frac{1}{i\eta} k_A = 0 \tag{27}$$

Solving Equation (27) for η we get the expression of the efficiency in function of the meshing efficiency ρ and the gear ratios k_A, k_B, i:

$$\eta = \frac{1}{i}\frac{1+k_A}{1+k_B/\rho} = \frac{1+k_B}{1+k_B/\rho} \tag{28}$$

3.1.2 Case 2: *i* > 0

In this case, the satellites S2 are smaller than S1, and the output ring gear B is smaller than the fixed ring gear F. As a result, the output and input shafts rotate in the same direction. We can observe the following relationships between the direction of torques and angular velocities:

$$|\omega_A| > |\omega_P| > |\omega_B| \tag{29}$$

$$\omega_A \omega_P > 0 \tag{30}$$

$$\omega_A \omega_B > 0 \tag{31}$$

$$\omega_A M_A > 0 \tag{32}$$

$$\omega_B M_B < 0 \tag{33}$$

$$M_B M_F < 0 \tag{34}$$

$$M_B M_A < 0 \tag{35}$$

Considering Equations (29) to (35), it is possible to identify input and output power contributions in Equation (13):

- $-M_F\, \omega_P$ is an output power contribution;
- $M_A\, (\omega_A - \omega_P)$ is an input power contribution;
- $M_B\, (\omega_B - \omega_P)$ is an input power contribution.

As a result, the meshing efficiency ρ can be written as a function of torques and gear ratios k_A and k_B:

$$\rho = \frac{|P_{out}|}{|P_{in}|} = \frac{|-M_F\omega_P|}{M_A(\omega_A-\omega_P)+M_B(\omega_B-\omega_P)} \tag{36}$$

$$\frac{1}{\rho} = \frac{M_A(\omega_A-\omega_P)}{M_F\omega_P} + \frac{M_B(\omega_B-\omega_P)}{M_F\omega_P} = \frac{M_A}{M_F}k_A + \frac{M_B}{M_F}k_B \tag{37}$$

Leveraging the definition of efficiency in the fixed reference frame (Equation (23)) and the equilibrium to rotation (Equation (12)), it is possible to write:

$$M_B = -\eta i M_A \tag{38}$$

$$M_F = (\eta i - 1) M_A \tag{39}$$

Then, substituting in Equation (37) and solving for η:

$$\eta = \frac{1}{i}\frac{k_A\rho+1}{k_B\rho+1} = \frac{(k_B+1)(k_A\rho+1)}{(k_A+1)(k_B\rho+1)} \tag{40}$$

3.2 EFFICIENCY IN AIDING LOAD CONDITION

The aiding load condition refers to the case when the load is applied on an actuator in the direction of motion. This operating condition is common in the mechanical drives of aerospace actuators, during mission phases such as retraction of flaps or opening of the landing gear [21]. To determine the aiding load efficiency of the considered compound planetary gearbox, we follow a path similar to that described in Section 3.1. The two cases $i < 0$ and $i > 0$ are considered in the following sections.

3.2.1 Case 1: *i* < 0

Assuming that the transmission can be back-driven, the input and output power contributions of Equation (13) are inverted in sign with respect to the opposing load condition of Section 3.1.1:

- $-M_F\,\omega_P$ is an output power contribution;
- $M_A\,(\omega_A-\omega_P)$ is an output power contribution;
- $M_B\,(\omega_B-\omega_P)$ is an input power contribution.

Then, the efficiency in the reference frame of the planet carrier can be written as:

$$\rho = \frac{|P_{out}|}{|P_{in}|} = \frac{M_A(\omega_A-\omega_P) - M_F\omega_P}{-M_B(\omega_B-\omega_P)} = -\frac{M_A}{M_B}\frac{k_A}{k_B} + \frac{M_F}{M_B}\frac{1}{k_B} \tag{41}$$

The efficiency in the fixed reference frame in the aiding load condition is defined as:

$$\eta = -\frac{iM_A}{M_B} \tag{42}$$

Computing M_A/M_B from Equation (42), substituting into Equation (41) and solving for M_F/M_B yields:

$$\frac{M_F}{M_B} = k_B\rho - \frac{\eta}{i}k_A \tag{43}$$

Substituting Equations (42) and (43) into Equation (26) and solving for η we get the expression of the gearbox efficiency in aiding load conditions:

$$\eta = i\,\frac{1+k_B\rho}{1+k_A} = \frac{1+k_B\rho}{1+k_B} \tag{44}$$

3.2.2 Case 1: *i* > 0

Similarly to Section 3.2.1, it is possible to identify input and output power contributions of Equation (13) for the case of $i > 0$, in aiding load condition. Assuming that the gearbox can be back-drive, the sign of all torques will be inverted with respect to the case considered in Section 3.1.2:

- $-M_F\,\omega_P$ is an input power contribution;
- $M_A\,(\omega_A-\omega_P)$ is an output power contribution;
- $M_B\,(\omega_B-\omega_P)$ is an output power contribution.

Then, the efficiency in the reference frame of the planet carrier can be written as:

$$\rho = \frac{|P_{out}|}{|P_{in}|} = \frac{M_A(\omega_A-\omega_P)+M_B(\omega_B-\omega_P)}{M_F\omega_P} = \frac{M_A}{M_F}k_A + \frac{M_B}{M_F}k_B \tag{45}$$

In the aiding load condition, leveraging the definition of overall efficiency of Equation (23) and the equilibrium to torques of Equation (12), it is possible to write:

$$M_B = -\frac{i}{\eta}M_A \tag{46}$$

$$M_F = \left(\frac{i}{\eta} - 1\right)M_A \tag{47}$$

Equations (46) and (47) can be substituted into Equation (45), and solving for η yields the expression for the efficiency in terms of gear ratios and meshing efficiency ρ:

$$\eta = \frac{\rho+k_B}{\rho+k_A}\,i = \frac{(1+k_A)(\rho+k_B)}{(1+k_B)(\rho+k_A)} \tag{48}$$

4 RESULTS

Table I summarizes the expressions for the opposing and aiding load efficiencies, η_D and η_I respectively, for the two analysed in Section 3. Numerical evaluation of the efficiency has been performed for two combination of numbers of teeth, to consider both cases of $i < 0$ and $i > 0$. The configurations were chosen to achieve a gear ratio similar in modulus, and the numbers of teeth of each wheel are reported in Table II

Table I – Analytical expressions of efficiency

Gearbox configuration	Opposing load efficiency, η_D	Aiding load efficiency, η_I
$i < 0$	$\dfrac{1+k_B}{1+k_B/\rho}$	$\dfrac{1+k_B\rho}{1+k_B}$
$i > 0$	$\dfrac{(k_B+1)(k_A\rho+1)}{(k_A+1)(k_B\rho+1)}$	$\dfrac{(1+k_A)(\rho+k_B)}{(1+k_B)(\rho+k_A)}$

Table II – Number of teeth of the two considered configurations

Gear ratio	i = -122	i = 124
z_A	20	21
z_B	61	62
z_F	60	63
z_{S1}	20	21
z_{S2}	21	20

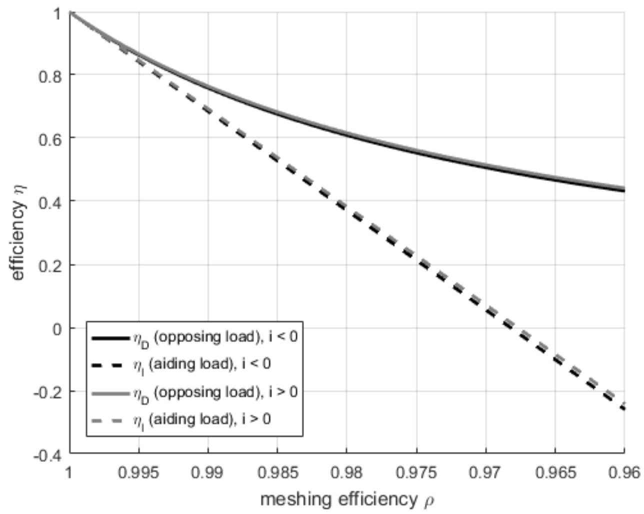


Figure 3 Opposing (solid line) and Aiding load (dashed line) efficiency for i = –122 (black) and i = 124 (grey).

Figure 3 shows the aiding and opposing load efficiencies of both configurations, as a function of the meshing efficiency ρ. Despite the different analytical formulation, the behaviour of the two gearboxes is very similar for a given gear ratio. With the parameters considered for this numerical evaluation, both the transmission can be back-driven (i.e. the aiding load efficiency is positive) if the meshing efficiency is higher than 96.8%.

Figure 4 shows the aiding load efficiency plotted against opposing load efficiency, for the two considered gearboxes. In both cases, the aiding load efficiency is positive if the opposing load efficiency is higher than about 50%.

4.1 COMPARISON WITH SIMILAR REDUCERS

The compound planetary gearbox considered in this work offers performances in terms of torque density and gear ratio similar to Harmonic Drives, and is suitable for similar applications: robotic joints, final reduction of electromechanical aircraft actuators, industrial automation. Then, a comparison with Harmonic Drives in terms of efficiency can help the choice of the most suitable drive for a given application.

Additionally, the efficiency of the considered compound planetary gearbox is compared with the planetary layouts of Figure 5: those are alternative, carrier-driven gearboxes that allow high reduction ratios within small dimensions.
The methodology described in Section 2 can be applied to an Harmonic Drive with fixed ring gear F and flexspline S, resulting in the following analytical expressions for the efficiencies:

$$\eta_D = \frac{1}{i}\frac{1}{1-k/\rho} \tag{49}$$

$$\eta_I = i(1-k\rho) \tag{50}$$

where:

$$i = \frac{1}{1-k} \tag{51}$$

and $k=z_F/\,z_S$. The same approach allows to estimate the efficiency of the layouts shown in Figure 5.
In both cases, assuming that P is the planet carrier (input shaft) and B is the output shaft, we can define the gear ratio in the reference frame of the planet carrier:

$$k = -\frac{\omega_B-\omega_P}{\omega_P} \tag{52}$$

As a result, the gear ratio is still given by Equation (51), and considering, as an example, the case for $i > 0$:

$$\eta_D = \frac{1-k}{1-k\rho} \tag{53}$$

$$\eta_I = \frac{1-k\rho}{1-k} \tag{54}$$

The efficiency of those reducers is compared with that of the compound planetary gearbox considered in this work, for different values of the meshing efficiency ρ. The result is shown in Figure 6.

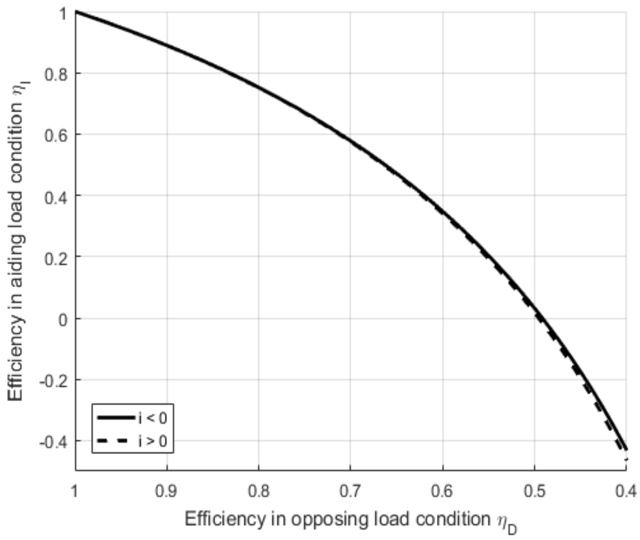


Figure 4 Aiding load efficiency vs Opposing load efficiency for *i* = –122 (solid line) and *i* = 124 (dashed line).

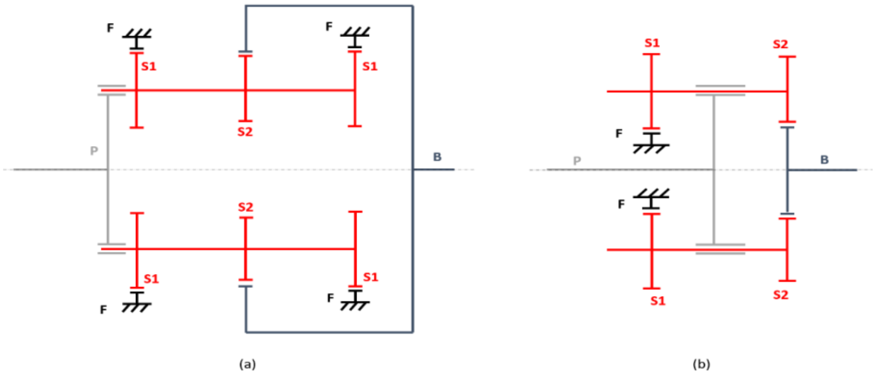


Figure 5 Some possible alternative configurations for a high gear ratio, carrier driven planetary reducer

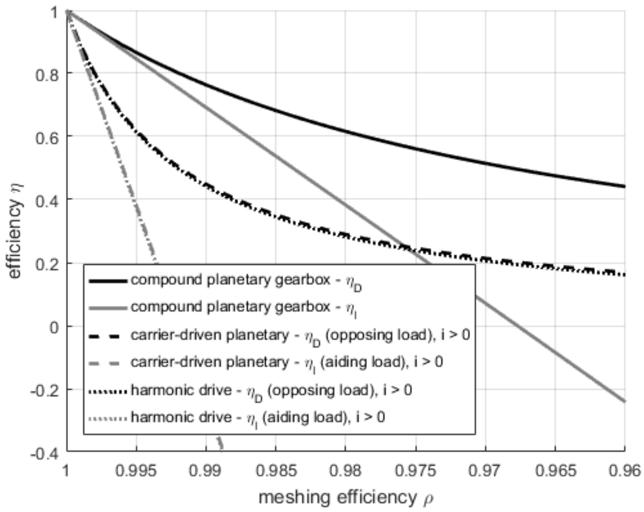


Figure 6 Comparison between the considered compound planetary gearbox, a Harmonic Drive, and the alternative layouts of Figure 5.

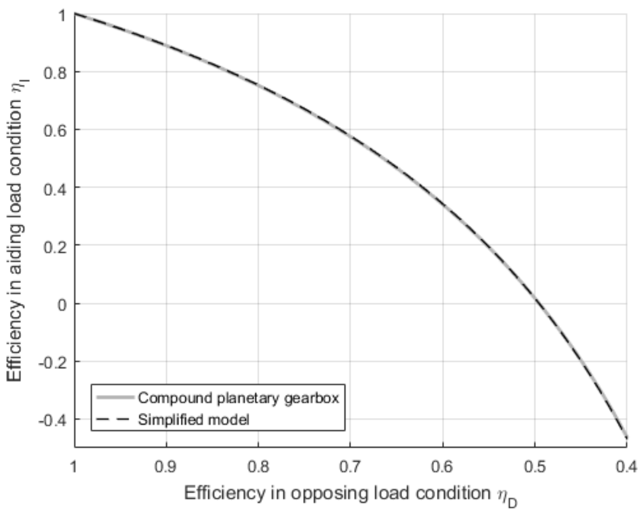


Figure 7 Comparison between the results for the compound planetary gearbox with *i* = 124 and the simplified model proposed by [14].

The compound planetary gearbox shows a slower decrease of both aiding and opposing load efficiencies with the efficiency of meshing, since the lower speed of the planet carrier reduces the amount of recirculating power through the satellites, with respect to the alternative gearboxes.
However, to compare correctly the different solutions, the different number of meshing gears must be taken into account. Typical values of ρ are in the range from 99.2% to 99.5% for each external meshing, and in the range from 99.5% to 99.8% for each internal meshing.

- The Harmonic Drive has a single internal meshing, so its meshing efficiency is in the range of 99.5% to 99.8%. This results in an opposing load efficiency in the range of 60% to 80%.
- The proposed planetary gearbox has two internal gear meshings and one external, with an overall meshing efficiency in the range of 98.2% to 99.1%. This results in an opposing load efficiency in the same range of the Harmonic Drive.
- The carrier driven reducers have two meshings, either external or internal, with with an overall meshing efficiency in the range of 98.4% to 99.6%. This results in an opposing load efficiency in the range of 35% to 65%.

Hence, the compound planetary gearbox considered in this work has performances in terms of efficiency comparable with Harmonic Drives, clearly superior to carrier-driven planetary drives. The design considered in this paper is easier to manufacture than an Harmonic Drive of similar performances, since it relies on wheels with a reduced number of teeth, and accepts lower manufacturing tolerances; the elimination of a flexible component (the flexspline of a Harmonic Drive) allows to achieve a higher transmission stiffness and better robustness to fatigue.
Additionally, the design can be easily back-driven even with a high reduction ratio. This may be useful for robotic application where compliant actuators are needed, and allows to estimate the external load from the motor current.

4.2 COMPARISON WITH A SIMPLIFIED MODEL

The results in terms of aiding and opposing load efficiency for the considered gearbox are compared with the simplified model proposed in [14]. This simplified model only provides a relationship between gear ratio, aiding load efficiency and opposing load efficiency, but not an estimate of the efficiency itself. The relationship is derived from a simple model of a lever with friction in the hinge, and is summarized by the following equation:

$$\eta_I = \frac{2i^2\eta_D - i^2 + i}{i^2\eta_D + i - \eta_D + 1} \tag{55}$$

The result of the comparison is shown in Figure 7 and highlights a very good matching between the curves, with an error in the order of 1%. Then, the simplified model of [14] can be applied in computationally light simulations of gearboxes characterized by the architecture considered in this work.

5 CONCLUSIONS

A particular configuration of planetary drive was analysed in terms of mechanical efficiency, dealing with aiding and opposing loads. The results were compared with the performance of alternative reducers suitable for similar applications, such as Harmonic Drives and carrier driven planetary gears. The configuration analysed in this work showed a significantly better efficiency than other compact reducers, comparable with Harmonic Drives, while relying on less strict tolerances and allowing for a higher stiffness. Additionally, the resulting relationship between aiding and opposing load efficiency was compared to a simplified model showing a good matching, and suggesting that the simplified model can be successfully employed to estimate aiding load efficiency of mechanical transmissions within computationally light numerical simulations.

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