An MILP Optimization Problem for Sizing Port Rail Networks and Planning Shunting Operations in Container Terminals

(Article begins on next page)

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***Abstract*—This paper proposes an optimization approach for sizing port rail networks and planning railway shunting operations by adopting a discrete-time model of the overall system. Firstly, a mixed-integer linear mathematical programming problem is defined in order to optimize shunting operations to be performed on the considered network by satisfying certain arrivals and departures of import and export flows. Moreover, the proposed procedure can be used to evaluate the capacity of a port rail network, in terms of maximum number of trains that can be managed over a certain time horizon, and to carry out what-if analyses aimed at testing different scenarios. The effectiveness of the proposed approach is shown by applying the optimization problem to a real case study referred to the port rail network of La Spezia Container Terminal located in Northern Italy. A computational analysis realized by varying the dimension and complexity of the problem instances is also reported in the paper to discuss the computational performance of the proposed model.**

***Note to Practitioners* — This paper addresses the problem of planning shunting operations and sizing rail networks in seaports. This problem is faced by defining a dynamical model and an associated optimization problem, in which the movements of rail cars inside a port rail network are described, both for the import and for the export flow. In the considered problem, many important and realistic aspects are modeled in detail, such as arrivals and departures of scheduled trains, sharing of locomotives among different areas of the network, number and capacity of different resources in the system, and so on. In the paper, the application to a real case study of an Italian port rail network is discussed, showing the effectiveness of the proposed methodology. Thanks to the general approach provided in the paper, this planning procedure could be applied to other real cases worldwide.**

***Index Terms*—Seaport container terminals; rail networks; optimization methods; MILP model.**

I. INTRODUCTION

The importance of rail freight transport for ensuring sustainability, reducing pollution and congestion and favoring intermodal transportation is well known [1]. However, this transport mode struggles against a certain number of critical issues, mainly regarding organizational and infrastructural aspects. Drawing a comparison with road transport, rail mode is affected by a higher number of constraints (such as weight limitations on rail cars and rail lines, or maximum gabarit). For these reasons, rail mode requires a very efficient planning in order to be really competitive with road transport.

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A survey on container processing in railway yards is presented in [2], from an operations research perspective. Specifically, the authors of [2] analyze the basic decision problems for two important yard types, i.e. conventional rail-road and modern rail-rail transshipment yards. For other surveys on rail transport refer to [3], [4], where optimization models for the most commonly studied rail transportation problems are presented, and to [5], basically focused on rail-road terminals and intermodal freight transport.

Seaports, representing intermodal exchange nodes for national and international networks, play a crucial role in the capacity of fostering a fast and efficient forwarding of goods in the worldwide logistics supply chains (see e.g. [6], [7], [8], [9], [10] for surveys on maritime container terminals and the main associated planning problems). In this perspective, the enhancement of rail transport in seaports is essential for the development of the logistic-transportation system of a country.

One of the challenges related to rail freight transport is represented by properly sizing the capacity of a port rail network in terms of maximum number of trains that can be managed by the overall system over a certain time horizon. The present paper faces this topic by providing an optimization approach which jointly considers the import and the export flows passing through a seaport node. In the literature, different optimization and simulation approaches have been developed for defining the capacity of a railway network, which is a crucial aspect for evaluations of new investments, as well as for an efficient management of the infrastructure [11]. Objective of this work is to provide logistic operators and rail shunting companies with a planning tool to be used in real contexts to size the capacity of rail networks, as well as to evaluate future investments. In other words, this type of planning tool, based on optimization techniques, is able to provide quantitative analyses to identify the most critical resources in the system, to compare different possible investments in order to evaluate the most profitable ones, and so on.

One of the first analytical approaches for sizing railway networks dates back to the Sixties [12], dealing with the problem of junction capacity. Other approaches are based on optimization techniques, as for instance [13], where a Mixed-Integer Linear Programming (MILP) model is formulated in order to evaluate the capacity of a railway station plant, by considering a single railway station for passenger transport. As an alternative to analytical methods, researchers have also studied railway networks by using simulation approaches. In [14] a mesoscopic simulation model is proposed for the analysis and evaluation of rail operations connected to freight trains in a railway network, the components of which are thought as interconnected systems of queues. In order to analyse the main bottlenecks of the port network in Northern Italy and to estimate the possible increase of maritime traffic, discrete-event simulation is used in [15]. The framework of Petri Nets is adopted in [16] to model railway networks, in terms of stations and tracks including sensors and semaphores, in order to automatically design the system controller.

Another branch of study related to rail transport regards the railyard dispatching problem. In particular, [17] provides a survey on the operational processes at shunting yards by classifying the approaches of the literature according to different sorting strategies. Several authors have faced the railyard dispatching problem; most of them apply fuzzy theory, such as [18] which presents an expanded yard model where several variables are considered fuzzy, via a multi-objective approach.

Other papers in the literature refer to the problem of optimally scheduling trains in a rail network. While passenger trains are normally planned long time before they begin operating, freight trains are more often scheduled on a real-time basis ([19], [20]). In [21], the Train Timetable Problem is taken into account, both in its nominal and robust version. In [22], a Branch and Bound algorithm is developed in order to reschedule trains in real time, so that the deviation from the original schedule can be minimized. In [23], the train schedule is realized by applying Model Predictive Control. In [24], a MILP model is formulated to define the best schedule of passenger trains on single and double tracked lines. The work in [25] solves the train scheduling problem by deciding the sequence of trains at parallel railway tracks. The authors put a special focus on solving deadlocks and avoiding multiple crane picks per container move by providing a mathematical program which is solved with Dynamic Programming and Beam Search heuristic. In [26], some solutions for the train scheduling problem and rail yards planning are studied. The train scheduling problem regards the assignment of trains and train times to a set of rail lines and station stops, whereas a yard planning is the detailed definition of the yards activities in terms of trains classification or assembly.

The work presented in this paper is devoted to model the import and export flows in a port rail network. The adopted dynamic model represents the movement of rail cars in the system: the presence of rail cars in a specific area is modeled as a buffer, and the dynamics of the system is given by discrete-time conservation equations. Such model takes inspiration from [27], in which a planning approach is proposed to optimize railway operations in a seaport terminal by adopting a queue-based discrete-time model. More specifically, in [27] a MILP problem is defined to optimize the timing of import trains and the use of the handling resources devoted to rail port operations. Compared with [27], the model presented in this paper is more general and extended, since both import and export flows are modeled, many specific operative constraints are added and the dynamic evolution of rail cars (instead of containers) is represented. Moreover, a wider port rail network is considered and a more detailed and comprehensive simulation analysis is reported. Other works adopting the same queue-based logic can be found in [28] and [29], in which the whole seaport container terminal is modeled by a system of queues and the optimization problem aims at defining the optimal utilization of terminal resources in order to minimize the transfer delays of containers.

The strength of the proposed model lies in the ability of accurately representing the port rail network, considering different types of rail cars and peculiar aspects of the considered system, whilst maintaining an aggregate flow-based modeling framework. Different aggregate models for container terminals have been proposed in the literature. Some works provide mathematical models to optimize logistic processes inside terminals, as in [30], [31]. In other cases, simulation models are developed in order to study and analyse management policies in terminals, as done for instance in [32], [33], [34], [35], [36]. Finally, Petri Nets models have been also considered for making performance evaluations of intermodal freight transport terminals [37], [38].

The contribution of this paper stands in providing a planning procedure for the port rail network considering both import and export flows. Firstly, the proposed scheme allows to take decisions on rail operations in terms of sequence and timing of all the shunting operations that have to be performed for satisfying arrivals and departures of import and export flows. Secondly, the proposed planning procedure can be used to evaluate the capacity of a port rail system in terms of maximum number of trains that can be correctly managed over a specific time horizon, given a certain level of productivity of the terminal resources. In [39], an optimization model for the same class of systems as the one considered in the present paper has been used for the specific purpose of evaluating the capacity of a port rail system by performing what-if analyses on several scenarios. In particular, the different scenarios refer to the possibility of varying the terminal resources such as the number of diesel locomotives and the productivity of the terminal equipment. The decisional level is, in the case of [39], moved from the tactical level addressed in the present paper to a strategic level in which the evaluation of different network configurations is the main objective.

The paper is organized as follows. In Section II the problem under consideration is analysed, while in Section III the discrete-time dynamic model of the port rail network is described in detail. The MILP problem for sizing the capacity of the system is formulated in Section IV. Some results based on a real case study related to a port in Northern Italy are discussed in Section V. Finally, concluding remarks are presented in Section VI.

II. Problem description

The present model considers both the import and export flows in a port rail network: specifically, the former represents the movement of freight trains from the seaport terminal to the hinterland, while the latter models the opposite flow.

As described in Fig. 1, the import flow is modeled starting from the wait of containers in the yard stacking area until their departure from external rail stations towards their final destinations. In particular, containers stored in the yard are

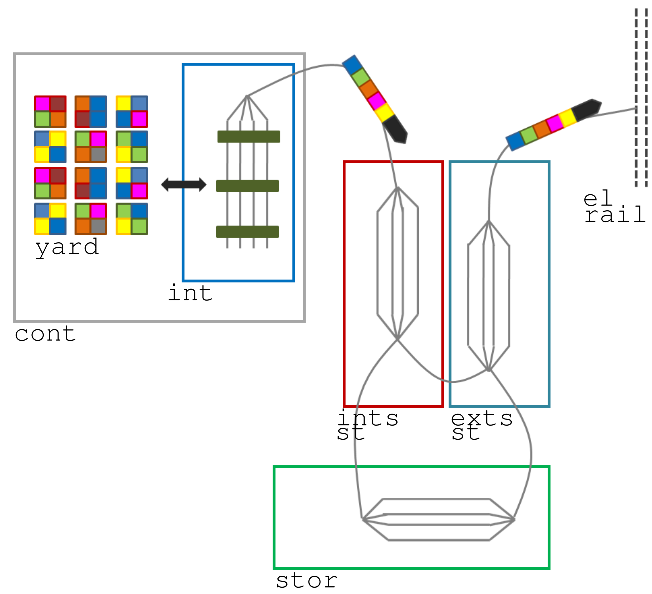

Fig. 1. The port rail network.

brought by terminal equipment to the rail tracks in the internal rail park (inside the port terminal) and loaded on rail cars usually by means of rail-mounted gantry cranes or reach stackers. Once the train is loaded, shunting operations by diesel locomotives are performed to move trains to one of the railway stations located outside the seaport terminal. These rail stations are distinguished in "internal" stations, if they are not directly connected with the electrified rail lines, and "external" stations, if they allow trains to depart by using the electric traction. In case the scheduled departure is not close in time or there is a high level of occupation of rail tracks inside the stations, trains can be moved to apposite storage parks (again with diesel locomotives) where they can wait for their departure towards the hinterland.

Similarly, the export flow is characterized by the following steps. Freight trains loaded with containers that will continue their trip by ship arrive at an external rail station by electrified line. Then, depending on the urgency of their loading on the ship, trains are brought through diesel traction to the internal rail park, by passing through an internal rail station, or to a rail storage park where they can wait before being transported to the internal rail park. Once trains have been transported inside the terminal, containers can be unloaded from rail cars and moved to the yard stacking area.

It is worth underlining that, inside the considered port rail network, diesel traction allows to move entire trains as well as groups of rail cars, while the arrivals and departures through electric lines are permitted only to complete trains.

The entire process is affected by a certain number of delays, related to both physical aspects and documentary practices. First of all, delays associated with shunting operations are considered; such delays primarily affect the storage parks, where there can be strong space limitations due to the presence of short tracks, so obliging to split trains and then recompose them. Other types of delays regard technical checks that are carried out in the stations, namely the verification of the train braking system due to the change of traction, technical checks on rail cars to test their correct weight patterns and the emission of legal documents necessary to allow trains to leave the network via the electrified line. While delays of the first kind primarily affect rail shunting companies, the second type is strongly influenced by the companies operating the rail transport service.

The model described in this paper represents the import and export flows in the port rail network in terms of dynamic evolution of rail cars, which are properly distinguished in different typologies, depending also on the railway company they belong to. Moreover, the operations of composing trains and dividing them in sets of rail cars are modeled: these operations are normally necessary in case the length of a full train is greater than the length of the tracks of a rail park. Besides, some constraints have been considered in the present problem, regarding for instance the number of available tracks in rail parks as well as the number of connecting tracks, the allowable length of each track, the limited number of diesel locomotives serving the different areas of the port rail network, and the maximum productivity of handling resources.

III. The Discrete-Time Dynamic Model

In the proposed model, the positions of rail cars inside the port rail network are modelled by means of buffers. The dynamic evolution of such buffers is described by discrete-time equations with sample time equal to Δt. Throughout the paper, the following notation is adopted: if a given variable z refers to the import flow, the corresponding variable associated with the export flow is denoted with $\bar{z}$.

A. Network structure and problem parameters

The port rail network is described through a graph (see Fig. 2), in which the nodes can be gathered in set $\mathcal{N} \cup \{0\}$, where $\mathcal{N}$ is the set of railway parks or stations, whereas 0 is a source node. This latter represents the yard stacking area where both import and export containers are present (in this model, containers are properly converted into rail cars since the model dynamics is referred to flows of rail cars in the network). The set $\mathcal{N}$ can be subdivided into four disjoint sets, i.e. $\mathcal{N} = \mathcal{N}^{\mathrm{T}} \cup \mathcal{N}^{\mathrm{S}} \cup \mathcal{N}^{\mathrm{IS}} \cup \mathcal{N}^{\mathrm{ES}}$. $\mathcal{N}^{\mathrm{T}}$ is the set of internal railway parks, present in the terminal, where rail cars are loaded/unloaded by terminal equipments; $\mathcal{N}^{\mathrm{S}}$ represents the storage parks, i.e. places where trains or groups of rail cars wait before being moved to other nodes; $\mathcal{N}^{\mathrm{IS}}$ is the set of internal stations, i.e. intermediate nodes; $\mathcal{N}^{\mathrm{ES}}$ is the set of external stations where trains arrive/leave by electrified line.

Referring to Fig. 2, the import flow is the following: from node 0 rail cars move to an internal rail park $n \in \mathcal{N}^{\mathrm{T}}$, from where they can go either to an internal station $n \in \mathcal{N}^{\mathrm{IS}}$ (and only later to an external one $n \in \mathcal{N}^{\mathrm{ES}}$) or directly to an external station; both from internal and external stations rail cars can be moved to a storage area $n \in \mathcal{N}^{\mathrm{S}}$, if there exists one. Rail cars leave the system only via external stations $n \in \mathcal{N}^{\mathrm{ES}}$. The export flow is in the opposite direction, from external stations to node 0. Moreover, let $\mathcal{S}_n^{\mathrm{I}}$ and $\mathcal{S}_n^{\mathrm{E}}$ indicate the set of successor nodes of node n, in import and in export

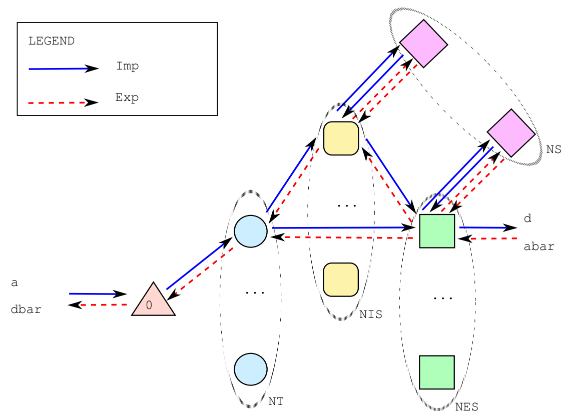

Fig. 2. The graph of the network.

respectively; analogously, $\mathcal{P}_n^{\mathrm{I}}$ and $\mathcal{P}_n^{\mathrm{E}}$ indicate the set of import and export predecessor nodes of node n. Since import and export flows are in opposite directions, it holds that $\mathcal{S}_n^{\mathrm{I}} = \mathcal{P}_n^{\mathrm{E}}$, $\mathcal{P}_n^{\mathrm{I}} = \mathcal{S}_n^{\mathrm{E}}$, $\forall n \in \mathcal{N}$.

Each node $n \in \mathcal{N}$ is modelled as a physical resource composed of a certain number of rail tracks: let $\mathcal{R}_n$ indicate the set of tracks in node n and $R_{n,m}$ the number of tracks connecting node n with node m in the network. Note that these connecting tracks are shared by import and export flows, then by definition it yields $R_{n,m} = R_{m,n}$, $\forall n, m \in \mathcal{N}$. The length of track $i \in \mathcal{R}_n$ of node n is indicated with $L_{n,i}$. The model also distinguishes between long and short rail tracks inside each node and, in particular, $\mathcal{R}_n^{\mathrm{L}} \subseteq \mathcal{R}_n$ and $\mathcal{R}_n^{\mathrm{S}} \subseteq \mathcal{R}_n$ indicate the set of long and short tracks of node $n \in \mathcal{N}$, respectively: the former type of tracks can contain a whole train, while only groups of rail cars can be present in short rail tracks. The number of rail cars composing an entire train is denoted with Q', while groups are composed of Q'' rail cars, with $Q'' < Q'$.

As explained above, the rail cars in the network can be of different types and can belong to different railway companies. Let C indicate the number of railway companies and $\mathcal{W}$ be the set of rail car types. The set $\mathcal{W}$ is partitioned into subsets according to the railway company, i.e. $\mathcal{W} = \mathcal{W}_1 \cup \mathcal{W}_2 \ldots \cup \mathcal{W}_C$. Moreover, let l^w denote the length of car type w.

Diesel locomotives are shared in specific areas of the network; areas are intended as sets of nodes of the considered network. Let H indicate the number of these areas and Λ_h the number of locomotives available in area h. Then, $\mathcal{N}_h \subseteq \mathcal{N}$, $h = 1, \ldots, H$, indicates the set of nodes of area h served by the Λ_h locomotives. The productivity of the handling means moving the equivalent rail cars from the yard stacking area to each terminal and vice versa is denoted with Γ_n, $n \in \mathcal{N}^{\mathrm{T}}$.

All the considered delays are supposed to be multiple of the sample time Δt. In particular, $\tau_{n,m}$, $n, m \in \mathcal{N}$, is the time required to cross the tracks between node n and node m. The parameter δ represents the time required to realize shunting operations in storage parks, where, as already said, there are several space limitations. The time required for technical checks and documentary practices on rail cars of type w is indicated with γ^w, $w \in \mathcal{W}$.

In node 0 import rail cars arrive and export rail cars leave. In particular, at a generic time t, the arrivals of import rail cars and the departures of export rail cars are given by quantities $a_0^w(t)$ and $\bar{d}_0^w(t)$, $w \in \mathcal{W}$, $t = 0, \ldots, T-1$, indicating the number of rail cars of type w arrived and left at time t, respectively. Note that if in a given time t no arrivals nor departures occur, these quantities are set to 0. Analogously, in the external stations, arrivals of export containers and departures of import containers occur, i.e. $d_{n,i}^w(t)$ and $\bar{a}_{n,i}^w(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}^{\mathrm{ES}}$, $i \in \mathcal{R}_n$, $t = 0, \ldots, T-1$.

B. Model variables

The number of rail cars, in import and in export respectively, of type w present in track i of node n at time t are denoted with $q_{n,i}^w(t)$ and $\bar{q}_{n,i}^w(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $t = 0, \ldots, T$. Analogously, referring to the source node, $q_0^w(t)$ and $\bar{q}_0^w(t)$, $w \in \mathcal{W}$, $t = 0, \ldots, T$ indicate the number of import and export rail cars of type w present at time t, respectively.

Inside the network, the movements of trains (composed of Q' rail cars) from a node to another one, in the import flow, are represented with a set of binary variables, i.e. $y_{n,i,m,j}^c(t)$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n^{\mathrm{L}}$, $m \in \mathcal{S}_n^{\mathrm{I}}$, $j \in \mathcal{R}_m$, $c = 1, \ldots, C$, $t = 0, \ldots, T-1$. Specifically, $y_{n,i,m,j}^c(t) = 1$ means that, at time t, an import train belonging to railway company c leaves track i of node n, being directed to track j of node m. In the same way, the binary variable $\bar{y}_{n,i,m,j}^c(t)$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n^{\mathrm{L}}$, $m \in \mathcal{S}_n^{\mathrm{E}}$, $j \in \mathcal{R}_m$, $c = 1, \ldots, C$, $t = 0, \ldots, T-1$, models the shift of trains in the export cycle. Note that, since the binary variables $y_{n,i,m,j}^c(t)$ and $\bar{y}_{n,i,m,j}^c(t)$ are related to entire trains, the set of tracks to which they refer is the set of long tracks.

Considering the same logic, it is possible to represent the movement of groups of rail cars (i.e. Q'' rail cars) from a node to another one, both in import and in export. In particular, binary variables $x_{n,i,m,j}^c(t) = 1$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{I}}$, $j \in \mathcal{R}_m$, $c = 1, \ldots, C$, $t = 0, \ldots, T-1$, indicate that, at time t, a group of rail cars of railway company c is moved from track i of node n to track j of node m. Analogously, for the export flow, binary variables $\bar{x}_{n,i,m,j}^c(t)$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{E}}$, $j \in \mathcal{R}_m$, $c = 1, \ldots, C$, $t = 0, \ldots, T-1$, are defined.

Moreover, it is necessary to associate, with each movement of trains or groups of rail cars, the corresponding number of rail cars actually moved. To this end, two sets of continuous variables are defined, for the import and export flow respectively. In particular, $r_{n,i,m,j}^w(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{I}}$, $i \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, represents the number of rail cars of type w that, at time t, leaves track i of node n, being directed to track j of node m. Analogously, $\bar{r}_{n,i,m,j}^w(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{E}}$, $i \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, is defined for the export flow.

Furthermore, similar variables are introduced for the connection with node 0, i.e. $r_{0,m,j}^w(t)$, $w \in \mathcal{W}$, $m \in \mathcal{S}_0^{\mathrm{I}}$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, representing the number of rail cars of type w that, at time t, leave the yard stacking area to go to track j of node m. Similarly, $\bar{r}_{n,i,0}^w(t)$, $w \in \mathcal{W}$, $n \in \mathcal{P}_0^{\mathrm{E}}$, $i \in \mathcal{R}_n$, $t = 0, \ldots, T-1$, have the same meaning for the export flow.

C. System dynamics

The dynamics of the overall transfer activities in the network can be described with conservation equations; in particular, each node is modelled by two conservation equations, for the import and export flow, respectively. Conservation equations for node 0 are the following:

$$q_0^w(t+1) = q_0^w(t) + a_0^w(t) - \sum_{m\in\mathcal{S}_0^{\text{I}}}\sum_{j\in\mathcal{R}_m} r_{0,m,j}^w(t) \qquad w\in\mathcal{W},\, t=0,\dots,T-1 \quad (1)$$

$$\bar{q}_0^w(t+1) = \bar{q}_0^w(t) + \sum_{n\in\mathcal{P}_0^{\text{E}}}\sum_{i\in\mathcal{R}_m} \bar{r}_{n,i,0}^w(t) - \bar{d}_0^w(t) \qquad w\in\mathcal{W},\, t=0,\dots,T-1 \quad (2)$$

In (1), it is shown that the number of import rail cars at node 0 increases depending on the arrivals and decreases if import rail cars are transferred to the internal railway parks by terminal equipment. On the contrary, as reported in (2), the number of export rail cars at node 0 increases if export rail cars are moved from the internal railway parks and decreases when they leave the terminal.

The dynamics of the internal rail parks is given by the following equations:

$$q_{n,i}^w(t+1) = q_{n,i}^w(t) + r_{0,n,i}^w(t) - \sum_{m\in\mathcal{S}_n^{\text{I}}}\sum_{j\in\mathcal{R}_m} r_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{T}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (3)$$

$$\bar{q}_{n,i}^w(t+1) = \bar{q}_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{E}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} \bar{r}_{m,j,n,i}^w(t-\tau_{m,n}) - \bar{r}_{n,i,0}^w(t) \qquad n\in\mathcal{N}^{\text{T}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (4)$$

The conservation equation (3) describes the dynamic evolution of the import rail cars in the internal rail parks: they increase when new rail cars are transferred from node 0 by specific handling systems and decrease when they are moved to other parks/stations. The opposite situation occurs for export rail cars in the internal rail parks, as described by (4). Note that in this latter case the time $\tau_{n,m}$ required to cross the tracks from other parks/stations to the internal rail parks is appropriately taken into account in (4).

The dynamic evolution of the rail storage parks can be written as:

$$q_{n,i}^w(t+1) = q_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{I}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} r_{m,j,n,i}^w(t-\tau_{m,n}) - \sum_{m\in\mathcal{S}_n^{\text{I}}}\sum_{j\in\mathcal{R}_m} r_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{S}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (5)$$

$$\bar{q}_{n,i}^w(t+1) = \bar{q}_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{E}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} \bar{r}_{m,j,n,i}^w(t-\tau_{m,n}) - \sum_{m\in\mathcal{S}_n^{\text{E}}}\sum_{j\in\mathcal{R}_m} \bar{r}_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{S}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (6)$$

In particular, (5) refers to import rail cars in storage areas, which increase when rail cars arrive from other parks/stations (taking into account the time required to cross the tracks) and decrease when they leave towards other parks/stations. An analogous behaviour occurs for export rail cars in storage areas, as described by (6).

The dynamics of internal stations is given by the following:

$$q_{n,i}^w(t+1) = q_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{I}}\cap\mathcal{N}^{\text{S}}:\tau_{m,n}\le(t-\delta)}\ \sum_{j\in\mathcal{R}_m} r_{m,j,n,i}^w(t-\tau_{m,n}-\delta) + \sum_{m\in\mathcal{P}_n^{\text{I}}\setminus\mathcal{N}^{\text{S}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} r_{m,j,n,i}^w(t-\tau_{m,n}) - \sum_{m\in\mathcal{S}_n^{\text{I}}}\sum_{j\in\mathcal{R}_m} r_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{IS}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (7)$$

$$\bar{q}_{n,i}^w(t+1) = \bar{q}_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{E}}\cap\mathcal{N}^{\text{S}}:\tau_{m,n}\le(t-\delta)}\ \sum_{j\in\mathcal{R}_m} \bar{r}_{m,j,n,i}^w(t-\tau_{m,n}-\delta) + \sum_{m\in\mathcal{P}_n^{\text{E}}\setminus\mathcal{N}^{\text{S}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} \bar{r}_{m,j,n,i}^w(t-\tau_{m,n}) - \sum_{m\in\mathcal{S}_n^{\text{E}}}\sum_{j\in\mathcal{R}_m} \bar{r}_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{IS}},\, w\in\mathcal{W},\, i\in\mathcal{R}_n,\, t=0,\dots,T-1 \quad (8)$$

In particular, the dynamics of import and export rail cars in internal stations described by (7) and (8) is very similar to the one related to rail storage parks and described by (5) and (6), respectively. The only difference is that, if the preceding node corresponds to a storage park, not only the time $\tau_{n,m}$ required to cross the tracks is considered but also the time δ required for the shunting operations is added. Finally, the conservation equations for the external stations are:

$$q_{n,i}^w(t+1) = q_{n,i}^w(t) + \sum_{m\in\mathcal{P}_n^{\text{I}}\cap\mathcal{N}^{\text{S}}:\tau_{m,n}\le(t-\delta)}\ \sum_{j\in\mathcal{R}_m} r_{m,j,n,i}^w(t-\tau_{m,n}-\delta) + \sum_{m\in\mathcal{P}_n^{\text{I}}\setminus\mathcal{N}^{\text{S}}:\tau_{m,n}\le t}\ \sum_{j\in\mathcal{R}_m} r_{m,j,n,i}^w(t-\tau_{m,n}) - \sum_{m\in\mathcal{S}_n^{\text{I}}}\sum_{j\in\mathcal{R}_m} r_{n,i,m,j}^w(t) - d_{n,i}^w(t+\gamma^w) \qquad n\in\mathcal{N}^{\text{ES}},\, i\in\mathcal{R}_n,\, w\in\mathcal{W},\, t=0,\dots,T-1 \quad (9)$$

$$\bar{q}_{n,i}^w(t+1) = \bar{q}_{n,i}^w(t) + \bar{a}_{n,i}^w(t) - \sum_{m\in\mathcal{S}_n^{\text{E}}}\sum_{j\in\mathcal{R}_m} \bar{r}_{n,i,m,j}^w(t) \qquad n\in\mathcal{N}^{\text{ES}},\, w\in\mathcal{W},\, i\in\mathcal{R}_n,\, t=0,\dots,T-1 \quad (10)$$

Analyzing (9), the import rail cars in external stations increase if rail cars arrive from other parks/stations (considering specifically storage parks for including δ) and decrease when they leave because of a train departure in the electrified rail line (in this case the time required for technical checks and documentary processes is taken into account). On the contrary, the export rail cars in external stations increase for train arrivals and decrease when rail cars are transferred to other stations/parks.

IV. The MILP Problem

In this section, the mathematical formulation for the considered planning problem is presented. Given the arrivals and departures of import and export flows, the planning problem allows to find the optimal sequence and timing of the shunting operations in the considered port rail network in order to minimize an appropriate cost function while respecting all the system constraints. This problem can be stated with the following mixed-integer linear formulation.

Given:

- the network structure $\mathcal{G}$=($\mathcal{N} \cup 0$, $\mathcal{A}$), and the sets of successor and predecessor nodes $\mathcal{P}^{\mathrm{I}}_n$, $\mathcal{S}^{\mathrm{I}}_n$, $\mathcal{P}^{\mathrm{I}}_n$, $\mathcal{S}^{\mathrm{I}}_n$, $n \in \mathcal{N}$;
- the set of types of rail cars $\mathcal{W}$ and its subdivision according to the railway company $c = 1, \ldots, C$, i.e. $\mathcal{W}_c$, the length l^w and the delay γ^w for rail car type $w \in \mathcal{W}$;
- the maximum productivity Γ_n of handling resources connecting the yard stacking area to the internal rail terminal $n \in \mathcal{N}^{\mathrm{T}}$;
- the set of tracks $\mathcal{R}_n$ for each node $n \in \mathcal{N}$, the length $L_{n,i}$ of track $i \in \mathcal{R}_n$ of node $n \in \mathcal{N}$, and the number of linking tracks $R_{n,m}$, $n, m \in \mathcal{N}$;
- the set of nodes $\mathcal{N}_h \subseteq \mathcal{N}$, corresponding to the area $h = 1, \ldots, H$, served by Λ_h diesel locomotives;
- the transfer time $\tau_{n,m}$ from node $n \in \mathcal{N}$ to node $m \in \mathcal{N}$;
- the arrivals and departures in node 0, i.e. $a^w_0(t)$ and $\bar{d}^w_0(t)$, $w \in \mathcal{W}$, $t = 0, \ldots, T-1$;
- the scheduled arrivals and departures of trains in external stations, i.e. $d^w_{n,i}(t)$ and $\bar{a}^w_{n,i}(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}^{\mathrm{ES}}$, $t = 0, \ldots, T-1$;
- the initial conditions $q^w_{n,i}(0)$ and $\bar{q}^w_{n,i}(0)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $q^w_0(0)$ and $\bar{q}^w_0(0)$, $w \in \mathcal{W}$;
- the cost weighting parameters α^w_0 and $\bar{\alpha}^w_0$, $w \in \mathcal{W}$, $\alpha^w_{n,i}$ and $\bar{\alpha}^w_{n,i}$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $\beta^w_{n,i,m,j}$ and $\zeta^w_{n,i,m,j}$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{I}}_n$, $j \in \mathcal{R}_m$, $\bar{\beta}^w_{n,i,m,j}$ and $\bar{\zeta}^w_{n,i,m,j}$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{E}}_n$, $j \in \mathcal{R}_m$;
- parameters Q', Q'' and δ;

find

- the state variables $q^w_{n,i}(t)$ and $\bar{q}^w_{n,i}(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $t = 1, \ldots, T$ and $q^w_0(t)$ and $\bar{q}^w_0(t)$, $w \in \mathcal{W}$, $t = 1, \ldots, T$;
- the binary decision variables $y^c_{n,i,m,j}(t)$, $c = 1, \ldots, C$, $n \in \mathcal{N}$, $i \in \mathcal{R}^{\mathrm{L}}_n$, $m \in \mathcal{S}^{\mathrm{I}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, $x^c_{n,i,m,j}(t)$, $c = 1, \ldots, C$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{I}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, $\bar{y}^c_{n,i,m,j}(t)$, $c = 1, \ldots, C$, $n \in \mathcal{N}$, $i \in \mathcal{R}^{\mathrm{L}}_n$, $m \in \mathcal{S}^{\mathrm{E}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, and $\bar{x}^c_{n,i,m,j}(t)$, $c = 1, \ldots, C$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{E}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$;
- the continuous decision variables $r^w_{n,i,m,j}(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{I}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, $\bar{r}^w_{n,i,m,j}(t)$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}^{\mathrm{E}}_n$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, $r^w_{0,m,j}(t)$, $w \in \mathcal{W}$, $m \in \mathcal{S}^{\mathrm{I}}_0$, $j \in \mathcal{R}_m$, $t = 0, \ldots, T-1$, and $\bar{r}^w_{n,i,0}(t)$, $w \in \mathcal{W}$, $n \in \mathcal{P}^{\mathrm{E}}_0$, $i \in \mathcal{R}_n$, $t = 0, \ldots, T-1$;

minimizing

$$\begin{aligned}
&\sum_{t=1}^{T} \sum_{w \in \mathcal{W}} \Bigg[\sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{R}_n} \alpha^w_{n,i} q^w_{n,i}(t) + \bar{\alpha}^w_{n,i} \bar{q}^w_{n,i}(t) \\
&\qquad + \alpha^w_0 q^w_0(t) + \bar{\alpha}^w_0 \bar{q}^w_0(t) \Bigg] \\
&+ \sum_{c=1}^{C} \sum_{t=0}^{T-1} \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{R}_n} \Bigg[\sum_{m \in \mathcal{S}^{\mathrm{I}}_n} \sum_{j \in \mathcal{R}_m} \beta^c_{n,i,m,j} y^c_{n,i,m,j}(t) \\
&\qquad + \zeta^c_{n,i,m,j} x^c_{n,i,m,j}(t) \\
&+ \sum_{m \in \mathcal{S}^{\mathrm{E}}_n} \sum_{j \in \mathcal{R}_m} \bar{\beta}^c_{n,i,m,j} \bar{y}^c_{n,i,m,j}(t) + \bar{\zeta}^c_{n,i,m,j} \bar{x}^c_{n,i,m,j}(t) \Bigg]
\end{aligned} \tag{11}$$

subject to the model dynamics given by (1)-(10), and

$$\sum_{w \in \mathcal{W}} l^w \left[q^w_{n,i}(t) + \bar{q}^w_{n,i}(t) \right] \le L_{n,i} \qquad n \in \mathcal{N},\ i \in \mathcal{R}_n,\ t = 0, \ldots, T-1 \tag{12}$$

$$\sum_{w \in \mathcal{W}} \sum_{i \in \mathcal{R}_n} r^w_{0,n,i}(t) + \bar{r}^w_{n,i,0}(t) \le \Gamma_n \qquad n \in \mathcal{N}^{\mathrm{T}},\ t = 0, \ldots, T-1 \tag{13}$$

$$\begin{aligned}
&\sum_{c=1}^{C} \sum_{i \in \mathcal{R}_n} \sum_{j \in \mathcal{R}_m} y^c_{n,i,m,j}(t) + x^c_{n,i,m,j}(t) \\
&\quad + \sum_{c=1}^{C} \sum_{i \in \mathcal{R}_m} \sum_{j \in \mathcal{R}_n} \bar{y}^c_{m,i,n,j}(t) + \bar{x}^c_{m,i,n,j}(t) \le R_{n,m} \\
&\qquad n \in \mathcal{N},\ m \in \mathcal{S}^{\mathrm{I}}_n,\ t = 0, \ldots, T-1
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\sum_{c=1}^{C} \sum_{m \in \mathcal{S}^{\mathrm{I}}_n} \sum_{j \in \mathcal{R}_m} y^c_{n,i,m,j}(t) + x^c_{n,i,m,j}(t) \\
&\quad + \sum_{c=1}^{C} \sum_{m \in \mathcal{S}^{\mathrm{E}}_n} \sum_{j \in \mathcal{R}_m} \bar{y}^c_{n,i,m,j}(t) + \bar{x}^c_{n,i,m,j}(t) \le 1 \\
&\qquad n \in \mathcal{N},\ i \in \mathcal{R}^{\mathrm{L}}_n,\ t = 0, \ldots, T-1
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\sum_{w \in \mathcal{W}_c} r^w_{n,i,m,j}(t) = Q' \cdot y^c_{n,i,m,j}(t) + Q'' \cdot x^c_{n,i,m,j}(t) \\
&\qquad c = 1, \ldots, C,\ n \in \mathcal{N},\ i \in \mathcal{R}^{\mathrm{L}}_n, \\
&\qquad m \in \mathcal{S}^{\mathrm{I}}_n,\ j \in \mathcal{R}_m,\ t = 0, \ldots, T-1
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\sum_{w \in \mathcal{W}_c} \bar{r}^w_{n,i,m,j}(t) = Q' \cdot \bar{y}^c_{n,i,m,j}(t) + Q'' \cdot \bar{x}^c_{n,i,m,j}(t) \\
&\qquad c = 1, \ldots, C,\ n \in \mathcal{N},\ i \in \mathcal{R}^{\mathrm{L}}_n, \\
&\qquad m \in \mathcal{S}^{\mathrm{E}}_n,\ j \in \mathcal{R}_m,\ t = 0, \ldots, T-1
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\sum_{c=1}^{C} \sum_{m \in \mathcal{S}^{\mathrm{I}}_n} \sum_{j \in \mathcal{R}_m} x^c_{n,i,m,j}(t) + \sum_{c=1}^{C} \sum_{m \in \mathcal{S}^{\mathrm{E}}_n} \sum_{j \in \mathcal{R}_m} \bar{x}^c_{n,i,m,j}(t) \le 1 \\
&\qquad n \in \mathcal{N},\ i \in \mathcal{R}^{\mathrm{S}}_n,\ t = 0, \ldots, T-1
\end{aligned} \tag{18}$$

$$\sum_{w \in \mathcal{W}_c} r^w_{n,i,m,j}(t) = Q'' \cdot x^c_{n,i,m,j}(t)$$
$$c = 1, \dots, C, \; n \in \mathcal{N}, \; i \in \mathcal{R}^{\mathrm{S}}_n,$$
$$m \in \mathcal{S}^{\mathrm{I}}_n, \; j \in \mathcal{R}_m, \; t = 0, \dots, T-1 \quad (19)$$

$$\sum_{w \in \mathcal{W}_c} \bar{r}^w_{n,i,m,j}(t) = Q'' \cdot \bar{x}^c_{n,i,m,j}(t)$$
$$c = 1, \dots, C, \; n \in \mathcal{N}, \; i \in \mathcal{R}^{\mathrm{S}}_n,$$
$$m \in \mathcal{S}^{\mathrm{E}}_n, \; j \in \mathcal{R}_m, \; t = 0, \dots, T-1 \quad (20)$$

$$\sum_{m \in \mathcal{S}^{\mathrm{I}}_n} \sum_{j \in \mathcal{R}_m} r^w_{n,i,m,j}(t) \le q^w_{n,i}(t)$$
$$w \in \mathcal{W}, \; n \in \mathcal{N}, \; i \in \mathcal{R}_n, \; t = 0, \dots, T-1 \quad (21)$$

$$\sum_{m \in \mathcal{S}^{\mathrm{E}}_n} \sum_{j \in \mathcal{R}_m} \bar{r}^w_{n,i,m,j}(t) \le \bar{q}^w_{n,i}(t)$$
$$w \in \mathcal{W}, \; n \in \mathcal{N}, \; i \in \mathcal{R}_n, \; t = 0, \dots, T-1 \quad (22)$$

$$\sum_{c=1}^{C} \sum_{n \in \mathcal{N}_h} \sum_{i \in \mathcal{R}_n} \sum_{m \in \mathcal{N}_h} \sum_{j \in \mathcal{R}_m} y^c_{n,i,m,j}(t) + \bar{y}^c_{n,i,m,j}(t)$$
$$+ x^c_{n,i,m,j}(t) + \bar{x}^c_{n,i,m,j}(t) \le \Lambda_h$$
$$h = 1, \dots, H, \; t = 0, \dots, T-1 \quad (23)$$

The objective function (11) is a weighted sum of rail cars present in the nodes, as well as of movements of trains and groups of rail cars in the linking tracks between nodes. In particular, by suitably tuning the weights associated with buffers, it is possible to privilege the presence of rail cars in specific areas of the network or reducing them in areas where there could be limitations of any kind to be considered. In this model, in order to represent real situations, the highest weights are assigned to the buffers associated with rail station tracks, because it is desirable that trains stop there as little as possible (both in import and in export); medium weights are related to internal rail tracks and the smallest weights are associated with the remaining buffers, i.e., the ones of the storage parks. As for the movements of trains and groups of rail cars in the linking tracks between nodes, they are minimized because a fixed cost must be paid when a movement occurs.

Constraints (12) impose that, at each time t, the total length of the rail cars on track i of node n does not exceed the track length. Constraints (13) guarantee that, at each time t, the productivity of the handling means between node 0 and the internal rail parks $n \in \mathcal{N}^{\mathrm{T}}$ is not greater than the corresponding maximum productivity. Constraints (14) impose that, at each time t, the number of trains or groups of rail cars moving between node n and node m is not greater than the number of linking tracks.

Constraints (15) impose that, at each time t, at most one train or group of rail cars can leave from each long track i of node n. Constraints (16), together with (15), ensure that, at each time t, the number of import rail cars of a given railway company c leaving from each long track i of node n to track j of node m is equal to 0, or equal to Q' (whole train), or equal to Q'' (group of rail cars). Similarly, constraints (17) refer to the export flow. Constraints (18)-(20) are analogous to (15)-(17), but referred to short tracks i of node n.

Constraints (21) and (22) guarantee that the quantity of rail cars of type w leaving track i of node n at time t, in import and in export respectively, is not greater than the quantity of rail cars present there at the same time. Finally, constraints (23) guarantee that, for a given area h where the same diesel locomotives are shared, the maximum number of possible movements realized at time t does not exceed the number of locomotives in that zone.

V. Application to La Spezia port rail system

The proposed planning procedure has been implemented in C♯, using Cplex 12.5 as a MILP solver and exploiting the IBM ILOG Concert library for building the model from the C♯ language. The proposed procedure has been applied to a real case, regarding the rail network of La Spezia port, located in Northern Italy. More specifically, the complex rail system of La Spezia Container Terminal (LSCT), run by Contship Italia Group, has been taken into account. Contship Italia is a leading logistic company in Italy, operating six maritime terminals and one dry hub for intermodal transportation. The company belongs to the German group Eurokai which is a European leader for terminal services and integrated logistics. LSCT represents the main container terminal in the port of La Spezia, occupying a surface of more than 270.000 m^2 over two piers (Fornelli and Ravano) and manages a dry port terminal, in Santo Stefano Magra, located 12 km far from the port. The annual throughput of the terminal has been of more than 1.300.000 TEUs in 2014, 35% of which moved by rail.

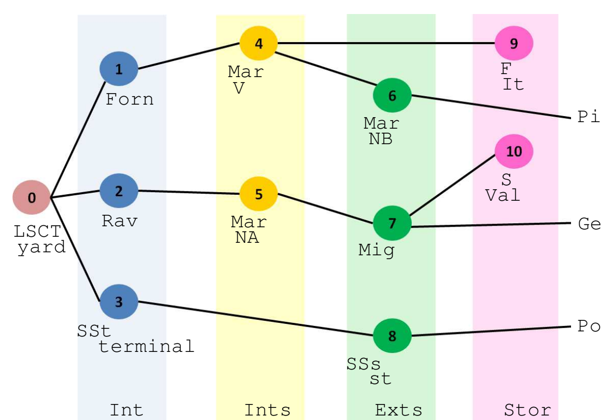

Fig. 3. The port rail network.

The rail activity to/from LSCT is provided by La Spezia Shunting Railways (LSSR) company. As depicted in Fig. 3, La Spezia port rail system is composed of 10 nodes, divided as follows:

- three internal rail parks, *Fornelli* and *Ravano* (located in LSCT terminal) and *Santo Stefano* dry terminal;
- two internal rail stations, *Marittima Vecchia* and *Marittima Nuova B*;
- three external rail stations, *Marittima Nuova A*, *Migliarina* and *Santo Stefano* station, which allow import trains

to merge the electrified rail line towards their destinations, and export trains to reach La Spezia port rail system;

- two rail storage parks, *Fascio Italia* and *Scalo Valdellora*, that allow to accommodate trains when there is not enough space in the rail stations, so acting as buffers.

In the following, first of all the present situation of La Spezia rail network is analysed: the optimization problem is applied to the considered case in order to validate it. Secondly, the proposed approach is used to size the capacity of the network, by analysing different scenarios with increasing number of arriving and departing trains. Finally, a computational analysis is reported in order to show the computational times necessary to solve the MILP problem for different settings.

A. The present case of La Spezia rail network

For representing the present situation of La Spezia rail network, a planning horizon of $T = 32$ time steps has been considered (corresponding to 8 hours, since the sample time Δt is equal to 15 minutes). In the following, the values of the main parameters of the model are reported:

- $|\mathcal{W}| = 4$ types of rail cars are taken into account, i.e. 3 TEU and 4 TEU rail cars (respectively 20 and 36 metre long), belonging to two railway companies $c = 1, 2$;
- the number of tracks for each node is as follows: $|\mathcal{R}_1| = |\mathcal{R}_5| = |\mathcal{R}_6| = 4$, $|\mathcal{R}_2| = |\mathcal{R}_3| = 2$, $|\mathcal{R}_4| = |\mathcal{R}_8| = |\mathcal{R}_9| = 11$, $|\mathcal{R}_7| = 9$, $|\mathcal{R}_{10}| = 18$;
- the lengths of tracks in each node are quite different and vary between 100 and 700 metres;
- the number of connecting tracks is always equal to 1, except for the connection between Marittima Vecchia and Fascio Italia, Marittima Vecchia and Marittima Nuova A, Migliarina and Scalo Valdellora, where 2 connecting tracks are present, i.e. $R_{4,9} = R_{4,6} = R_{7,10} = 2$;
- the number of rail cars composing a train is $Q' = 16$, whereas $Q'' = 8$;
- the delays $\tau_{n,m}$, $n, m \in \mathcal{N}$, vary depending on the couple of nodes and are between 1 and 4 (i.e. between 15 minutes and 1 hour);
- the other delays are $\gamma^1 = 9$ and $\gamma^2 = 12$, corresponding respectively to more than 2 hours and 3 hours.

Regarding the weights in the cost function, as aforementioned, the idea is to weigh more the critical areas of the network where rail cars should stop for the shortest possible time, whereas lower weights are assigned to storage areas or areas with a bigger space availability. The weights associated with the movements of trains or groups of rail cars in the network are much higher than those associated with buffers, taking into account that they represent fixed costs. It is worth noting that all these weights have not a physical meaning but only their relative values are meaningful. The following values have been considered in the problem for the import flow (the values of the export flow are exactly the same):

- $\alpha_0^w = 5$, $w \in \mathcal{W}$;
- $\alpha_{n,i}^w = 20$, $w \in \mathcal{W}$, $n = 1, \ldots, 3$, $i \in \mathcal{R}_n$;
- $\alpha_{n,i}^w = 25$, $w \in \mathcal{W}$, $n = 4, \ldots, 8$, $i \in \mathcal{R}_n$;
- $\alpha_{n,i}^w = 15$, $w \in \mathcal{W}$, $n = 9, 10$, $i \in \mathcal{R}_n$;
- $\beta_{n,i,m,j}^w = 500$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{I}}$, $j \in \mathcal{R}_m$;9.5cm
- $\zeta_{n,i,m,j}^w = 1000$, $w \in \mathcal{W}$, $n \in \mathcal{N}$, $i \in \mathcal{R}_n$, $m \in \mathcal{S}_n^{\mathrm{I}}$, $j \in \mathcal{R}_m$.

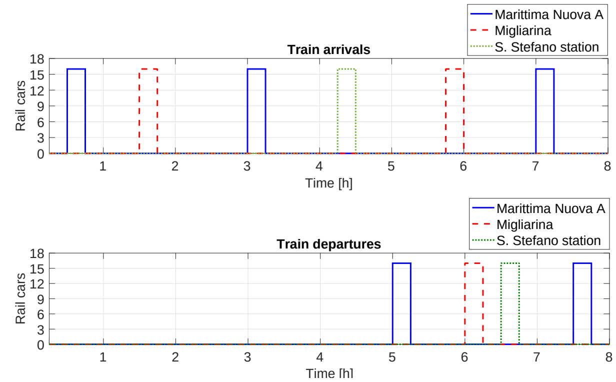

Fig. 4. Arrivals and departures - present case.

In the considered planning horizon equal to 8 hours, 6 train arrivals and 4 train departures are considered, as it currently happens in La Spezia rail network during the night. The considered train arrivals and departures are depicted in Fig. 4, specifically distinguished for the three external rail stations.

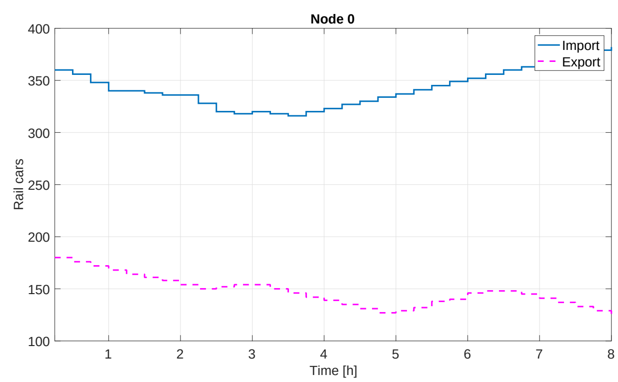

Fig. 5. Rail cars in node 0 - present case.

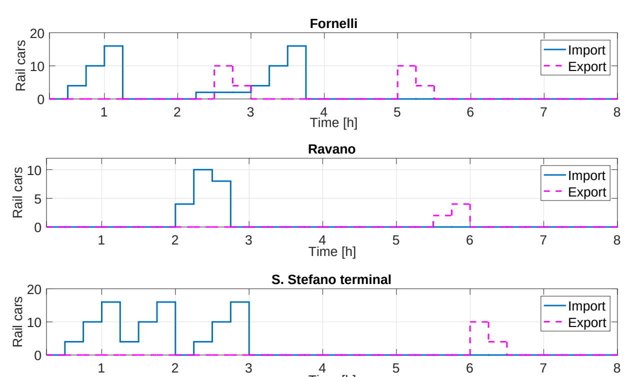

Fig. 6. Rail cars in the internal railway parks (nodes 1, 2, 3) - present case.

The dynamic evolutions of buffers in each node are reported in Figs. 5-9, for node 0, for the internal railway parks, for the

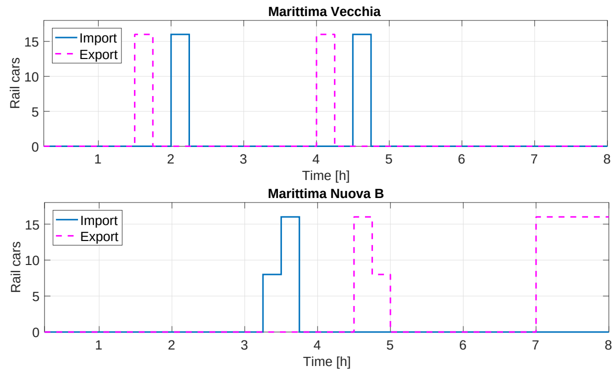

Fig. 7. Rail cars in the internal stations (nodes 4 and 5) - present case.

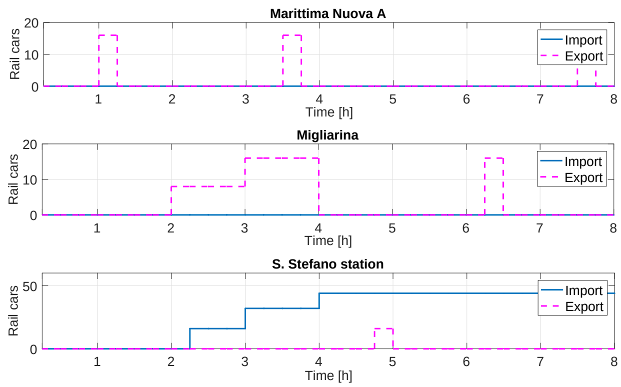

Fig. 8. Rail cars in the external stations (nodes 6, 7 and 8) - present case.

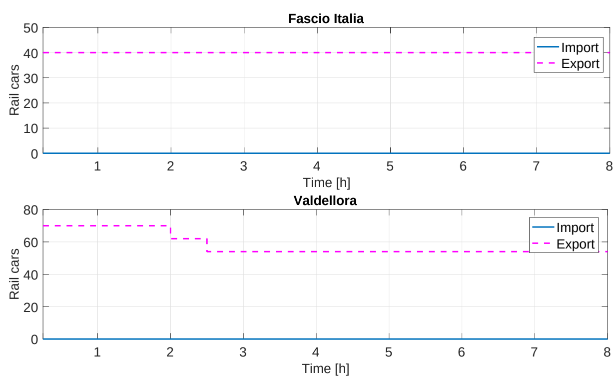

Fig. 9. Rail cars in the storage parks (nodes 9 and 10) - present case.

internal rail stations, for the external rail stations, and for the storage parks, respectively. In each graph, both the rail cars in import and in export are shown. As expected, the buffers characterized by lower weights in the cost function are those in which a higher number of rail cars is present. The behaviors of these buffers are quite realistic if compared with real data of the rail network of La Spezia.

B. Evaluation of the system capacity

As mentioned earlier, the optimization problem described in this paper not only can be used to plan the timing of shunting operations in a port rail network, but it can also be applied for evaluating the capacity of such a system. In order to do that, cases with increasing number of trains are examined.

In this specific case of La Spezia network, to make a detailed analysis of the system capacity, three different scenarios have been analyzed, all corresponding to a horizon of 8 hours:

- *Scenario 1*: the number of train arrivals is almost equal to the number of train departures;
- *Scenario 2*: the number of train arrivals is much higher than the number of train departures;
- *Scenario 3*: the number of train arrivals is much lower than the number of train departures.

For each scenario, different system configurations have been considered, in which the number of trains varies between 10 and 14. For each scenario and each system configuration, 10 random instances have been generated and solved. The randomization is associated with the initial conditions, the number of arrivals and departures, and the time instants in which such arrivals and departures occur. For instance, when considering the case with 10 trains, in Scenario 1 the number of train arrivals is uniformly distributed between 4 and 6, in Scenario 2 it is uniformly distributed between 7 and 9, in Scenario 3 between 1 and 3 (the number of train departures is then obtained as 10 minus the number of arrivals). As for the initial conditions, they have been generated randomly, uniformly distributed between a lower and an upper value, which are different for each buffer, considering realistic values for the La Spezia rail network determined by analysing real data profiles. These lower and upper values have been maintained fixed for the different scenarios.

For each scenario and for each number of trains, Table I reports the number of feasible instances (over the total 10 generated instances), as well as the sample mean value and the sample standard deviation of the optimal values of the objective function. For each scenario, Table I includes only the results of the system configurations in which at least one feasible instance has been found. It can be noted that by increasing the number of trains, the number of feasible instances decreases.

TABLE I
SCENARIO ANALYSIS.

Scenario	♯ trains	♯ feas. inst.	Cost (mean)	Cost (std. dev.)
1	10	10	164155.5	4479.3
1	11	9	165477.3	4354.7
1	12	7	167131.0	6440.9
1	13	3	164255.0	12333.1
2	10	10	169362.9	5235.6
2	11	9	173369.8	7719.2
2	12	6	172111.0	7142.9
2	13	6	178224.0	7329.3
2	14	2	175049.0	3396.9
3	10	7	157669.4	2659.9
3	11	2	154832.5	2105.1

By exploiting the information about the number of feasible instances, it is possible to determine the capacity of the system, i.e. the number of trains that the system is able to manage on average. For each scenario and for a given number of trains, the *feasibility percentage* has been computed as the number of feasible instances over the total number of 10. This index represents a sort of probability that the system can manage the corresponding number of trains in the considered scenario. The value of the feasibility percentage with increasing number of trains, in the three scenarios, is shown in Fig. 10. Besides observing that, increasing the number of trains, the number of feasible instances decreases, it can also be noted that Scenario 2 has more chances of feasibility than the others, while Scenario 3 is surely the one characterized by the lowest feasibility percentage.

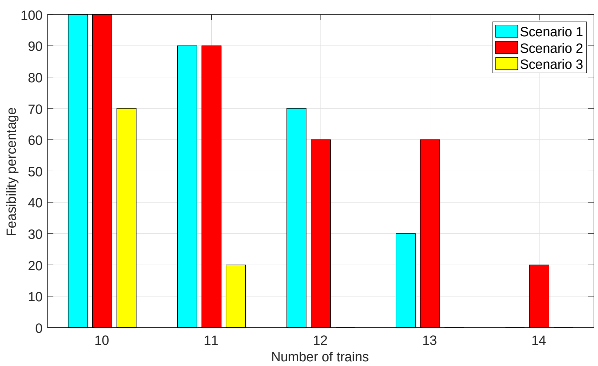

Fig. 10. Feasibility percentage in each scenario.

Note that with this type of analysis it is possible, under specific conditions, to size the port rail network in terms of number of trains that it can manage. Analysing Fig. 10, it can be argued that, in case for instance Scenario 2 is the most probable to happen, the system is likely to be able to manage 11 (or even 12) trains in 8 hours.

C. Computational analysis

The computational analysis has the objective of evaluating the computational times required to solve the problem when its dimensions increase or when the instances become more difficult to be solved. In particular, in the considered optimization problem, the number of train arrivals and departures seems a relevant factor in making the problem more difficult to be solved (an instance with a low number of trains is trivial, whereas it becomes harder and harder to find a solution if the number of trains increases). Secondly, one of the most important parameter affecting the problem dimensions is the length of the planning horizon T. An analysis has been then carried out to evaluate the computational times depending on these two parameters, the number of trains and the length of the planning horizon. For these experiments a pc Intel(R) Celeron(R) CPU B800 @ 1.50GHz with an installed RAM of 2.00 GB has been used.

We have first of all analysed the difficulty of the problem when the number of trains increases, with a fixed value for the planning horizon, i.e. $T = 32$ (corresponding to 8 hours). In particular, 6 groups of instances have been considered with number of trains increasing from 2 to 12. For each group, 5 instances have been randomly generated (all of them are characterized by 409448 variables and 75360 constraints since the number of train arrivals and departures does not influence the instance dimensions). Table II reports, for each instance

TABLE II
COMPUTATIONAL ANALYSIS FOR INSTANCES WITH $T = 32$.

Group	Instance	♯ trains	CPU time [s]
1	1	2	678
1	2	2	739
1	3	2	489
1	4	2	276
1	5	2	593
2	1	4	758
2	2	4	579
2	3	4	1098
2	4	4	845
2	5	4	963
3	1	6	1187
3	2	6	1092
3	3	6	854
3	4	6	604
3	5	6	952
4	1	8	1146
4	2	8	1242
4	3	8	952
4	4	8	1652
4	5	8	1589
5	1	10	2305
5	2	10	1276
5	3	10	2043
5	4	10	1958
5	5	10	1623
6	1	12	1679
6	2	12	2045
6	3	12	2223
6	4	12	1801
6	5	12	1592

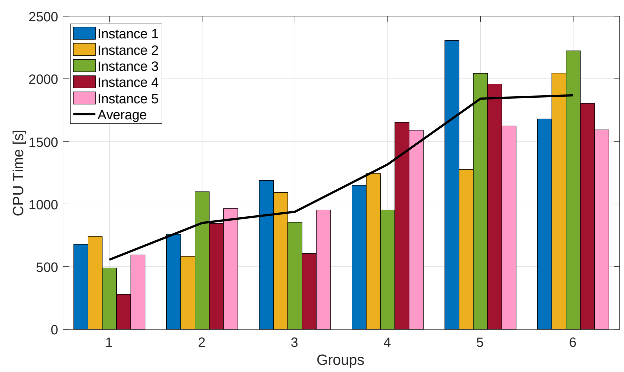

Fig. 11. CPU time for instances with $T = 32$.

and each group, the CPU time required by the solver to find the optimal solution and Fig. 11 plots these values, as well as the profiles of the average CPU time for each instance group. Note that the computational times grow when the number of trains increases but they never exceed 40 minutes. The same instances have also been solved setting a time limit for the solver of 10 minutes. In this case, only 4 instances are optimally solved but, for all the remaining instances, the optimality gap is lower than 7%, indicating a good quality of the obtained solution even with a short time limit.

Similarly, other groups of instances have been analysed, considering a fixed number of trains, equal to 8, and varying the planning horizon. Specifically, 4 groups of instances (and again 5 instances per group) have been generated and solved, with T varying from 24 (i.e. 6 hours) to 48 (i.e. 12 hours).

TABLE III
COMPUTATIONAL ANALYSIS FOR INSTANCES WITH 8 TRAINS.

Group	Instance	T	♯ variables	♯ constraints	CPU time [s]
1	1	24	307240	56520	882
1	2	24	307240	56520	750
1	3	24	307240	56520	723
1	4	24	307240	56520	1049
1	5	24	307240	56520	801
2	1	32	409448	75360	1146
2	2	32	409448	75360	1242
2	3	32	409448	75360	2091
2	4	32	409448	75360	1652
2	5	32	409448	75360	1589
3	1	40	511656	94200	1632
3	2	40	511656	94200	1808
3	3	40	511656	94200	1404
3	4	40	511656	94200	1334
3	5	40	511656	94200	1943
4	1	48	613864	113040	2130
4	2	48	613864	113040	1905
4	3	48	613864	113040	1643
4	4	48	613864	113040	1894
4	5	48	613864	113040	1749

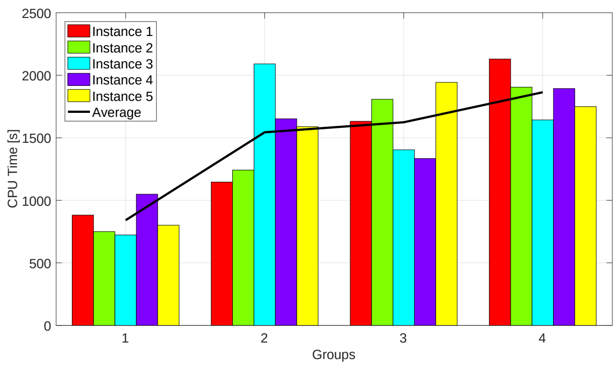

Fig. 12. CPU time for instances with 8 trains.

Table III reports, for each instance and group, the value of T, the number of variables and constraints, and the CPU time to obtain the optimal solution. The CPU times and the profiles of the average values for each group are also displayed in Fig. 12. For all the instances the solver requires between 10 and 40 minutes to find the optimal solution, considering that the highest computational times are required for the instances with larger values of T. Besides, setting a time limit of 10 minutes for the solver, none of the instances are optimally solved but all of them are characterized by low optimality gaps, lower than 5%.

Concluding, it can be observed that the proposed optimization problem is solvable in acceptable computational times for the dimensions shown in the paper. The computational times to find the optimal solutions are always lower than 40 minutes and, in any case, after 10 minutes of computation good solutions are provided by the solver. For larger dimensions, higher computational times will be required or more powerful computers should be used. Nevertheless, remind that this is a planning approach suitable for medium-term planning, which is not a real-time setting, hence the required computational times could be acceptable to take the decision.

VI. Conclusion

This paper has proposed an optimization approach in order to size the maximum number of trains, both in import and in export, that can be correctly managed by a rail network, according to the given productivity of handling resources devoted to rail operations inside the terminals. This planning approach can also be used for determining the optimal configuration of the import and export port rail cycles in terms of planning of times, handling sequences and shunting operations. The application to a real case study, i.e. an Italian port rail network, has shown the effectiveness of the proposed methodology, both for planning the shunting operations in the network and for determining the capacity of the system.

Thanks to the general model provided in the paper, the proposed planning approach could be applied to other real cases worldwide. Besides, the optimization-based framework described in the paper could be a very useful tool for logistic and rail shunting companies which have to manage complex rail networks. In particular, the proposed approach is able to include very specific characteristics of the real operation of a rail network while maintaining a rather aggregate model which describes the system dynamics quantitatively through discrete-time equations. One of the benefits of adopting this tool for the management of a real system is related to the possibility of realizing what-if analyses and scenario investigations in order to evaluate future investments or reorganisation policies.

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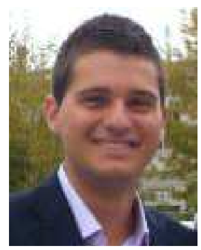

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