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Influence of the geometrical shape on the structural behavior of diagrid tall buildings under lateral and torque actions

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A B S T R A C T

Diagrid structural systems are more and more exploited worldwide for the realization of tall buildings, due to their versatility, their capability to realize complex-shaped constructions, and their efficiency in limiting lateral displacements. Plenty of research has been carried out in the last decade aimed at analyzing the structural behavior of these systems, mainly considering buildings with square or rectangular floor plans and commonly investigating only the lateral deformability of the structure. In this paper, we investigate the influence of some geometrical parameters on the structural response of diagrid tubular buildings, under both lateral and torque actions. To this aim, we make use of a matrix-based method (MBM), which was recently developed for the structural analysis of generic diagrid structures. Various building configurations, differing for the aspect ratio, the inclination of the external diagonals, and the floor plan shape, are considered and the structural solutions which allow to minimize the lateral displacements and torsional rotations are thoroughly surveyed.

1. Introduction

Nowadays the increasing growth of global population associated with the intense urbanization phenomenon claims for more spaces intended for housing and office services within the cities. This inevitably leads governors and designers to consider a more efficient and rational usage of the city land, which is causing a tireless growing and development of tall buildings (Ibrahim, 2007). Although tall buildings can provide a positive answer to the spacing problem and they can constitute successful solutions in defining the skyline of modern cities, they pose a variety of sustainability challenges which must be taken into account throughout the whole design and construction process (Elnimeri and Gupta, 2008; Ali and Armstrong, 2010; Al-Kodmany, 2018). The sustainability of a tall building should be investigated considering three main dimensions, namely the social, economic and environmental dimension (Al-Kodmany, 2018), and it should be addressed via a multidisciplinary approach involving various disciplines. From a purely Structural Engineering point of view, aiming to the sustainability of a tall building implicates to find the optimal design solutions which allow to use the minimum amount of material while enhancing the global performance of the structure (Ali and Moon, 2007). Such performance is usually evaluated by controlling the lateral displacements, since the need to limit the lateral deformability of tall structures plays a pivotal role in the design phases and has a strong impact in defining the principal resisting elements.

Among the different solutions adopted to increase the structural performance and sustainability of tall buildings, diagrid systems have been one of the most widely exploited in the last decade (Asadi and Adeli, 2017; Liu et al., 2018; Korsavi and Maqhareh, 2014). These are composed by tubular truss systems which are located on the exterior of the building and allow to reach high structural performances by exploiting the axial resisting mechanism of inclined mega-diagonals. Moreover, being composed by an assembly of triangular modules on the building façade, they allow to realize complex-shaped structures achieving noteworthy architectural effects (Al-Kodmany and Ali, 2016), like in the case of the Swiss Re Tower in London, the Tornado Tower in Doha, etc.

Many researchers have dealt with the structural behavior and performance of diagrid systems. Moon et al. firstly proposed an analytical methodology for the preliminary design of diagrid tubes and showed that there exists an optimal inclination of the external diagonals in order to minimize the lateral displacement (Moon et al., 2007). Subsequently, complex-shaped diagrid systems, such as twisted, tilted and freeform diagrid towers, were also analyzed by means of Finite Element (FE) calculations (Moon, 2011). Zhang et al. explored the optimal diagonal inclination in diagrid tube buildings composed of straight diagonals with gradually varying angles (Zhang et al., 2012), whereas Montuori et al.

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investigated the influence of stiffness- and strength-based criteria on the design of the diagonal members (Montuori et al., 2013). Real case studies, such as the Hearst Tower in New York, the Swiss Re Tower in London and the Guangzhou West Tower in Guangzhou, were also surveyed by Mele et al. via simplified hand calculations (Mele et al., 2014). The influence of some geometrical patterns was also investigated on the structural behavior (Montuori et al., 2014a; Angelucci and Mollaioli, 2017; Tomei et al., 2018; Mele et al., 2019; Mirniazmandan et al., 2018), and special secondary bracing systems were also proposed by Montuori et al. in order to minimize the effect of local structural issues which could undermine the building performance, such as the stability of interior columns and the excessive inter-story drift (Montuori et al., 2014b).

As for the methodologies which have been used for the structural analysis of diagrid buildings other than FE calculations, Liu and Ma proposed an analytical approach, based on the modular method, in order to calculate the shear and bending rigidity of polygonal diagrid tubes (Liu and Ma, 2017). More recently, Lacidogna et al. developed a matrix-based method (MBM), for the investigation of generic diagrids, in order to obtain information not only regarding the lateral deformability of the diagrid structure but also concerning its torsional behavior (Lacidogna et al., 2019). In particular, the MBM was developed by the authors with the aim of providing a methodology, more expeditious than FE modelling and analysis, which allowed to obtain the fundamental information on the global behavior of the diagrid structure. The MBM was set up within a more general analytical framework, called General Algorithm, which was extensively developed by some of the authors in recent years in order to perform the structural analysis of tall buildings. By means of the General Algorithm, the interaction between various vertical resisting elements could be deeply analyzed, including unconventionally-shaped structures (Carpinteri et al., 2014), open- and closed-section shear walls (Carpinteri et al., 2010, 2016) and buildings of different height (Carpinteri et al., 2012; Lacidogna, 2017). The General Algorithm also allowed to investigate real case studies (Carpinteri et al., 2013; Nitti et al., 2019).

As can be found out by analyzing the literature concerning diagrid systems, a lot of research has been carried out regarding diagrid tubes made up of square floor plans, but little attention has been paid when it comes to different floor shapes, such as polygonal or circular floor plans (Mirniazmandan et al., 2018; Liu and Ma, 2017). Moreover, although plenty of calculations has been performed regarding lateral displacements, the analysis of the diagrid torsional behavior has received no consideration at all. Here, we make use of the recently developed MBM in order to investigate the influence of some geometrical parameters, such as the diagonal inclination and the floor plan shape, on the structural response of diagrid buildings under both lateral and torque actions. In particular, after performing the analysis for four different building aspect ratios, we show that the diagonal inclination plays the key role in governing the diagrid behavior as far as lateral displacements and torsional rotations are concerned, whereas minor differences are observed when changing the floor plan shape, especially when it comes to the lateral displacements.

2. Methodology

In this Section, the fundamentals of the MBM, used to perform the structural analysis of the diagrid structures, are briefly recalled. The details of the different generated diagrid buildings, obtained by changing the total height of the building, the floor plan shape and the inclination of the external diagonals, are also shown.

2.1. The matrix-based method (MBM) for the structural analysis

The MBM is based on the following assumptions, which are meant to simplify the mathematical formulation while allowing to capture the global structural behavior: the diagonals are supposed to be only subjected to axial force, remaining into the linear elastic regime; the floors included within the triangular modules are neglected, thus the local bending and shear deformations of the diagonals are not taken into account; the considered floors, i.e. the ones lying at the end of the pinned diagonals, are assumed to remain plane after deformation, so that they can be treated as rigid bodies in the space characterized by six degrees of freedom (Lacidogna et al., 2019). The structure, which is considered into a three-dimensional reference system XYZ, is subjected to concentrated forces and moments, acting at the level of the floor centroids, which are grouped into the $6N$ generalized force vector $\{F\}$, being N the number of floors. Accordingly, the building undergoes floor displacements and rotations, which can be grouped into the $6N$ generalized displacement vector $\{\delta\}$. The linear structural problem can then be formulated through the following matrix relation:

$$\begin{Bmatrix} \{F_x\} \\ \{F_y\} \\ \{M_z\} \\ \{M_x\} \\ \{M_y\} \\ \{F_z\} \end{Bmatrix} = \begin{bmatrix} [K_{F_x\delta_x}] & [K_{F_x\delta_y}] & [K_{F_x\phi_z}] & [K_{F_x\phi_x}] & [K_{F_x\phi_y}] & [K_{F_x\delta_z}] \\ [K_{F_y\delta_x}] & [K_{F_y\delta_y}] & [K_{F_y\phi_z}] & [K_{F_y\phi_x}] & [K_{F_y\phi_y}] & [K_{F_y\delta_z}] \\ [K_{M_z\delta_x}] & [K_{M_z\delta_y}] & [K_{M_z\phi_z}] & [K_{M_z\phi_x}] & [K_{M_z\phi_y}] & [K_{M_z\delta_z}] \\ [K_{M_x\delta_x}] & [K_{M_x\delta_y}] & [K_{M_x\phi_z}] & [K_{M_x\phi_x}] & [K_{M_x\phi_y}] & [K_{M_x\delta_z}] \\ [K_{M_y\delta_x}] & [K_{M_y\delta_y}] & [K_{M_y\phi_z}] & [K_{M_y\phi_x}] & [K_{M_y\phi_y}] & [K_{M_y\delta_z}] \\ [K_{F_z\delta_x}] & [K_{F_z\delta_y}] & [K_{F_z\phi_z}] & [K_{F_z\phi_x}] & [K_{F_z\phi_y}] & [K_{F_z\delta_z}] \end{bmatrix} \begin{Bmatrix} \{\delta_x\} \\ \{\delta_y\} \\ \{\phi_z\} \\ \{\phi_x\} \\ \{\phi_y\} \\ \{\delta_z\} \end{Bmatrix} \tag{1}$$

In Eq. (1), $\{F_x\}$, $\{F_y\}$nd $\{F_z\}$ represent the vectors containing respectively the floor forces along X, Y and Z direction, $\{M_z\}$ is the vector of the floor torque moments, whereas $\{M_x\}$ and $\{M_y\}$ represent the vectors containing respectively the out-of-plane floor moments along X and Y axes. As for the displacements, $\{\delta_x\}$, $\{\delta_y\}$ and $\{\delta_z\}$ represent the vectors containing the displacements along X, Y and Z direction respectively, $\{\phi_z\}$ is the vector of the in-plane torsional rotations, whereas $\{\phi_x\}$ and $\{\phi_y\}$ represent the vectors containing the out-of-plane rotations along X and Y axes respectively. The global stiffness matrix is a symmetric $6N \times 6N$ matrix, which is reported in Eq. (1) by a partition based on the six degrees of freedom of each floor. Each $N \times N$ submatrix stands for the stiffness matrix which links each force/moment vector to each displacement/rotation vector. Given the properties of the diagrid building, i.e. the structure geometry and diagonals' properties, each submatrix is analytically computed by applying unitary displacements/rotations to selected floors and calculating the total reaction forces/moments arising at the other floors. For more details about the procedure for the evaluation of the stiffness matrices, the reader can refer to (Lacidogna et al., 2019).

2.2. The generated models with changing geometrical parameters

In order to study the influence of the geometrical shape on the structural behavior of diagrid systems, various buildings with different geometrical parameters (diagonal inclination and floor plan shape) were investigated and analyzed by means of the MBM. For each structural model, some geometrical and material parameters were kept constant, which are the following ones: inter-story height equal to 3.5 m, total floor area equal to 900 m^2, diagonals' elastic modulus equal to 210 GPa and cross-sectional area equal to 380 cm^2 for all the diagonals (Table 1). Note that, in real diagrid structures, diagonals usually exhibit tapered cross sections towards the top of the building. For sake of completeness, this case has also been considered and the results are reported in the Appendix. As is shown in that section, considering a different distribution for the diagonal cross-sectional area does not affect the main conclusions

Table 1
Main parameters of the generated diagrid buildings.

Parameter	Value
Inter-story height [m]	3.5
Total floor area [m^2]	900
Diagonals' elastic modulus [GPa]	210
Diagonals' cross-sectional area [cm^2]	380
Total building height [m] (total number of floors [−])	126 (36), 168 (48), 210 (60), 252 (72)

Table 2
Variable geometrical parameters of the different structures.

Structure	Floor plan shape	Number of intra-module floors [−]	Diagonal angle [°]
SQ.1	Square	1	34.99
SQ.2		2	54.46
SQ.3		3	64.54
SQ.4		4	70.35
SQ.6		6	76.61
SQ.12		12	83.21
HE.1	Hexagon	1	36.97
HE.2		2	56.40
HE.3		3	66.11
HE.4		4	71.63
HE.6		6	77.51
HE.12		12	83.68
OC.1	Octagon	1	37.57
OC.2		2	56.98
OC.3		3	66.57
OC.4		4	72.00
OC.6		6	77.77
OC.12		12	83.82
CI.1	Circle	1	38.37
CI.2		2	57.73
CI.3		3	67.17
CI.4		4	72.48
CI.6		6	78.11
CI.12		12	83.99

of the present analysis.

Four different heights of the building were taken into account, namely 126 m, 168 m, 210 m and 252 m, which correspond to four different numbers of floors, i.e. 36, 48, 60 and 72 respectively (Table 1). For each building height, four different floor plan shapes were investigated, namely square, hexagonal, octagonal and circular. Six different diagonal inclinations were adopted by considering different numbers of floors included within the diagonal module, namely 1, 2, 3, 4, 6 and 12 intra-module floors. Therefore, twenty-four diagrid models were investigated for each building height (Table 2), for a total of ninety-six structures. Each diagrid module was comprised of twenty-four diagonals, placed all over the exterior of the building. The details of the investigated models with changing geometrical parameters are also observable from Fig. 1.

Thus, each structure was assumed to be subject to an horizontal load of 30 kN/m along the X axis and a torque load of 70 kNm/m, uniformly distributed along the height of the building, as shown in Fig. 2. The distributed loads were then correspondingly converted into concentrated forces and moments acting at the floor level, and the lateral displacements and torsional rotations of the floor were finally obtained by the application of the MBM. It is worth to note that this particular load distribution is not affecting the analysis results. In the Appendix, the case of a reverse triangle load pattern is also reported and, as is shown in that section, considering a different load distribution is found not to affect the main conclusions of the present analysis.

3. Results and discussion

In this Section the results arising from the application of the MBM to the generated diagrid structures are reported, for the four different heights of the building.

3.1. 36-Story building

In Table 3 and Fig. 3, the results are shown for the twenty-four diagrid structures referring to the 36-story 126 m-high building. Table 3 reports the total number of diagonal modules along the height of the building (which only depends on the number of intra-module floors), the resulting total mass of the diagonals, and the lateral displacements and torsional

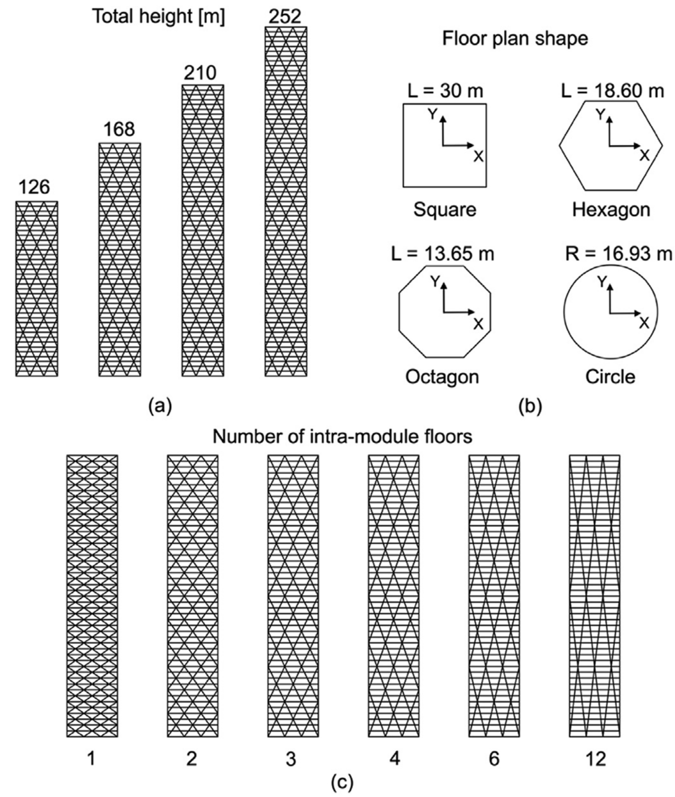

Fig. 1. Geometry of the generated diagrid buildings: (a) four different total heights; (b) four different floor plan shapes; (c) six different diagonal inclinations.

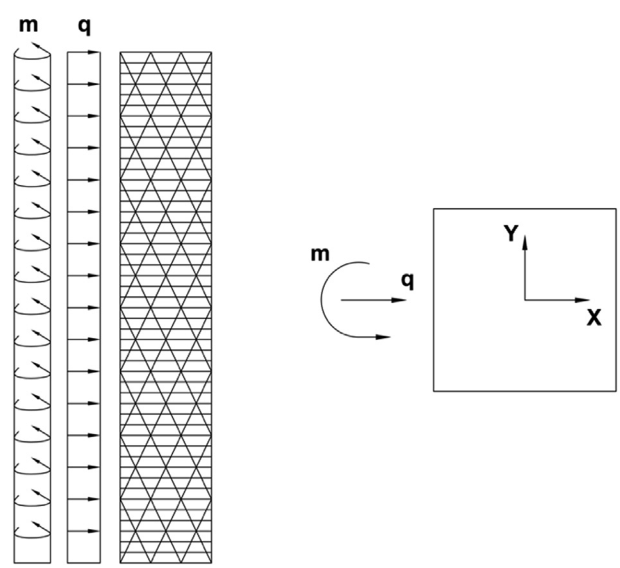

Fig. 2. Uniform load pattern: *q* refers to the distributed horizontal load, *m* to the distributed torque moments.

rotations evaluated at the top of the structure. To better visualize the results, the displacements and the rotations are also displayed in Fig. 3. In Fig. 3a–b the obtained lateral displacements are reported depending on the number of intra-module floors and the diagonal inclination, respectively, whereas in Figs. 3c and 3d the torsional rotations are shown.

As can be observed from Table 3 and Fig. 3, the optimal solution to minimize lateral displacements corresponds to the configuration with three intra-module floors for each floor plan shape (Fig. 3a). Correspondingly, the optimal diagonal inclination to minimize the lateral

Table 3
Displacements and rotations for the 36-story building (minimum values of displacements and rotations for each floor plan shape are in bold).

Structure	Total number of modules [–]	Diagonals' total mass [ton]	Lateral displacement [m]	Torque rotation [E–4 rad]
SQ.1	36	1563	0.186	**0.34**
SQ.2	18	1101	0.071	0.47
SQ.3	12	993	**0.060**	0.77
SQ.4	9	952	0.063	1.21
SQ.12	3	903	0.207	9.29
HE.1	36	1490	0.168	**0.29**
HE.2	18	1076	0.070	0.44
HE.3	12	980	**0.062**	0.75
HE.4	9	944	0.066	1.18
HE.6	6	918	0.090	2.44
HE.12	3	902	0.235	9.28
OC.1	36	1470	0.159	**0.28**
OC.2	18	1069	0.069	0.43
OC.3	12	977	**0.062**	0.74
OC.4	9	942	0.068	1.18
OC.6	6	917	0.094	2.43
OC.12	3	902	0.251	9.26
CI.1	36	1443	0.153	**0.27**
CI.2	18	1060	0.068	0.43
CI.3	12	927	**0.063**	0.74
CI.4	9	939	0.069	1.19
CI.6	6	915	0.097	2.48
CI.12	3	901	0.264	9.45

displacement stands in the range 64°–67° (Fig. 3b). By examining Fig. 3a and Table 3, it is evident how the solutions related to two and four intra-module floors are not far from the optimal condition, leading to lateral displacements just 10% higher than the minimum ones. However, increasing the diagonal inclination, the total amount of employed material decreases (Table 3). A balance between the need to limit the lateral displacement and reduce the amount of employed material is then needed, when selecting the optimal structural solution in the preliminary design stages. In the optimal diagonal configuration (three intra-module floors), no significant differences can be observed when changing the plan shape. The absolute minimum displacement corresponds to the square shape (60 mm) while the highest arises from the circular one (63 mm), which is just 5% higher. The influence of the specific plan shape is otherwise important far from the optimal diagonal configuration, e.g. when considering one intra-module floor or more than six floors included within the diagonal module. In this case, as shown in Fig. 3a and b, changing the floor plan geometry can lead to not negligible differences in terms of lateral flexibility (up to 30% difference).

Different conclusions can be drawn when looking at the torsional flexibility of the building. In fact, the optimal solution to minimize torsional rotations corresponds to one intra-module floor, which is related to the minimum diagonal inclination (Table 3, Fig. 3c and d). This is due to the fact that torsional rigidity is related to the shear rigidity of the diagonal modules, and the latter has already been shown to achieve the highest value for low diagonal inclinations, close to 35° (Moon et al., 2007). Note that, although the configuration associated with one intra-module floor is optimal to reduce torsional rotations, it is the one which exhibits the highest amount of employed material and leads to lateral displacements much higher than the optimal ones (Table 3). The results of the calculations also show that the optimal floor plan geometry to withstand torque actions corresponds to the circular shape. In fact, among the structures with the optimal diagonal inclination (one intra-module floor), the circular building exhibits the lowest torsional rotations (2.7×10^{-5} rad), the other ones providing higher values (up to 26% higher for the square building).

3.2. 48-Story building

The results obtained for the 48-story 168 m-high diagrid buildings are shown in Table 4 and Fig. 4. In this case, the optimal solution to minimize lateral displacements is found to be associated to four intra-module floors for the square floor plan geometry and three intra-module floors for the other plan shapes (hexagonal, octagonal and circular). Accordingly, the optimal diagonal inclination is found to lie in the range 64°–70° (Fig. 4b). The case of the square building demonstrates that, by increasing the aspect ratio of the building, higher diagonal inclinations are expected in order to minimize the lateral displacement. Again, as in the case of the 36-story building, the influence of the specific plan geometry is significant only in the region which is far from the optimal solution, e.g. for one, six or twelve intra-module floors (Fig. 4a), and it leads to negligible differences in the region of the optimal diagonal inclination (3.5% difference).

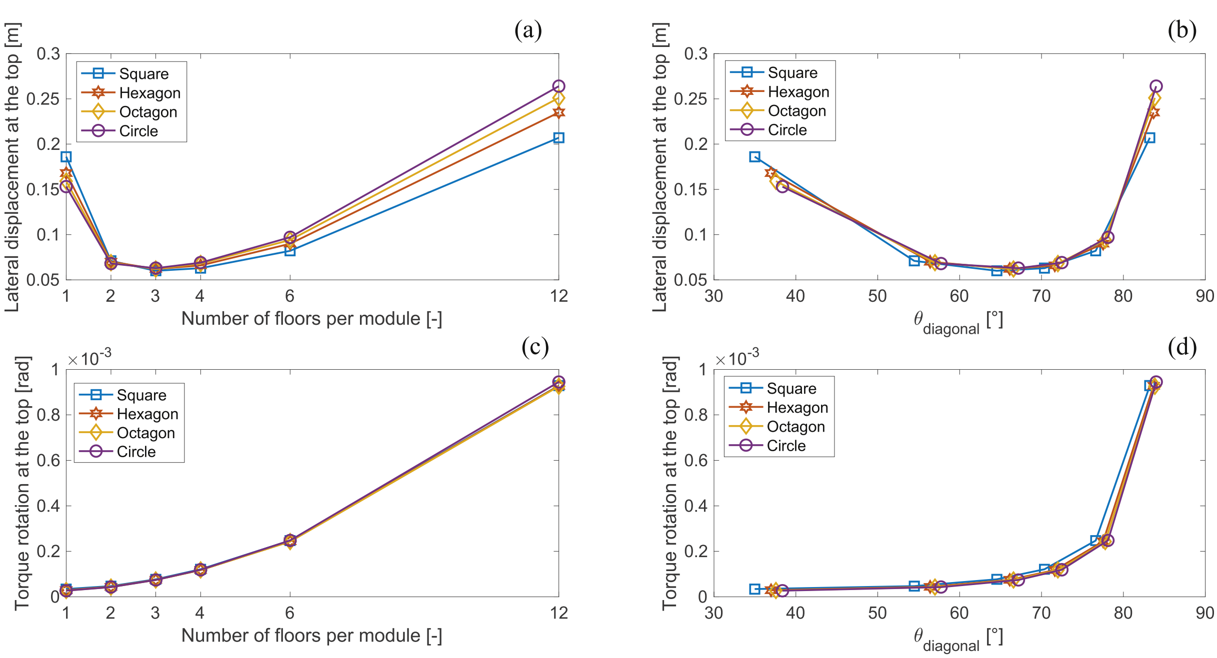

Fig. 3. Displacements and rotations for the 36-story building: (a–b) lateral displacements; (c–d) torsional rotations.

Table 4
Displacements and rotations for the 48-story building (minimum values of displacements and rotations for each floor plan shape are in bold).

Structure	Total number of modules [−]	Diagonals' total mass [ton]	Lateral displacement [m]	Torque rotation [E−4 rad]
SQ.1	48	2084	0.578	**0.60**
SQ.2	24	1469	0.214	0.83
SQ.3	16	1324	0.171	1.37
SQ.4	12	1269	**0.168**	2.15
SQ.6	8	1228	0.193	4.39
SQ.12	4	1204	0.414	16.52
HE.1	48	1987	0.522	**0.52**
HE.2	24	1434	0.209	0.78
HE.3	16	1307	**0.174**	1.32
HE.4	12	1259	0.175	2.11
HE.6	8	1224	0.212	4.35
HE.12	4	1202	0.466	16.49
OC.1	48	1960	0.494	**0.49**
OC.2	24	1425	0.203	0.76
OC.3	16	1302	**0.172**	1.31
OC.4	12	1257	0.176	2.09
OC.6	8	1223	0.218	4.33
OC.12	4	1202	0.494	16.46
CI.1	48	1925	0.474	**0.48**
CI.2	24	1413	0.201	0.76
CI.3	16	1297	**0.173**	1.32
CI.4	12	1253	0.179	2.12
CI.6	8	1221	0.225	4.41
CI.12	4	1202	0.518	16.81

As far as the torsional behavior is concerned, in line with the outcomes of the 36-story building, the optimal solution to reduce torsional rotations corresponds to the configuration with one intra-module floor and the circular plan shape (Table 4, Fig. 4c and d).

3.3. 60-Story building

In Table 5 and Fig. 5, the results are displayed which are related to the 60-story 210 m-high diagrid buildings. In this case, the configurations associated to four intra-module floors are found to be the optimal ones in order to minimize lateral displacements for both the floor plan geometries. Again, the influence of the specific plan shape is not negligible only when considering one or more than six intra-module floors (Fig. 5a). In this case, the diagonal inclination associated to the minimum lateral displacements is found to lay in the range 70°–72° (Fig. 5b). As can be seen, increasing the aspect ratio of the building leads to higher values of the optimal diagonal angle.

Analyzing the results related to the torsional flexibility, the optimal solution to minimize the torsional rotations involves again considering only one intra-module floor and the circular floor geometry (Table 5, Fig. 5c and d).

3.4. 72-Story building

Finally, the results arising from the analysis of the 72-story 252 m-high buildings are shown in Table 6 and Fig. 6. In this case, the same outcomes observed for the 60-story structures are found: the best configurations which minimize the lateral displacements imply four intra-module floors for all the plan geometries (Fig. 6a), the optimal diagonal angle lies in the range 70°–72° (Fig. 6b) and the differences among the different floor plan shapes are not negligible just for one, six or twelve intra-module floors (Fig. 6a). Again, as far as the torsional behavior is concerned, the one intra-module floor circular building is the most capable one to withstand torque actions, since it provides the lowest torsional deformability (Table 6, Fig. 6c and d).

3.5. Influence of the total height of the building

As described in Sections 3.1-3.4, the total height of the building has an influence mostly on defining the optimal structural configurations to minimize lateral displacements. In fact, as recalled in Table 7, by increasing the total height of the building, the number of intra-module floors which leads to the minimum lateral displacements increases from three to four, for each plan shape. As a consequence, the optimal range for the diagonal inclination increases from 64°-67° to 70°–72°. This is due to the fact that both shear and bending rigidity compete to define the lateral stiffness of the building. As shown by Moon et al. (2007), the shear rigidity of the diagrid modules reaches the highest value for a diagonal inclination of about 35° and it decreases significantly for higher diagonal angles; contrariwise, bending rigidity is maximum if the diagonal angle is 90° and decreases for lower inclinations. By the competition of shear and bending rigidity, the optimal solution is usually

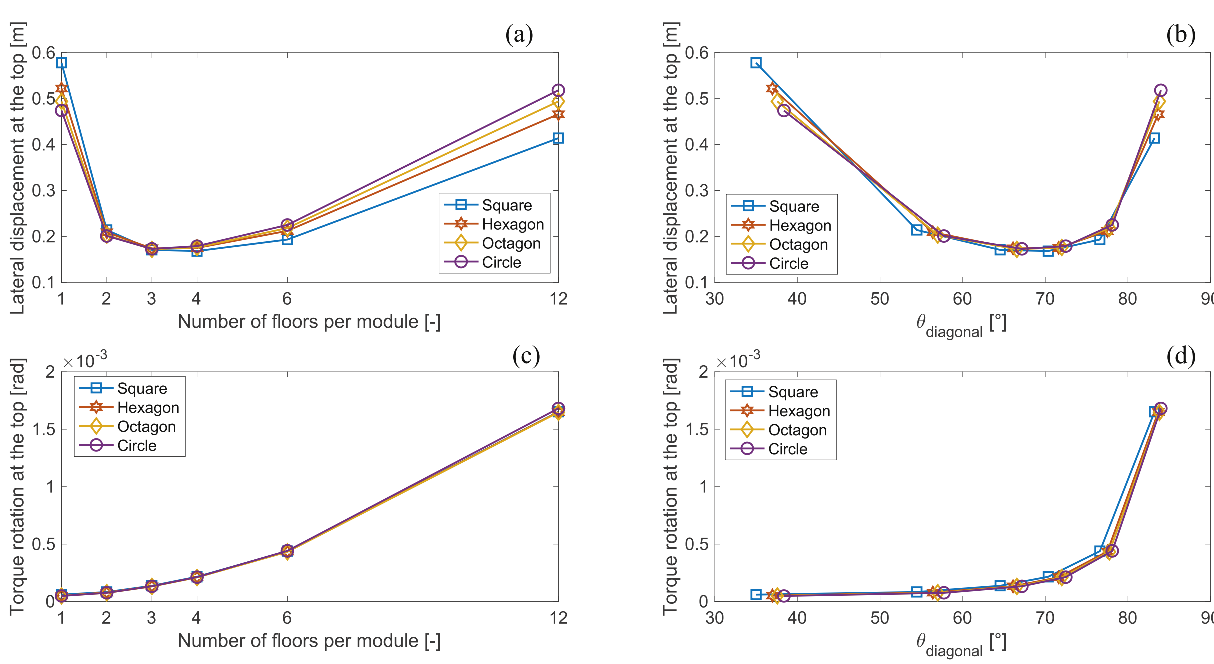

Fig. 4. Displacements and rotations for the 48-story building: (a–b) lateral displacements; (c–d) torsional rotations.

Table 5
Displacements and rotations for the 60-story building (minimum values of displacements and rotations for each floor plan shape are in bold).

Structure	Total number of modules [–]	Diagonals' total mass [ton]	Lateral displacement [m]	Torque rotation [E−4 rad]
SQ.1	60	2605	1.402	**0.93**
SQ.2	30	1836	0.509	1.30
SQ.3	20	1655	0.394	2.15
SQ.4	15	1586	**0.375**	3.36
SQ.6	10	1536	0.410	6.86
SQ.12	5	1504	0.744	25.81
HE.1	60	2484	1.263	**0.81**
HE.2	30	1793	0.495	1.22
HE.3	20	1634	0.399	2.07
HE.4	15	1574	**0.387**	3.29
HE.6	10	1530	0.437	6.80
HE.12	5	1503	0.828	25.77
OC.1	60	2450	1.196	**0.78**
OC.2	30	1782	0.481	1.19
OC.3	20	1628	0.393	2.05
OC.4	15	1571	**0.386**	3.27
OC.6	10	1528	0.443	6.77
OC.12	5	1503	0.870	25.72
CI.1	60	2406	1.148	**0.75**
CI.2	30	1767	0.476	1.19
CI.3	20	1621	0.395	2.06
CI.4	15	1567	**0.392**	3.31
CI.6	10	1527	0.455	6.89
CI.12	5	1502	0.909	26.26

Table 6
Displacements and rotations for the 72-story building (minimum values of displacements and rotations for each floor plan shape are in bold).

Structure	Total number of modules [–]	Diagonals' Total mass [ton]	Lateral displacement [m]	Torque rotation [E−4 rad]
SQ.1	72	3126	2.896	**1.34**
SQ.2	36	2203	1.040	1.88
SQ.3	24	1985	0.793	3.09
SQ.4	18	1904	**0.738**	4.84
SQ.6	12	1843	0.771	9.88
SQ.12	6	1805	1.239	37.17
HE.1	72	2981	2.611	**1.16**
HE.2	36	2152	1.010	1.75
HE.3	24	1961	0.799	2.98
HE.4	18	1889	**0.759**	4.74
HE.6	12	1836	0.813	9.79
HE.12	6	1804	1.367	37.11
OC.1	72	2940	2.468	**1.12**
OC.2	36	2138	0.979	1.72
OC.3	24	1954	0.785	2.95
OC.4	18	1885	**0.752**	4.70
OC.6	12	1834	0.820	9.75
OC.12	6	1803	1.425	37.04
CI.1	72	2888	2.368	**1.08**
CI.2	36	2120	0.968	1.71
CI.3	24	1945	0.788	2.97
CI.4	18	1880	**0.761**	4.77
CI.6	12	1832	0.838	9.92
CI.12	6	1803	1.483	37.82

found between these two angle values, depending on the building aspect ratio. Since shear behavior prevails for lower buildings and bending behavior for taller buildings, increasing the total high of the building leads to an increasing predominance of bending rigidity over shear rigidity. Therefore, by increasing the height of the building, the diagonal inclination which provides the lowest lateral displacement exhibits higher values (Table 7, Figs. 3-6b).

Contrariwise, no competition between shear and bending rigidity occurs when dealing with the torsional behavior because, as mentioned above, this is governed only by the shear rigidity of the diagrid modules. For this reason, the diagonal inclination which leads to the lowest torsional rotations is always found to be the lowest one, in the range 35°–38° (Table 7).

With regards to the influence of the floor plan geometry on the structural response, as shown in Sections 3.1-3.4, this is usually found to be very small when the diagonal inclination lies in the optimal range. Although the differences are usually lower than 5%, it is interesting to note that the configurations which lead to the minimum lateral displacements are always associated to the square buildings (Tables 3–7). At first sight, this result seems in contrast with the findings of Mirniazmandan et al. (2018), where the buildings with square geometry were not included in the list of the most performant solutions for the limitation of the lateral displacements. However, this difference mainly arises from the different choice of keeping different geometrical parameters constant

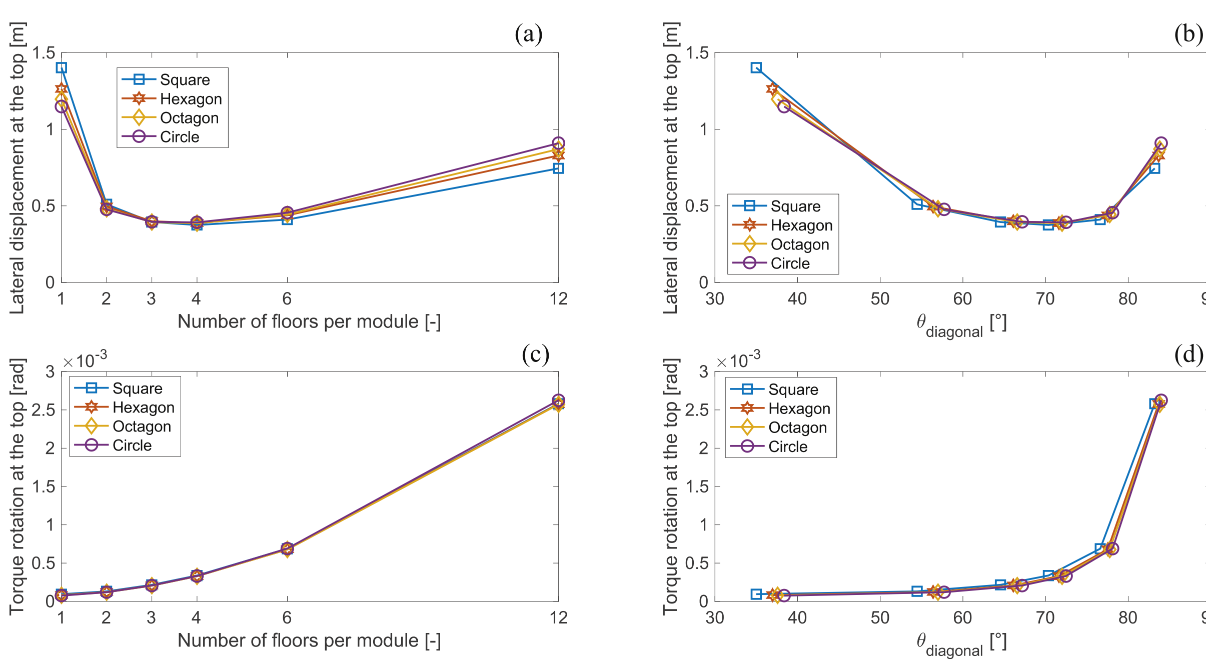

Fig. 5. Displacements and rotations for the 60-story building: (a–b) lateral displacements; (c–d) torsional rotations.

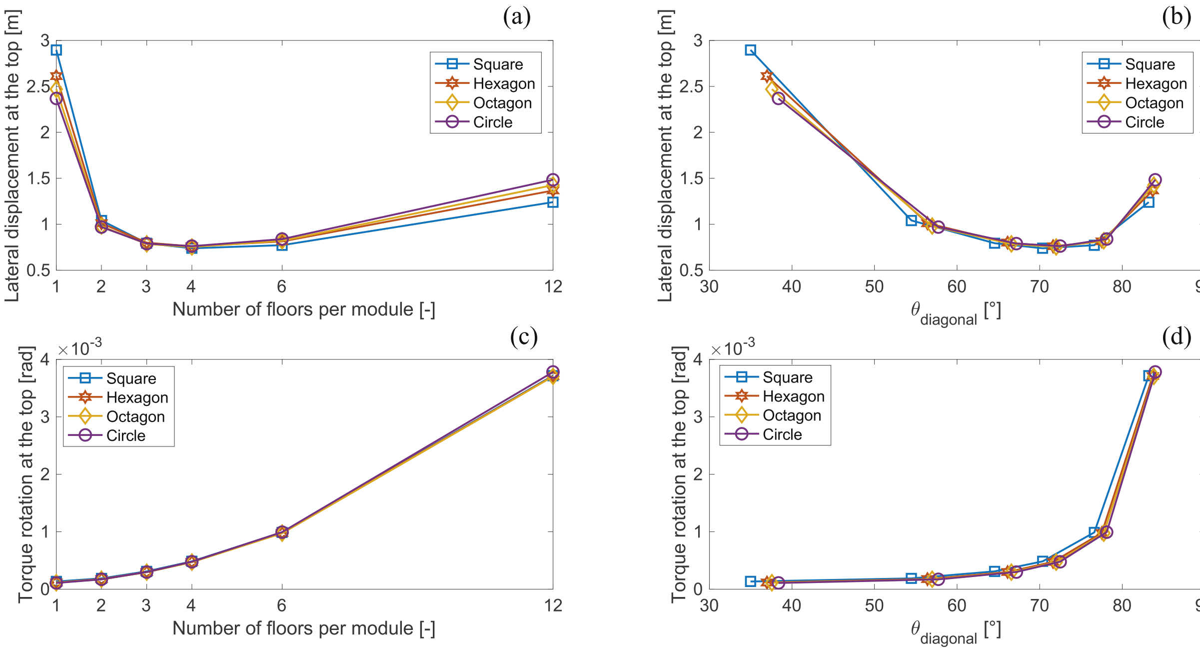

Fig. 6. Displacements and rotations for the 72-story building: (a–b) lateral displacements; (c–d) torsional rotations.

Table 7
Optimal number of intra-module floors and diagonal inclination to minimize lateral displacements and torsional rotations (configurations providing the absolute minimum displacements and rotations are in bold).

Total height of the building [m]		126	168	210	252
Number of intra-module floors which minimizes lateral displacements [−]	**Square**	**3**	**4**	**4**	**4**
	Hexagon	3	3	4	4
	Octagon	3	3	4	4
	Circle	3	3	4	4
Diagonal inclination which minimizes lateral displacements [°]	**Square**	**64.54**	**70.35**	**70.35**	**70.35**
	Hexagon	66.11	66.11	71.63	71.63
	Octagon	66.57	66.57	72.00	72.00
	Circle	67.17	67.17	72.48	72.48
Number of intra-module floors which minimizes torsional rotations [−]	Square	1	1	1	1
	Hexagon	1	1	1	1
	Octagon	1	1	1	1
	Circle	**1**	**1**	**1**	**1**
Diagonal inclination which minimizes torsional rotations [°]	Square	34.99	34.99	34.99	34.99
	Hexagon	36.97	36.97	36.97	36.97
	Octagon	37.57	37.57	37.57	37.57
	Circle	**38.37**	**38.37**	**38.37**	**38.37**

when changing the floor plan shapes. As a matter of fact, in the present work, we choose to keep the total floor area constant (this being one fundamental parameter for architectural purposes), whereas Mirniazmandan et al. (2018) decided to keep the total external perimeter constant in their calculations. Choosing different geometrical parameters to be constant leads to different results in terms of floor dimensions. For example, by taking the circle geometry as the reference, considering the external perimeter constant leads to obtain a square geometry which is 12% smaller than it would be in the case of considering the total floor area constant. Since the base dimensions play a key role in governing the stiffness of the building, as they strongly affect the bending rigidity, this difference is the one which make our results deviate from the ones of Mirniazmandan et al. Anyway, from both our analysis and the one of Mirniazmandan et al., it is evident how the geometrical characteristic which mostly affects the lateral flexibility of the diagrid is the diagonal inclination, whereas the influence of the plan shape geometry is less evident. Conversely, far from the optimal number of intra-module floors, the differences between the different floor plan shapes is found to be significant; for a number of intra-module floors lower than the optimum ones the optimal geometry is usually associated to the circular plan shape, whereas for higher numbers of intra-module floors the square plan geometry is the one providing the highest stiffness of the building (Figs. 3a-6a). The hexagonal and octagonal plan geometries always exhibit structural responses in between.

Regardless the total height of the building, the optimal configuration which leads to the highest torsional stiffness is always associated to the circular plan geometry with the lowest inclination of the diagonals (one intra-module floor). As already remarked above, this is due to the torsional mechanism of the diagrid structure, which only involves the shear rigidity of the diagonal modules, and not their bending rigidity as in the case of lateral deformability. So far, all the researchers have focused their attention only on the limitation of the lateral displacements, not considering the torsional rotations (Moon et al., 2007; Moon, 2011; Zhang et al., 2012; Montuori et al., 2013, 2014a; Mele et al., 2014, 2019; Angelucci and Mollaioli, 2017; Tomei et al., 2018; Mirniazmandan et al., 2018). Sometimes, torque actions can be particularly severe, e.g. in the case of a strong asymmetry in the resisting elements placed in the interior of the building which leads to a not negligible eccentricity between the mass and stiffness centroids of the floors. In these cases, the torsional rotations induced by these actions should be taken into account. Unfortunately, in the present analysis, we have shown that a unique diagonal inclination which minimizes the lateral displacement and the torsional rotation at the same time does not exist. Therefore, when adopting the diagonal inclination which minimizes the lateral displacements, attention should be paid to the corresponding torsional rotations, as they might create problems to the façade elements as well.

4. Conclusions

The influence of geometrical parameters, such as the inclination of external diagonals, the floor plan shape, and the building aspect ratio, on the structural response of diagrid tall buildings was investigated. In particular, a previously developed matrix-based method (MBM) was used in order to perform the structural analysis of diagrid tube structures, under both horizontal forces and distributed torque moments. Lateral displacements and torsional rotations were calculated by the MBM for a variety of diagrid structures, with changing geometrical parameters, and the following conclusions were drawn.

The diagonal inclination is the main geometrical parameter affecting the structural behavior of the building. For the investigated diagrid structures, its optimal values are found to lie in the range 64°–72° in order to minimize lateral displacements and these increase when the aspect ratio of the building increases, due to the competition between shear and bending rigidity. Contrariwise, the diagonal inclinations which provide the highest torsional stiffness are always found in the range 35°–38° and do not depend on the total height of the building, since torsional behavior is only affected by the shear rigidity of the diagrid modules.

The influence of the floor plan geometry was also investigated, by considering four different plan shapes for each structure, i.e. square, hexagonal, octagonal, and circular, by keeping the floor area constant. It is observed that the specific plan geometry does not affect significantly the structural response, when the diagonal inclination is in the optimal range for limiting the lateral displacements. In these cases, the differences among the adoption of different plan shapes is found to be lower than 5% for all the investigated buildings. Contrariwise, significant differences can be found when far from the optimal diagonal inclinations. In this case, adopting different floor geometries leads to bigger differences in terms of lateral displacements (up to 25%). As far as the torsional behavior is concerned, when the diagonal inclination is optimized to minimize torsional rotations, the circular plan geometry is always found to be the most suitable for withstanding torque actions.

The outcomes reported in this paper showed that the structural solutions which lead to the minimum lateral displacements and torsional rotations are not the same. Furthermore, besides limiting the structure deformability, it is essential for sustainability purposes to minimize the amount of employed material as well. Other parameters to take into account should also be the axial loads in the diagonals and the story drifts. For these reasons, a multi-objective multi-parameter approach is going to be developed in future work to address this problem. Various geometrical parameters are going to be considered to find the *best* diagrid solutions among numerous *good* solutions, in order to find a compromise to limit both lateral displacements, torsional rotations, amount of employed material, diagonal axial loads and story drifts.

Finally, it is needed to remark that the outcomes shown in this paper are based on the linear elastic regime of the diagrid structure. Although this is common in the literature (Moon et al., 2007; Moon, 2011; Zhang et al., 2012; Montuori et al., 2013, 2014a, 2014b; Mele et al., 2014, 2019; Angelucci and Mollaioli, 2017; Tomei et al., 2018; Mirniazmandan et al., 2018; Liu and Ma, 2017), nonlinearities might play a crucial role, especially under moderate or severe earthquake motions. As a matter of fact, some diagonals may enter into plastic state and the force distribution within the diagrid may change drastically, affecting the structural response. All these aspects should be taken into account for a more detailed design and analysis of diagrid structure, which was not the aim of the present analysis. For this reason, future research works will also consider enriching the MBM in order to investigate the structural response of the building in the nonlinear regime, considering both geometrical and material nonlinearities.

Declaration of competing interest

The Authors declare no conflict of interest.

Appendix

A1. Effect of considering a variable cross-sectional area for the diagonals along the height of the building

In this section we report the results obtained when considering the diagrid buildings, with different floor shapes and diagonal inclinations, with a variable cross-sectional area of the diagonals along the height of the building. Specifically, the cross-sectional areas of the diagonals belonging to the ground module and the top module were assumed to be 1000 cm^2 and 100 cm^2, respectively. A gradual linear interpolation was considered for the other modules. Except for this parameter, all the parameters reported in Table 1 were used for the diagrid structures. In Fig. A1 the results are shown for the four buildings. In Tab. A1, the best solutions to minimize the lateral displacements and torsional rotations are reported. As can be observed from the results, the main conclusions which were drawn when considering a uniform distribution of the cross-sectional areas are still valid. Therefore, considering a different diagonal cross-sectional area distribution is found not to affect the main outcomes of the present analysis.

Tab. A1
Optimal number of intra-module floors and diagonal inclination to minimize lateral displacements and torsional rotations (configurations providing the absolute minimum displacements and rotations are in bold) – Variable cross-sectional area.

Total height of the building [m]		126	168	210	252
Number of intra-module floors which minimizes lateral displacements [−]	Square	**3**	**3**	**4**	**4**
	Hexagon	3	3	4	4
	Octagon	3	3	3	4
	Circle	3	3	3	4
Diagonal inclination which minimizes lateral displacements [°]	Square	**64.54**	**70.35**	**70.35**	**70.35**
	Hexagon	66.11	66.11	71.63	71.63
	Octagon	66.57	66.57	72.00	72.00
	Circle	67.17	67.17	72.48	72.48
Number of intra-module floors which minimizes torsional rotations [−]	Square	1	1	1	1
	Hexagon	1	1	1	1
	Octagon	1	1	1	1
	Circle	**1**	**1**	**1**	**1**
Diagonal inclination which minimizes torsional rotations [°]	Square	34.99	34.99	34.99	34.99
	Hexagon	36.97	36.97	36.97	36.97
	Octagon	37.57	37.57	37.57	37.57
	Circle	**38.37**	**38.37**	**38.37**	**38.37**

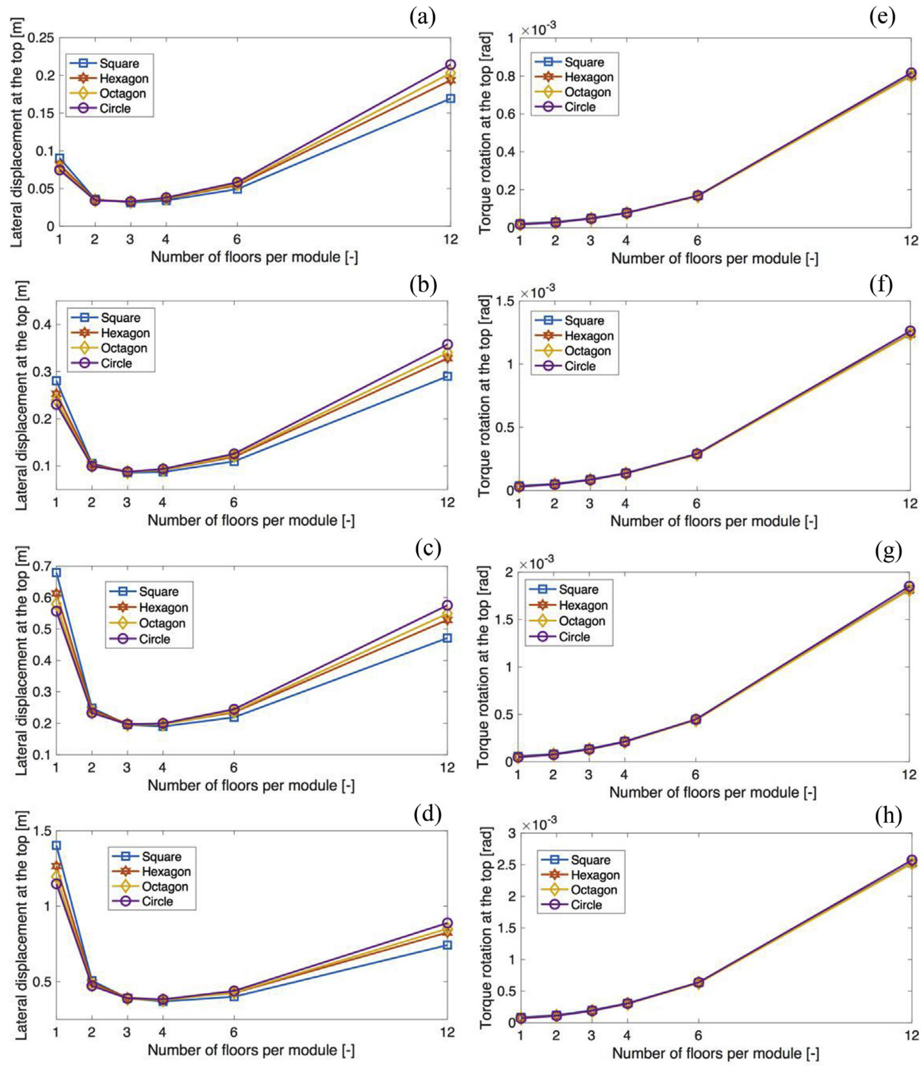

Fig. A1. Displacements (a-d) and torsional rotations (e-h) for the four buildings – Variable cross-sectional area.

A2. Effect of considering a reverse triangle load pattern along the height of the building

In this section we report the results obtained when considering the diagrid buildings, with different floor shapes and diagonal inclinations, subjected to a reverse triangle load pattern along the height of the building. Specifically, as shown in Fig. A2, the maximum values for the lateral and torque distributed load were assumed 30 kN/m and 70 kNm/m, respectively, at the top of the building. A linear decrease was applied up to the ground floor. Except for this condition, all the parameters reported in Table 1 were used for the diagrid structures. In Fig. A3 the results are shown, for the four buildings. In Tab. A2, the best solutions to minimize the lateral displacements and torsional rotations are reported. As can be observed from the results, the main conclusions which were drawn when considering a uniform load distribution along the height of the building are still valid. Therefore, considering a different load scheme is not found to affect the main outcomes of the present analysis.

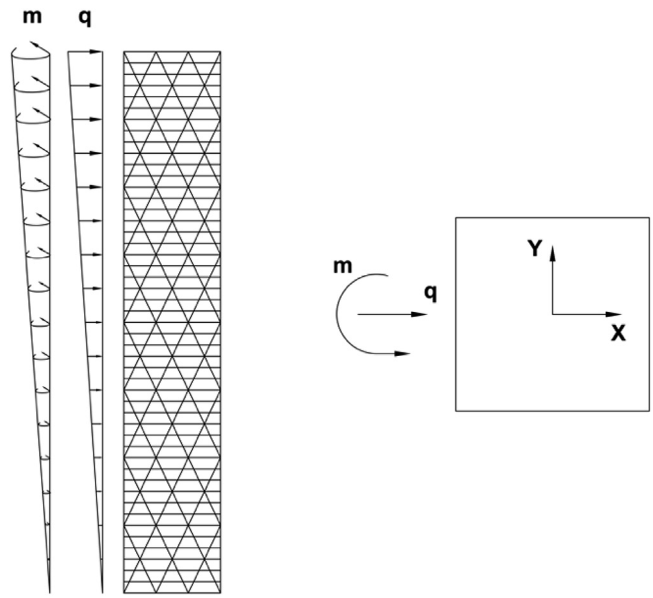

Fig. A2. Reverse triangle load pattern: *q* refers to the distributed horizontal load, *m* to the distributed torque moments.

Tab. A2
Optimal number of intra-module floors and diagonal inclination to minimize lateral displacements and torsional rotations (configurations providing the absolute minimum displacements and rotations are in bold) – Reverse triangle load.

Total height of the building [m]		126	168	210	252
Number of intra-module floors which minimizes lateral displacements [−]	Square	**3**	**4**	**4**	**4**
	Hexagon	3	3	4	4
	Octagon	3	3	4	4
	Circle	3	3	4	4
Diagonal inclination which minimizes lateral displacements [°]	Square	**64.54**	**70.35**	**70.35**	**70.35**
	Hexagon	66.11	66.11	71.63	71.63
	Octagon	66.57	66.57	72.00	72.00
	Circle	67.17	67.17	72.48	72.48
Number of intra-module floors which minimizes torsional rotations [−]	Square	1	1	1	1
	Hexagon	1	1	1	1
	Octagon	1	1	1	1
	Circle	**1**	**1**	**1**	**1**
Diagonal inclination which minimizes torsional rotations [°]	Square	34.99	34.99	34.99	34.99
	Hexagon	36.97	36.97	36.97	36.97
	Octagon	37.57	37.57	37.57	37.57
	Circle	**38.37**	**38.37**	**38.37**	**38.37**

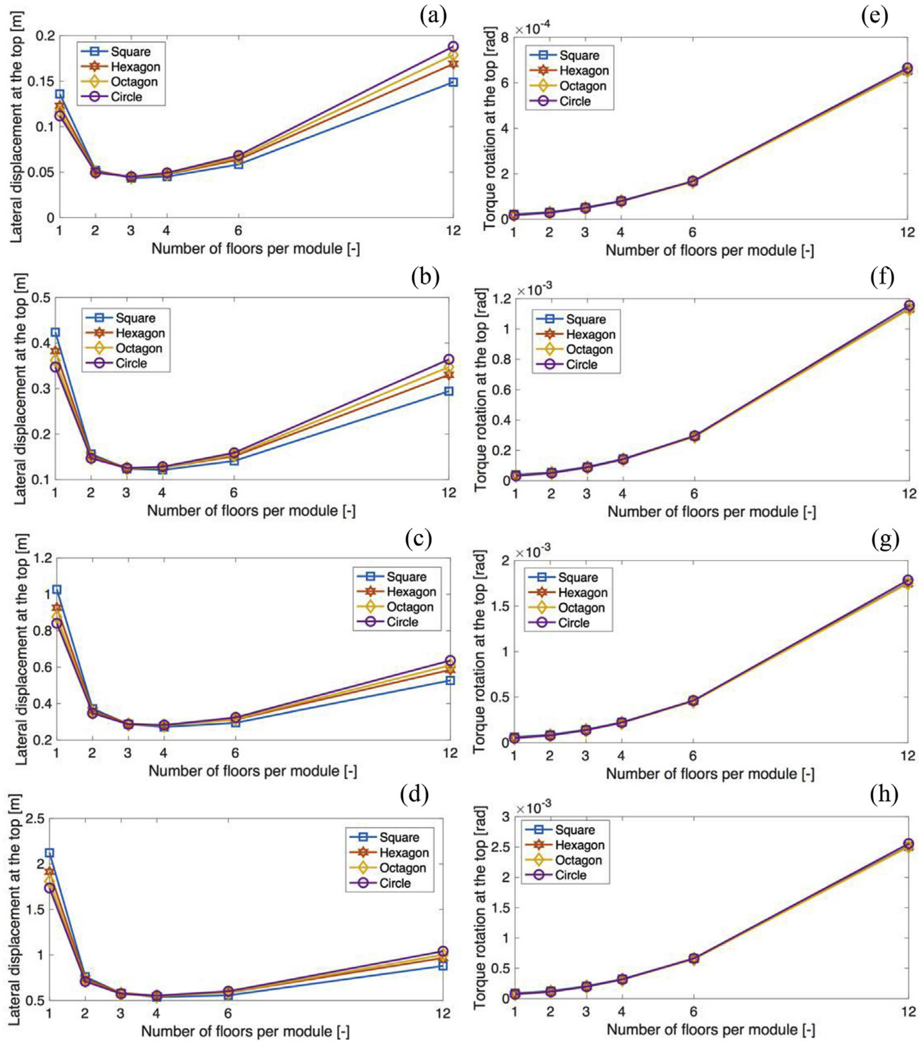

Fig. A3. Displacements (a-d) and torsional rotations (e-h) for the four buildings – Reverse triangle load.

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