High-Dimensional Parameterized Macromodeling with Guaranteed Stability

# High-Dimensional Parameterized Macromodeling with Guaranteed Stability

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***Abstract*—We introduce a Radial Basis Function (RBF) parameterized macromodeling algorithm, specifically designed for high-dimensional parameters. As opposed to standard approaches, the adopted RBF model representation has the potential to scale very favorably when the number of model parameters increases, since the number of model coefficients is not related to the dimension of the embedding parameter space. A transmission-line example with up to seven parameters is used to demonstrate the proposed approach.**

## I. Introduction

Macromodeling tools and algorithms allow to cast complex electromagnetic structures as simple automatically-derived behavioral equivalent circuits. This fact led to an ubiquitous adoption of macromodels in both academia and industry in various application fields. This is true in particular in the area of Signal and Power Integrity, where macromodels of interconnects, components and subsystems have been demonstrated to enable fast numerical verification at the system level.

Parameterized macromodels add one layer of complexity. The resulting behavioral equivalents aim at embedding in closed form some approximate dependence of the system response on external parameters, which may be related, e.g., to geometry, materials or temperature. Such closed-form dependence can be exploited since the early stages of the design in performing what-if, monte-carlo or sensitivity studies, as well as optimization under prescribed performance objectives.

Several algorithms have been proposed for the construction of parameterized macromodels. Focusing on data-driven methods, which construct the models starting from samples of frequency responses in the parameter space, we can cite interpolation-based methods [1], which construct the models by connecting non-parameterized (univariate) models through special interpolation schemes in the parameter space, and fitting-based methods [2], which assume a given dependence on the model on both frequency and parameters. Based on this dependence, a (possibly iterative) least squares approach provides the unknown model coefficients. Practically all documented results on both classes of methods are limited to a restricted number of parameters (usually up to 2 or 3). This is due to the apparently unavoidable curse of dimensionality that affects both model representation and particularly the identification algorithms, whose complexity scales exponentially with the dimension of the parameter space.

This paper wants to overcome this limiation by breaking the dependence of the model complexity on the parameter space dimension. The only possible approaches that may provide a viable solution to this problem are those methods based on unstructured (mesh-free) representations. Among these techniques, we propose a sparse model structure based on a Radial Basis Function (RBF) expansion. When combined with a rational (barycentric) frequency-domain approximation, this approach leads to parameterized models with a complexity that depends solely on the behavior of the multivariate responses to be approximated, and not on the dimension of the parameter space. As an additional byproduct, based on using positive interpolation kernels (Gaussian RBF's), we show that a uniform model stability can be achieved with much simpler (and fewer) constraints than for standard approaches. This is demonstrated using a transmission-line example with up to seven parameters, whose model requires as few as 28 coefficients to represent the response variations in the 7-dimensional parameter space.

## II. Background

Let us start considering a $P$-port electromagnetic structure characterized through its frequency dependent scattering responses. Additionally, assume the structure to be parameterized by some external (real-valued) parameters $\vartheta^1, \ldots, \vartheta^\rho$, collected in a vector $\boldsymbol{\vartheta} \in \boldsymbol{\Theta}$. Aiming at synthesizing a surrogate model $\mathbf{H}(s, \boldsymbol{\vartheta})$ that reproduce the true system responses $\breve{\mathbf{H}}_{k,m} = \breve{\mathbf{H}}(s_k, \boldsymbol{\vartheta}_m)$, we enforce

$$\mathbf{H}(s_k, \boldsymbol{\vartheta}_m) \approx \breve{\mathbf{H}}_{k,m} \tag{1}$$

at a discrete set of $\bar{k}$ frequencies $s_k = \mathrm{j}\omega_k$ and $\bar{m}$ parameter samples $\boldsymbol{\vartheta}_m$, available from repeated field-solver runs.

The general adopted model structure is standard [2]

$$\mathbf{H}(s, \boldsymbol{\vartheta}) = \frac{\mathbf{N}(s, \boldsymbol{\vartheta})}{\mathsf{D}(s, \boldsymbol{\vartheta})} = \frac{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} \mathbf{R}_{n,\ell}\ \xi_\ell(\boldsymbol{\vartheta})\ \varphi_n(s)}{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} r_{n,\ell}\ \xi_\ell(\boldsymbol{\vartheta})\ \varphi_n(s)} \tag{2}$$

where $\varphi_n(s) = (s - q_n)^{-1}$ is the partial fraction basis associated with the fixed basis poles $\{q_n\}$ and $\xi_\ell(\boldsymbol{\vartheta})$ are suitable multi-variate basis functions capturing the parameter dependence. The model coefficients $\mathbf{R}_{n,\ell},\ r_{n,\ell}$ are computed by the so-called *Parameterized Sanathanan-Koerner* (PSK) algorithm [2], [3], which iteratively solves the following optimization problem

$$\min \left\| \frac{\mathbf{N}^\mu(\mathrm{j}\omega_k, \boldsymbol{\vartheta}_m) - \mathsf{D}^\mu(\mathrm{j}\omega_k, \boldsymbol{\vartheta}_m)\, \breve{\mathbf{H}}_{k,m}}{\mathsf{D}^{\mu-1}(\mathrm{j}\omega_k, \boldsymbol{\vartheta}_m)} \right\|_F^2 \tag{3}$$

for $\mu = 1, 2, \ldots$, until the coefficient estimates stabilize. The most advanced PSK implementations guarantee the uniform stability of the synthesized model enforcing the following *Positive-Realness* condition on the denominator (see [4], [5])

$$\Re\{\mathsf{D}(s, \boldsymbol{\vartheta})\} > 0, \ \forall s = \mathrm{j}\omega, \boldsymbol{\vartheta} \tag{4}$$

As originally introduced in [2], it is a well-established technique to define the parameter-dependent basis function $\xi_\ell(\boldsymbol{\vartheta})$ as product of $\rho$ univariate basis functions $\xi_\ell^\nu(\vartheta)$ (usually first-kind Chebychev polynomials) defined along each direction in the parameter space as

$$\xi_\ell(\boldsymbol{\vartheta}) = \prod_{\nu=1}^{\rho} \xi_\ell^\nu(\vartheta^\nu) \tag{5}$$

where $\ell = (\ell_1, \ldots, \ell_\rho)$ with $\ell_i = 1, \ldots, \bar{\ell}_i$ is a $\rho$-dimensional multi-index.

Employing the model structure (2), we see that the model complexity (number of coefficients to be estimated) scales exponentially with the number of parameters $\rho$ as $N_c = (P^2 + 1) \cdot (\bar{n} + 1) \cdot \bar{\ell}_1 \cdot \ldots \cdot \bar{\ell}_\rho$. Thus, in case of high-dimensional parameter spaces, the PSK procedure becomes intractable in terms of memory resources. The primary limitation lies in definition (5), that constrains the multi-variate basis functions $\xi_\ell(\boldsymbol{\vartheta}) : \mathbb{R}^\rho \to \mathbb{R}$ to be defined as a cartesian (tensor) product of scalar functions $\xi_\ell^\nu : \mathbb{R} \to \mathbb{R}$. In the following section, we present a novel approach to overcome this limitation, extending the "standard" PSK scheme to high dimensional settings.

## III. Beyond the Cartesian Product

In the context of data-driven interpolation, methods based on kernels and, in particular, on Radial Basis Functions, have proven to be extremely successful, especially in case of high-dimensional non-linear problems [6], [7]. Simply speaking, a function is defined RBF if it is symmetric with respect to a "center" and translation-invariant (for more details see [8]). The most notable example is the well known *Gaussian RBF*, defined as $r \cdot e^{-\varepsilon \|\boldsymbol{\vartheta} - \boldsymbol{\vartheta}_0\|^2}$, where the scalar $r$ defines its amplitude, $\boldsymbol{\vartheta}_0$ denotes the center and $\varepsilon$ is a free hyper-parameter denoted as "shape factor". In this work, we adopt

$$\xi_\ell(\boldsymbol{\vartheta}) = e^{-\varepsilon \|\boldsymbol{\vartheta} - \boldsymbol{\vartheta}_\ell\|_2^2}, \ \ell = 1, \ldots, \bar{\ell} \tag{6}$$

as the parameter-dependent basis functions in (2). Note that the above $\xi_\ell(\boldsymbol{\vartheta})$ is a multi-variate function with a scalar argument (2-norm $\|\cdot\|_2$), instead of a multi-linear combination of univariate functions. Therefore, the number of model coefficients reduces to $N_{\text{RBF}} = (P^2 + 1) \cdot (\bar{n} + 1) \cdot \bar{\ell}$, achieving a model compression $\frac{N_c}{N_{\text{RBF}}}$ that increases and may become considerable as the number of parameters $\rho$ grows. This result is graphically shown in Figure 1 for the example discussed in Sec. IV. The figure compares the model complexities $N_c$ and $N_{\text{RBF}}$ for increasing number of parameters (due to memory limitations, $N_c$ values are extrapolated for $\rho > 3$). It is clear that, although for small $\rho$ the two approaches are comparable,

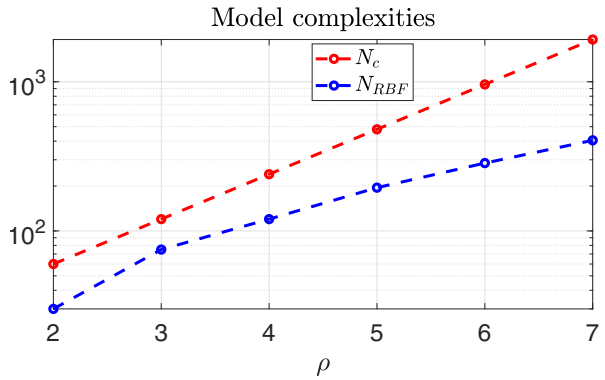


Fig. 1: Comparison between model complexities: red line refers to cartesian product of univariate basis functions while blue line is related to RBFs

as the dimensionality grows less RBF basis functions are required to obtain comparable model accuracies.

The adoption of positive-definite RBF kernels ($\xi_\ell(\boldsymbol{\vartheta}) > 0 \ \forall \boldsymbol{\vartheta}$, satisfied by Gaussian RBFs) greatly simplifies the enforcement of the Positive-Realness condition (4), which we recall being a sufficient condition for ensuring that the parameter-dependent model poles are uniformly stable throughout the parameter space. In fact, the standard approach as originally presented in [4] would require to detect all the frequency-parameter combinations at which $\mathsf{D}(s, \boldsymbol{\vartheta})$ has a negative real part. This task has a complexity that scales exponentially with the number of parameters (see [9]). Conversely, defining $\xi_\ell(\boldsymbol{\vartheta})$ as in (6), greatly simplifies the enforcement of condition (4), which becomes fully equivalent to embedding the following linear inequality constraints in each PSK iteration (3) for real and complex conjugate poles with associated coefficients $r_n$ and $r'_n \pm \mathrm{j} r''_n$, respectively,

$$\begin{cases} r_{n,\ell} > 0, & q_n \in \mathbb{R}, \\ -\alpha_n r'_{n,\ell} \pm \beta_n r''_{n,\ell} > 0, & q_n = \alpha_n \pm \mathrm{j}\beta_n \in \mathbb{C}. \end{cases}$$

Therefore, both model structure and computational cost of the model identification algorithm are uncorrelated with the number $\rho$ of parameters, depending only on the number of required basis functions. As Fig. 1 shows, this number can be very small, provided that the model responses are smooth in the parameter space.

## IV. Example

We illustrate the performances of the presented approach on a transmission line network made of four cascaded lossy line sections with three internal loaded stubs (see [10] for further details), whose responses depend on seven external parameters, namely the three load reflection coefficients $\Gamma \in [0.5, \ 0.9]$, the three (independent) stub lengths $d_s \in [0.006, 0.007]$ mm and the (identical) inner line lengths $d_l \in [0.098, 0.01]$ mm. The remaining line lengths are fixed to 0.01 mm. Raw scattering responses, each composed of $\bar{k} = 1000$ frequency samples in the band $[0, \ 5]$ GHz, are generated

with an in-house solver for $\bar{m} = 400$ different parameter combinations, arranged according to a Latin Hypercube [11] distribution. A guaranteed (uniformly) stable model is synthesized with $\bar{n} = 14$ parameterized poles and only $\bar{\ell} = 14$ radial basis functions. These RBF's are centered in correspondence with a (randomly defined) subset of the available raw datasets and their shape parameters are set all equal to $\varepsilon = 0.1$. The top and middle panels of Fig. 2 compare, for a representative set of parameter combinations, the model responses (red-dashed lines) with respect to the available raw data (solid blue line) at ports (1,1) and (2,1), while the bottom panel depicts the data-model error for all $\bar{m}$ frequency/parameter combinations. The worst case RMS error is $2.1 \times 10^{-2}$ and the maximum absolute error is $6.0 \times 10^{-2}$. These results confirm the potential of proposed model representation to extract accurate and stable high-dimensional parameterized models, a task that was unfeasible with any existing structured macromodeling approaches.

## V. Conclusion

In this paper, we documented a novel parameterized macromodeling strategy with guaranteed uniform stability, specifically designed to handle high-dimensional parameter spaces. The key enabling factor for handling a large number of independent parameters, to be embedded in a closed form in the model responses, is the adoption of Radial Basis Functions (RBF) to capture the parameter variability. This choice, which has been widely adopted in many other fields, drastically reduces the number of model unknowns to be estimated, leading to major improvements in term of computational efforts. Additionally, the use of positive-definite basis functions considerably simplifies the stability enforcement procedure that, in contrast with previous approaches, does not require expensive adaptive sampling steps. A relevant test-case of a 7-parameter transmission line network demonstrates the performance of the proposed algorithm.

This paper presented only preliminary results. Work is under way to assess the robustness of proposed method in presence of noise and/or inaccuracy due to EM solver, and to compare the performance with other types of RBFs, including the case of different $p$-norms in the exponent of (6).

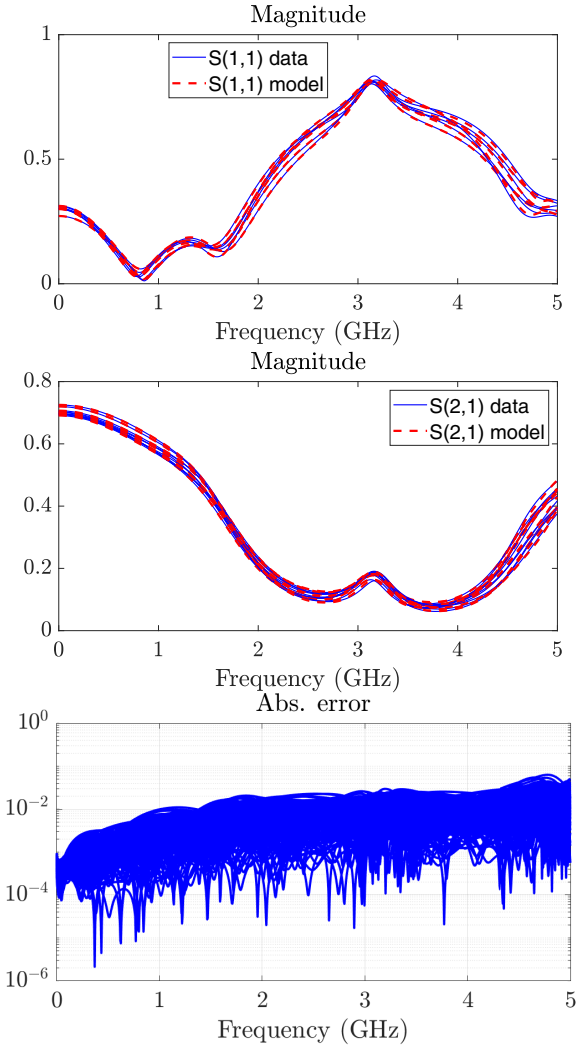


Fig. 2: Top and middle panels: comparison between data (blue solid lines) and model responses (red dashed lines) for ports (1,1) and (2,1). Bottom panel: model-data absolute error evaluated for all the available frequency/parameter combinations.

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