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Accurate estimation of prediction models for operator-induced defects in assembly manufacturing processes

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Abstract

The presence of defects in industrial manufacturing may compromise the final quality and cost of a product. Among all possible defect causes, human errors have significant effects on the performances of assembly systems. Much research has been conducted in recent years focusing on the problem of defect generation in assembly processes, considering the close connection between assembly complexity and human errors. It was observed that the relationship between the average number of defects introduced during each assembly phase and the related assembly complexity follows a power-law relationship. Accordingly, many authors proposed a data logarithmic transformation in order to linearize the mathematical model. However, as has already been discussed in literature, when the model is retransformed in the original form a significant bias may occur, leading to completely wrong predictions. In this paper, the bias due to the logarithmic transformation of models for predicting defects in assembly is analysed and discussed. Two alternative methods are proposed and compared to overcome this drawback: the use of a bias correction factor to the retransformed fitted values and a power-law nonlinear regression model. The latter has proved to be the best approach to predict defects with few non-repeated data and affected by high variability, such as in the case under study.

Keywords: quality control; assembly process; defect generation; logarithmic transformation; nonlinear regression model

1 Introduction

Much research has been conducted in recent years focused on the problem of defect generation in manufacturing, since the presence of defects in the final product may affect its quality and cost to a great extent. The sources of these defects may be extremely different, according to the product typology and the production context. Many studies have focused on the identification of the factors that may cause defects, considering e.g., Ishikawa diagrams, in which the sources of defects can be classified into several categories: machines, methods, materials, people, measurements and environment (Ishikawa 1976). The use of these cause-and-effect diagrams may help to improve the product design and prevent the occurrence of defects. The importance of the identification of defects has already been assessed in other research, mainly in the field of assembly manufacturing processes. Assembly, which is one of the activities that constitutes the manufacturing of complex products, together with the acquisition of raw materials, processing, functional testing, etc., is crucial for the cost and quality performance (Vandebroek, Lan, and Knapen 2016; Xiaoqing, Bo, and Shuchun 2010). Recently, a growing body of literature has recognized the importance of the role of quality control in the assembly production context, since the product life cycle requires a faster response and a lower defect rate. As a result, assembly quality control is considered to be one of the most crucial issues in modern manufacturing environments (Zhong, Liu, and Shi 2010; Ferrer 2007). Assembly defects have been classified into four categories: improper design, defective part, variance in the assembly system (induced by the changes in the
plan/schedule/arrangement of a machine, fixture, tooling, etc.), and operator mistakes (Su, Liu, and Whitney 2010). In recent years, much research has focused on the first three categories, and some useful assembly quality control technologies and management approaches have been developed (Zhang and Luk 2007; Gearbox, Pawar, and Mukhopadhyay 2015; Zheng 2000; Ping, Hua, and Guanlong 2008; He and Kusiak 1997; Evans, Evans, and Yu 1997; Vandevelde et al. 2018; Qin, Cudney, and Hamzic 2015; Chiang and Su 2003).

As far as the fourth category is concerned, there is a large volume of published studies that have described the significant impact of human errors on the performance of assembly systems, which is sometimes higher than that of technological errors (Shin, Wysk, and Rothrock 2006; Kang et al. 2018; Le, Qiang, and Liangfa 2012; Báez et al. 2014; Saptari, Leau, and Mohamad 2015; Su, Liu, and Whitney 2010; Shibata 2002; Kolus, Wells, and Neumann 2018; Krugh et al. 2016a; Genta, Galetto, and Franceschini 2018; Xiaoqing, Bo, and Shuchun 2010; Falck et al. 2017; Caputo, Pelagagge, and Salini 2017). Research in the field of semiconductor products has shown that 25% of the total assembly errors are induced by human mistakes (Shibata 2002). Another study has demonstrated that operator errors account for 20% of the total defects in copier assembly (Su, Liu, and Whitney 2010). These high percentages suggest that more attention should be paid to operator-induced assembly defects, and that reducing the number of operator mistakes is a central problem for assembly manufacturing processes. For instance, Caputo et al. (2017) developed a quantitative model to assess the probability of errors and the correction costs of errors in part feeding systems for assembly lines, in order to compare alternative part feeding policies and identify corrective measures. Genta et al. (2018) used some defect prediction models to obtain a priori predictions of the probability of occurrence of defects in order to design effective inspection strategies for short-run productions.

In the past few years, various studies in this field have focused on the close relationship between assembly complexity and human mistakes. Some prediction models of the operator-induced assembly defect rate have been proposed on this basis. Hinckley (Hinckley 1994; Hinckley and Barkan 1995) empirically found that defects per unit were positively correlated with the total assembly time and negatively correlated with the number of assembly operations. In later studies, Shibata (2003; 2002) detailed Hinckley’s model by subdividing the product assembly process into a series of manufacturing operations, called by Su et al. (2010) “workstations”, which were defined through operation standard sheets. A certain number of job elements (Aft 2000), i.e., elementary operations, were identified at each workstation. In addition, Shibata introduced a design-based assembly complexity factor because he had noted that the time related measures might not capture all the sources of defects (Shibata 2002). In line with Hinckley and Shibata’s research in the field of semiconductor products, Su et al. (2010) developed a new defect generation mathematical model to match the characteristics of copier assembly. Moreover, Antani (2014) successfully tested the hypothesis that manufacturing complexity could be considered to reliably predict product quality in mixed-model automotive assembly. The manufacturing complexity he proposed incorporated variables driven by design, process and human factors. Krugh et al. (2016a; 2016b) adapted the method developed by Antani for use with automotive electromechanical connections in a large complex system. Falck et al. (2017) proposed a method for the
predictive assessment of basic manual assembly complexity in which he developed a tool to predict and control operator-induced quality errors.

Most of the above mentioned defect prediction methods rely on a power law relationship between the average number of defects introduced during each assembly phase and the related assembly complexity (Shibata 2002; Su, Liu, and Whitney 2010; Hinckley 1994). Accordingly, many authors have proposed a logarithmic transformation of data to linearize the mathematical model. However, the fact that the defect rate in each assembly phase is usually extremely low can be extremely critical. In fact, when a logarithmic transformation is applied, a bias may occur, especially for very low values, and this can lead to completely erroneous predictions.

The problem of the bias introduced after a data transformation, which is a very common approach in predictive model building, has been formally addressed in the scientific literature by some authors such as Land (1974), Miller (1984), Taylor (1986), and Sakia (1988; 1990). Specifically, when a transformation is applied to data, and it is also invertible, the fitted value in transformed units may be evaluated using the inverse of the transformation. Despite retransformed model being in the appropriate units of measurement for interpretation, it is generally recognised that such retransformation can induce significant distortion of expected values (Land 1974; Miller 1984). More recently, a number of papers have addressed the issue of the retransformation bias by estimating prediction intervals in the original units of observation after fitting a linear model to an appropriately transformed response variable (Perry and Walker 2015; Perry 2018a; Perry 2018b).

The bias and the related consequences introduced by the logarithmic transformation applied to defect prediction models is discussed in the present paper, taking the case study proposed by Su et al. (2010) as an example. More in detail, Su et al. (2010), unaware of the error introduced by retransforming the logarithmic function in the power-law model for predicting assembly defects, stated that the prediction models previously developed in the literature were not suitable. Accordingly, they developed new models in the case of the assembly of copiers.

The present study, in addition to demonstrating that the models of generation of defects existing in the scientific literature, developed by Hinckley (Hinckley 1994) and Shibata (2002), are also applicable to the assembly of electromechanical components such as copiers, aims to contribute to the improvement of these prediction models for operator-induced assembly defects by analysing and comparing two alternative methods to overcome this drawback. The first approach is based on the application of a bias correction factor to the retransformed fitted value. The second method uses a power-law nonlinear regression model for obtaining reliable estimates of defects. In the case of a small set of data, often not repeated, and affected by great variability, as in the case of operator-induced assembly defects, it has been demonstrated that nonlinear regressions are preferable to linear models with the bias correction factor as more accurate in predicting defects.

The paper is arranged as follows. First, the most diffused defects prediction models are reviewed in Section 2. The bias occurring when predicting defects in assembly manufacturing after the linearization of the models is presented in Section 3. In Section 4, the consequences of the logarithmic transformation bias in models of
defect generation are discussed. In Section 5, two methods to correct the bias are introduced and compared. Finally, Section 5 summarises the main findings of the paper and reports the final conclusions.

2 Defect prediction models

The most diffused models used to predict operator-induced defects in assembly processes are reviewed in this section.

2.1 Hinckley’s Model

Hinckley, studying defects generated in semiconductor products, empirically found that the defects per unit (DPU) were positively correlated with the total assembly time and negatively correlated with the number of assembly operations (Hinckley 1994; Hinckley and Barkan 1995). He defined the assembly complexity factor (Cf) as:

\[ Cf = TAT - t_0 \cdot TOP \]  

where \( TAT \) is the total assembly time for the entire product, \( TOP \) is the total number of assembly operations, and \( t_0 \) is the threshold assembly time. The threshold assembly time was defined as the time required to perform the simplest assembly operation, which required a finite time for its execution. Hence, no defects are supposed to exist under the threshold assembly time. Hinckley found that the logarithms of the complexity and the corresponding defect rate showed a positive linear correlation, as can be observed in the following equation:

\[ \log DPU = K \cdot \log Cf - \log C \]  

where \( C \) and \( K \) are coefficients obtained from a linear regression analysis.

2.2 Shibata’s Model

Shibata (2002; 2003) applied Hinckley’s model to the case of the assembly of Sony’s home audio products, detailing it by subdividing the product assembly process into a series of workstations (Su, Liu, and Whitney 2010), defined through operation standard sheets. A certain number of job elements (Aft 2000), i.e., elementary operations, were identified in each workstation.

Shibata defined the process-based complexity factor of a generic workstation \( i \) as follows:

\[ Cf_{p,i} = \sum_{j=1}^{N_{a,i}} SST_{i,j} - t_0 \cdot N_{a,i} = TAT_i - t_0 \cdot N_{a,i} \]  

where \( N_{a,i} \) is the number of job elements in workstation \( i \), \( SST_{i,j} \) is the time spent on job element \( j \) in workstation \( i \), \( TAT_i \) is the total assembly time relevant to workstation \( i \), and \( t_0 \) is the threshold assembly time (Shibata 2002). It should be noted that the assembly times, \( SST_{i,j} \), are determined according to Sony Standard Time (SST), a time estimation tool commonly used for electronic products. SST is used to set the standard process time and estimate the required labour cost. Therefore, the Shibata prediction model is based on the standard times in which the operators should complete each job element, rather than the actual times of some specific operators.
Time standards allow accurate estimates to be obtained, but a good understanding of the system and significant practical experience are required (Aft 2000).

Correlation relationships between the process-based assembly complexity factor and DPU for each workstation \(i\) were derived as follows:

\[
\log DPU_i = K \cdot \log Cf_{p,i} - \log C
\]

\[
DPU_i = \frac{Cf_{p,i}^K}{C}
\]

where \(C\) and \(K\) are two regression coefficients obtained by applying linear regression to experimental data, according to the model in Eq. (2) (Shibata 2002).

In addition, Shibata remarked that the time related measures may not be able to capture all the sources of defects. For this reason, he also defined a design-based assembly complexity factor as follows:

\[
Cf_{b,i} = \frac{K_D}{D_i}
\]

where \(K_D\) is an arbitrary coefficient for calibration with process-based complexity; \(D_i\) refers to the ease of assembly (EOA) of workstation \(i\), which is evaluated by means of the design method for assembly/disassembly cost-effectiveness (DAC) developed by Sony Corporation (Yamagiwa 1988).

Shibata found that the correlation relationships between design-based complexity and \(DPU\) can be expressed as follows:

\[
\log DPU_i = b \cdot \log Cf_{D,i} + \log a
\]

\[
DPU_i = a \cdot Cf_{D,i}^b
\]

where \(a\) and \(b\) are again coefficients that may be obtained by means of linear regression applied to experimental data, according to the model in Eq. (7) (Shibata 2002).

At this point, by combining Eqs. (4) and (7), Shibata derived the following bivariate prediction model:

\[
\log DPU_i = k_1 \cdot \log Cf_{p,i} + k_2 \cdot \log Cf_{D,i} + \log k_3
\]

where \(k_1, k_2, k_3\) are again regression coefficients that may be obtained by linear regression (Shibata 2002).

It is worth noting that Eq. (9) can also be written as:

\[
DPU_i = k_3 \cdot Cf_{p,i}^{k_1} \cdot Cf_{D,i}^{k_2}
\]

where \(k_1, k_2, k_3\) may be obtained more correctly by means of a power-law nonlinear regression.

In fact, as will be demonstrated in Section 3, the logarithmic transformation applied to low values may introduce a significant bias into the linear regression, with resulting dramatic errors in the predictions (Osborne 2010).
2.3 Su’s Model

The aim of the work of Su et al. (2010) was to determine whether Shibata’s model was also suitable for the copier industry. Once they had collected all the data, they made a regression analysis using SPSS® 13.0 software and selecting the power regression option. Consequently, a prediction model was derived, according to Eq. (9). The authors reported that the $R^2$ value of the obtained bivariate model was only 0.257, instead of 0.7 (the value obtained in Shibata’s study for the case of audio equipment assemblies). This finding implied that Shibata’s model was not appropriate for copiers (Su, Liu, and Whitney 2010). However, as will be demonstrated in Section 3, this conclusion was the consequence of an improper and misleading use of the SPSS® 13.0 software.

According to the unsatisfactory result of the prediction obtained using Shibata’s model, Su et al. decided to redesign the assembly complexity factor evaluation methods to better satisfy the requirements of copiers (Su, Liu, and Whitney 2010). Specifically, a new process-based assembly complexity factor was formulated by considering Fuji Xerox Standard Time instead of Sony Standard Time and by integrating the time variation (Su, Liu, and Whitney 2010). Furthermore, the design-based complexity factor was redesigned by using the weights and the degree of difficulty associated to 11 design parameters chosen as criteria to evaluate the complexity of the design (Su, Liu, and Whitney 2010). As in the studies of Shibata (2002; 2003), Su et al. (2010) tested the correlation between each redesigned assembly complexity factor and the $DPU$, showing that the best regression function, in both cases, was a cubic polynomial model (Su, Liu, and Whitney 2010). In addition, the redesigned process- and design-based complexity factors were also integrated in a new bivariate prediction model, whose behaviour was confirmed to be again cubic (Su, Liu, and Whitney 2010). All the three new predictive models derived by Su et al. (2010) showed a significant increase in $R^2$ values compared to those obtained using the relative models proposed by Shibata, as shown in Table 1.

Table 1 – Comparison of the $R^2$ values of the regression models applied to copier assembly, using Shibata and Su approaches (Su, Liu, and Whitney 2010).

<table>
<thead>
<tr>
<th></th>
<th>Process-based complexity factor</th>
<th>Design-based complexity factor</th>
<th>Bivariate model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction using Shibata’s model</td>
<td>$R^2 = 0.169$</td>
<td>$R^2 = 0.153$</td>
<td>$R^2 = 0.257$</td>
</tr>
<tr>
<td>Prediction using Su’s model</td>
<td>$R^2 = 0.755$</td>
<td>$R^2 = 0.663$</td>
<td>$R^2 = 0.793$</td>
</tr>
</tbody>
</table>

3 The bias of logarithmic transformation in models of defect generation

Most of the assembly defect prediction models proposed in literature apply logarithmic transformation to data in order to linearize the relationship between the $DPU$s and the related assembly complexity factors (Hinckley
1994; Shibata 2002; Su, Liu, and Whitney 2010). However, the extremely low defect rate at each assembly step can result in an undesirable effect. In fact, when a logarithmic transformation is applied, a bias can occur, especially for very low values, and this can lead to erroneous predictions. In this section, in order to demonstrate the bias of the logarithmic transformation in defect prediction models for assembly processes, the case study proposed by Su et al. (2010) has been adopted as an example.

As previously pointed out, in the work of Su et al. (2010), the correlation between the process-based complexity factor and the design-based complexity factor with $DPU$ was analysed using the approach proposed by Shibata, both separately and in a bivariate model. The results of the regressions on copier data and the related $R^2$, as obtained by Su et al. using the SPSS® 13.0 software, with the power regression option proposed by the software (Su, Liu, and Whitney 2010) (SPSS Inc. 2004), are reported in Table 2.

**Table 2 – Prediction models obtained by Su et al. (2010) for copier assembly using the approach proposed by Shibata and selecting the power regression option in SPSS® 13.0.**

<table>
<thead>
<tr>
<th>n.</th>
<th>Model</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$DPU_i = 6.42 \cdot 10^{-5} Cf_{P,i}^{2.51}$</td>
<td>0.169</td>
</tr>
<tr>
<td>(2)</td>
<td>$DPU_i = 0.0785 Cf_{D,i}^{3.534}$</td>
<td>0.153</td>
</tr>
<tr>
<td>(3)</td>
<td>$\log(DPU_i) = 1.525 \cdot \log(Cf_{P,i}) + 1.935 \cdot \log(Cf_{D,i}) - 3.432$</td>
<td>0.257</td>
</tr>
</tbody>
</table>

**Figure 1** – Curve fitting of $DPU$ versus $Cf_P$ using logarithmic transformation and linear regression (see model (1) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).
Figure 2 – Curve fitting of DPU versus $C_{f_D}$ using logarithmic transformation and linear regression (see model (2) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).

Figure 3 – DPU residuals versus fitted values using logarithmic transformation and linear regression and considering $C_{f_P}$ as the predictor (see model (1) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).
Figure 4 – DPU residuals versus fitted values using logarithmic transformation and linear regression and considering CfD as the predictor (see model (2) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).

The models in Table 2 could appear correct to a hasty reader. However, as shown in Figure 1 and Figure 2, which illustrate the data and the fitting curves of model (1) and (2) in Table 2, the regression curves are shifted downwards, compared to the data dispositions, and do not seem to properly fit the data. Furthermore, the residual analysis of DPU versus the fitted values, considering CfP,i as the predictor (see Figure 3), and that of DPU versus the fitted values, considering CfD,i as the predictor (see Figure 4), using the logarithmic transformation and the linear regression with copier assembly data (Su, Liu, and Whitney 2010), show that the residual averages deviate from zero and several outliers appear. This is further confirmed by the very low values of $R^2$ (see Table 2). This bias must be due to the algorithm that is used by SPSS® 13.0 (2004) for the calculation of the regression curve. In this software, the algorithm used for the calculation of the power regression model is based on the following equation:

$$E(Y_t) = \beta_0 \cdot t^{\beta_1}$$

where $Y_t$ is the observed series ($t=1,2,\ldots,n$) and $E(Y_t)$ is the expected value of $Y_t$. Furthermore, it is assumed that this nonlinear model (as well as the Compound, S, Growth and Exponential models) may be expressed in linear form by applying a logarithmic transformation, as shown in Eq. (12) (SPSS Inc. 2004):

$$\log(Y_t) = \log(E(Y_t)) + \varepsilon_t$$

with $\varepsilon_t$ ($t=1,\ldots,n$) being independently identically distributed $N(0,\delta^2)$.

Eq. (12) may also be rewritten as:
\[ \log(Y_i) = \log(\beta_0) + \beta_1 \cdot \log(t) + \epsilon_i \]  

(13)

Once the power model has been expressed in linear form (see Eq. (13)), linear regression computational techniques can be applied.

It should be noted that this approach is also automatically implemented in other commercial software packages, such as, for example, Microsoft Excel (Liengme 2008).

As an example, Figure 5 shows the approach pursued by the software whereby \( \log(DPU) \) versus \( \log(C_f_P) \) (see model (1) in Table 2) is plotted.

By applying the linear regression model of Eq. (13) to the data in Figure 5, the following regression curve is obtained:

\[ \log DPU_i = 2.51 \cdot \log C_{f,P,j} - 9.65 \]  

(14)

Eq. (14) can be transformed into the following power-law model:

\[ DPU_i = 6.42 \cdot 10^{-5} C_{f,P,j}^{2.51} \]  

(15)

which corresponds to model (1) anticipated in Table 2.

The same approach can be applied to the other two models, that is, models (2) and (3) in Table 2.

At this point, a question arises: why do the fitting curves obtained by applying the approach proposed by Shibata (i.e., with logarithmic transformation) to copier assembly not seem to fit the data properly? By analysing the data in Figure 1 and Figure 5 carefully, it can be noticed that the points that refer to the workstations where the DPU's are close to zero (see Figure 1) result to be more spaced-out downwards as one moves along the log-log scale (see Figure 5). On the other hand, the points corresponding to the workstations

**Figure 5 – Curve fitting for \( \log(DPU) \) versus \( \log(C_f_P) \).**
with high DPU values become less spaced-out. The effect of the logarithmic transformation of the data is in fact that of both a downwards expansion of the positions of the workstations with low DPU values and of a compression of those of the workstations with high DPU values. Consequently, the regression curve is heavily biased towards low values (i.e., the lower part of the graphs in Figure 5 and, consequently, of that in Figure 1). In fact, the logarithmic transformation has a significant bias effect when applied to numbers between 0 and 1, as in the case of DPU.

The problem caused by data transformations has been extensively studied in the literature (Land 1974; Miller 1984; Perry 2018a; Perry 2018b; Perry and Walker 2015; Sakia 1990; Sakia 1988). Formally speaking, the bias in the retransformed model can be explained using Jensen's Inequality. More specifically, Jensen's Inequality states that for a random variable Z:

\[ E[h(Z)] \geq h(E[Z]) \]  

where \( h(\cdot) \) is a convex function (if \( h(Z) \) is concave, then the inequality is reversed) (Perry 2018b). A simple example involves the square-root transformation from the original units \( Y \) to the transformed units \( Z: \sqrt{Y} = Z \sim D(\mu, \sigma^2) \), where \( D \) is a distribution (often assumed normal). Thus, \( Y = Z^2 = h(Z) \) denotes its inverse. Then:

\[ E[Y] = E[h(Z)] = E[Z^2] = \mu^2 + \sigma^2 > h(E[Z]) = h(\mu) = \mu^2. \]  

Thus, the retransformation bias for a square-root transformation is \( E[Y] - h(E[Z]) = \sigma^2 \), or the variance of the transformed response, i.e., \( Var(Z) \).

In order to correct this bias, several authors have proposed reduced-bias point estimators (Taylor 1986; Sakia 1990; Sakia 1988; Perry 2018a; Perry 2018b; Perry and Walker 2015).

4 Consequences of logarithmic transformation bias on the prediction of assembly defects

The obtained results suggest that using the proposed nonlinear models leads to a more accurate defect prevision, compared with models which apply a logarithmic transformation to data, as demonstrated in the case of copier assembly (Su, Liu, and Whitney 2010). One of the main consequences of the bias resulting from the linearization is that erroneous conclusions may be drawn. For instance, Su et al. (2010), unaware of the logarithmic transformation problem, suggested that the method proposed by Shibata (2002) was not suitable for their case study concerning assembly defect prediction. Accordingly, they redesigned new evaluation methods of the assembly complexity factors and, as a result, they developed a new prediction model (see Section 2). However, the predictions obtained applying the nonlinear regressions in the form proposed by Shibata (2002), as reported in Table 3, reveal that Shibata’s model was also suitable for copier products and, more in general, for electro-mechanical products. For these reasons, the new prediction models proposed by Su et al. (2010) (see Eqs. (14), (15) and (16)) are not necessary and, additionally, as a result of their polynomial structure, fit perfectly from a mathematical point of view, but are difficult to interpret from the physical problem perspective.

5 Alternative approaches to correct the logarithmic bias
5.1 Bias correction factor

According to previous studies it can be demonstrated that, in order to correct the bias introduced when retransforming the linearized function in the power-law form, a bias correction factor can be easily applied to the retransformed fitted value (Perry 2018a; Perry 2018b; Perry and Walker 2015). Specifically, the natural logarithmic function which allows to transform the original units $DPU$ into the transformed units $Z$ can be formalized as: $\log DPU = Z \sim N(\mu, \sigma^2)$. Then, the inverse transformation is: $DPU = e^Z = h(Z)$. Therefore, the exact closed-form expression for $E[DPU]$ in this case is easily shown to be:

$$E[DPU] = E[h(Z)] = E[e^Z] = e^{(\mu + \sigma^2/2)} = e^\mu \cdot e^{\sigma^2/2} > h(E[Z]) = h(\mu) = e^\mu$$

(17)

Thus, the retransformation bias for a natural logarithmic transformation is:

$$E[DPU] - h(E[Z]) = e^\mu \cdot (e^{\sigma^2/2} - 1)$$

(18)

Accordingly, the reduced-bias fitted values of the untransformed response would be computed as: $e^{\hat{\mu}} \cdot e^{\hat{\sigma}^2/2}$, where $\hat{\mu}$ is the estimator for the mean of the transformed variable $\log DPU$ and $\hat{\sigma}$ is the estimator for the standard deviation of $\log DPU$ in homoschedastic conditions.

The multiplicative bias correction factor, $e^{\sigma^2/2}$, derived from Eq. (18), was applied to the set of data concerning the copier assembly (Su, Liu, and Whitney 2010). The fitted values obtained from the regressions specified in Table 2 have been corrected and the resulting curve fittings (see Figures 6 and 8) and residual plots (see Figures 7 and 9) are reported.

![Figure 6](image)

**Figure 6** – Experimental data of copier assembly (Su, Liu, and Whitney 2010) and curve fitting of the reduced-bias fitted values of $DPU$ versus $Cf_p$ (obtained from model (1) in Table 2).
Figure 7 – \textit{DPU} residuals versus reduced-bias fitted values considering $C_f$ as the predictor (obtained from model (1) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).

Figure 8 – Experimental data of copier assembly (Su, Liu, and Whitney 2010) and curve fitting of the reduced-bias fitted values of \textit{DPU} versus $C_f$ (obtained from model (2) in Table 2).
Figure 9 – DPU residuals versus reduced-bias fitted values considering CfP as the predictor (obtained from model (2) in Table 2) with copier assembly data (Su, Liu, and Whitney 2010).

As can be seen from Figures 6 and 8, the curves obtained by applying the bias correction factor fit very well with the experimental data up to a certain threshold (up to 2 for CfP and before the value of 0.25 for CfD). As a result, residuals above these thresholds are higher than those obtained for lower complexity values (see Figures 7 and 9). The reasons for this upward shift of the curve from a certain threshold onwards must be sought in the actual structure of the experimental data. In fact, in the work of Su et al. (2010), the data set is small (less than thirty experimental data), and there are also few replicated data, especially for the higher values of complexity. Moreover, the data are characterized by an intrinsic internal variability. The combination of these factors justifies the above-mentioned trend of the curves.

The same approach was applied to the bivariate model, i.e., the model which include both the process and the design-based complexity factors (see model (3) of Table 2), and the related results will be discussed in the Section 5.3.

5.2 Power-law nonlinear regression model

In order to overcome the bias introduced by the logarithmic transformation, an alternative approach based on a nonlinear regression model is here proposed. In other words, the bias problem can be overcome by applying power-law regression models to a set of data with low DPU values, such as in the case of a copier assembly (Su, Liu, and Whitney 2010). To this aim, the three models in Table 2 have been redefined and, using the Minitab® software, the corresponding power regression models become:

\[
DPU_j = k_1 \cdot Cf_{P,j}^{k_2}
\]
\[ DPU_i = k_3 \cdot Cf_{D,i}^{k_4} \] (20)

\[ DPU_i = k_5 \cdot Cf_{P,i}^{k_6} \cdot Cf_{D,i}^{k_7} \] (21)

where \( k_3, k_2, k_3, k_4, k_5, k_6, k_7 \) are regression coefficients obtained from power-law nonlinear regression models. The method used in Minitab\textsuperscript{®} to determine the least squares estimation is the Gauss-Newton method. This method uses a linear approximation for the expectation function in order to iteratively improve an initial guess \( \theta^0 \) for \( \theta \), and the method then keeps improving the estimates until the relative offset falls below a prescribed threshold (Bates and Watts 1988).

First, according to the Gauss-Newton method, the new regression model for \( DPU \) versus \( Cf_p \) (using the data in Figure 1) is obtained, as shown in Figure 10. Similarly, the data in Figure 2 are re-analysed and the new curve fitting for \( DPU \) versus \( Cf_D \) is reported in Figure 11. In Figure 10 and 7, the 95\% confidence and prediction interval bands are represented around the fitted curve.

**Figure 10** – Curve fitting for \( DPU \) versus \( Cf_p \) using power-law nonlinear regression with data produced by Su et al. (2010) (see Eq. (19)).
Figure 11 – Curve fitting for DPU versus $Cf_D$ using power-law nonlinear regression with data produced by Su et al. (2010) (see Eq. (20)).

Figure 12 – Residuals of DPU versus fitted values using power-law nonlinear regression, considering $Cf_D$ as a predictor and using data produced by Su et al. (2010) (see Eq. (19)).
Figure 13 – Residuals of $DPU$ versus fitted values using power-law nonlinear regression, considering $Cf_0$ as a predictor and using data produced by Su et al. (2010) (see Eq. (20)).

As far as the bivariate model is concerned, the regression coefficients were estimated using the Minitab® software, and the related 3D surface plot is reported is illustrated in Figure 14. Furthermore, in Figure 15 the residuals of the bivariate nonlinear regression model can be visualized.

Figure 14 – 3D surface plot of $DPU$ against $Cf_P$ and $Cf_D$ (experimental points of the workstations pertaining to the copier assembly process (see also Su et al. (2010)) and the theoretical model (see Eq. (21)).
5.3 Comparison between methods

In order to compare the two different methods, i.e., the linear regression with a bias correction factor and the power-law nonlinear regression, with respect to the simple linear regression, all the regression curves are reported in Table 3 in the original units. Models (1a), (2a) and (3a) refers to the re-transformed model after having performed the linear regression, models (1b), (2b) and (3b) refers to the re-transformed model after having performed the linear regression with a bias correction factor. Finally, models (1c), (2c) and (3c) refers to the nonlinear regression model.

At this point, a consideration must be expressed on goodness-of-fit statistic to assess the adequacy of the models. When dealing with nonlinear regression, using the $R^2$ as a goodness-of-fit statistic is not recommended (Spiess and Neumeyer 2010; Kvålseth 1983; Bates and Watts 1988). In fact, $R^2$ is based on the underlying assumption that the regression is linear. Instead, in nonlinear regression, the addition of the residual sum of squares and the regression sum of squares is not equal to the total sum of squares (Draper and Smith 1998; Bates and Watts 1988; Devore 2011). As a result, $R^2$ for nonlinear models may not fall between 0 and 100%. This phenomenon was demonstrated by Spiess and Neumeyer (2010). They performed thousands of simulations and confirmed that using $R^2$ to evaluate the fit of nonlinear models leads to incorrect conclusions (Spiess and Neumeyer 2010). Kvålseth proved that, for the case of nonlinear models, such as power models and the exponential models frequently used in behavioural sciences, the $R^2$ measure is often subject to incorrect calculations and misinterpretations, thus producing potentially misleading results (Kvålseth 1985; Kvålseth 1983).

In order to overcome this misleading use of $R^2$, the authors suggest using the $S$ value as a goodness-of-fit statistic. The $S$ value, known both as the standard error of the regression and as the standard error of the
estimate, represents the average distance that the observed values fall from the regression line, according to Eq. (22):

\[ S = \sqrt{\frac{RSS}{N - P}} \]  

where \( RSS \) is the sum of squared residuals (according to Eq. (23)), \( N \) is the number of observations and \( P \) is the number of free parameters.

\[ RSS = \sum_{i=1}^{N} (e_i)^2 = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \]  

where \( i \) falls between 1 and \( N \), i.e., the total number of observations; \( e_i \) is the residual of an observation; \( y_i \) is the \( i \)-th observed response variable and \( \hat{y}_i \) is the \( i \)-th fitted response.

Therefore, in order to evaluate the goodness of fit of the nonlinear regression models, smaller values of \( S \) are desirable because in this way the observations would fall closer to the fitted line. In addition, the regression residuals should be randomly distributed, because in this way the nonlinear model would not present a systematic bias (Devore 2011).

**Table 3** – Comparison between predictions, \( S \) values (to be multiplied by \( 10^{-4} \)) and \( D \) values (to be multiplied by \( 10^{-6} \)) obtained by applying (a) the linear regression after the logarithmic transformation of data, (b) the linear regression after the logarithmic transformation of data with the bias correction factor, (c) power-law nonlinear regression, (c*) generalized Poisson power-law nonlinear regression to the data produced by Su et al. (2010).

<table>
<thead>
<tr>
<th>n.</th>
<th>Predictor</th>
<th>Regression curve</th>
<th>( S ) value (( \times 10^{-4} ))</th>
<th>( D ) value (( \times 10^{-6} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a)</td>
<td>( CF_P )</td>
<td>( DPU_i = 6.42 \cdot 10^{-5} CF_P )</td>
<td>6.46</td>
<td>9.18</td>
</tr>
<tr>
<td>(1b)</td>
<td>( CF_P )</td>
<td>( DPU_i = 1.53 \cdot 10^{-4} CF_P )</td>
<td>9.49</td>
<td>19.80</td>
</tr>
<tr>
<td>(1c)</td>
<td>( CF_P )</td>
<td>( DPU_i = 1.88 \cdot 10^{-4} CF_P )</td>
<td>5.43</td>
<td>6.49</td>
</tr>
<tr>
<td>(2a)</td>
<td>( CF_D )</td>
<td>( DPU_i = 8.66 \cdot 10^{-3} CF_D )</td>
<td>8.02</td>
<td>13.51</td>
</tr>
<tr>
<td>(2b)</td>
<td>( CF_D )</td>
<td>( DPU_i = 1.97 \cdot 10^{-1} CF_D )</td>
<td>15.80</td>
<td>52.15</td>
</tr>
<tr>
<td>(2c)</td>
<td>( CF_D )</td>
<td>( DPU_i = 1.24 \cdot 10^{-2} CF_D )</td>
<td>6.49</td>
<td>8.83</td>
</tr>
<tr>
<td>(3a)</td>
<td>( CF_P ) and ( CF_D )</td>
<td>( DPU_i = 2.74 \cdot 10^{-6} \cdot CF_{P,j} \cdot CF_{D,j} )</td>
<td>11.10</td>
<td>25.89</td>
</tr>
<tr>
<td>(3b)</td>
<td>( CF_P ) and ( CF_D )</td>
<td>( DPU_i = 1.44 \cdot 10^{-3} \cdot CF_{P,j} \cdot CF_{D,j} )</td>
<td>18.80</td>
<td>74.25</td>
</tr>
<tr>
<td>(3c)</td>
<td>( CF_P ) and ( CF_D )</td>
<td>( DPU_i = 2.86 \cdot 10^{-5} \cdot CF_{P,j} \cdot CF_{D,j} )</td>
<td>3.36</td>
<td>2.38</td>
</tr>
<tr>
<td>(3c*)</td>
<td>( CF_P ) and ( CF_D )</td>
<td>( DPU_i = 7.68 \cdot 10^{-5} \cdot CF_{P,j} \cdot CF_{D,j} )</td>
<td>-</td>
<td>2.47</td>
</tr>
</tbody>
</table>

As can be seen from Table 3, in which the \( S \) value of each model, calculated according to Eq. (22), is reported, the nonlinear regression models have a lower \( S \) value than the two linear regression models performed after
the logarithmic transformation of data (see $S$ value of models (1c), (2c) and (3c) compared to others. This result shows that, on average, the nonlinear regression models leads to smaller residuals with respect to other models. More specifically, plotting on the same graph the models (1a), (1b) and (1c) (see Figure 16) and models (2a), (2b) and (2c) (see Figure 17), it is evident that nonlinear regression models fit better with the experimental data. In fact, although the models in which the bias correction factor is introduced (see models (1c), (2c) and (3c)) are moved upwards compared to the respective models without correction (see models (1a), (2a) and (3a)) and improve residual plots as the residual averages no longer deviate from zero (compare Figures 3 and 4 with Figures 7 and 9), they present some peculiarities. Focusing on Figure 16, it is noticeable that up to the value 1.7 of $Cf_P$ the two alternative models used to correct the bias introduced by the linearization, i.e., model (1b) and (1c) exactly match. Above this value, model (1c) moves upwards. Similarly, in Figure 17, up to the value 0.25 of $Cf_D$, the models (2b) and (2c) are very similar. However, from this threshold onwards the model (2c) deviates from the other. As anticipated in Section 5.1, the upward shift of the linear models with bias correction factors (models (1c), (2c) and (3c)) is to be attributed to the small dataset, to the lack of replicated data, especially for higher values of complexity and the intrinsic internal variability of data. Accordingly, the nonlinear regression models are to be preferred when having data similar to those used in the work of Su et al. (2010).

![Figure 16](image_url)  
**Figure 16** – Comparison of curve fitting for $DPU$ versus $Cf_P$ using linear regression (model (1a) in Table 3), linear regression with bias correction factor (model (1b) in Table 3), and power-law nonlinear regression (see model (1c) in Table 3) with data produced by Su et al. (2010).
Figure 17 – Comparison of curve fitting for $DPU$ versus $C_{f_D}$ using linear regression (model (2a) in Table 3), linear regression with bias correction factor (model (2b) in Table 3), and power-law nonlinear regression (see model (2c) in Table 3) with data produced by Su et al. (2010).

Finally, analysing in detail the best suitable models for this dataset, i.e., the non-linear models, and their residual plots (see Figures 12, 13 and 15), it is evident that heteroscedasticity occurs. However, heteroscedasticity is physiological for counts, as in this case for $DPU$s, as variability increases with the growth of $DPU$s. It is worth remarking that the introduction of both the design-based and the product-based complexity factor in the bivariate prediction model (model (3c) in Table 3) contributes to reducing the heteroscedasticity with respect to the models with single predictors (models (3a) and (3b) in Table 3). Indeed, Figure 15 reveals that the maximum residual values are significantly reduced in comparison to those of models (3a) and (3b), illustrated in Figures 12 and 13.

In such a case, namely when the least squares assumption of constant variance in the residuals is violated due to the use of count data in the response, a generalized nonlinear regression approach can be performed using a response distribution that has the characteristics of the variance as a function of the mean, a property commonly associated with count data (Seber and Wild 1989). Among all possible response distributions, the most appropriate for this case study are the Poisson and the Negative Binomial distributions or the Gamma distribution as a continuous response distribution. Consequently, the generalized power-law nonlinear regressions were performed, and the estimates of the model parameters obtained using the three different response distributions were not significantly different from each other. In the model (3c*) in Table 3, by way of example, the parameters of the bivariate model estimated using the generalized Poisson nonlinear regression are reported. In addition, the residuals of the performed generalized Poisson nonlinear regression are shown in Figure 18. Since the generalized nonlinear regression is a refinement of the classical regression, the authors decided to implement it exclusively for the bivariate model, which is the complete model to be used for
predicting defects that presents the lowest heteroscedasticity, as mentioned above. However, it should be noted that the generalized nonlinear regression could also have been used for estimating the parameters of the models (3a) and (3b) in Table 3. When residuals do not follow a Normal distribution, as in the case of the generalized nonlinear regression model, the use of a standard residuals-based measurement, such as $S$ value, may be inappropriate (McCullagh and Nelder 1989). For that reason, in Table 3, the deviance value $D$ of each model is reported in order to compare the model $(3c^*)$ with the others. Indeed, the deviance is a goodness-of-fit statistic that generalizes the sum of squares of residuals in ordinary least squares to cases where model-fitting is achieved by maximum likelihood, as for generalized linear models (McCullagh and Nelder 1989). In the case of normal distribution of residuals, the deviance is calculated using Eq. (23). For the generalized Poisson power-law nonlinear regression of model $(3c^*)$, $D$ value is calculated according to Eq. (24):

$$D = \sum_{i=1}^{N} 2 \cdot (y_i \cdot \log \frac{y_i}{\hat{y}_i} - y_i + \hat{y}_i)$$

As shown in Table 3, the nonlinear bivariate models $(3c)$ and $(3c^*)$ have very close deviance values, which are lower than those of all the other models.

![Residuals of $DPU$ versus fitted values using generalized Poisson power-law nonlinear regression, considering $Cf_P$ and $Cf_D$ as predictors with data produced by Su et al. (2010) (see model $(3c^*)$ in Table 3).]

**Figure 18** – Residuals of $DPU$ versus fitted values using generalized Poisson power-law nonlinear regression, considering $Cf_P$ and $Cf_D$ as predictors with data produced by Su et al. (2010) (see model $(3c^*)$ in Table 3).

6 Conclusion

Defect prevention and elimination are increasingly being adopted in the manufacturing field, since defects can affect the final quality and cost of products to a great extent. A lower defect rate is required above all in the assembly process, where the continuously shortening product life cycles require a faster response speed as well
as a high level of product quality. In this situation, assembly quality control is becoming one of the most demanding problems in the modern manufacturing environment. Specific studies concerning the causes of assembly defects have shown that operator errors account for high percentage of the total defects. For instance, Shibata (2002) proposed a model to predict defects in the semiconductor product field, and Su et al. (2010), on the basis of the former model, focused on the manufacturing field of copier assembly processes. These models are based on the relationship between the average number of defects introduced during each assembly phase and the related assembly complexity factors, which follows power law relationship. For this reason, many authors have proposed a logarithmic transformation of data in order to linearize the relationship model. However, the most critical aspect of such an approach is that the defect rates are often very low. Therefore, if a logarithmic transformation is applied, a bias may occur, especially for very low values, and this in turn can lead to dramatically erroneous predictions.

The bias of the logarithmic transformation that may distort defect predictions has been analysed and discussed in this paper. This study has shown that the bias can lead to dramatically wrong conclusions, as in the case of the work of Su et al. (2010). In fact, these authors were led to erroneously affirm that Shibata’s model was not suitable for electromechanical products. In order to overcome this bias, two alternative methods are analysed and compared: the use of a bias correction factor to correct the fitted values of the linear regression, performed after the logarithmic transformation of data, and a (generalized) power-law nonlinear regression model for obtaining reliable estimates of defects. The use of nonlinear models has proved to be more accurate in predicting defects in the case of a few data, often not repeated, and affected by high variability, as in the case of copier assembly processes (Su, Liu, and Whitney 2010). Accordingly, the implementation of these models can improve defect previsions to a great extent and confirm the validity of the approach proposed by previous authors, such as Shibata (2002). Although applying a bias correction factor may be easier and immediate for practitioners, performing nonlinear regression models, which are more complicated from a computational point of view, has become straightforward thanks to the automatic implementation in commonly used software, such as Minitab®, extensively used also in the business environment.

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