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Gianluca D'Antonio, Giulia Bruno, Emiliano Traini, Franco Lombardi

Politecnico di Torino, Torino, Italy (e-mail: {gianluca.dantonio, giulia.bruno, emiliano.traini, franco.lombardi}@polito.it).

Abstract: Automation systems to support warehouse activities are a topic largely debated. Critical decisions on the technology to be adopted and the basic system attributes must be taken in the design phase; to support designers, several methodologies have been defined to evaluate performance indicators such as throughput, cycle time and equipment utilization. Conversely, smaller attention has been paid so far in the estimation of the energy necessary to perform the scheduled activities. Nonetheless, this topic is gaining popularity as the impact of renewable energy sources is increasing and the adoption of complex energy grids is gaining popularity. This paper is aimed at proposing an analytical model to evaluate the energy consumption of Autonomous Vehicle Storage and Retrieval Systems (AVS/RS), which exhibit higher flexibility and lower energy consumption compared to the traditional AS/RS. The proposed model is able to estimate the average energy consumption by accounting for realistic criteria adopted to store and retrieve unit loads. The proposed approach is validated by comparing the analytical estimation to the output of a simulation model.

Keywords: Warehouse automation; Autonomous vehicle storage and retrieval system; Analytical and numerical modelling; Energy consumption modelling; Performance analysis

1. INTRODUCTION

The harsh global competition has pulled an increased consideration for the efficiency of logistics. Warehouse operations are usually labour intensive and require large spaces; thus manufacturers increasingly demand automation systems for storing and retrieving goods with high levels of flexibility, high reconfigurability and, possibly, low energy consumption (Azadeh et al., 2017). The overall market of automation systems for warehouses is estimated to be greater than USD 9 Billion by 2023, with a CAGR close to 7.4% between 2017 and 2023 (Markets & Markets, 2017). This result is significantly impacted by the increasing success of e-commerce organizations, which business is based on storing millions of unique products as well as on managing a highly variable demand.

Autonomous Vehicle Storage and Retrieval Systems (AVS/RS) are an automation solution for warehouses that is gaining popularity. They are based on racks with transversal aisles supported by a set of different, autonomous vehicles integrated with each other: shuttles travel through the aisles and change the operating level by means of a lifting table. In deep-lane applications, the shuttle is also equipped with a satellite capable to enter rack channels and store or retrieve unit loads (ULs). AVS/RS enable better space exploitation, increased storage capacity, higher flexibility, lower energy consumption, compared to the traditional Automated Storage and Retrieval Systems (AS/RS). Nonetheless, AVS/RS exhibit a higher design complexity: in order to fully exploit all the potentialities of such systems, appropriate methods are necessary to evaluate the expected performances and support designers and users in taking appropriate decisions.

To date, the efforts of academic literature and standardization organizations have been mainly devoted to performance indicators such as cycle time, system throughput, or equipment utilization; fewer attention has been paid to the energy consumption. Further, the available research mainly focus on systems supporting single- or double-depth racks, with limited space exploitation. Based on these limitations, the present paper aims to extend the state of the art by presenting a novel mathematical model capable to assess the expected energy consumption of a deep-lane AVS/RS in a variety of realistic deployment scenarios.

The remainder of this paper is organised as follows. Recent literature concerning AVS/RS and the operating principles of such systems are discussed in Section 2. The analytical model to estimate energy consumption is defined in Section 3. The experimental plan for model validation is presented in Section 4; the results are discussed in Section 5. Finally, conclusive remarks and outlines for future work are presented in Section 6.

2. BACKGROUND

2.1 Literature review

The first scientific paper on AVS/RS has been authored by Malmborg (2002), who developed an analytical model to evaluate cycle time, throughput and vehicles utilization of a tier-to-tier AVS/RS. The term "cycle" denotes that vehicles movement starts and finishes at the same location; "tier-to-tier" denotes systems where a vehicle changes the operating level through a lifting table, and is opposed to "tier-captive" configurations where each vehicle is assigned to a determined rack tier. The work has been extended (2003) to estimate the best trade-off between single command (SC) and dual command (DC) cycles in which one and two ULs are involved, respectively.

Since then, AVS/RS performance has been evaluated by means of the queuing theory to include also the waiting times (Kuo et al., 2007) and to investigate the effect of properly pairing S/R tasks (Fukunari and Malmborg, 2008). Later, numerical improvements enabled to reduce the computational time (Fukunari and Malmborg, 2009) and approximation techniques have been adopted to deal with non-Poissonian processes. The queueing approach has also been used by Roy et al. (2012) to model the lift and the vehicles as independent queues interacting with each other. A standardization effort has been recently made: the standard FEM 9.860 (2017) provides guidelines to evaluate the travel time of an AVS/RS supporting single- or double-depth racks. The capability to travel along deep-lane racks and improve space efficiency has been captured by Manzini et al. (2016), who provided a model to evaluate duration of SC and DC cycles.

All the mentioned works consider random, uniform S/R policies. D'Antonio et al. (2018) first enriched analytical cycle time evaluation with the capability to consider different policies in deep-lane racks for multi command (MC) cycles involving an arbitrary number of ULs.

Further work has been done in the field of the Shuttle-Based Storage and Retrieval Systems (SBS/RS), which perform tasks similar to AVS/RS. Lerher et al. defined an analytical model to evaluate the average duration of SC (Lerher et al., 2015) and DC cycles (Lerher, 2016) for a single-depth rack. Simulation approaches enabled to evaluate the performance of a tier-captive SBS/RS with multiple elevators in SC cycles and support designers in the early-stage design by self-generating and evaluating different alternative system compositions (Ning et al., 2016).

The evaluation of AVS/RS energy consumption has emerged as a research topic more recently. Tappia et al. (2015) assessed energy consumption and environmental impact of both AS/RS and AVS/RS, finding that the latter technology has a better performance due to its greater energy efficiency per cycle. Akpunar et al. (2017) defined an analytical model to evaluate the engine power necessary to move vehicles and transport ULs and, in turn, the required energy. The model has been applied to tier-captive, single-depth AVS/RS to assess the energy consumption of SC cycles in a variety of rack topologies. The work has been extended (Ekren et al., 2018) to estimate average values and cycle time for both travel time and energy consumption of a SBS/RS performing SC cycles. Bruno et al. (2016) adopted a simulation approach to compare the energy performance of an AVS/RS with a traditional crane-based system.

Research in the estimation of energy consumption is still in its early stages. The research gap that this paper aims to fill is the definition of an analytical model capable to estimate the average energy consumption for an AVS/RS in realistic deployment scenarios. In particular, the approach presented in the following is able to: (i) consider the criteria adopted for ULs storage and retrieval; (ii) model MC cycles. These two capabilities are crucial to fully exploit AVS/RS flexibility: such systems may be equipped with a lift capable of simultaneously transporting more than a single UL, and this feature is not exploited in SC and DC cycles.

2.2 Autonomous Vehicle Storage and Retrieval Systems

As introduced, an AVS/RS relies on a rack made of an arbitrary number of tiers; each of them is equipped with a transversal aisle that provides access to the channels with arbitrary depth. The involved vehicles can be classified as follows: (i) a lift operating on the vertical direction that provides access to the tiers; (ii) a shuttle travelling through the aisles; (iii) a satellite storing and retrieving ULs within the channels. The shuttle can change the operating tier through the lift. In turn, the satellite is transported by the shuttle in front of the channel where storage or retrieval actions need to be made. The whole system interacts with the external world by means of a bay where ULs to be stored wait to enter the rack according to a First In First OUT (FIFO) policy (see Fig. 1). Elevators capable to simultaneously host more than a single UL can be adopted: therefore, in case multiple storages have to be made quickly, the additional capacity of the lifting table can be saturated to reduce the travels to and from the bay. Further, if storage tasks have to be made on the same tier, simultaneous autonomous movements can be planned for the shuttle and the satellite. Normally, the shuttle is idle on the aisle while the satellite is storing an UL in the channel. This waiting time can be better exploited by requiring the shuttle to go to the lift, load the next UL and move back towards the channel where

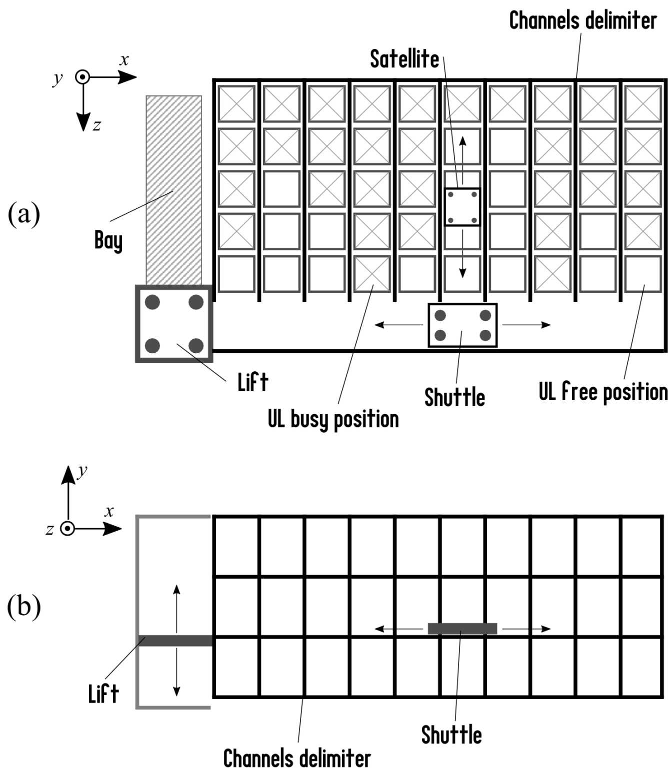

Fig. 1. Schematic representation of a rack and an AVS/RS system: top (a) and front (b) views.

the satellite has been left. In this way, different storage or retrieval tasks can be performed at the same time to increase the overall throughput, increase system flexibility and minimize energy consumption. This capability can be symmetrically deployed for retrieval tasks.

3. ANALYTICAL MODEL FOR ENERGY EVALUATION

The model presented in this section enables to evaluate the average energy need of an AVS/RS made of one lift, one shuttle and one satellite. Although it is a basic configuration, the capabilities mentioned in the previous Section (simultaneous shuttle-satellite activities, extended lift capacity) enable to increase system flexibility and obtain good performances even with a limited economic investment. The approach adopted for system modeling introduced in (D'Antonio et al., 2018) is enriched with a methodology for energy estimation inspired to (Ekren et al., 2018). In the following, the variables and a synthesis of the assumptions are presented; then, the model is presented.

Rack topology. Without loss of generality, the rack is supposed to be symmetrical: all the aisles provide access to the same number of channels, and each channel has the same number of storage positions. Therefore, the following parameters are defined: N_x is the number of channels in each aisle; N_y is the number of levels; N_z is the number of storage positions in each channel.

Vehicles performance. Vehicles acceleration (a_x, a_y, a_z) and maximum speed (v_x, v_y, v_z) are to be provided in input. The subscript denotes the axis assigned to each vehicle. Accelerations and decelerations are supposed to be equal.

These parameters enable to evaluate the duration of the acceleration and the distance travelled in this time span by each vehicle. As an example, along the x axis the acceleration lasts T_x and the distance d_x is travelled:

$$T_x = \frac{v_x}{a_x}; \qquad d_x = \frac{a_x T_x^2}{2} \tag{1}$$

The travel time T_W in which energy is required has different expressions depending on whether the distance x_i to be traveled is lower or greater than d_x. In the former case, energy is required only during the acceleration. Conversely, if $x_i \geq d_x$, energy is also required during the steady-state speed travel.

$$T_W(x_i) = \begin{cases} \sqrt{\frac{x_i}{a_x}}, & x_i < d_x \\ \frac{v_x}{a_x} + \frac{x_i - 2d_x}{v_x} & x_i \geq d_x \end{cases} \tag{2}$$

Cycle parameters. The cycle to be evaluated is described by the following parameters: I and O are, respectively, the number of ULs to be stored and retrieved in the cycle; $U = I + O$ is the overall number of involved ULs; S is the number of switches from a storage to a retrieval task.

ULs positioning. The S/R criteria are defined in terms of discrete probability distributions:

- $\mathbf{a} = \{a_i\} = \{\mathbb{P}(x = x_i)\}, i = 1, \ldots, N_x$, describes the probability to store or retrieve an UL for each of the positions available along the x-axis, i.e. along the different channels;
- $\mathbf{b} = \{b_j\} = \{\mathbb{P}(y = y_j)\}, j = 1, \ldots, N_y$, describes the S/R probability for each of the positions available along the y-axis, i.e. the different tiers;
- $\mathbf{c} = \{c_k\} = \{\mathbb{P}(z = z_k)\}, k = 1, \ldots, N_z$, describes the S/R probability for each of the positions available along the z-axis, i.e. within a channel.

3.1 Vehicles energy consumption

Due to the different operating trajectories, in the following we introduce the model for the shuttle and the satellite separately from the model for the lift. As the mass above the vehicles may change according to the operating conditions, we first evaluate the energy necessary to move a unit mass.

Shuttle and satellite model. For any movement performed by the shuttle, the overall energy spent is given by three contributions: acceleration, deceleration and steady state speed.

During the acceleration, the required unit mass power is given by:

$$p_{A,x} = (a_x f_r + g c_r) \frac{v_{top,x}}{\eta}, \tag{3}$$

where f_r is the resistance factor for masses with variable speed, g is the gravity acceleration, c_r is the friction coefficient, η is the efficiency of the power transmission system, and $v_{top,x}$ is the speed achieved at the end of the acceleration.

Similarly, the unit mass power required during the braking phase is equal to:

$$p_{B,x} = (a_x f_r - g c_r) \frac{v_{top,x}}{\eta}, \tag{4}$$

Last, the unit mass power required during the constant speed travel is:

$$p_{C,x} = g c_r \frac{v_x}{\eta}. \tag{5}$$

These results enable to evaluate the energy required to move a unit of mass for a distance equal to x_i:

$$w(x_i) = \begin{cases} p_{A,x} \frac{T_W(x_i)}{2} + p_{B,x} \frac{T_W(x_i)}{2}, & x_i < d_x \\ p_{A,x} T_x + p_{B,x} T_x + p_{C,x} (t(x_i) - 2T_x), & x_i \geq d_x \end{cases} \tag{6}$$

Although on a different axis, the satellite moves on the horizontal plan, too; therefore, the same reasoning can be used to evaluate the power required to travel a distance z_k:

$$w(z_k) = \begin{cases} p_{A,z} \frac{T_W(z_k)}{2} + p_{B,z} \frac{T_W(z_k)}{2}, & z_k < d_z \\ p_{A,z} T_z + p_{B,z} T_z + p_{C,z} (t(z_k) - 2T_z), & z_k \geq d_z \end{cases} \tag{7}$$

Lift model. The model for the energy spent by the lift while going against the gravitational acceleration is also based on the three contributions. During both the initial acceleration and the braking phases, the required power is equal to:

$$p_{A,y} = p_{B,y} = (a_y f_r + g)\,\frac{v_{top,y}}{\eta}. \tag{8}$$

Conversely, during the maximum speed travel, the required power is:

$$p_{C,y} = g\frac{v_y}{\eta}. \tag{9}$$

Therefore, the overall energy per mass unit spent by the lift to travel the distance y_j is:

$$w(y_j) = \begin{cases} p_{A,y}\dfrac{T_W(y_j)}{2} + p_{B,y}\dfrac{T_W(y_j)}{2}, & y_j < d_y \\ p_{A,y}T_y + p_{B,y}T_y + p_{C,y}\left(t(y_j) - 2T_y\right), & y_j \geq d_y \end{cases} \tag{10}$$

The case of lift traveling accordingly with the gravitational acceleration is not modeled, as no energy must be provided to the system.

Variables. The probability distributions **a**, **b**, and **c** enable to evaluate the following average unit mass energy required by the vehicles:

$$w_x = \mathbb{E}\left[w\left(x\right)\right] = \sum_{i=1}^{N_x} a_i w\left(x_i\right); \tag{11}$$
$$w_y = \mathbb{E}\left[w\left(y\right)\right] = \sum_{j=1}^{N_y} b_j w\left(y_j\right);$$
$$w_z = \mathbb{E}\left[w\left(z\right)\right] = \sum_{k=1}^{N_z} c_k w\left(z_k\right).$$

The average unit mass energy that a shuttle requires to move along the aisle starting from the lift is given by w_x; w_y is the average energy per mass unit that the lift needs to travel from the bay along the vertical direction; w_z is average energy per mass unit necessary for the satellite to move along the channel from its entrance to the target point. The corresponding average distances travelled in each direction are denoted $(\hat{x}, \hat{y}, \hat{z})$.

In MC cycles (i.e. $U > 1$) the most appropriate sequencing to minimize cycle time consists in performing all the retrievals after finishing all the storage duties. This situation is described by setting $S = 1$; the average energy per mass unit spent for this displacement is denoted by $w_{\delta x}$, which is given by the following weighted mean:

$$w_{\delta x} = \mathbb{E}\left[w\left(|\hat{x} - x|\right)\right] = \sum_{i=1}^{N_x} a_i w\left(|\hat{x} - x_i|\right). \tag{12}$$

3.2 AVS/RS energy consumption

The overall energy consumption for a cycle involving an arbitrary number of ULs and tiers is given by the following contributions.

Lift energy. As stated, the lift requires energy while traveling against the gravitation acceleration and involves the masses of lift, shuttle, satellite and ULs to be stored. No energy is used in the opposite direction. Therefore, the average energy spent for travels along the y axis is:

$$W_y = \left(m_{lift} + m_{sh} + m_{sat} + I m_{UL}\right) w_y \tag{13}$$

Shuttle energy: for each UL to be stored, the shuttle travels from the lift to the target channel (involving shuttle and UL masses) and vice versa (involving shuttle mass only). The retrieval movements are made symmetrically. Further, in the first and the last movements from/to the lift, the satellite is also carried. Two additional cases may occur for shuttle travels between two different channels: (1) storages following the first one as well as retrievals following the first one: the load is given by shuttle, satellite and UL masses; (2) switch from storage to retrieval: the load is given by shuttle and satellite masses. In this case, two travels to/from the lift made by the shuttle are saved. Therefore, the average energy spent for travels along the x axis is:

$$\begin{aligned} W_x = {} & \left[\left(2m_{sh} + m_{UL}\right)U + 2m_{sat} - 2S m_{sh}\right] w_x + \ldots \\ & + \left(I - 1 + O - 1\right)\left(m_{sh} + m_{sat} + m_{UL}\right) w_{\delta_x} + \\ & + S\left(m_{sh} + m_{sat}\right) w_{\delta_x} \end{aligned} \tag{14}$$

Satellite energy: for each UL to be stored, the satellite has to travel from the channel entrance to the target position with a load given by satellite and UL masses, and viceversa with a load equal to satellite mass. The retrieval task is made symmetrically. Therefore, the average energy spent for travels along the z axis is:

$$W_z = U\left(2m_{sat} + m_{UL}\right) w_z \tag{15}$$

The overall mathematical model for energy estimation is given by the sum of the three contributions:

$$W = W_x + W_y + W_z \tag{16}$$

4. MODEL VALIDATION

In order to validate the models over a large number of configurations, two factors have been varied in the study. First, the type of cycle has been varied to take into account sets of activities involving up to 4 ULs. Second, three S/R criteria have been tested; namely:

- Random. The position to store or retrieve an UL is chosen randomly, according to a uniform distribution. This is the most common criterion in literature.
- Closest Floor (CF). The position of the UL is selected according to the following hierarchy: (a) the tier closest to the bay; (b) the channel closest to the lift hosting the same type of item;
- Closest Channel (CC). The UL position is selected according to the following hierarchy: (a) the channel closest to the lift hosting the same type of item; (b) the tier closest to the bay.

Thus, the experimental plan consists of 12 different scenarios. More details are provided in Table 1.

The simulations are performed by means of a Matlab routine. Model initialisation is made by filling the rack to 100% of its capacity and, subsequently, by running a set of retrieval orders to achieve a fill-ratio equal to 50%. Then, system observation began: a sequence of 20,000 ULs to be

Table 1. Summary of the experimental plan adopted for model validation.

Cycles	Cycle 1	$I = 1,\ O = 0,\ S = 0$
	Cycle 2	$I = 1,\ O = 1,\ S = 1$
	Cycle 3	$I = 2,\ O = 1,\ S = 1$
	Cycle 4	$I = 2,\ O = 2,\ S = 1$
S/R Criteria	1. Random	
	2. Closest Channel (CC)	
	3. Closest Floor (CF)	
Rack size	x	11 channels = 16.5 m
	y	10 levels= 20 m
	z	19 UL positions = 22.8 m
	Capacity	2090 ULs
Vehicles	$a_x = 0.5$ m/s^2	$v_x = 2.0$ m/s
	$a_y = 0.3$ m/s^2	$v_y = 0.2$ m/s
	$a_z = 0.5$ m/s^2	$v_z = 1.2$ m/s
$g = 9.81$ m/s^2; $c_r = 0.01$; $\eta = 0.80$; $f_r = 1.15$		

stored or retrieved is generated. The same sequence is used for all the case-studies, and the S/R orders ratio is kept equal to 1 to simulate a steady state scenario. For each case, 10 repetitions have been performed, each of them based on a different sequence of ULs. To be closer to the real deployment scenarios, the rack is assumed to store 4 different classes of ULs, and each channel is dynamically allocated for a single type of UL: any class of item can be stored into an empty channel; conversely, the units stored in the channel have to be of the same type.

Among the simulations output, the distributions **a**, **b**, **c**, and the average energy need have been saved. The discrete probability distributions are then provided in input to the mathematical model and the energy estimation is compared with the results of the simulations.

5. RESULTS

The results comparison is shown in Table 2: the minimum and the maximum values of the average energy need are provided, for both the analytical models and the corresponding simulations, for each positioning criterion and type of cycle. The relative differences are shown in Figure 2.

From a general perspective, the results show that the error in analytical estimation never exceeds 3%. In cycle 1, which includes the easiest tasks, the analytical model provides an almost exact evaluation; in cycle 2, the deviation of the model is always within ±3%. These are the most investigated cycles in literature. The adoption of simultaneous shuttle-satellite movements is effectively modeled, as represented in cycles 3 and 4.

The results also highlight the importance of properly modeling the S/R criteria. The graphical representation in Figure 3 shows that non-random criteria lead to improved energy consumption. Further, an appropriate pairing of S/R activities on the same tier can be convenient, as the energy spent for each moved UL tends to decrease.

6. CONCLUSIONS AND FURTHER WORK

In the present paper, an original analytical model to estimate the energy performance of an AVS/RS in realistic

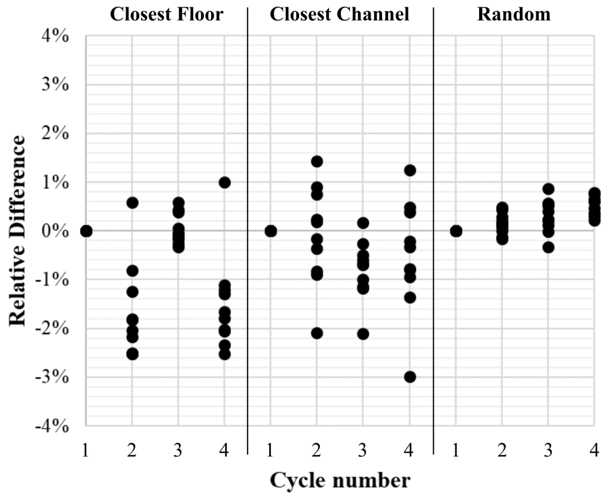

Fig. 2. Relative differences between the results provided by the analitical model and the simulations.

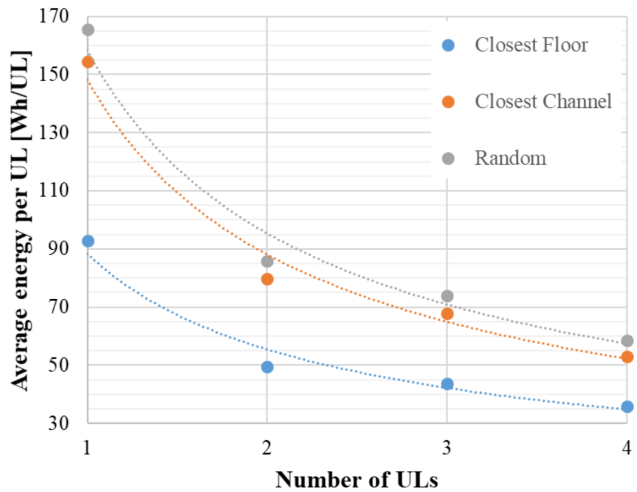

Fig. 3. Average energy consumption in case of MC cycles involving different ULs on the same tier.

scenarios has been presented. At the moment of writing this contribution, to the best of the authors' knowledge, no paper in literature has provided a model capable to deal with deep-lane racks and to consider storage/retrieval policies different from random, uniform distributions. Further, the model extends the state of the art by enabling

Table 2. Comparison between the average energy values evaluated through the analytical model and the simulations.

		Energy need mean values [Wh]		
		CF	CC	Random
Cycle 1	An	80.72-105.19	140.55-170.58	157.66-173.25
	Sim	80.72-105.19	140.55-170.58	157.66-173.25
Cycle 2	An	86.98-111.68	145.38-175.17	163.87-179.44
	Sim	84.85-110.78	144.87-173.63	164.68-179.65
Cycle 3	An	116.27-147.50	186.80-223.57	212.22-231.54
	Sim	116.33-147.11	184.68-223.95	212.48-232.30
Cycle 4	An	128.10-159.87	195.45-231.74	224.17-243.42
	Sim	125.52-158.13	193.61-230.97	225.91-244.94

the evaluation of MC cycles, while most of the literature focuses on SC and DC cycles.

The results presented in section 5 show that the analytical technique is an effective tool for the evaluation of AVS/RS performance: the maximum relative error - with respect to a set of simulations - is usually below 3%. These models aim to support AVS/RS designers in the evaluation of system performance over a wide range of scenarios. The model can be easily implemented in an Excel spreadsheet to obtain a tool capable to help designers in taking reliable decisions. Here, the probability distributions introduced in section 3 have been taken from the simulations just for the sake of comparison; in normal design activities, the probability distributions may be any set of weights with a (normalised) sum equal to 1.

The plot in Figure 3 shows that, to best exploit AVS/RS features, designers should properly model both the S/R criteria and MC cycles; conversely, the analysis is usually limited to SC and DC cycles randomly managed.

Additional efforts must be made to enlarge the configurations embraced by this kind of analysis: the considered system could perform cycles involving different tiers, as well as systems consisting of more than one shuttle, in a tier-to-tier configuration must be taken into account.

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