

Constrained rational fitting for D-stable model order reduction

*Original*

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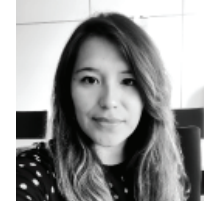
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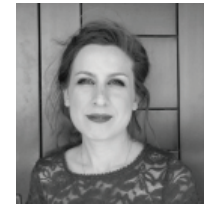
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## Interactive Digital Environments for Cultural Heritage and Museums. Building a digital ecosystem to display hidden collections

The paper presents the final outcome of a project carried out through the collaboration between Politecnico di Torino and Museo Egizio of Turin. The main aim of the project has been to make available a small series of artefacts that are part of the museum collection through their virtual reproduction. The research investigates procedures that are used for the dissemination of cultural heritage in museum context. This practices and the use of media and the web for dissemination purposes became part of a contemporary digital ecosystem that involves heritage institutions and museums. The paper describes the workflows used to develop different kind of outputs using the same content, and how it is possible to reuse digital resources for the communication and the visualisation of cultural heritage in an attractive way through the use of the latest visualisation technologies and web applications. The final stage of the proposed research, in ad-

dition to the others already developed solution, consists on an edutainment web application that assists the user in the discovery of historical iconography associated with digital models, with the intent to educate on the understanding of the drawn space and to visualise some contents of the Museo Egizio of Turin: the 'Expedition models of Egyptian Architecture'. The digital ecosystem developed for the project consists on a set of digital data of historical documentation and the digital replica of the museum collection: a set of wood maquettes representing ancient Egyptian buildings. The task, carried out in collaboration with the VHLab, CNR ISPC (Istituto di Scienze del Patrimonio Culturale), allows to narrate the meeting of two different cultures, cultural institutions and science of representation, creating a new reasoned storytelling.

**Keywords:**  
Collection Information Modeling; Virtual Reality; Immersive experience; Museum collection; Egypt

## INTRODUCTION

Computer graphics and digital imaging are changing the relationship between users and digital environments. In the cultural heritage field, ICT applications have become a new medium of communication for museum collections and their stories.

Communicate cultural and educational content through drawing, rendering, mapping, and modeling can ensure a wider access to high quality and scientific cultural content.

In the past, the use of design and models was already the means by which voyagers communicate the architectural evidence of distant and unknown countries. This is the case of the research carried on in collaboration with the Museo Egizio of Turin. The main aim of the research was to make available a small collection of artefacts through their virtual reproductions promoting the use and reuse of digital content for different purposes.

The final output consists of a digital ecosystem where digitised cultural resources can be reused for innovative and diverse outcomes in the museum sector. The methodology used considers different outputs to create interactive digital environments for cultural heritage and museums collection that nowadays are partially inaccessible to the larger public. The work describes the final outcome of a project going beyond the process of digitisation and where the multiplicity of the outputs finds justification into the potential of each of them to disseminate different knowledge useful to reach a wider and various public.

Starting from historical research and going through digitisation, the proposed methodology is based on a multidisciplinary approach where professionals and academics with different skills jointly elaborate guidelines to model data and knowledge aiming at a comprehensive documentation of 3D collection asset.

The methodology proposed is articulated in different phases:

- collection of data (historical and archival documentation related to the museum collection)
- digital acquisition (for both documents and objects)

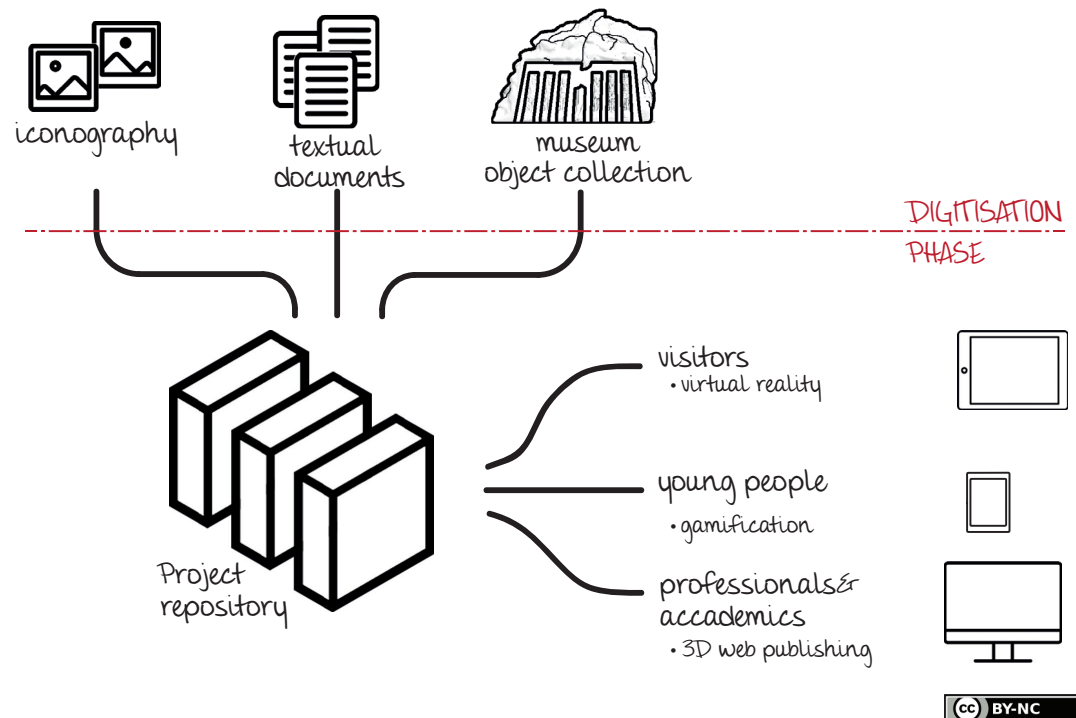
- multidisciplinary approach for sharing content and metadata online (relational database schema based on CIDOC Conceptual Reference Model - CRM)
- use of 3D content in research, entertainment and education (visualisation tools and applications).

Digital technologies offer new ways to interact with the digital world and education: the novelty aspect of the project lies in the direct relation between the necessity of documenting and researching about objects still not much studied and the complementary requirement of bringing together different expertise, both in terms of form and content, in order to foresee different narrative scenarios mainly targeted at the visitors.

The developed outcomes are the following, considering different users and their use:

- research: an interactive web presentation of high-resolution 3D models with their associated information (historical and archival documentation)
- edutainment: a web-based serious game that permits to put in relation drawings with a simplified geometrical model of the object. The result is the necessity of new interactive models addressed to the involvement of the younger through processes of gamification to approach different educational environments.
- entertainment: virtual reality application to experience the museum collection at its primary exhibition design.

Fig. 1 - Digitised content and different outputs within the project



## THE IMAGE OF THE EGYPTIAN TEMPLES THROUGH THE DRAWINGS OF JEAN JACQUES RIFAUD

Between the 18th century and the 19th century, a great number of scientists, alone or joining expedition teams, went to Egypt to take over some souvenirs to sell, and to record that unknown world with different skillfulness and instruments. Jean Jacques Rifaud (1786-1852) was one of them, but referring to his experience to Nubia and Egypt, there are two travels overlapping in time: the first is his solo travel from 1805 to 1827 (Italy, Spain, Mediterranean Islands, Asia Minor, Egypt and Nubia); the second is the expedition coordinated by Bernardino Drovetti in Nubia in 1816-1817, as the architect of the group. His work abroad Europe has been reported in three different kinds of textual sources: the published Tableau (Rifaud, 1830a), the travel journals and his handwritten notes [1]. He narrated not only the place he visited and what he saw, but he explained what he drew and how too.

Though he is better known for his involvement in the Drovetti travel in the Nubian area, his drawings were part of a broader aspiration. The Chevalier Rifaud celebrated for his Travels in Egypt, Nubia, and the neighbouring countries, in which he spent twenty-two years, has brought with him to Amsterdam a collection of more than six thousand drawings made on the spot, and embracing everything connected with art that presented itself to his view [...] [2]. Rifaud huge iconographical production was his main key to introduce himself to European potential subscribers and academics that had to examine his work abroad. The transport to Europe represented a great difficulty for the findings of Rifaud. All the items were moved from Alexandria of Egypt to Livorno in Italy first. In several documents, Rifaud and also Carlo Vidua, at the time entrusted to check the collection of Drovetti acquired by the Savoy family, reported the bad conditions of the depots, both in Egypt and in Italy, especially for the humidity, dangerous for the drawings and the manuscripts, conserved in tinplate boxes [3].

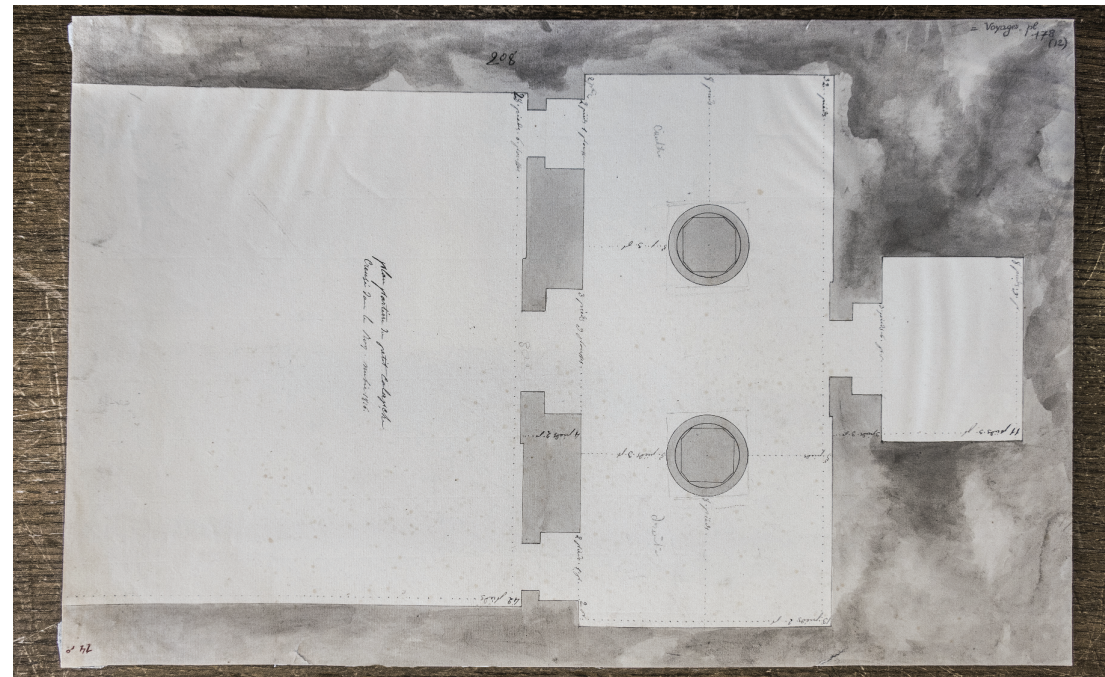
Rifaud took with him the drawings in his way back

to Europe, anticipating the arrival of the other part of the collection that remained in Alexandria of Egypt longer. His plan was to find funding to publish all his Egyptian and Nubian work, five volumes of texts and an atlas of *in folio planches*, figuring an oversized collection of illustrated plates. In his *Prospectus* [4], he stated about a schedule of sixty issues for the plates, foreseeing a two-years overall time frame for their printing.

The *corpus* of drawings of Rifaud about the monuments [5] is a heterogeneous collection of sketches made on papers with different dimensions and with drawing techniques that let also think to different hands, as Bruwier noted with a consideration on all Rifaud iconographical production (Bruwier, 2014, 12). They show plans, sections

and elevations of several of the temples visited by Rifaud. Some of the drawings present improbable perspective views, giving picturesque sceneries of temples in ruins, occasionally populated by some natives. Some others give also dimensional information about the temples, most frequently in their plan representation. Certain give also representation of shadows. Some of those lithographs were published from 1830, several years after the realisation of the drawings, together with others that focused on different themes (e.g. folkloristic, naturalistic) and have been realised from different artists [6]. Within the project, the lithographs have been the historical documentation chosen as starting point for the web application, mainly for their homogeneity in representation for all the temples,

Fig. 2 - Jean-Jacques Rifaud, 'Plan of the Temple of Beit el-Wali'. Drawing. Genève, Bibliothèque de Genève, Ms. fr. 1602/1, f. 14r.



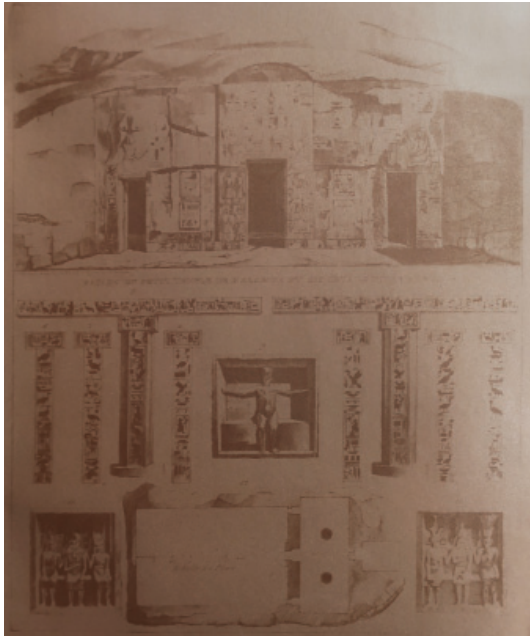


Fig. 3 - Winter, 'Elevation, plan and particulars of the Temple of Beit el-Wali'. Lithograph from the original drawing of J. J. Rifaud, Impr. lith. de Lacroix. Planche 178, in 'Voyage en Égypte, en Nubie, et lieux circonvoisins, depuis 1805 jusqu'en 1827' by J. J. Rifaud, photographic copy conserved at Museo Egizio of Turin of the original copy conserved at Fondation Égyptologique Reine Élisabeth.

and also for the more intelligible understanding of the same instead of the first original drawings [7]. The 'Expedition models of Egyptian Architecture' were part of the Drovetti collection and arrived between December 1823 and the beginning of 1824 (Lo Turco, Giovannini & Mafriaci 2019). From his memories [8] there are several references to Rifaud professional training and background as a sculptor of models in wood and his knowledge was surely useful also in the Drovetti expedition. The *maquettes* arrived at the 'Regio Museo di Antichità ed Egizio' in Turin and were allocated at its ground floor. The first configuration was in the nowadays

<http://disegnarecon.univaq.it>

'Gallery of the Kings' (*Sala II* in the 19th c., *sala I* in the 20th c.), and in the beginning of the 20th century, the first reallocation of the models was carried out in the near 'Room 1' (*Sala I* in the 19th c., *sala II* in the 20th c.) [9], where they remained until WWII. After the war, the models were exhibited in the 'gipsoteca', and then they were moved in a dedicated underground room in the '70s (Curto, 1976, 22). After this frame, the models were divided between the museum depots and three of those were recently exhibited in the Nubian room. The *corpus* of drawings seems to be the basis for the construction of the models, according to several similarities between the two kinds of representation of the same monuments (Einaudi, 2016, 505), and this fundamental correlation is highly considered in the narrative structure of the web application of the case study.

#### FROM DIGITISATION TO 3D WEB PUBLISHING

Within this project part of the research focused on museum collections and their digitisation. In the "Declaration of Cooperation on Cultural Heritage" (2019) the role of digitisation is declared as a medium to make cultural heritage accessible to all. The case study refers to fifteen 19th c. wooden ar-

chitectural models of Egyptian monuments. The digitisation of the artefacts (a small museum collection composed by 14 *maquettes*) has been the first step of the workflow. The digital acquisition combined mixed techniques and was carried on in parallel with historical investigation process (Lo Turco, Giovannini & Mafriaci 2019).

One of the aims of the project was to make available and accessible a hidden museum collection, partially exhibited to the public in the Museo Egizio and partially conserved in the Museum depots. The D.M. 21 February 2018 titled «Adozione dei livelli minimi uniformi di qualità per i musei e i luoghi della cultura di appartenenza pubblica e attivazione del Sistema museale nazionale» imposes the observance of the minimum standards for a proper organisation of museums depots, by giving directions in matters of organisation and conservation of not displayed artefacts, following criteria of operability and security. The document describes also parts of the improvement objectives considering that the organised hidden heritage could be let consultable on request and publicly available for special events.

Over the years, cultural heritage has become an important part of the digital environment that takes advantage of the opportunities offered by the

Fig. 4 - The Nubian Room of the Museo Egizio of Turin. Showcase with maquettes of Temple of Tafa, Temple of Beit el-wali and Temple of Dakka.



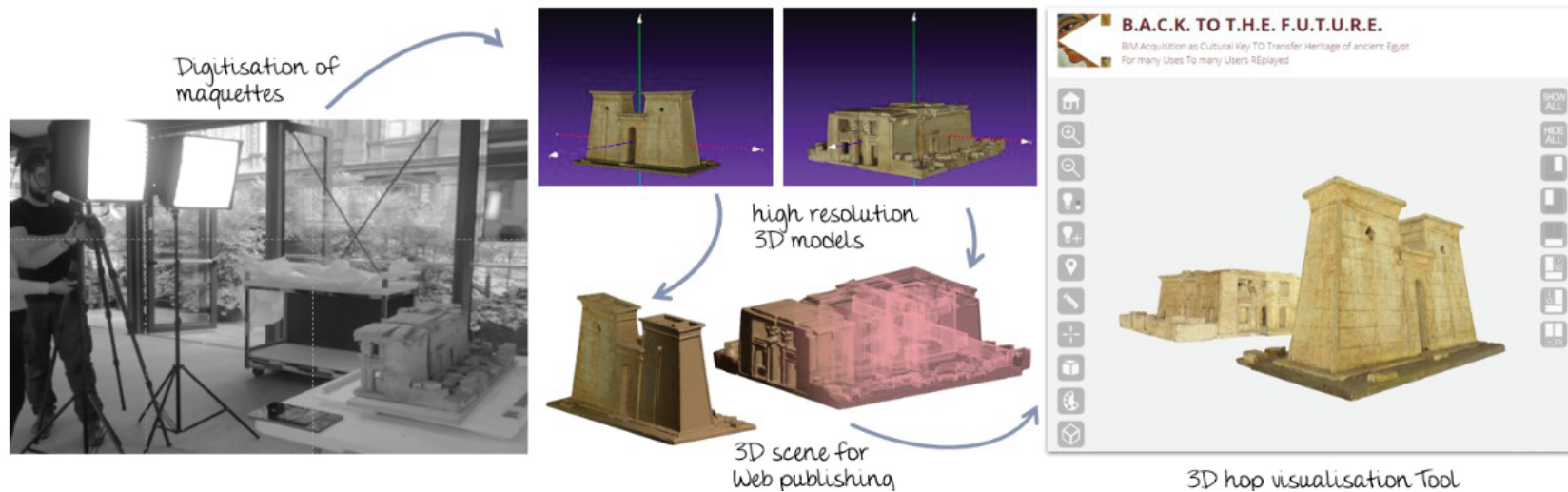


Fig. 5 - models acquired using photogrammetry techniques ready for 3D web publishing. Temple of Dakka.

web and ICT technologies. In recent years, moreover, we have moved from an information age focused on data and content to an electronic age based mainly on the use of smartphones and mobile devices that allows an increasingly massive use of online content anytime and anywhere.

This revolution has also brought a transformation in the academic sector by creating new modes of knowledge formation enabled by networked, digital environments [Burdick et. al 2012].

The potential of new technologies can be nowadays used to enhance access to heritage for different kinds of users from professionals to citizens developing new methodologies to use interactive technologies such as virtual and augmented reality for cultural applications.

Different outputs require different methodologies then, the pipeline developed takes into account different needs of technologies used.

The digitisation phase concerned all sources available, historical and archival documentation (textual documentation and drawings) and objects (most of the object were composed of two parts).

The output of the first phase is a heterogeneous set of documentation that needed to be organised and structured to be more accessible, and re-usable for different communication contents.

Enable the use and re-use of content means keep in mind FAIR principles [Wilkinson et al. 2016] that provide guidance for scientific data management and stewardship and are relevant to all stakeholders in the current digital ecosystem. Research assets of this project were annotated with rich metadata [Lo Turco, Calvano & Giovannini, 2019], using museum community standard ISO 21127:2014 also known as the CIDOC Conceptual Reference Model [CRM] [10].

Different users generate diverse communication strategies, then in this project it is possible to see how documentation and images collaborated to the creation of 3D contents.

In recent years we witnessed large advancements within presentation and dissemination of interactive 3D scenes on desktop and mobile browsers through Web3D technologies (see for instance SketchFab platform [11]).

One of these platforms, was developed by the Visual Computing Lab of ISTI-CNR. The main difference from other commercial solutions is the possibility to manage 3D high resolution models. 3DHOP [12], 3D Heritage Online Presenter is an open-source software package for the creation of interactive Web presentations of 3D models, oriented to the Cultural Heritage field [Potenziani et al. 2015].

Another open-source framework [ATON [13]] - developed by CNR ISPC - is already employed in several Web3D projects including 3D landscape presentation within European cloud-based infrastructures [Fanini, Pescarin & Palombini, 2019], augmentation of museum collections on mobile devices [Barsanti et al. 2018] and WebVR analytics [Fanini & Cinque, 2019a; Fanini & Cinque 2019b]. Thanks to recent open specifications such as WebVR/WebXR [14] for instance, immersive VR experiences (using consumer HMDs) are becoming easier to deploy through common web browsers [MacIntyre & Smith, 2019], without requiring any additional plugin or software. Web3D applications

are also becoming very appealing for the mobile world, due to users not being forced to install third-party apps from stores nor require additional components to inspect 3D models embedded in a web page. Because of its inherent openness and accessibility, the web can thus represent a valid solution to enable “universal” Web-Apps that automatically adapt to user devices without requiring additional software. The developed solutions within this project used different technologies to generate different kinds of outputs where different necessity of users defines different approaches. Starting from the needs of researchers, academics and museum professionals, a high resolution 3D model for each part of the *maquettes* of the collection was generated using digital tools such as 3D scanning and photogrammetry to capture the structural aspects of the objects (Spreafico, Patrucco & Calvano 2019).

Basic web pages, created using 3DHOP and html, allowed to customize a front-end interface where 3D models were associated with digitised historical documentation. The web portal allows, thanks to 3DHOP, to visualize a multiresolution 3D scene with the possibility to interact with 3D model using simple operation (sections, measurements, different point of views explorations). Starting from the default template available and the consultation of their associated information within this structure, it is possible to identify other resources related to the original artefact as the plates assembled from the original drawings by J. J. Rifaud, sculptor involved in the Drovetti expedition in Egypt. The developed platform was designed for professionals use, with an open repository in github [15] (Mafriçi & Giovannini, 2019). After this first output other Web3D solutions were proposed and investigated for the engagement of

visitors. The following solutions were developed for edutainment using web-based 3D applications and gamification. The main web-app aims to assist a broader public to advanced cultural content, through an entertaining process of investigation, useful to prompt the curiosity and to broaden the boundaries of knowledge. The developed Web3D puzzle put the user into a digital ecosystem where he can play with the digitised heterogeneous documentation.

### 3D MODELLING FROM HISTORICAL SOURCES

A different purpose needs a different 3D model, then to develop the 3D puzzle application a low-poly model was generated starting from iconographical and historical sources: as *pilot* for the development of the App was chosen the Temple of Beit el-wali, first because an omogeneous iconographical documentation was available and also because its *maquette* is shown at the Nubian Room of the Museo Egizio of Turin.

The temple of Beit el-Wali was represented by J. J. Rifaud into three planches, encoded as pl. 178, pl. 179 and pl. 180 in the edition de Munich of his *Voyage en Egypte, en Nubie, et Lieux Circonvoisins, depuis 1805 jusqu'en 1827*.

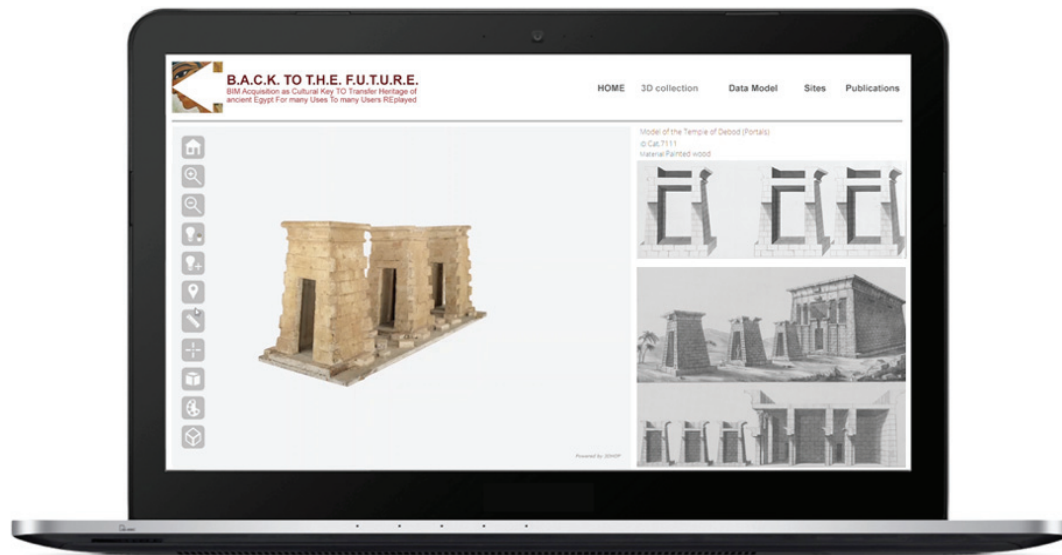
The Temple was chosen because it is the most well documented building in J. J. Rifaud drawings and planches and the use of his representations allowed to cover with textures the most important architectural elements of its digital replica.

The pl. 178 is entitled *Facade du petit temple e Kalabche et ses details interieur (Nubie)* and it represents starting from the top of the layout, the exterior facade of the rock-temple and some details about the interior elements; below, two pillars and their architrave and three niches.

These details are enriched by the drawing of hieroglyphics and the representation of some Egyptian gods. at the bottom of the layout there is also a plan with measurements and a graphic scale.

The pl. 179 and 180 are drawn in the same paper, they are named *subjects hiéroglyphiques des deux murs collatéraux du petit temple de klabche creusé dans le roc* and their represents the internal facades of the courtyard that precedes the

Fig. 6 - 3Dhop visualization tool inside a web-page. Temple of Debod.



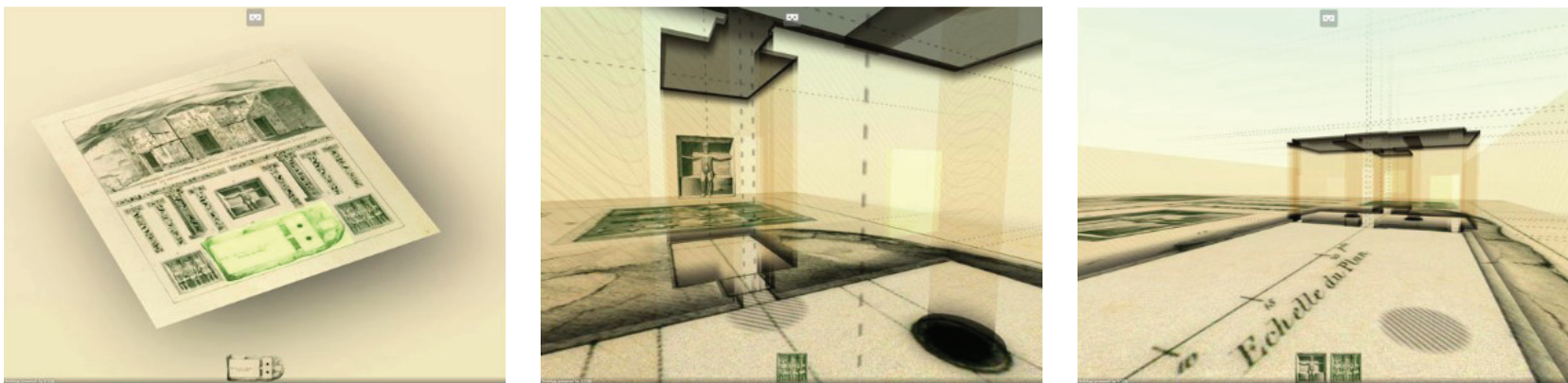


Fig. 7 - Stages of the 3D puzzle performed on common web browsers with solvers (green) and pieces to solve (bottom). Temple of Beit el-Wali.

main facade of the temple (represented in pl. 178). All drawings were used as textures to create *ad-hoc* materials for the 3D model and missing textures were generated starting from the original color of paper of the planches.

#### GRAPHIC FIELDS AND KNOWLEDGE, THE WEB APP AS A LEARNING TOOL

The final phase of this project investigates solutions offered by web technologies to develop web applications for edutainment and possible use of gamification for creation of learning tools.

The collaboration with the VHLab allowed the investigation and development of online 3D solutions for gamification through the recontextualization of selected elements in the temple of Beit el Wali. The application permits to put in relation the lithographs of J. J. Rifaud with a simplified geometrical model of the temple. The application created is aimed not only at the category of children, but at all those who want to approach a knowledge of inaccessible artefacts. The accessibility of a cultural product derives from a double level of knowledge:

- the knowledge of the language for reading the intrinsic content of the product;
- knowledge of the specific content of which the product is an expression.

The first condition of accessibility is expressed through an analysis of the artefacts proposed in this work. The object under investigation is an architecture (the temple) which in the web app is represented by the historical drawings which are a first level of reality discretization, using a graphic code linked to the method of representation adopted and with which the flat images are generated. They are mostly orthogonal projections which, in the works presented, leave some exceptions to the author of the drawn elaborations.

The drawing is defined by John Klaus Koenig as an instrument capable of “denote” and at the same time “connote” “a homogeneous group of images, traced or reported on a two-dimensional surface” [16] (Koenig, 1964). We then move from the three-dimensional graphic field to the two-dimensional graphic field through an operation of knowledge: the knowledge of the method’s rules, whether it be orthogonal projection, axonometry, perspective. The web app developed as part of this research, addresses in an elementary way this category of problems: the understanding of an existing space, albeit located in a different context from the real, starting from its 2D graphics reduction. The game is in the contemporary sense as a useful means of learning, the game is then given a character of seriousness and is thought of as one

of the main keys to trigger the imagination, able to create simplified worlds where to build hypotheses that stimulate the understanding of phenomena. Today the gap between entertainment and education has disappeared and this has led to the growing confirmation of video games as a possible educational medium (McLuhan, 1967). The process of re-evaluating playful activity started with Paul Gee, who proposes it as a model of implicit learning: through experience, users acquire the rules of the game and therefore knowledge (Gee, 2003). Among the different purposes, the game developed aims at learning the graphic code of the elaborates used in the app, facilitating a direct relationship between two-dimensional and three-dimensional models. The approach to the understanding of space derives from the particular aspect of the web app that, compared to other games, proposes the interaction with two-dimensional images instead of with 3D objects. The user is initially in front of Beit el Wali’s drawn table; the document is fragmented in the different details that are collected on a sliding menu. The game starts by dragging the plan in a highlighted area of the graphic field in a part of the display and at this point, in the same position the 3D model, that represents the spatiality of the model, appears. The model is geometric, made bare of material and

chromatic indications, useful just as a support to recompose the following two-dimensional pieces. The 3D model was previously built using the same drawings actively used in the game, becoming a second level of synthesis of the real model, but still sufficient for the creation of an immersive space in which the user moves in a familiar way. The switch from one vision to another helps to compare two identities in a narrative logic. On one hand, through an orthographic view the user is confronted with a synthetic view of the entire model (the drawings), on the other, a first person mode allows the user to explore the three-dimensional graphic field, in which there are suggestions for actions to be taken (drag the fragments of the drawing on the corresponding parts of the 3D model). The narrative construction just illustrated is an effective way to reveal and understand the graphic code that allows the passage from the two-dimensional field to the three-dimensional field.

The second condition of accessibility linked to the knowledge of the specific contents of which the product is an expression, has to be built in the game through the correct use of storytelling. The web app offers a narration linked to multimedia materials: digitized images, animations and models, so it is more correct to talk about Digital Storytelling (Robin, 2006: p. 709). The narrative structure of the webapp is summarized through the following steps:

- Introduction from the museum display cases where three models are exhibited;
- Zoom in on a model and switch to the app;
- View in third person constrained to the movements of the camera, video that shows the entire table to proceed with the fragmentation into pieces useful for the game;
- Third person view free (possibility to orbit), activation of the game by dragging the plant in a highlighted area of the graphic field;
- View in third person, growth of the model and passage in first person;
- First-person navigation within the 3D model;
- Insertion of the table fragments in the right highlighted positions.

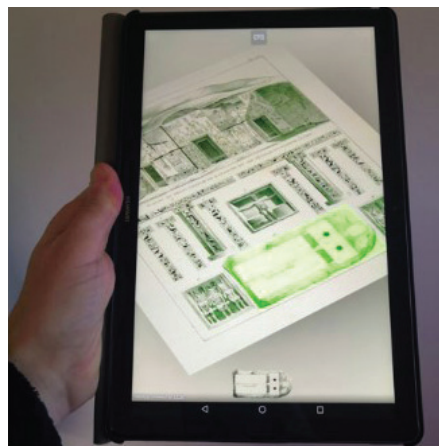
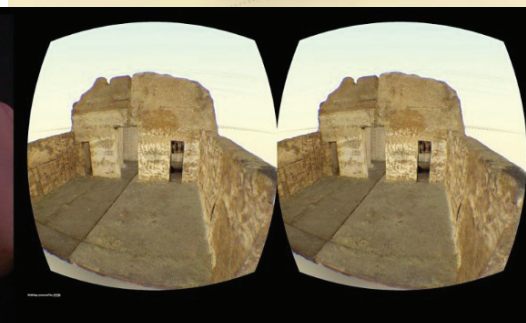
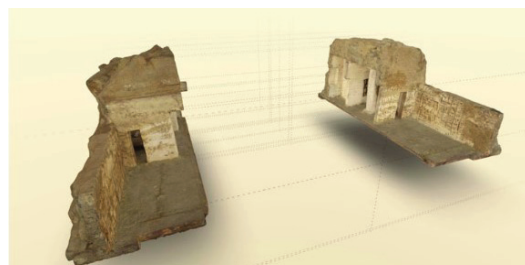


Fig. 8 - Multi-touch interface for mobile devices: constrained orthographic navigation ( left) and first person (right).

Fig. 09 - The final stage of the 3D puzzle, where the actual wooden model is shown with the animation of the two halves (top); multi-touch interaction on mobile device (bottom left) and immersive VR exploration using HMD (bottom right).



The end of the game is determined by placing all the details in the correct positions, contemplating a prize that is related to the time of solution of the 3D puzzle. Once the required actions have been solved, there is the insertion of a moment of choice: to continue a new reconstruction or to deepen the subject just concluded by investigating other digitised resources. Once the game is over, in fact, it is possible to access a web space dedicated to the exploration of a new resource, the digital model of the wooden *maquette* of the temple, subject of the game. In this space the user can orbit the surveyed model and interrogate it in different parts highlighted to access an advanced knowledge of the contents of the object.

### 3D WEB APPLICATION FOR EDUTAINMENT AND GAMIFICATION

The main goal is the creation of a serious game (3D puzzle) in the form of a scalable and *cross-device* Web-App (mobile, desktop and immersive VR) based on modern web standards - without any installation required for the final user. The application goal is to put in relation the drawings of J. J. Rifaud and subsequent elaborations with a simplified geometrical model of the temple. The user is invited to spatially place the elements (historical orthographic projections) in correct locations, starting from the plans and settling then as a 3D puzzle. The end of the game envisages the visualisation of the original museum wooden models through the identification of the correct topology of the graphical elaborations.

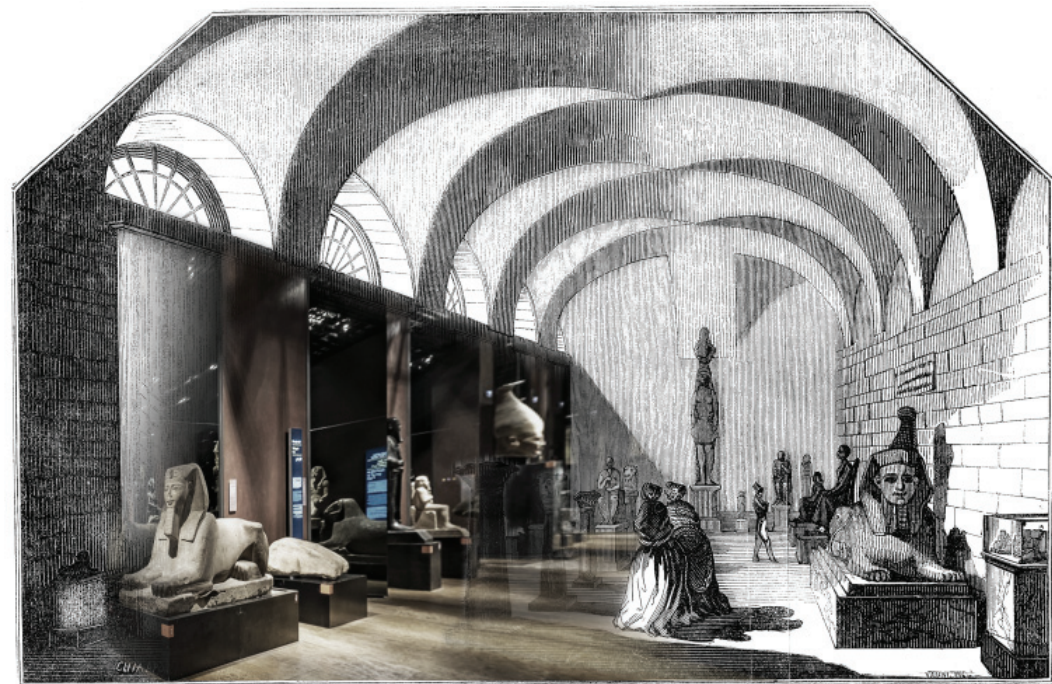
The 3D puzzle (Web-App) for the temple of Beit el Wali was developed on top of open-source ATON framework, since it provides *built-in* functionalities for scene-graph manipulation (hierarchies, node transformations, etc.), support for multi-touch and immersive VR devices, two navigation modes (orbit and first-person) and viewpoint interpolation - including smooth transitions from orthographic to perspective views and vice versa. Furthermore, it offers a separate graph of shape descriptors (basic 3D geometries generally used for semantic queries (Demetrescu & Fanini 2017)

that allowed in this context to deploy “solvers” (3D shapes where the player has to place the correct piece) into the virtual 3D space.

The web-app presents a set of items (historical orthographic projections from the drawings of J. J. Rifaud) that user has to spatially relocate in the 3D space, supporting the recontextualization task. The client component for the players leverages on ATON features to present 3D content and provide a responsive front-end (web page) that automatically adapts to device capabilities. For instance, on desktop PCs players can interact using a common mouse; on mobile devices a multi-touch interface is offered and on HMDs (WebVR/XR) a different interface is automatically adopted. On mobile devices, the multi-touch interaction is performed

following common guidelines (Moscovich 2007) to manipulate the virtual camera and to explore the 3D environment. A *drag&drop* approach is employed to solve elements, leveraging on 3D placeholders (“solvers”) with appropriate on-screen sizes to facilitate the user task (Motti, Vigouroux & Gorce 2015). The first part of the game shows a brief introductory screen and allows the user to get familiar with the interface by solving a single item. The following steps are performed in a first-person mode, where the player has to explore a limited area (the drawing blueprints of the temple) and solve remaining pieces. Regarding first-person navigation, a double-tap gesture is used to move into specific locations within the area. The same approach is automatically adopted on

Fig. 10 - The original exhibition display vs the nowadays “Gallery of the Kings”



(Una sala del Museo Egizio)

HMDs (WebVR/XR) with proper transition speeds (Bozgeyikli et al. 2016; Boletsis 2017; Buttussi & Chittaro 2019) to avoid motion sickness and to offer a comfortable experience to final users.

Once the last piece is solved, the actual wooden model is shown and the two halves animated - allowing the player to understand the consistency of the original maquette. The player can explore/inspect the 3D model and submit his/her score: such score is computed taking also into account elapsed time, thus guaranteeing diverse performances by different players. For a more technical visualization, a link to the corresponding 3D model on 3D-hop is also offered to the user.

The server-side component of the web-app is based on Node.js [17]: it is responsible for serving content (3D models and pieces, shaders, user interface elements, sounds, etc.) to final users, and also to maintain a persistent scoreboard in the form of a json database. The latter also includes a per-user sorted list of solved pieces, allowing game analysts to further investigate players' performances. A json configuration file is also employed to customize the application and suit specific requirements (e.g. museums needs, etc.). It is also possible to deploy the Web-App on cheap, low-cost hardware (such as SBCs [18]), kiosks or local hotspots where internet connection is not available.

The knowledge is carried by the experience, then the development of an App requires different levels of elaboration. First, the learning process is guided by the discovery of the informative and iconographic apparatus of the original temple, even if simplified through drawings and architectural scaled models previously digitised. Then, the app guides the user to the knowledge and the comprehension of the three-dimensional space, placing the visitor in a participatory process based on ICT technologies.

## CONCLUSIONS

The illustrated application is a contribution to the knowledge of themes connected to a specific collection of the Museo Egizio of Turin. Digital technologies modified the existing relationships between

the different actors of a museum experience: the museum building, the collected objects, the visitors. In the contemporary museum, the exhibition is the moment in which the visitor meets the displayed contents. During the exhibition, the collections are settled in the rooms of the museum building following a narration made by the curator. Digital technologies broaden these conditions implementing the relationship between the container and the content, giving the opportunity to introduce artefacts in time and space. The future perspectives of the research are addressed to this direction.

The museum modified in time its architectonic configuration, changing uses, spaces and displays. In the 19th century, the fifteen *maquettes* of the 'Expedition models of Egyptian Architecture' found place in the 'Regio Museo di Antichità ed Egizio'. Starting from some historical sources [19] and using the digital models acquired, it is possible to build an immersive environment that presents the time frame in which the models have been publicly exhibited for the first time, clearly documenting that fundamental meeting between different cultures. A final step will be the monitoring of the application, carrying out extensive user testing of the web-app within specific venues and public events to identify player performances and assess usability on mobile devices maximising the 'physical meets digital' dimension [20].

## NOTES

[1] The main original documentation related to Jean-Jacques Riffaud travel is conserved at the Bibliothèque de Genève (BGE). The travel journals inventoried and accessible are those from 1821, considering that he travelled to Nubia in 1816-1817.

[2] The 'Advertising of Foreign Quarterly Review', refers to one of the European city-stop of the 'promotional tour' that Riffaud made when he came back to Europe from his long travel abroad. In 'Staffordshire Advertiser', Saturday 06 February 1836, 22, 4.

[3] 'L'humidité est également funeste aux collections, et l'on n'y saurait assez prendre garde dans les magasins, aux bords du Nil, comme en Europe-dans les lazarets. On doit tenir enfin les dessins et les manuscrits dans des boîtes en fer-blanc; ils y sont moins froissés, et sont en outre plus aisés à transporter.' (Riffaud, 1830a, 119)

[4] Riffaud, J.-J. (1830b). Prospectus of the Voyage en Égypte, en Nubie, et lieux circonvoisins, depuis 1805 jusqu'en 1827. Paris: Imprimerie de Crapelet. It is a publication, not circulated until the second edition of 1836, to invite opinions about his work and to tell all the material ready for print. This note has been done in 'Avant-propos' of n.a. (1829). Rapports faits par les diverses académies et sociétés savantes de France, sur les ouvrages et collections rapportées de l'Égypte et de la Nubie par M. Riffaud. Paris: Imprimerie de Crapelet.

[5] The drawings of the planches of the Monuments of Nubia are conserved at BGE, Ms. fr. 1602/1, ff. 1-64.

[6] For an exhaustive state of the art on the different lithographs of Riffaud see Claes, 2014.

[7] For research purposes, the document used is: original plate in Société royale d'Archéologie, d'Histoire et de Folklore de Nivelles et du Brabant wallon (SANiv), published in M.-C. Bruwier, W. Claes, & A. Quertinmont (Ed.), «*La Description de L'Égypte*» de Jean-Jacques Riffaud (1813-1826) (p.131). Bruxelles: Éditions Safran.

[8] A part of his handwritten documents, both correspondences and autobiographical texts have been collected and archived in three volumes, BGE, Ms. Supplements.

[9] The rooms of the Museum have different denominations in the inventories and in the guides of the museum.

[10] <http://www.cidoc-crm.org/>

[11] <https://sketchfab.com/>

[12] <http://3dhop.net/>

[13] <http://osiris.itabc.cnri.it/scenebaker/index.php/projects/aton/>

[14] <https://www.w3.org/TR/webxr/>

[15] <https://github.com/backtothefuture/backtothefuture.github.io>

[16] Koenig points out that the graphic sign is not limited to denote something, as it happens with the signs of the language of the word, but it connotes something, it represents something, so the image is an iconic sign.

[17] <https://nodejs.org/>

[18] Single-board Computers, for instance Raspberry Pi (<https://www.raspberrypi.org/>)

[19] Among the 19th c. sources: 'Una sala del Museo Egizio'. Engraving in Stefani, G., Mondo, D. *Torino*

*e suoi dintorni - guida storico-artistica, amministrativa e commerciale* (pp. 168-169). Torino: Carlo Schieppati, 1852.

[20] Massimiliano Lo Turco wrote the paragraph "Introduction", Paolo Piumatti "Conclusions", Michele Calvano "Graphic fields and knowledge, the web application as a learning tool", Noemi Mafriaci "The image of the Egyptian temples through the drawings of Jean Jacques Rifaud", Elisabetta C. Giovannini "From digitisation to 3D web publishing", Andrea Tomalini "3D modeling from historical sources" and Bruno Fanini "3D web app for edutainment and gamification".

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