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Invited Paper

# Experimental realization of robust weak measurements

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## ABSTRACT

Despite being very influential on both foundations and applications of quantum mechanics, weak values are still somewhat controversial. Although there are some indications that weak values are physical properties of a single quantum system, the common way weak values are presented is statistical: it is commonly believed that for measuring weak values one has to perform many weak measurements over a large ensemble of pre- and postselected particles. Other debates surround the anomalous nature of weak value and even their quantumness. To address these issues, we present some preliminary data showing that anomalous weak values can be measured using just a single detection, i.e. with no statistics. In our experiment, a single click of a detector indicates the weak value as a single photon property, which moreover lies well beyond the range of eigenvelues of the measured operator. Importantly, the uncertainty with which the weak values is measured is smaller than the difference between the weak value and the closet eigenvalue. This is the first experimental realization of robust weak measurements.

**Keywords:** weak value, weak measurement, protective measurement, quantum metrology

## 1. INTRODUCTION

A milestone in the development and understanding of the implications of weak measurement was the paper by Aharonov, Albert, and Vaidman[1] which identified the results of weak measurement of physical variables in terms of their weak values. Weak measurements have opened up a new avenue of research of quantum phenomena.[2–5] With time it has become evident that one can take advantage of weak measurements to create a variety of novel quantum applications, such as wave function measurements,[3] weak-value amplification,[6–8] exquisitely sensitive sensors,[9–12] quantum random walks,[13] and superoscillations.[14] The latter are especially interesting in the context of metrology and imaging as they allow for super-resolution.[15–18] Applications to light-matter interactions[19] and many-body systems[20] have been recently discussed as well. In recent years there is also a growing number of works utilizing sequential weak values[21,22] for both practical[23] and foundational[?,24] purposes.

A key feature of weak values underlying most of the aforementioned works is their *anomalous* nature, i.e. their ability to reside outside the spectrum of the measured operator. Weak values can in fact be much larger than the largest eigenvalue of the corresponding observable (as we shall demonstrate below), and can also be

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complex (even when the measured operator is Hermitian). This anomalous nature has been shown to be related to violations of the Leggett-Garg inequality[25] and to quantum contextuality.[26,27]

Significant experimental and conceptual achievements employing weak measurements continue to appear, yet several questions remain. There has been, for instance, a debate regarding the utility of post-selection and weak measurements for detection and estimation tasks,[28,29] which seems to be significantly clarified by some recent papers such as.[8,30] In addition, it was recently shown how to infer weak values without weak measurements,[31] thereby disconnecting the two concepts, and how to obtain anomalous weak values without post-selection.[32,33] And indeed, the meaning of weak values is still a controversial issue.[26,31,34–48] Loosening the coupling in the weak measurement procedure made the uncertainty of a single measurement outcome much larger than the outcome itself. Thus, anomalous weak values were found in the past only after averaging over a large number of readings of the measurement pointer. Although averaging is a common practice in various measurement protocols, postselection is not regarded as such, and therefore the soundness of the statistical analysis was debated.[46,49–55]

Here we will report preliminary data of the first experimental realization of *robust weak measurement*, i.e. a measurement scheme in which a single detector click yields the weak value of a single photon, without the need of using statistics. Moreover, the weak value turns out to be anomalous, i.e. residing outside the range of eigenvalues of the measured observable.

Our measured observable had eigenvalues between -7 and 7. The predicted weak value of this operator in a certain pre- and postselected system was 18.7, and our single click measurement gave rise to $21.4 \pm 4.5$, see Fig.1. We find this outcome surprising, because for a preselected-only system, the expectation value of the corresponding variable was only 2.2.

Since our current measurement procedure relies, to some extent, on our previous demonstration of protective measurement,[56] it would be instructive to present it as well.

## 2. PROTECTIVE MEASUREMENT

Protective measurement[57,58] is an outgrowth of the study of weak measurements. The technique allows one to infer expectation values and eventually the wavefunction itself using only a single particle, rather than a large ensemble. Unlike strong (projective) measurements, this kind of measurement negligibly changes the wavefunction during the process thanks to a protection mechanism. Protection of the state in the case of discrete non-degenerate spectrum of energy eigenstates was shown to be a consequence of energy conservation when the measurement is sufficiently slow and weak,[58] i.e. in the adiabatic limit. Alternatively, protection of the wavefunction can be achieved by performing a dense set of projective measurements, which is indeed the technique we used in[56] for measuring the operator $P = |H\rangle\langle H| - |V\rangle\langle V|$ of a (heralded) single photon. We used a sequence of 7 weak couplings between $P$ and a measuring pointer (the transverse momentum of the photon) interleaved with 7 protection stages where we projected on the initial polarization. Eventually, if the photon survived all projection stages, we could ensure upon its detection with a SPAD array that: (1) its polarization state has not changed, and (2) the spatial shift of the measuring pointer (which was reflected by the hitting point on the SPAD array) corresponds, within reasonable error bars, to the expectation value of $\sigma_z$, see Fig.2. This result defies the standard view of the expectation value as a statistical property of large ensembles. But protective measurement provides not only a conceptual conundrum – it may also have practical merits. In[56] we examined protective measurement as a supporting tool for quantum metrology. Our numerical simulation showed that the insertion of $M$ protection stages leads to an increase proportional to $\sqrt{M}$ in the precision of the measurement, compared to a standard projective measurement.

## 3. ROBUST WEAK MEASUREMENT

By modifying the experimental setup, Fig.3, in,[56] we next realized robust weak measurements. Robust weak values were traditionally obtained as rare cases of post-selection for a large system of similarly prepared particles.[59] In accordance with the definition of weak values, and due to the rare post-selection, the robust weak value lies well outside the range of eigenvalues of the measured operator and therefore leads to amplification. We wished to obtain the robust weak values of a single particle in a not-so-weak coupling regime,[47,60] and with a not-so-rare post-selection. The latter choices were made in order to increase the efficiency of the protocol while still observing an anomalous value.

To achieve these goals, we created again, in the same repetitive way, a weak coupling between the polarization of the photon and its transverse momentum. The difference from the above scheme for realizing protective measurements was given by the repeated pre- *and* post-selection of the photon's polarization state. Eventually the position of the photon within the spatially resolving detector (an electro-multiplied CCD camera operating in the photon counting mode) determined the weak value of the sum of polarization variables of the same photon at 7 different times (provided that all pre- and post-selected stages succeeded).

The experimental results are presented in Fig.3. A single photon click indicates the anomalous weak value 21.4 which fits the theoretical value of 18.7 and lies far outside the range of eigenvalues of the measured operator.

## 4. CONCLUSION

We presented two experiments: The first realization of protective measurements and preliminary data of the first realization of robust weak measurements. Both of these demonstrations broaden the standard meaning of measurements in quantum mechanics and moreover challenge the statistical foundations underlying the quantum formalism. We have shown that the expectation value and the (anomalous) weak value can be determined based on a single click of the detector. This implies that the weak value should be understood as a single-particle, non-statistical property. Furthermore, our outcomes were achieved with reduced amount of uncertainty compared to other measurement techniques, suggesting novel practical implications for quantum metrology.

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## REFERENCES

[1] Aharonov, Y., Albert, D. Z., and Vaidman, L., "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," *Physical review letters* **60**(14), 1351 (1988).

[2] Steinberg, A. M., "Quantum measurement: A light touch," *Nature* **463**(7283), 890 (2010).

[3] Lundeen, J. S., Sutherland, B., Patel, A., Stewart, C., and Bamber, C., "Direct measurement of the quantum wavefunction," *Nature* **474**(7350), 188 (2011).

[4] Tamir, B. and Cohen, E., "Introduction to weak measurements and weak values," *Quanta* **2**(1), 7–17 (2013).

[5] Dressel, J., Malik, M., Miatto, F. M., Jordan, A. N., and Boyd, R. W., "Colloquium: Understanding quantum weak values: Basics and applications," *Reviews of Modern Physics* **86**(1), 307 (2014).

[6] Pang, S., Alonso, J. R. G., Brun, T. A., and Jordan, A. N., "Protecting weak measurements against systematic errors," *Phys. Rev. A* **94**, 012329 (Jul 2016).

[7] Dixon, P. B., Starling, D. J., Jordan, A. N., and Howell, J. C., "Ultrasensitive beam deflection measurement via interferometric weak value amplification," *Phys. Rev. Lett.* **102**, 173601 (Apr 2009).

[8] Jordan, A. N., Martínez-Rincón, J., and Howell, J. C., "Technical advantages for weak-value amplification: When less is more," *Phys. Rev. X* **4**, 011031 (Mar 2014).

[9] Li, D., Shen, Z., He, Y., Zhang, Y., Chen, Z., and Ma, H., "Application of quantum weak measurement for glucose concentration detection," *Applied optics* **55**(7), 1697–1702 (2016).

[10] Zhang, Y., Li, D., He, Y., Shen, Z., and He, Q., "Optical weak measurement system with common path implementation for label-free biomolecule sensing," *Optics letters* **41**(22), 5409–5412 (2016).

[11] Qiu, X., Xie, L., Liu, X., Luo, L., Li, Z., Zhang, Z., and Du, J., "Precision phase estimation based on weak-value amplification," *Applied Physics Letters* **110**(7), 071105 (2017).

[12] Li, D., He, Q., He, Y., Xin, M., Zhang, Y., and Shen, Z., "Molecular imprinting sensor based on quantum weak measurement," *Biosensors and Bioelectronics* **94**, 328–334 (2017).

[13] Aharonov, Y., Davidovich, L., and Zagury, N., "Quantum random walks," *Physical Review A* **48**(2), 1687 (1993).

[14] Berry, M. V., "Evanescent and real waves in quantum billiards and gaussian beams," *Journal of Physics A: Mathematical and General* **27**(11), L391 (1994).

[15] Berry, M. V. and Popescu, S., "Evolution of quantum superoscillations and optical superresolution without evanescent waves," *Journal of Physics A: Mathematical and General* **39**(22), 6965 (2006).

[16] Berry, M. V. and Dennis, M. R., "Natural superoscillations in monochromatic waves in d dimensions," *Journal of Physics A: Mathematical and Theoretical* **42**(2), 022003 (2009).

[17] Huang, F. M. and Zheludev, N. I., "Super-resolution without evanescent waves," *Nano Letters* **9**(3), 1249–1254 (2009).

[18] Wong, A. M. H. and Eleftheriades, G. V., "An optical super-microscope for far-field, real-time imaging beyond the diffraction limit," *Scientific Reports* **3**, 1715 EP – (04 2013).

[19] Aharonov, Y., Cohen, E., Carmi, A., and Elitzur, A. C., "Extraordinary interactions between light and matter determined by anomalous weak values," *Proc. R. Soc. A* **474**(2215), 20180030 (2018).

[20] Aharonov, Y., Cohen, E., and Tollaksen, J., "Completely top–down hierarchical structure in quantum mechanics," *Proceedings of the National Academy of Sciences* , 201807554 (2018).

[21] Mitchison, G., Jozsa, R., and Popescu, S., "Sequential weak measurement," *Phys. Rev. A* **76**, 062105 (Dec 2007).

[22] Piacentini, F. et al., "Measuring incompatible observables by exploiting sequential weak values," *Phys. Rev. Lett.* **117**, 170402 (Oct 2016).

[23] Thekkadath, G. S., Giner, L., Chalich, Y., Horton, M. J., Banker, J., and Lundeen, J. S., "Direct measurement of the density matrix of a quantum system," *Phys. Rev. Lett.* **117**, 120401 (Sep 2016).

[24] Georgiev, D. and Cohen, E., "Sequential weak values probe finite coarse-grained virtual feynman histories," *arXiv preprint arXiv:1709.08479* (2017).

[25] Avella, A., Piacentini, F., Borsarelli, M., Barbieri, M., Gramegna, M., Lussana, R., Villa, F., Tosi, A., Degiovanni, I. P., and Genovese, M., "Anomalous weak values and the violation of a multiple-measurement leggett-garg inequality," *Physical Review A* **96**(5), 052123 (2017).

[26] Pusey, M. F., "Anomalous weak values are proofs of contextuality," *Phys. Rev. Lett.* **113**, 200401 (Nov 2014).

[27] Piacentini, F. et al., "Experiment investigating the connection between weak values and contextuality," *Phys. Rev. Lett.* **116**, 180401 (May 2016).

[28] Ferrie, C. and Combes, J., "Weak value amplification is suboptimal for estimation and detection," *Physical review letters* **112**(4), 040406 (2014).

[29] Vaidman, L., "Comment on "weak value amplification is suboptimal for estimation and detection"," *arXiv preprint arXiv:1402.0199* (2014).

[30] Magaña-Loaiza, O. S., Harris, J., Lundeen, J. S., and Boyd, R. W., "Weak-value measurements can outperform conventional measurements," *Physica Scripta* **92**(2), 023001 (2016).

[31] Cohen, E. and Pollak, E., "Determination of weak values of quantum operators using only strong measurements," *Physical Review A* **98**(4), 042112 (2018).

[32] Abbott, A. A., Silva, R., Wechs, J., Brunner, N., and Branciard, C., "Anomalous weak values without post-selection," *Quantum* **3**, 194 (2019).

[33] Cohen, E., "Quantum measurements-yet another surprise," *Quantum Views* **3**, 27 (2019).

[34] Leggett, A. J., "Comment on "how the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100"," *Phys. Rev. Lett.* **62**, 2325–2325 (May 1989).

[35] Peres, A., "Quantum measurements with postselection," *Phys. Rev. Lett.* **62**, 2326–2326 (May 1989).

[36] Aharonov, Y. and Vaidman, L., "Aharonov and vaidman reply," *Phys. Rev. Lett.* **62**, 2327–2327 (May 1989).

[37] Duck, I. M., Stevenson, P. M., and Sudarshan, E. C. G., "The sense in which a "weak measurement" of a spin-1/2 particle's spin component yields a value 100," *Phys. Rev. D* **40**, 2112–2117 (Sep 1989).

[38] Ferrie, C. and Combes, J., "Weak value amplification is suboptimal for estimation and detection," *Phys. Rev. Lett.* **112**, 040406 (Jan 2014).

[39] Jordan, A. N., Martínez-Rincón, J., and Howell, J. C., "Technical advantages for weak-value amplification: When less is more," *Phys. Rev. X* **4**, 011031 (Mar 2014).

[40] Pang, S., Dressel, J., and Brun, T. A., "Entanglement-assisted weak value amplification," *Phys. Rev. Lett.* **113**, 030401 (Jul 2014).

[41] Dressel, J., "Weak values as interference phenomena," *Phys. Rev. A* **91**, 032116 (Mar 2015).

[42] Vaidman, L., "Weak value controversy," *Philos. T. Roy. Soc. A* **375**(2106) (2017).

[43] Kastner, R. E., "Demystifying weak measurements," *Found. Phys.* **47**(5), 697–707 (2017).

[44] Cohen, E., "What weak measurements and weak values really mean: Reply to kastner," *Found. Phys.* **47**, 1261–1266 (Jun 2017).

[45] Sinclair, J., Hallaji, M., Steinberg, A. M., Tollaksen, J., and Jordan, A. N., "Weak-value amplification and optimal parameter estimation in the presence of correlated noise," *Phys. Rev. A* **96**, 052128 (Nov 2017).

[46] Vaidman, L., Ben-Israel, A., Dziewior, J., Knips, L., Weißl, M., Meinecke, J., Schwemmer, C., Ber, R., and Weinfurter, H., "Weak value beyond conditional expectation value of the pointer readings," *Physical Review A* **96**(3), 032114 (2017).

[47] Piacentini, F. et al., "Investigating the effects of the interaction intensity in a weak measurement," *Sci. Rep.* **8**(1), 6959 (2018).

[48] Sokolovski, D. and Akhmatskaya, E., "An even simpler understanding of quantum weak values," *Ann. Phys.* **388**, 382 – 389 (2018).

[49] Ferrie, C. and Combes, J., "How the result of a single coin toss can turn out to be 100 heads," *Phys. Rev. Lett.* **113**, 120404 (Sep 2014).

[50] Brodutch, A., "Comment on "how the result of a single coin toss can turn out to be 100 heads"," *Phys. Rev. Lett.* **114**, 118901 (Mar 2015).

[51] Cohen, E., "A comment on 'how the result of a single coin toss can turn out to be 100 heads'," *arXiv preprint arXiv:1409.8555* (2014).

[52] Ipsen, A. C., "Disturbance in weak measurements and the difference between quantum and classical weak values," *Phys. Rev. A* **91**, 062120 (Jun 2015).

[53] Mundarain, D. F. and Orszag, M., "Quantumness of the anomalous weak measurement value," *Phys. Rev. A* **93**, 032106 (Mar 2016).

[54] Romito, A., Jordan, A. N., Aharonov, Y., and Gefen, Y., "Weak values are quantum: you can bet on it," *Quant. Stud.: Math. Found.* **3**, 1–4 (Apr 2016).

[55] Sinclair, J., Spierings, D., Brodutch, A., and Steinberg, A. M., "Interpreting weak value amplification with a toy realist model," *Phys. Lett. A* **383** (2019).

[56] Piacentini, F., Avella, A., Rebufello, E., Lussana, R., Villa, F., Tosi, A., Gramegna, M., Brida, G., Cohen, E., Vaidman, L., et al., "Determining the quantum expectation value by measuring a single photon," *Nature Physics* **13**(12), 1191 (2017).

[57] Aharonov, Y. and Vaidman, L., "Measurement of the schrödinger wave of a single particle," *Physics Letters A* **178**(1-2), 38–42 (1993).

[58] Aharonov, Y., Anandan, J., and Vaidman, L., "Meaning of the wave function," *Physical Review A* **47**(6), 4616 (1993).

[59] Aharonov, Y., Albert, D. Z., Casher, A., and Vaidman, L., "Surprising quantum effects," *Physics Letters A* **124**(4-5), 199–203 (1987).

[60] Dziewior, J., Knips, L., Farfurnik, D., Senkalla, K., Benshalom, N., Efroni, J., Meinecke, J., Bar-Ad, S., Weinfurter, H., and Vaidman, L., "Universality of local weak interactions and its application for interferometric alignment," *Proceedings of the National Academy of Sciences* **116**(8), 2881–2890 (2019).

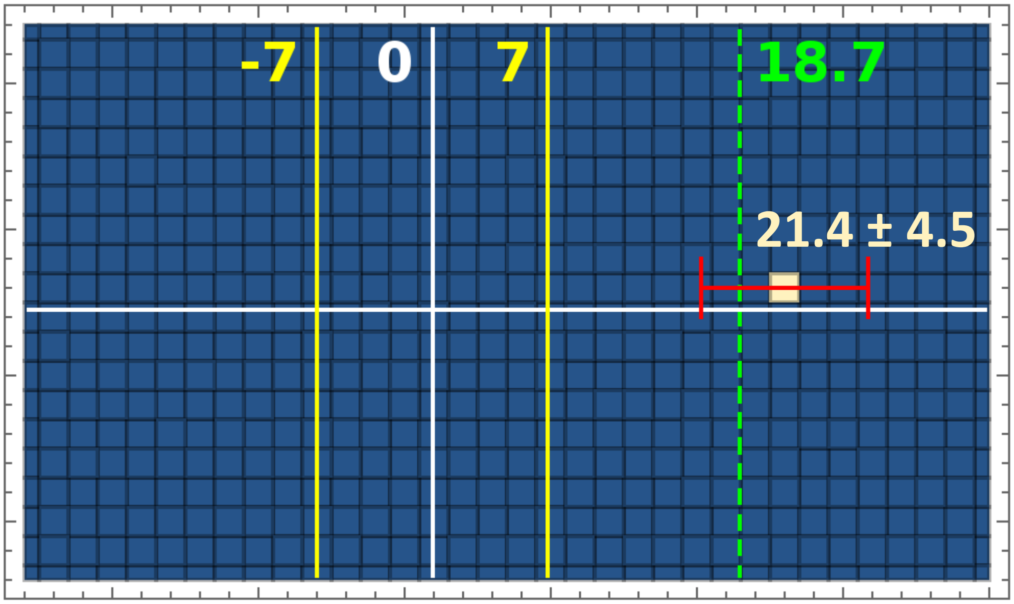


Figure 1. Robust weak measurement: single photon experiment. The yellow solid lines indicate the measured observable eigenspectrum bounds $[-7, 7]$, while the expected anomalous weak value (18.7) is highlighted by the green dashed line on the right. The firing pixel, in white, allows estimating a weak value of $21.4 \pm 4.5$, with the uncertainty (horizontal red bars) given by the width of the final spatial distribution of the photon itself.

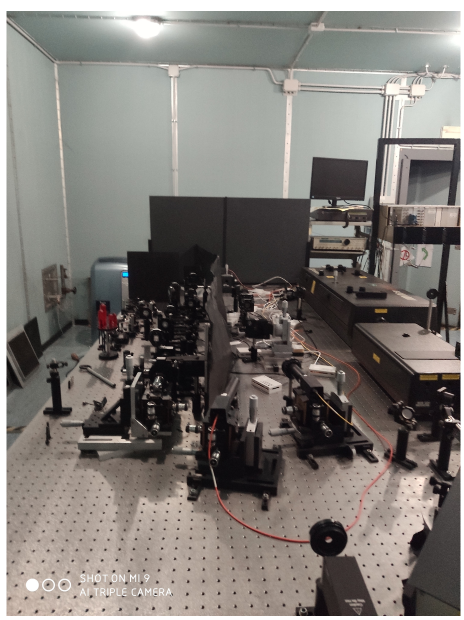


Figure 2. This picture presents our setup. On the right hand side the laser source. On the left hand side in foreground the PDC source, in background the detection system.

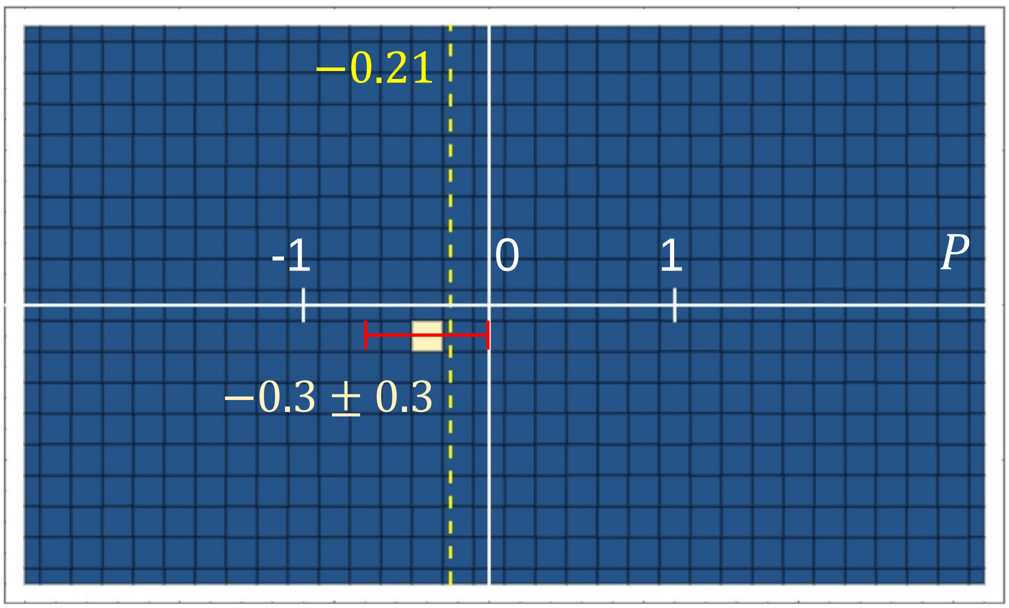


Figure 3. Protective measurement: single photon experiment. The yellow dashed line indicates the theoretically-expected value of the photon polarization expectation value (-0.21), while the white square shows the position of the single photon click, yielding the experimental value $\langle P \rangle = -0.3$. The uncertainty on that value, given by the width of the Gaussian spatial photon distribution, is $\pm 0.3$ (indicated by the red bars).