All the wing contributions to C evaluation

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All the Wing Contributions to C_{l_β} Evaluation

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Abstract

Introduction

The derivative $C_{l_\beta} = \partial C_l / \partial \beta$, that is the derivative of the rolling moment coefficient with respect to the sideslip angle, goes under the name of *Dihedral Effect*. This aerodynamic derivative is important, as widely known, in the study of the side-directional motion of the aircraft, and in particular in the determination of its dynamic characteristics concerning Dutch roll and spiral mode.

The dihedral angle of the wing, even if it gives its name to this derivative (which actually only explains the fact that the aircraft is subjected to a rolling moment L when it is invested by a sideslip angle β), is just one of the elements contributing to C_{l_β}.

Another wing element contribution is the interference with the fuselage.

The swept wings also contribute to C_{l_β}. A sweep-back angle Λ supplies a speed component V_n, which is aerodynamically useful because it is normal for the quarter line chord, and is different on the two semi-wings. This situation generates, like in the above-mentioned cases, a lift difference for the two semi-wings and therefore a rolling moment. C_{l_β} in this case is negative with sweep-back angles.

Even if a straight wing is invested by a sideslip angle, it can produce a rolling moment. In fact, two wing sections equidistant from the symmetrical plane have different distances from the global U-vortex wing system and particularly from the vortices of the tip wing. This is the reason why the induced speed and therefore the induced angle of attack is on the average different on the two semi-wings. This event always causes a lift difference of the two semi-wings, therefore generating a rolling moment. In this case the contribution to C_{l_β} is positive together with a positive sideslip angle.

Another wing element contribution to C_{l_β} is the shape of the tip of the wing. Anyway, this contribution can be omitted even when the wing has winglets.

Besides the wing, the vertical tail contributes considerably, as we know, to the C_{l_β}.

Among these contributions, three of them have the same order of size: swept wing angle, dihedral wing angle and vertical tail which can reach the same tenth (obviously with angles of a certain entity and conventional vertical tail). The wing aerodynamic induction contribution is of a smaller size (and in any case it depends on the C_L of flight), while the wing-body relative position contribution is of two orders smaller and the contribution of the shape of the tip of the wing is even of three orders smaller.

Studies Carried Out on the Matter

Two previous studies (presented at ICAS Congress)[5, 6], were designed to fine-tune a method for evaluating the contribution of the aerodynamic induction of the wing on the dihedral effect. For this purpose we have used a method which takes into account, also through integration along the span, the different circulation of the two semi-wings, each with a different flow situation, again in the presence of a sideslip angle. This method is Anderson's method modified by us for wings of whatever plant form and with any geometric twist.

Problem Formulation

Anderson's method doesn't provide for high swept angle wings and so we have now extended this method to sweep-back or sweep-forward angle wings and to wings with dihedral angle.

This method is based on the study of wings with finite aspect ratio, according to Prandtl classic model. When the wing geometry is known, all the other aerodynamic quantities can be deducted from the Γ circulation in every wing section. The Γ function can be determined by solving the Prandtl integral-differential equation:

$$\Gamma(y) \;=\; k(y)\,\pi\,V_\infty\,c(y)\cdot[\alpha_a(y)\;-\;\alpha_i(y)]$$

where the k coefficient, which adjusts the theoretical value 2π of the lift angular coefficient, will be a function of y if the wing has a variable section. The other terms, which can be easily understood, can be found in the *List of symbols.*

α_i is the value of the induced angle of attack, which has the following expression:

$$\alpha_i(y) \;=\; \frac{1}{4\pi V_\infty}\int_{-b/2}^{b/2}\frac{\frac{d\Gamma'}{dy'}}{y-y'}\,dy'$$

where the y co-ordinate is referred to the point where the induction is calculated and the y' is relative to the inducting vortex.

This equation is not easy to be solved, so that different methods of calculations have been devised, and they have led to the solution of the problem with excellent results, even if with some approximations. In this work, the method which has been used follows the Anderson process in its initial part, as modified in my previous work (see [5]) to eliminate some limitations and to adapt it to any shape of wings and to any twist laws.

The Anderson method replaces the Γ function with its series development and stops the series at a certain N value.

The first problem we have faced has been the determination of the N value, in order to obtain good results avoiding lengthy calculations. After few experiments, the number of the N sections and the criterion of angle subdivision have been defined. This criterion provides an increase of the subdivisions of the wing tips, which are the areas where the biggest calculation problems occur.

The aim of this research being the calculation of the dihedral effect on any type of wings, some calculation programs have been formulated. They had to allow for the geometrical shape of the wing and particularly the tapering, the aspect ratio, the twisting and the wing tip shape; moreover, it has been possible to prolong the treatment and also discuss about swept angles wings and dihedral angles wings.

This program can consider any type of wing tip geometry. By changing the wing geometry, the value of the coefficients of the N system of N equations in N unknown values also changes. Consequently the value of the other parameters which have been calculated will also be modified: for example, the wing lift coefficient and the rolling moment coefficient.

The calculation process which has been adopted allows for the fact that the induction of the wake vortices affects the lift coefficient, and therefore the rolling moment coefficient. For the calculation of the induced angle of attack, we have allowed for the Biot-Savart formula, which provides the induced speed value on a generic point P from a stream vortex whose intensity is $d\Gamma$:

$$\vec{V}_P \;=\; \int_s \frac{d\vec{\Gamma}\wedge\vec{r}}{4\,\pi\,r^3}\,\sigma\,ds$$

About a generic wing, we must consider the contribution of the wake vortices as well as of the adherent vortices to the induction. If the wings are straight, the adherent vortices will result to be "packaged" on the quarter line chord on the basis of Prandtl scheme, so that there will be no contribution of this type of vortices to the total induction. About the swept wings and/or wings with dihedral angle and/or wings whose quarter line chord is not linear, it must be said that this contribution is present, but the following treatment will not allow for it. In fact, because of the symmetry between the two semi-wings, the adherent vortices of any semi-wing induce one another in the same way, apart from the fact that the wing is invested or not by a sideslip angle. Therefore, the contributions of the two semi-wings to the rolling moment are both equal and opposed, and consequently there is no global contribution to the dihedral effect.

About the calculation of α_i, we must consider 3 contributions for every semi-wing:

1. Induction of a semi-wing on the other one.
2. Induction of a semi-wing on itself $\Rightarrow$ contribution of the vortices on the right of the point concerned.
3. Induction of a semi-wing on itself $\Rightarrow$ contribution of the vortices on the left of the point concerned.

The calculation process of the induction on the two semi-wings is the same, but we can find some differences on those equations that have be applied because of the different aerodynamic conditions of the two semi-wings when they are invested by a lateral wind ($\beta \neq 0$). These differences are only of geometrical type, so that the considerations about the right semi-wing can be easily applied to the left one: it is only necessary to give great care in calculating angles and distances.

We then wanted to verify, by using a substantially analogous method, the results obtained up to this point by following the aforementioned way.

Always using the Prandtl vortical scheme and the Biot-Savart formula to calculate the vortices induction in a generic point, we have made a direct calculation of α_i along the span. In order to do this, with a generic initial distribution of α_i, it is necessary to calculate the adherent vorticity Γ, from this the wake one e then the new distribution of α_i, by which, knowing the apparent angle of attack α_a on the wing, one can calculate the new adherent Γ, and so on, through iterative calculation, one can rapidly know, for each β taken into account, the effective trend of α_i along the span and from this the effective trend of c_l along the span.

The C_{l_β} trends, varying of the geometric characteristics of the wing, are comparable, with this second calculation program, to the previous ones.

Results and Numerical Examples

The following figures show an example of these trends.

The results are presented in the shape of curves showing the rolling moment coefficient C_l as a function of β for different α (see the example in figure), and in the summary curves that show C_{l_β} as a function of lift coefficient C_L (see the example in figure), being, as we know, two of these three contributions to C_{l_β} (dihedral wing angle, swept wing angle and aerodynamic induction) dependent on the different circulation of the two semi-wings and therefore dependent on the angle of attack (swept wing angle and aerodynamic induction).

Another type of representation included in the paper is that which combines, for a wing with certain characteristics (Aspect ratio, Wing taper ratio, Twist angle, etc.), all C_{l_β} for a predetermined range of Λ and γ. Such representation can be tridimensional or isolevel (see the enclosed figures).

<u>List of Symbols</u>

A	Aspect ratio
b	Wing span
$c(y)$	Local chord

Figure 1: Induced angle of attack along the wing span.

Figure 2: Lift coefficient along the wing span.

Figure 3: C_l as a function of β for different α.

Figure 4: C_{l_β} as a function of C_L for various configurations.

Figure 5: C_{l_β} as a function of Λ and γ for a certain configuration (tridimensional).

Figure 6: C_{l_β} as a function of Λ and γ for a certain configuration (isolevel).

C_l	Rolling moment coefficient
C_{l_β}	Dihedral effect $= \partial C_l / \partial \beta$
L	Rolling moment
r	Wing taper ratio $= c_t / c_r$
$\vec{r}$	Distance of P from vortex
s	Vortex co-ordinate
V	Wind speed $= V_\infty$
y	Generic co-ordinate of a wing section $= b/2 \cos\theta$
y'	Vortex co-ordinate
α	Wing angle of attack
α_a	Section apparent angle of attack
α_i	Section induced angle of attack
β	Sideslip angle
γ	Dihedral wing angle
$\Gamma(y)$	Circulation along the wing span
ε^{rad}	Twist in radiants from root to tip
Λ	Swept wing angle
σ	Vortex section

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