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Low-complexity Greedy Algorithm in Compressed Sensing for the Adapted Decoding of ECGs

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***Abstract*—The balanced weighted orthogonal matching pursuit (bWOMP) algorithm for recovering signals in compressed sensing (CS) based system is presented as a specialized recovering tool for Electrocardiograph (ECG) signals. Being based on the standard OMP approach, bWOMP is a lightweight reconstruction algorithm both in terms of complexity and memory footprint. Furthermore, the concept of weighting is introduced in the algorithm by exploring a prior knowledge on ECG signals. Experimental results show a performance increase of about 10 dB with respect to the standard OMP approach, and also an increase with respect to the decoding approaches considered as the state-of-the-art. In this case the gain could be as high as 4 dB with respect to the best of currently known decoding approaches.**

I. Introduction

Compressed Sensing (CS) is a recently introduced paradigm [1] capable under some assumptions to perform a sub-Nyquist sampling, i.e., to acquire an input signal with a number of measurements smaller with respect to what expect according to its bandwidth. This interesting feature, paired with a simple sampling mechanism (projection of the input signal on a set of typically random sampling waveforms) has received increasing attention in particular in the area of biomedical signal processing. Properties of these signals, in fact, allow CS to exploit all its capabilities. A typical field of interest is given by Body Area Sensor Networks (BASNs) [2], [3] where many micro-power sensor nodes take advantage from the very simple CS-based sensing architecture. This is why, in the recent literature, efforts have concentrated on improving CS acquisition (f.i. exploiting *rakeness* [4][1]) and it is already possible to find many biomedical signals acquisition systems based on CS [7], [8], [9], [10].

Such as sampling advantage has a cost: the complexity of the recovery stage, which typically relies on the solution of a convex optimization problem [11]. Even if this may not be critical (decoding in BASNs is executed on a central gateway where energy is not an issue), reducing the energy required for decoding is a trending topic and many reconstruction algorithms have been proposed which allow a large reduction in the computational cost. In particular, *greedy* algorithms [12], [13] are worth mentioning, which are based on an iterative approach that generates an intermediate and approximate solution converging, with a speed varying with the algorithm, to the correct one after a number of steps.

Furthermore, some papers have recently proposed algorithms specialized over a specific class of biosignals (ECG f.i. as in [14], [15]). These approaches, exploiting some signal statistical properties, are able to boost reconstruction quality.

The aim of this paper is to propose a new specialized reconstruction algorithm, namely bWOMP, based on the standard Orthogonal Matching Pursuit (OMP) method developed in [12], that is one of the fastest and simplest reconstruction algorithm known [16], and modified to include specialization for ECG signals. In detail, we exploit prior information on the ECG signal statistic characterization as in the the Weighted ℓ_1 Minimization (WLM) method proposed in [15].The outcome is an algorithm that is still as fast and simple as the original OMP approach, but capable of increasing reconstruction quality with respect to OMP (in average, of $10\,\mathrm{dB}$) and also when compared to WLM (in some cases up to $4\,\mathrm{dB}$).

The paper is organized as follows. Section II introduces the basic concepts of CS. In Section III we summarize the OMP and the WLM decoding approach, and describe the proposed bWOMP algorithm. In Section IV we show performance results for ECG signal decoding. Finally, we draw the conclusion.

II. CS Fundamentals

Let $x \in \mathbb{R}^n$ be an instance of a discrete-time signal, defined by its n Nyquist-rate samples. CS can be used to sample x under the following assumptions. First, let x be κ-sparse, i.e., a proper orthonormal n-dimensional *sparsity basis* $S \in \mathbb{R}^{n\times n}$ exists, in which any instance $x = S\xi$, where $\xi \in \mathbb{R}^n$ has no more than $\kappa \ll n$ non-zero components. We refer to the κ non-zero components of ξ as the signal support. Then, let $A \in \mathbb{R}^{m\times n}$ be a *sensing matrix* such that A posses properties required by CS theory [11] and $B = AS$.

With this, it is possible to get a representation of x by means of $m < n$ measurements only, by means of the linear projection of x over a sensing matrix

$$y = Ax + \nu = B\xi + \nu \tag{1}$$

where $y \in \mathbb{R}^m$ is a vector collecting the m measurements, and ν is an additive disturbance term used to model non-

[1] One exploits the statistical features of input signals to increase signal reconstruction quality by suitably design matched acquisition sequence. This is similar to what happens in (chaos-based) DS-CDMA communication, where chip waveforms, spreading sequence statistics and rake receivers taps are jointly selected to collect (rake) as much energy as possible at the received side [5][6].

Algorithm 1 Pseudocode for OMP.

Require: $y \in \mathbb{R}^m$ vector of measurements
Require: $B = AS \in \mathbb{R}^{m \times n}$ sensing operator in the sparse domain
$\hat{\xi}_0 \leftarrow$ empty vector {initial guess}
$r_0 \leftarrow y$ {error in reproducing measurements from initial guess}
$\Phi_0 \leftarrow$ empty matrix
$i \leftarrow 1$
repeat
  $j = \arg\max_k |\langle r_{i-1}, b_k \rangle|$ {look for the most correlated row of B}
  $\Phi_i \leftarrow [\Phi_{i-1}\ b_j]$
  $\hat{\xi}_i = \arg\min_{\xi_i \in \mathbb{R}^i} \|y - B\xi_i\|_2$
  $r_i \leftarrow y - B\hat{\xi}_i$ {new error in reproducing measurements}
  $i \leftarrow i + 1$
until convergence
$\hat{\xi}$ is the sparse vector whose non-null elements are given by $\hat{\xi}_i$

idealities such as the quantization error or the signal noise. The decoding stage takes the m measurements and reconstruct the input signal as $\hat{x} = S\hat{\xi}$, where $\hat{\xi}$ is the solution of the optimization problem

$$\hat{\xi} = \arg\min_{\xi} \|\xi\|_1 \quad \text{s.t. } \|B\xi - y\|_2 < \varepsilon \tag{2}$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ are the standard ℓ_1 and ℓ_2 norms, and ε bounds the effects of ν. Such an approach is called basis pursuit with denoising (BPDN). In other words, $\hat{\xi}$ is found by looking at vectors ξ that solve (1) with a proper tolerance. Being an ill-posed problem, multiple solutions exists; CS states that the correct one is the sparsest [11]. Sparsity is generally promoted by the ℓ_1 norm instead of the typically computationally intractable count of non-zero components given by ℓ_0 norm.

The interest in CS is mainly due to the fact that, in order to ensure the required properties of A, it is enough to randomly draw its elements as independent and identically distributed random variables. In this case, correct reconstruction is guaranteed if $m \geq \mathcal{O}(\kappa \log(n/\kappa))$ [1].

III. Signal Recovery Techniques

A. OMP and the Greedy Recovery Approach

Solving (2) is a computationally complex task. To allow simpler signal reconstruction, many algorithms have been proposed relying on greedy approaches that iteratively promote sparsity by observing intermediate and approximate solutions. Despite being less rigorous than any straightforward approach, they ensure a lower complexity and hence lower computational costs, with an asymptotic solution that, given some assumptions, is a very good approximation of the real one.

In this paper we consider *Orthogonal Matching Pursuit* (OMP) introduced in [12]. This is one of the most common iterative approaches, due to its low cost both in terms of complexity (i.e., energy) and memory footprint [16].

The working principle can be summarized as follows. Let us indicate with b_j the j-th column of $B = AS$, and with $\hat{\xi}_i \in \mathbb{R}^i$ and $r_i = y - B\hat{\xi}_i \in \mathbb{R}^n$ the approximate intermediate solution at the i-th step and the residual error between actual measurements and that generated by $\hat{\xi}_i$, respectively. Let also

Algorithm 2 Pseudocode for bWOMP, where the differences with respect to OMP have been highlighted.

Require: $y \in \mathbb{R}^m$ vector of measurements
Require: $B = AS \in \mathbb{R}^{m \times n}$ sensing operator in the sparse domain
$\hat{\xi}_0 \leftarrow$ empty vector {initial guess}
$r_0 \leftarrow y$
$\Phi_0 \leftarrow$ empty matrix
$i \leftarrow 1$
repeat
  $\boxed{j = \arg\max_k \left|(1-\gamma)\langle r_{i-1}, b_k \rangle + \gamma \langle r_{i-1}, w_{k,k} b_k \rangle\right|}$
  $\Phi_i \leftarrow [\Phi_{i-1}\ b_j]$
  $\hat{\xi}_i = \arg\min_{\xi_i \in \mathbb{R}^i} \|y - B\xi_i\|_2$
  $r_i \leftarrow y - B\hat{\xi}_i$ {new error in reproducing measurements}
  $i \leftarrow i + 1$
until convergence
$\hat{\xi}$ is the sparse vector whose non-null elements are given by $\hat{\xi}_i$

be $\Phi_i \in \mathbb{R}^{n \times i}$ a matrix for temporary data. By pursuing $r_i \to 0$, we get the desired solution.

In detail, starting with Φ_0 an empty matrix, $\hat{\xi}_0$ an empty vector and $r_0 = y$, in each iteration OMP looks for the column of B that is the most strongly correlated with r_{i-1}. Indicating with j the index of this column, b_j is simply found by looking for the largest (in module) scalar product $\langle r_{i-1}, b_k \rangle$, $k = 1 \ldots n$, or equivalently by looking for the element with largest value (in module) of the vector $B^\top r_{i-1}$. Then, b_j is added as the i-th column of the auxiliary matrix Φ_i, and $\hat{\xi}_i$ is computed as the vector that minimizes r_i as solution of a least squares problem. When convergence is achieved, $\hat{\xi} \in \mathbb{R}^n$ is simply obtained as the sparse vector whose non-null entries are the elements of $\hat{\xi}_i$ taken in the correct order.

The pseudocode for OMP can be found in Algorithm 1, while a more detailed description in [12].

Interesting, from a computational point of view the OMP code could be simplified observing that r_i is always orthogonal to Φ_i. As a consequence, the least squares minimization problem in each step can be solved with marginal computational cost by using a modified Gram-Schmidt algorithm exploiting a companion system regulated by the orthonormalized matrix $\hat{\Phi}_i$ that has to be constructed step by step along with the Φ_i.

B. WLM and the Adapted Recovery Approach

Many different reconstruction algorithms specialized over a proper class of signals have been proposed so far in the literature with the aim of increasing reconstruction quality using some priors on the signal.

In this paper we focus on the Weighted ℓ_1 Minimization (WLM), that is a decoding algorithm specialized for ECGs proposed in [15] and based on prior information on ECG statistic characterization. In more detail, in the WLM approach the optimization problem (2) is replaced by

$$\hat{\xi} = \arg\min_{\xi} \frac{1}{2}\|B\xi - y\|_2^2 + \lambda \|W\xi\|_1 \tag{3}$$

where λ is a normalization value (set to $\lambda = 0.1$ according to authors' suggestion) and W is an $n \times n$ diagonal matrix

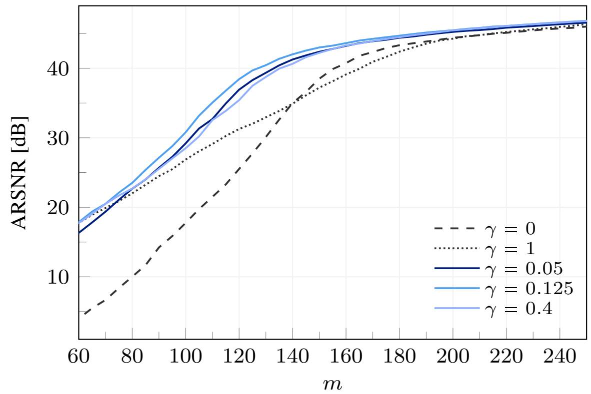

Fig. 1. bWOMP reconstruction performance in terms of ARSNR as a function of m, for a few values of γ.

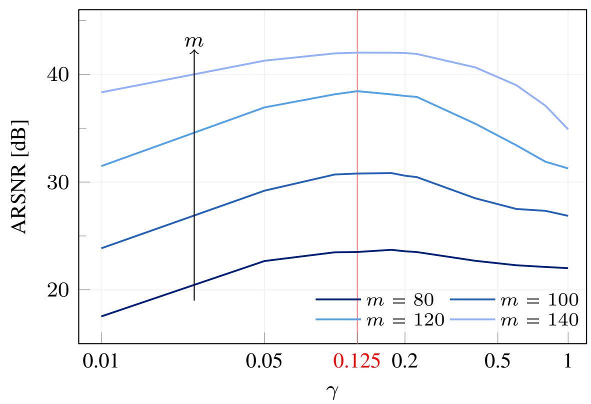

Fig. 2. bWOMP reconstruction performance in terms of ARSNR as a function of γ, for a few values of m.

whose entries are related to the probability of each column of S to be included in the support of an ECG signal [15].

The intuition is to minimize both quantities $\|B\xi - y\|_2^2$ (that is constrained in (2) by ε), and $\|W\xi\|_1$, that is the ℓ_1 norm of ξ weighted by W. The parameter λ set the trade-off between the two quantities to be minimized. As shown in [15], the approach is capable of outperforming many other known decoding approaches.

C. The Proposed bWOMP Approach

We propose in this section the balanced weighted OMP approach. The basic idea is the same as in WLM: give to components of S that are more frequently observed in signal representation a better chance to be present in the reconstructed signal, but using an iterative reconstruction algorithm as simple as OMP.

This is obtained with a very small and computationally negligible modification of the original OMP algorithm. In detail, instead of looking for the column of B that is the most strongly correlated with r_{i-1}, we introduce the weighting concept by means of a diagonal matrix W as in WLM, and compute the column of B to be added as

$$j = \arg\max_k |(1-\gamma)\langle r_{i-1}, b_k\rangle + \gamma\langle r_{i-1}, w_{k,k} b_k\rangle|$$

where $w_{k,k}$ is the (k,k)-th elements of W, or more practically by looking at the largest elements (in module) of the vector

$$(1-\gamma)B^\top r_{i-1} + \gamma W B^\top r_{i-1}$$

where in both cases γ is a parameter regulating the importance of the W matrix in the choice. Note that $\gamma = 0$ is the standard OMP approach.

IV. Results

In this section, by means of results of Montecarlo simulations, we analyze performance of the proposed bWOMP approach and compare it with that of other state-of-the-art CS reconstruction algorithms.

In all cases the simulation setting is the following. The input signal is a synthetic ECG generated as in [17], with an average heart-rate of 60 bpm, sampled at 360 Hz. The signal is then quantized using 11 bits in order to better emulate a realistic system.

The quantized ECG signal is then encoded by a CS system using $n = 512$ and different values of m. A is a random antipodal matrix where $+1$ and -1 occur with the same probability. Finally, signal has been reconstructed using the algorithms under test. The sparsity basis considered in reconstruction is the Symlet-6 wavelet basis, allowing us to use the weight matrix W computed as in [15].

As the main figure of merit to express reconstruction quality, we use Average Reconstruction SNR (ARSNR), defined as the average observed value of the reconstruction SNR

$$\text{RSNR} = 20\log_{10}\left(\frac{\|x\|_2}{\|x - \hat{x}\|_2}\right)$$

over all the different simulated instance. Provided values of ARSNR has been computed by averaging over 500 different combinations of x and A.

Figure 1 shows the ARSNR achieved by bWOMP as a function of m for different values of γ. The case $\gamma = 0$ (OMP decoding) has been included. bWOMP shows an outstanding advantage with respect to OMP, with performance better than 10 dB and more for all values of m for which ARSNR has not reached the saturation level imposed by the input signal quantization noise. Interestingly, $\gamma = 0.125$ ensures better performance with respect to any other value.

The optimum γ value is confirmed by Figure 2, showing the ARSNR achieved by bWOMP as a function of γ for different values of m. All the plots in the figure present a non-monotonic behavior, with a maximum achieved for $\gamma \approx 0.125$.

An example of the ECG signal reconstructed by bWOMP with $\gamma = 0.125$ and $m = 140$ is depicted in Figure 3, with RSNR $= 42.4$ dB.

Finally, we propose in Figure 4 the performance comparison between bWOMP and the decoding approaches currently

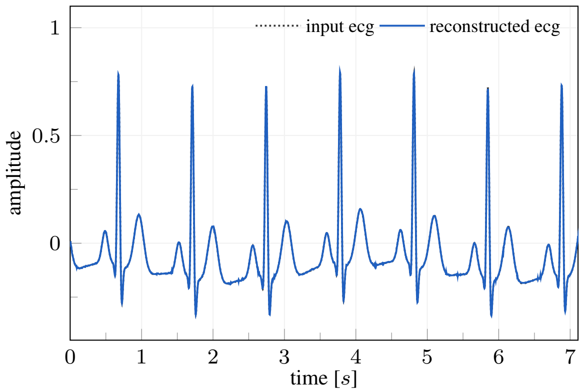

Fig. 3. Five consecutive windows of input signal (dotted line) with the corresponding reconstructed signal by bWOPM (solid line). Reconstruction refers to $m = 140$ that correspond to a compression ratio equal to 3.66.

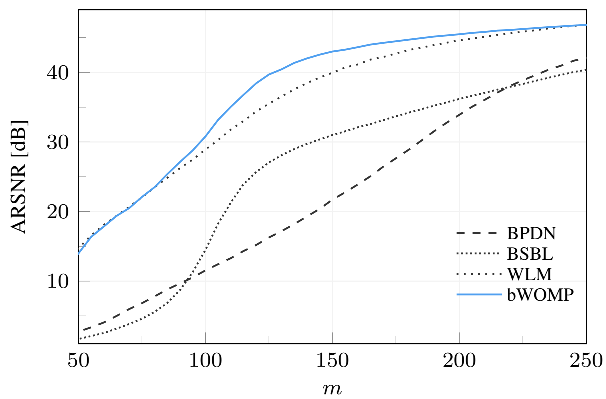

Fig. 4. Performance comparison in terms of signal reconstruction quality for the basic pursuit with denoising (BPDN), block sparse Bayesian learning (BSBL), weighted ℓ_1 minimization (WLM) and weighted OMP (bWOMP) approaches.

considered as the state-of-the-art for ECG signals. In detail, we have considered the BPDN problem solved by means of the SPGL1 toolbox[2], and the two adapted recovery approaches BSBL and WLM. The Block Sparse Bayesian learning (BSBL) is an approach proposed in [14] that exploits the block sparsity hypothesis to correctly decode signal in ECG applications. In the comparison, the value $\gamma = 0.125$ has been used for bWOMP.

The advantage of bWOMP with respect to the standard, signal-agnostic BPDN and the adapted BSBL is remarkable. Furthermore, bWOMP also outperforms WLM. For small and large values of m there is a minor advantage, that however grows to $4\,\mathrm{dB}$ for intermediate m values.

V. Conclusion

In this paper a reconstruction algorithm for CS systems, namely bWOMP, is introduced as an approach adapted for ECG signals. Being based on the iterative and greedy OMP algorithm, it presents low computational complexity and memory footprint. However, at the same time exploits a prior on the statistics of ECG signals similar to that used in the WLM approach. With this, bWOMP is shown to increase performance in signal decoding quality, in average, by $10\,\mathrm{dB}$ with respect to the standard OMP. Furthermore, performance are also improved with respect to the WLM, with an increase evaluated in up to $4\,\mathrm{dB}$.

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