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Multiple Wedges Diffraction in Propagation Problems using the Generalized Wiener-Hopf Technique

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Abstract— **In this work, in order to accurately predict diffraction phenomena in propagation problems, we introduce the analysis of the scattering of multiple wedges using the semi-analytical method known as Generalized Wiener-Hopf Technique. The analysis is of interest to correctly model path-loss in real-life scenarios for wireless communications.**

Keywords— Wedges, Wiener-Hopf method, Integral equations, Electromagnetic diffraction, Near-field interactions, Propagation.

I. INTRODUCTION

Accuracy in study of diffraction problems is of great interest in wireless propagation and security applications. In particular when the intensity of field is a sensible issue, the propagation model needs to take into account the correct modelling of diffraction phenomena. In this paper we consider the diffraction by multiple perfectly electrically conducting (PEC) wedges in separated objects as in Fig. 1.a or in joined objects as in Fig. 1.b. Both configurations have been studied in literature. In particular for the separated wedges we report a wide literature listed in [1], while for the joined wedges we acknowledge the works [2-6] and references therein. In wireless propagation, often, the modelling scheme of the interaction among objects or sharp structures make reference to ray-tracing or iterative schemes like iterative physical optics. Usually a far field assumption among the edges of the wedges is taken into account.

In this work we introduce the analysis of the scattering of multiple wedges using the semi-analytical method known as Generalized Wiener-Hopf Technique (GWHT) [7] to improve the estimation of field in presence of diffraction by multiple wedges. The Generalized version of the Wiener-Hopf technique is now able to study new classes of problems for its capability to handle complex scattering problems constituted of angular, rectangular and layer shapes made by different materials (see [8]-[11] and references therein). Our mathematical model is comprehensive i.e. it takes into account the entire structure in one shot and it is independent on geometrical distance of edges. Moreover, the Generalized Wiener-Hopf equations (GWHEs) obtained by the application of the method are defined in the spectral domain and their solution in terms of spectral transformation of the field components contains all the physical properties of the problem.

In general, the GWHEs cannot be solved exactly. To overcome this limitation we resort to the Fredholm Factorization [12]. The Fredholm factorization allow to obtain semi-analytical solutions of GWHEs of a given problem with high accuracy and efficiency. The complete solution procedure consists of the following steps: 1) deduction of GWHEs, 2) Fredholm factorization, 3) analytic continuation of the approximate solution, 4) evaluation of field components via inverse spectral transformation and asymptotics.

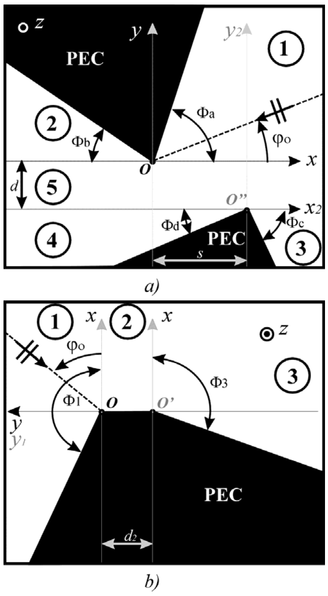

Fig. 1. Scattering by multiple wedges: a) separated wedges, b) joined wedges

II. FORMULATION

The procedure starts form subdividing the geometry of the problem into sub-regions which are homogeneous in geometry and material as for example the angular regions 1,2,3,4 and the layer region 5 in Fig. 1.a, while in Fig. 1.b we have the angular regions 1 and 3 and the half-layer region 2. For each sub-region we deduce the relevant GWHEs. In this paper we need the model of three kind of sub-regions. The GWHEs are written in terms of unilateral Laplace transforms and to model this problem we make use: radial Laplace transforms, Fourier transforms and their split into unilateral forms. All this quantities will be interpreted as WH unknowns.

Making reference to region 1, taking into account a cylindrical reference system, the radial Laplace transforms for the Ez-polarized case at $\varphi=0,\Phi$ are

$$V_+(\eta)=\int_0^\infty E_z(x,0)e^{j\eta x}dx,\ I_+(\eta)=\int_0^\infty H_x(x,0)e^{j\eta x}dx \quad (1)$$

$$I_{a+}(-m)=\int_0^\infty H_\rho(\rho,\Phi)e^{-jm\rho}d\rho$$

With reference to region 5, taking into account a Cartesian reference system, the Fourier transforms (2) for the Ez-polarized case at y=0,-d are splitted into unilateral transforms yielding (3)-(6) with shift factor s [13] in (5)-(6):

$$v(\eta,y)=\int_{-\infty}^\infty E_z(x,y)e^{j\eta x}dx,\ i(\eta,y)=\int_{-\infty}^\infty H_x(x,y)e^{j\eta x}dx \quad (2)$$

$$V_{1+}(\eta)=\int_0^\infty E_z(x,0)e^{j\eta x}dx,\ I_{1+}(\eta)=\int_0^\infty H_x(x,0)e^{j\eta x}dx \quad (3)$$

$$V_{1\pi+}(\eta)=\int_{-\infty}^0 E_z(x,0)e^{-j\eta x}dx,\ I_{1\pi+}(\eta)=-\int_{-\infty}^0 H_x(x,0)e^{-j\eta x}dx \quad (4)$$

$$V_{2+}(\eta)=e^{-j\eta s}\int_s^\infty E_z(x,-d)e^{j\eta x}dx,\ I_{2+}(\eta)=e^{-j\eta s}\int_0^\infty H_x(x,-d)e^{j\eta x}dx \quad (5)$$

$$V_{2\pi+}(\eta)=e^{j\eta s}\int_{-\infty}^s E_z(x,-d)e^{-j\eta x}dx,\ I_{2\pi+}(\eta)=-e^{j\eta s}\int_{-\infty}^s H_x(x,-d)e^{-j\eta x}dx \quad (6)$$

For region 2 of Fig. 1.b, taking into account a Cartesian reference system, we define the odd-Fourier transforms

$$\hat{F}(\alpha)=2j\int_0^\infty f(x)\sin(\alpha x)dx=F_+(\alpha)-F_+(-\alpha), \quad (7)$$

that can be used to model the field components in the half-slab/layer region as usually done for the layered regions.

A. Angular region

Without loss of generality in this sub-section we report the GWHE relevant for sub-region 1 at Ez polarization:

$$Y_c(\eta)V_+(\eta)-I_+(\eta)=-I_{a+}(-m) \quad (9)$$

with $m=-\eta\cos\Phi+\xi\sin\Phi$, $\xi=\sqrt{k^2-\eta^2}$, $Y_c(\eta)=\xi(\eta)/k\,Z_o$. Similar equations hold for the other angular sub-regions taking into account the different reference systems and spectral unknowns. By applying Fredholm factorization we obtain an integral representation that eliminates the minus unknowns defined on the PEC face:

$$I_+(\eta)=\mathcal{Y}V_+(\eta)+I_{ca}(\eta) \quad (10)$$

$$\mathcal{Y}=Y_c(\eta)[...]+\frac{1}{2\pi j}\int_{-\infty}^\infty\left(\frac{Y_c(\eta')}{\alpha(\eta')-\alpha(\eta)}\frac{d\alpha}{d\eta'}-\frac{Y_c(\eta)}{\eta'-\eta}+\right)[...]d\eta' \quad (11)$$

With $\alpha(\eta)=-k\cos[(\pi/\Phi)\arccos(-\eta/k)]$, Φ is obtuse, and I_{ca} depends on the sources. This integral representation can be interpreted as a constitutive equation of an equivalent network model of Norton kind. We need to pay particular attention in case of acute aperture angles Φ [14]. In this case we need to add extra terms to the model [9] due to the mathematical properties of the kernel known as sectional analytic functions.

B. Slab/Layer region

Transmission line modelling can be used to model multi-layered region [7]. In our case the following equivalent network model constituted of two ports holds [13]:

$$-I_{1+}(\eta)+I_{1-}(\eta)=Y_{11}(\eta)[V_{1+}(\eta)+V_{1-}(\eta)]+Y_{12}(\eta)[V_{2+}(\eta)+V_{2-}(\eta)]e^{j\eta s} \quad (12)$$
$$e^{j\eta s}[I_{2+}(\eta)+I_{2-}(\eta)]=Y_{21}(\eta)[V_{1+}(\eta)+V_{1-}(\eta)]+Y_{22}(\eta)[V_{2+}(\eta)+V_{2-}(\eta)]e^{j\eta s}$$

with $Y_{ii}(\eta)=-jY_o(\eta)\cot[\xi(\eta)d]$, $Y_{ij}(\eta)=jY_o(\eta)/\sin[\xi(\eta)d]$.

The application of Fredholm factorizations to (12) yields a system of integral representations that can be interpreted as a 4-port Norton equivalent network.

C. Half-slab/layer region

The classical transmission line modelling for multilayered regions can be extended to half-layers by considering odd and even Fourier transforms. For our problem (PEC boundary conditions in region 2 of Fig. 1.b) we use odd-Fourier transforms at Ez polarization. By noting that $\hat{F}(\eta)=F_+(\eta)-F_+(-\eta)$ and applying Fredholm factorization we obtain a two port equivalent network model.

III. Solution of the Problems

The problems represented in Fig. 1 can be interpreted as connection of equivalent network models as reported in the previous sections. By coupling the models and eliminating the equivalent current unknowns $\boldsymbol{I}$ we obtain a system of a system of Fredholm integral equations of second kind with only voltage unknowns $\boldsymbol{V}$. Solution is performed by analytical-numerical techniques in terms of approximate spectra of the voltages. Asymptotic estimation via UTD of the total field is performed from the spectra. Further details on the formulation, numerical validations and results will be shown during the presentation.

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