

The Scattering by Two Inverted Staggered PEC Half-Planes Loaded by a Dielectric Layer

*Original*

The Scattering by Two Inverted Staggered PEC Half-Planes Loaded by a Dielectric Layer / Daniele, V.; Lombardi, G.; Zich, R. S. - ELETTRONICO. - 2018 IEEE Antennas and Propagation Society International Symposium and USNC/URSI National Radio Science Meeting, APSURSI 2018 - Proceedings:(2018), pp. 2257-2258. (Intervento presentato al convegno 2018 IEEE Antennas and Propagation Society International Symposium and USNC/URSI National Radio Science Meeting, APSURSI 2018 tenutosi a The Westin Boston Waterfront, usa nel 2018) [10.1109/APUSNCURSINRSM.2018.8609170].

*Availability:*

This version is available at: 11583/2784774 since: 2020-01-24T11:47:03Z

*Publisher:*

Institute of Electrical and Electronics Engineers Inc.

*Published*

DOI:10.1109/APUSNCURSINRSM.2018.8609170

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

IEEE postprint/Author's Accepted Manuscript

©2018 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collecting works, for resale or lists, or reuse of any copyrighted component of this work in other works.

(Article begins on next page)

# The Scattering by Two Inverted Staggered PEC Half-Planes Loaded by a Dielectric Layer

Vito Daniele, Guido Lombardi, Rodolfo S. Zich

Politecnico di Torino- ISMB

Torino, Italy

vito.daniele@polito.it, guido.lombardi@polito.it, rodolfo.zich@torinowireless.it

**Abstract**—The scattering of a plane electromagnetic wave by two inverted perfectly electrically conducting (PEC) half-planes loaded by a dielectric layer is studied by using the Wiener-Hopf technique in the spectral domain. The procedure to obtain the solution is based on the reduction of the factorization problem to Fredholm integral equations of second kind. The structure is of interest in antenna technologies, electromagnetic compatibility and electromagnetic shielding. The study of structural singularities allows also the estimation of surface waves and leaky waves for practical engineering applications.

**Keywords**—Electromagnetic scattering, Diffraction, Wiener-Hopf method, Integral equations, Circuitual modelling, Stratified regions, Wedge, Step, Analytical-numerical methods, Antenna technologies, Electromagnetic compatibility, Electromagnetic shielding, Surface Waves and Leaky Waves.

## I. INTRODUCTION

In this paper we consider the classical canonical scattering problem constituted of two inverted staggered Perfectly Electrically Conducting (PEC) half planes separated by a dielectric layer.

Cartesian coordinates as well as polar coordinates will be used to describe the problem. Two origins are considered, see Fig. 1: the edge of the upper half-plane is chosen as origin  $O$  for coordinates  $(x,y,z)$  and  $(\rho, \varphi, z)$ , while the edge of the lower half-plane is chosen as origin  $O'$  for coordinates  $(x_2, y_2, z_2)$  and  $(\rho_2, \varphi_2, z_2)$ . The two reference systems are related through the following relations  $x_2 = x - s$ ,  $y_2 = y + d$ .

Fig.1 shows the two PEC half planes with zero thickness respectively defined by  $x < 0, y = 0, -\infty < z < \infty$  and  $x > s, y = -d, -\infty < z < \infty$ ; the problem is with translational symmetry along the  $z$  axis.

In the following we will consider  $s$  as the staggered parameters along  $x$  and  $d$  as the distance along  $y$  between the two half-planes. While  $d$  can assume only positive values,  $s$  can be either positive as in Fig. 1 or negative. In the last case we have that the two half-planes partially overlap.

Three regions are identified: region 1 is the half space region delimited by  $y > 0$  filled by free-space, region 2 is the rectangular finite region with  $-d < y < 0$  and characterized by the dielectric with relative dielectric permittivity  $\epsilon_r$  and region 3 is the half space region delimited by  $y < -d$  filled by free-space.

For the sake of simplicity, the structure is illuminated by an E-polarized plane wave from region 1 with azimuthal direction  $\varphi = \varphi_0$  ( $0 < \varphi_0 < \pi$ ) and with propagation constant  $k$ :

$$E_z^i = E_0 e^{jk\rho \cos(\varphi - \varphi_0)} \quad (1)$$

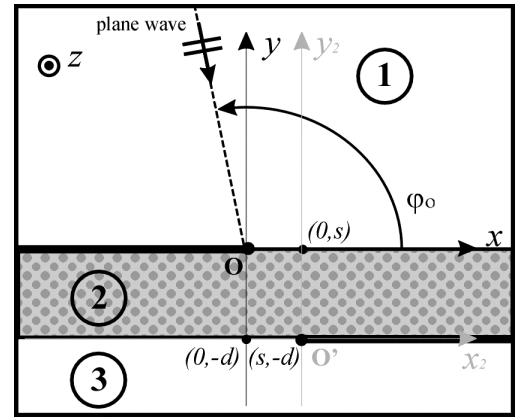


Fig. 1. Scattering by Two Inverted Staggered PEC Half-Planes Loaded by a Dielectric Layer

The literature shows several works that are related to this problem. In particular [1-2] study the problem in absence of dielectric layer via matrix Wiener-Hopf equations. These formulations highlight the presence of problematic exponential behavior of the spectral unknowns depending on the staggered parameter  $s$ . Other similar problems are described in [3-6].

With the reference to the present structure under investigation, in this work, we formulate the problem in terms matrix Wiener-Hopf equations in the spectral domain.

The procedure to obtain the solution is based on the reduction of the factorization problem to Fredholm integral equations of second kind. This technique is effectively applied in several scattering problems and described in [7,8].

The spectral formulation of the problem reported in the following section will take into consideration the interaction at near field of the edges of the two edges located at  $(x,y)=(0,0)$  and  $(x,y)=(s,-d)$  and the presence of the dielectric layer by using a unique entire model.

The solution steps are based on: 1) formulation via the Wiener-Hopf (WH) technique [8] with the help of network modelling [9-11], 2) the Fredholm factorization [7,8] and 3) asymptotic estimations of electromagnetic field [12].

## II. WIENER-HOPF FORMULATION AND STEPS FOR THE SOLUTION

In region 1 the following WH equation (2) holds [8,13-14] in terms WH unknowns (3) defined at  $y=0$ .

$$I_{a-}(\eta) + I_{1+}(\eta) = Y_{\infty}(\eta)V_{1+}(\eta) \quad (2)$$

$$\begin{aligned} V_{1+}(\eta) &= \int_0^{\infty} E_z(x, 0) e^{j\eta x} dx, \\ I_{1+}(\eta) &= \int_0^{\infty} H_x(x, 0) e^{j\eta x} dx \\ I_{a-}(\eta) &= \int_{-\infty}^0 H_x(x, 0_+) e^{j\eta x} dx \\ V_{1\pi+}(\eta) &= \int_{-\infty}^0 E_z(x, 0_+) e^{-j\eta x} dx = 0 \\ I_{1\pi+}(\eta) &= -\int_{-\infty}^0 H_x(x, 0_-) e^{-j\eta x} dx \end{aligned} \quad (3)$$

with  $\xi(\eta) = \sqrt{k^2 - \eta^2}$ ,  $Y_{\infty}(\eta) = \xi(\eta) / k Z_0$  and the free space impedance  $Z_0$ . Note the PEC boundary condition at  $x < 0$ ,  $y = 0$ .

The dielectric region 2 with PEC boundary conditions at  $x < 0$ ,  $y = 0$  and  $x > s$ ,  $y = -d$  is modelled through the following spectral equation (4) [8,11,15] in terms Fourier transforms (5) according to the transmission line modelling of plane stratified media

$$\begin{aligned} i(\eta, 0_-) &= -Y_{11}(\eta)v(\eta, 0) - Y_{12}(\eta)v(\eta, -d) \\ i(\eta, -d_+) &= Y_{21}(\eta)v(\eta, 0) + Y_{22}(\eta)v(\eta, -d) \end{aligned} \quad (4)$$

$$\begin{aligned} v(\eta, y) &= \int_{-\infty}^{\infty} E_z(x, y) e^{j\eta x} dx \\ i(\eta, y) &= \int_{-\infty}^{\infty} H_x(x, y) e^{j\eta x} dx \end{aligned} \quad (5)$$

Note that  $Y_{11}(\eta) = Y_{22}(\eta) = -jY_d(\eta) \cot[\xi_d(\eta)d] = Y_l(\eta)$  and  $Y_{12}(\eta) = Y_{21}(\eta) = Y_m(\eta) = j \frac{Y_d(\eta)}{\sin[\xi_d(\eta)d]}$ ,  $\xi_d(\eta) = \sqrt{\epsilon_r k^2 - \eta^2}$ ,  $Y_d(\eta) = \xi_d(\eta) / k Z_0$ .

By defining the following WH unknowns (6) at  $y = -d$  we rewrite (4) as (7).

$$\begin{aligned} V_{2+}(\eta) &= \int_0^{\infty} \dot{E}_z(x_2, 0) e^{j\eta x_2} dx_2 = e^{-j\eta s} \int_s^{\infty} E_z(x, -d) e^{j\eta x} dx = 0 \\ I_{2+}(\eta) &= \int_0^{\infty} \dot{H}_x(x_2, 0_+) e^{j\eta x_2} dx_2 = e^{-j\eta s} \int_s^{\infty} H_x(x, -d_+) e^{j\eta x} dx \\ I_{d+}(\eta) &= \int_0^{\infty} \dot{H}_x(x_2, 0_-) e^{j\eta x_2} dx_2 = e^{-j\eta s} \int_s^{\infty} H_x(x, -d_-) e^{j\eta x} dx \\ V_{2\pi+}(\eta) &= \int_{-\infty}^0 \dot{E}_z(x_2, 0) e^{-j\eta x_2} dx_2 = e^{j\eta s} \int_{-\infty}^s E_z(x, -d) e^{-j\eta x} dx, \\ I_{2\pi+}(\eta) &= -\int_{-\infty}^0 \dot{H}_x(x_2, 0_+) e^{-j\eta x_2} dx_2 = -e^{j\eta s} \int_{-\infty}^s H_x(x, -d_+) e^{-j\eta x} dx \\ -I_{1+}(\eta) + I_{1\pi+}(-\eta) &= Y_{11}(\eta)V_{1+}(\eta) + Y_{12}(\eta)\exp(j\eta s)V_{2\pi+}(-\eta) \\ I_{2+}(-\eta) - I_{2\pi+}(\eta) &= Y_{21}(\eta)\exp(j\eta s)V_{1+}(-\eta) + Y_{22}(\eta)V_{2\pi+}(\eta) \end{aligned} \quad (6)$$

Finally region 3 is modelled similarly to region 1 with the WH equation

$$-I_{d+}(-\eta) + I_{2\pi+}(\eta) = Y_{\infty}(\eta)V_{2\pi+}(\eta) \quad (8)$$

As described previously the factorization problem of the matrix WH equations (2), (7), (8) is reduced to Fredholm integral equations by eliminating the current unknowns via contour integrations. The unknowns in the system of Fredholm integral equations are the voltages  $V_{1+}(\eta)$ ,  $V_{2\pi+}(\eta)$ . Solution is performed by analytical-numerical techniques in terms of approximate spectra of the voltages.

Asymptotic estimation via UTD [12] of the total field in region 1 and 3 is performed via the voltages. The study of the structural singularities of the spectra [10] allows also the estimation of surface waves and leaky waves for practical engineering applications. Numerical validations and results will be shown during the presentation.

## ACKNOWLEDGMENT

This work is partially supported by Politecnico di Torino and Istituto Superiore Mario Boella (ISMB), Torino, Italy.

## REFERENCES

- [1] I.D. Abrahams, "Scattering of sound by two parallel semi-infinite screens," *Wave Motion*, 9, pp. 289-300, 1987
- [2] I.D. Abrahams, G.R. Wickham, "The scattering of water waves by two semi-infinite opposed vertical walls," *Wave Motion*, 14, pp. 145-168, 1991
- [3] S. C. Kashyap, "Diffraction characteristics of a slit formed by two staggered parallel planes," *Journal of Mathematical Physics*, 15:11, pp.1944-1949, 1974
- [4] S. C. Kashyap, "Diffraction characteristics of a slit formed by two staggered parallel planes," *Journal of Mathematical Physics*, 15:11, pp.1944-1949, 1974
- [5] A. Büyüksaksoy, E. Topsakal, M. Idemen, "Plane wave diffraction by a pair of parallel soft and hard overlapped half-planes," *Wave Motion*, 20, pp. 273-282, 1994
- [6] J.-P. Zheng, K. Kobayashi, "Diffraction by a semi-infinite parallel-plate waveguide with sinusoidal wall corrugation: combined perturbation and wiener-hopf analysis," *Progress In Electromagnetics Research B*, 13, pp. 75-110, 2009
- [7] V.G. Daniele, G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," *Radio Sci.*, 42: RS6S01, pp. 1-19, 2007.
- [8] V.G. Daniele, R.S. Zich, *The Wiener Hopf method in Electromagnetics*, Scitech Publishing, 2014
- [9] V.G. Daniele, G. Lombardi, R.S. Zich, "Network representations of angular regions for electromagnetic scattering" *Plos One*, 12(8):e0182763, pp. 1-53, 2017.
- [10] V.G. Daniele, G. Lombardi, R.S. Zich, "The electromagnetic field for a PEC wedge over a grounded dielectric slab: 1. Formulation and validation," *Radio Science*, 52, pp. 1-20, 2017
- [11] L.B. Felsen, N. Marcuvitz, *Radiation and Scattering of Waves*, Englewood Cliffs, NJ: Prentice-Hall; 1973
- [12] R.G.Kouyoumjian, P.H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE*, 62, pp. 1448-1461, 1974
- [13] V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle", *SIAM Journal of Applied Mathematics*, 63, pp. 1442-1460, 2003
- [14] V.G. Daniele, G. Lombardi, "The Wiener-Hopf technique for impenetrable wedge problems," in *Proc. Days Diffraction Int. Conf.*, Saint Petersburg, Russia, pp. 50-61, Jun. 2005.
- [15] V.G. Daniele, *Diffraction by two wedges*, Report DET-2014-1, 2014, available at <http://personal.delen.polito.it/vito.daniele>