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Vito Daniele, Guido Lombardi, Rodolfo S. Zich
Politecnico di Torino- ISMB
Torino, Italy
vito.daniele@polito.it, guido.lombardi@polito.it, rodolfo.zich@torinowireless.it

***Abstract*— This paper presents our recent efforts in developing an effective technique based on Generalized Wiener-Hopf Technique (GWHT) to deal with the canonical diffraction problem constituted of two separate perfectly electrically conducting (PEC) wedges. The model considers the near field interaction between the two wedges.**

Keywords—Electromagnetic scattering, Diffraction, Wiener–Hopf method, Integral equations, Circuital modelling, Stratified regions, Wedge, Analytical–numerical methods.

I. Introduction

Recently, the Generalized Wiener Hopf technique (GWHT) [1-2] has demonstrated its efficiency in dealing and solving complex diffraction problems (see for example) [3-8] and references therein. Complex canonical problems constituted of PEC wedges coupled with planar regions and angular regions can now be approached by this technique.

In particular in this paper we will consider the rather general case of the scattering problem constituted by two PEC wedges illuminated by a plane wave as illustrated in Fig.1.

Cartesian coordinates as well as polar coordinates will be used to describe the problem. Two origins are considered, see Fig. 1: O with coordinates (x,y,z) is located at the edge profile of the upper wedge, and O' with coordinates (X, Y,z) is located at the edge profile of the lower wedge. The two reference systems are related through the following relations X=x-s, Y=y+d. Five regions are identified: the four angular regions *a,b,c,d* and the layered region *e*, see Fig. 1.

The application of the Generalized Wiener-Hopf Technique consists of five steps:

1) Deduction of the Generalized Wiener-Hopf Equations (GWHEs) of the problem
2) Reduction of the GWHEs to Fredhom integral equations (FIEs)
3) Solution of the Fredholm integral equations
4) Analytical continuation of the numerical solution of the FIEs
5) Evaluation of the different possible field components: reflected and refracted plane waves, multiple reflected and refracted components, surface waves, lateral waves, leaky waves, diffraction coefficients, mode excitations, near field and so on.

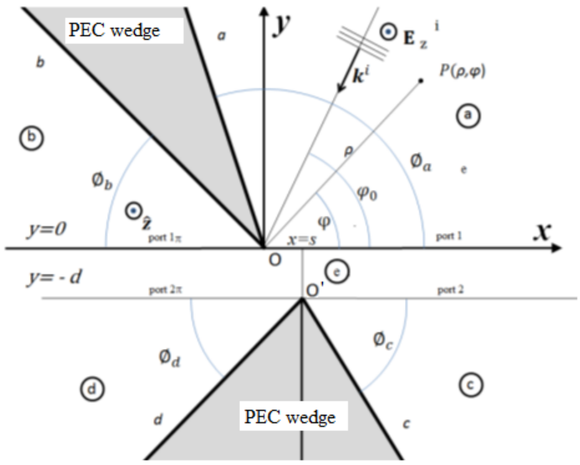

Fig. 1. Scattering by two PEC wedge

In practice steps 2) ,3) and 4) substitute the fundamental steps of the classical Wiener-Hopf (WH) technique, i.e. the factorization of the kernel of the system of WH equations. These steps constitutes the method that we call Fredholm factorization [9-10].

For space reasons in this paper we focus only on the first three steps of the procedure. In particular for this task we resort to circuital modelling of the free space regions that surround the two PEC wedges. While circuital modelling for electromagnetic problems with rectangular geometries immersed in planar stratified regions is presented and discussed in [10], the circuital representation of angular regions is a recent novelty [11].

Without loss of generality, we consider an incident field constituted of an E- polarized plane wave having the following components

$$E_z^i = E_o e^{jk\rho\cos(\varphi-\varphi_o)},\ 0<\varphi_o<\Phi_a \qquad (1)$$

$$H_\rho^i = (1/Z_o)\sin(\varphi-\varphi_o)E_o e^{jk\rho\cos(\varphi-\varphi_o)} \qquad (2)$$

where k and Z_o are respectively the propagation constant and the impedance of the free space.

II. Definitions

With reference to Fig. 1 we also use the polar coordinates $(\rho_a,\varphi_a),(\rho_b,\varphi_b)$ with origin O for regions a and b, and the coordinates $(\rho_c,\varphi_c),(\rho_d,\varphi_d)$ with origin O' for regions c and d. According to the two Cartesian reference systems we define $\dot{\mathbf{E}}(X,Y)=\mathbf{E}(x,y),\dot{\mathbf{H}}(X,Y)=\mathbf{H}(x,y)$ and the following spectral quantities for port 1, 2, 1π, 2 π located respectively at (x>0,y=0), (x>s,y=-d), (x<0,y=0), (x<s,y=-d):

$$V_{1+}(\eta)=\int_0^{\infty}E_z(x,0)e^{j\eta x}dx, I_{1+}(\eta)=\int_0^{\infty}H_x(x,0)e^{j\eta x}dx \quad (3)$$

$$\begin{aligned} V_{2+}(\eta)&=\int_0^{\infty}\dot{E}_z(X,0)e^{j\eta X}dX=e^{-j\eta s}\int_s^{\infty}E_z(x,-d)e^{j\eta x}dx,\\ I_{2+}(\eta)&=\int_0^{\infty}\dot{H}_x(X,0)e^{j\eta X}dX=e^{-j\eta s}\int_0^{\infty}H_x(x,-d)e^{j\eta x}dx \end{aligned} \quad (4)$$

$$V_{1\pi+}(\eta)=\int_{-\infty}^{0}E_z(x,0)e^{-j\eta x}dx, I_{1\pi+}(\eta)=-\int_{-\infty}^{0}H_x(x,0)e^{-j\eta x}dx \quad (5)$$

$$\begin{aligned} V_{2+}(\eta)&=\int_0^{\infty}\dot{E}_z(X,0)e^{j\eta X}dX=e^{-j\eta s}\int_s^{\infty}E_z(x,-d)e^{j\eta x}dx,\\ I_{2+}(\eta)&=\int_0^{\infty}\dot{H}_x(X,0)e^{j\eta X}dX=e^{-j\eta s}\int_0^{\infty}H_x(x,-d)e^{j\eta x}dx \end{aligned} \quad (6)$$

In the following we also use the spectral quantities:

$$v(\eta,y)=\int_{-\infty}^{\infty}E_z(x,y)e^{j\eta x}dx, i(\eta,y)=\int_{-\infty}^{\infty}H_x(x,y)e^{j\eta x}dx \quad (7)$$

$$I_{h+}(\eta)=\int_0^{\infty}H_x(\rho_h,\varphi_h)e^{j\eta\rho_h}d\rho_h \quad (8)$$

where $h=a,b,c,d.$

III. Angular Regions

In this section we make reference only to the angular region a. Similar considerations apply to the other angular regions b,c,d. The following GWHE holds [1-2]

$$Y_c(\eta)V_+(\eta)-I_+(\eta)=-I_{a+}(-m) \quad (9)$$

with $m=-\eta\cos\Phi+\xi\sin\Phi$, $\xi=\sqrt{k^2-\eta^2}$, $Y_c(\eta)=\xi(\eta)/kZ_o$.

The resulting Norton model is of the form [8,11]:

$$I_+(\eta)=\mathscr{Y}V_+(\eta)+I_{ca}(\eta) \quad (10)$$

$$\mathscr{Y}=Y_c(\eta)[...]+\frac{1}{2\pi j}\int_{-\infty}^{\infty}\left(\frac{Y_c(\eta')}{\alpha(\eta')-\alpha(\eta)}\frac{d\alpha}{d\eta'}-\frac{Y_c(\eta)}{\eta'-\eta}+\right)[...]d\eta' \quad (11)$$

with $\alpha(\eta)=-k\cos\left[\frac{\pi}{\Phi_a}\arccos(-\frac{\eta}{k})\right]$, Φ_a is obtuse, and I_{ca} depends on the sources. The integral kernel in (11) is a compact operator that needs extra structural terms when Φ_a is acute [11].

IV. Layered Regions

As reported in [10] the layer is modelled through transmission line theory, in particular in this case we obtain the following two port model:

$$\begin{aligned} &-I_{1+}(\eta)+I_{1-}(\eta)=Y_{11}(\eta)[V_{1+}(\eta)+V_{1-}(\eta)]+Y_{12}(\eta)[V_{2+}(\eta)+V_{2-}(\eta)]\mathrm{e}^{j\eta s}\\ &\mathrm{e}^{j\eta s}[I_{2+}(\eta)+I_{2-}(\eta)]=Y_{21}(\eta)[V_{1+}(\eta)+V_{1-}(\eta)]+Y_{22}(\eta)[V_{2+}(\eta)+V_{2-}(\eta)]\mathrm{e}^{j\eta s} \end{aligned} \quad (12)$$

with $Y_{ii}(\eta)=-jY_o(\eta)\cot[\xi(\eta)d]$, $Y_{ij}(\eta)=jY_o(\eta)/\sin[\xi(\eta)d]$.

The application of Fredholm factorizations to (12), see [8], yields a system of integral representations that relate the voltage and current unknowns as usually obtained in a 4-port Norton representation, see (13) where $\mathcal{Y}_{ij}^{e,a}$ are sum of algebraic and compact integral operators and I_{ci} depend on sources.

$$\begin{pmatrix} I_{1+}(\eta)\\ I_{2+}(\eta)\\ I_{1\pi+}(\eta)\\ I_{2\pi+}(\eta) \end{pmatrix}=\begin{bmatrix} \mathcal{Y}_{11}^e & \mathcal{Y}_{12}^e & \mathcal{Y}_{11}^a & \mathcal{Y}_{12}^a\\ \mathcal{Y}_{21}^e & \mathcal{Y}_{22}^e & \mathcal{Y}_{21}^a & \mathcal{Y}_{22}^a\\ \mathcal{Y}_{11}^a & \mathcal{Y}_{21}^a & \mathcal{Y}_{11}^e & \mathcal{Y}_{21}^e\\ \mathcal{Y}_{12}^a & \mathcal{Y}_{22}^a & \mathcal{Y}_{12}^e & \mathcal{Y}_{22}^e \end{bmatrix}\begin{pmatrix} V_{1+}(\eta)\\ V_{2+}(\eta)\\ V_{1\pi+}(\eta)\\ V_{2\pi+}(\eta) \end{pmatrix}+\begin{pmatrix} -I_{c1}(\eta)\\ -I_{c2}(\eta)\\ -I_{c3}(\eta)\\ -I_{c4}(\eta) \end{pmatrix} \quad (13)$$

V. THE FREDHOLM INTEGRAL EQUATION OF THE PROBLEM

The circuital representations of the angular regions (9) and of the layered region (13) allows the study of the coupling of the four free space angular regions through the layered region. In fact by eliminating the currents of the ports, we obtain a system of Fredholm integral equations of second kind with only voltage unknowns. Solution is performed by analytical-numerical techniques in terms of approximate spectra of the voltages. Asymptotic estimation via UTD of the total field in the angular regions is performed from the spectra. Numerical validations and results will be shown during the presentation.

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References

[1] V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle", SIAM Journal of Applied Mathematics. Vol 63, pp. 1442-1460, 2003

[2] V.G. Daniele, G. Lombardi, "The Wiener-Hopf technique for impenetrable wedge problems," in Proc. Days Diffraction Int. Conf., Saint Petersburg, Russia, pp. 50–61, Jun. 2005.

[3] V.G. Daniele, G. Lombardi G, "The Wiener-Hopf Solution of the Isotropic Penetrable Wedge Problem: Diffraction and Total Field," IEEE Trans. Antennas Propagat., vol. 59, n. 10, pp. 3797-3818, 2011

[4] V.G. Daniele, "Electromagnetic fields for PEC wedge over stratified media: Part I," Electromagnetics, vol. 33, n. 3, pp. 179-200, 2013.

[5] V.G. Daniele, G. Lombardi G, "Arbitrarily oriented perfectly conducting wedge over a dielectric half-space: diffraction and total far field," IEEE Trans. Antennas Propagat., vol. 64, n. 4, pp. 1416-1433, 2016.

[6] V.G. Daniele, G. Lombardi, R.S. Zich, "The electromagnetic field for a PEC wedge over a grounded dielectric slab: 1. Formulation and validation," *Radio Science*, **52**, pp. 1-20, 2017.

[7] V.G. Daniele, G. Lombardi, R.S. Zich, "The electromagnetic field for a PEC wedge over a grounded dielectric slab: 2. Diffraction, Modal Field, SurfaceWaves, and Leaky Waves," *Radio Science*, **52**, pp. 1-18, 2017.

[8] V.G. Daniele, Diffraction by two wedges, Report DET-2014-1, 2014, available at http://personal.delen.polito.it/vito.daniele

[9] V.G. Daniele, G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," Radio Sci., **42:** RS6S01, pp. 1-19**,** 2007.

[10] V.G. Daniele, R.S. Zich, The Wiener Hopf method in Electromagnetics, Scitech Publishing, 2014

[11] V.G. Daniele, G. Lombardi, R.S. Zich, "Network representations of angular regions for electromagnetic scattering" Plos One, 12(8):e0182763, pp. 1-53, 2017.