Robust Attitude Control Systems for Small Satellites

IAC-19,B4,6A,6,x52897

Robust Attitude Control Systems for Small Satellites

M. Dentis[a], E. Capello[a]*, G. Scirè[c], L. De Filippis[c] and G. Guglieri[a]

[a] *Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Italy*
[b] *Department of Mechanical and Aerospace Engineering, CNR-IEIIT, Politecnico di Torino, Italy*
[c] *SITAEL spa, Via San Sabino, 21, Zona Industriale, 70042 Mola di Bari BA*

* Corresponding Author

Abstract

In this paper, an Attitude and Orbit Control System (AOCS) robust flight software has been designed and tested. The proposed attitude control law is based on a Tube-based Robust Model Predictive Control (TRMPC). The proposed TRMPC control system, differently from a Classical MPC, is able to effectively manage system uncertainties and external disturbances according to a proper design of the "tube" and system constraints. Indeed, a TRMPC controller is robust, by definition, against system disturbances, provided that such disturbances are bounded by a well-defined convex set. In addition, a proper design of the "tube" by a Linear Matrix Inequalities (LMI) approach ensure robustness against system uncertainties as well. For example, uncertainties in the spacecraft inertia and limited, and bounded, orbital disturbances, typical of a Low Earth Orbit (LEO) environment, can be effectively managed by a TRMPC-based AOCS. The effectiveness of the proposed control system is shown for two different spacecraft, in a precise pointing telecom maneuver.

Keywords: MPC controller, small satellite, AOCS

1. Introduction

In the last decades, there where an incremental trend in designing and launch small satellites for Earth Observation applications [1]. Depending by sensors for remote sensing chosen for the specific application, Attitude Determination and Control (ADC) requirements may be very different, and control algorithm shall implement different technologies depending by hardware capabilities, intended as computational performance and sensors accuracy. Pointing requirements may be very restrictive and the ADC System/Attitude and Orbit Control System (ADCS/AOCS) shall be compliant with pointing requirements, ensuring satisfaction of such requirements through implementation of robust control algorithm and designing of a proper actuation system. Complexity of ADCS may increase in mission scenarios including a fleet of small satellites working together as a federated system [2].

A critical issue in designing an ADCS is that it should be able to cope all uncertainties that affect the ideal model of the spacecraft and the space environment of which it is designed to work in. Sources of uncertainties may be different: sensors, actuators, external disturbances are all elements that cannot be perfectly modeled without including a set of uncertainties typical of their working principle. Degradation in attitude determination due to aging of sensors and deficiencies in actuation due to thermo-mechanical deformations concur in performance degradation, and may affect both short and long term life of the spacecraft. Performance degradation may be mitigated by improving redundancy of sensors and actuators, if applicable, and by a robust design of navigation and control algorithms.

In addition, the robust design of the ADCS of small satellites should include a certain degree of autonomy, specifically for Low Earth Orbit (LEO) application, in which the communication interval between the spacecraft and the ground control center is very limited and mainly used for download of scientific data, including telemetry data and upload of the set of orbital corrections and mission objective update.

The development of a complete orbital simulator is mandatory for a satisfactory implementation of GNC algorithms for the proposed ADCS. Attitude control is mainly actuated by reaction wheels or different momentum exchange devices, and they require to be de-saturated supported by a different set of actuators, e.g. reaction control thrusters or magnetotorquers. Accurate actuator models shall include non-nominal behavior such as mounting errors, defects in nozzles (if applicable) and non-nominal moment of inertia for momentum exchange devices, simulating production errors. Non-linearities, such has response delays and hysteresis, shall be included as well. Particular attention shall be focused in modeling sensors: bias errors, noise, mounting errors, range limitation and/or performance degradation shall be included in sensors models, in order to be able to consider a large set of uncertainties the ADCS has to counteract in terms of control performance and state estimation.

In this paper, a robust MPC is implemented for two different spacecraft. Advantage of using a TRMPC control law with respect to a classical MPC is that control robustness is ensured introducing a simple static feedback gain, which is the implementation of the tube. In addition, compared with classical Proportional-Integral-Derivative (PID) control laws, a TRMPC controller is able to inherently manage system constraints, without using complex tuning techniques.

The proposed TRMPC control law has been tested and validated for different mission scenarios considering LEO orbits and considering different small spacecraft platforms. In addition, different reaction wheels configurations have been investigated, in order to support the system design of the spacecraft platforms: specifically, a pyramidal and tetrahedral reaction wheels configurations have been considered.

Effectiveness of the proposed TRMPC control law is also investigated in a reaction wheels failure scenario: according to system requirements, the proposed control law must ensure mission accomplishment in case of one wheel failed, considering a proper Fault Detection, Isolation and Recovery (FDIR) procedure. Extensive simulations have been executed, supported by a detailed orbital simulator which implements LEO orbital disturbances, such as magnetic dipole, residual aerodynamic torque, gravitational torque and solar radiation pressure torque.

2. Model Description

Different category of satellite platform have been used in order to assess the control law performance. Particularly, two class of platform have been considered: (i) a mini and (ii) a micro platform. These two platforms have different solar panels configurations, in order to cope with the mission power needs. Some parameters, i.e. the satellite inertia and the exposed area, are almost the same within each class.

The micro platform has a satellite total mass of 58 kg and principal inertia component, considering the deployed solar panel configuration, of 1.55, 3.08 and 3.4 Kgm^2 respectively for Ixx, Iyy and Izz.

The mini platform, instead, has a total mass for 200kg while the principal inertia component are respectively 60,60 and 80 Kgm^2 for Ixx, Iyy and Izz.

A complete model of the external disturbances is also included, in which the aerodynamic drag, the solar pressure and the gravity gradient are described.

For example, the aerodynamic force [4] acting on an orbiting object can be evaluated by

$$F_D = \frac{1}{2}\rho V^2 A_{ref} C_D \tag{2}$$

where ρ is the atmospheric density, V is the orbital velocity, A_{ref} is the reference area, usually selected as the frontal area which impact the incoming flow, C_D is the drag coefficient. Historically, the drag coefficient is set $C_D = 2.2$

3. Typical Mission Scenarios

Different mission scenarios are usually considered for small satellites. In the following, a deeply discussion on these scenarios is provided.

In the assessed optical earth observation mission a panchromatic optical payload with a FOV of 1° was selected to define the mission constraints, which lead the orbital and attitude design. In this frame in order to improve the payload GSD a very low earth orbit was chosen and in order to have the optimal sun illumination a sun-synchronous orbit with LTDN 10:30 at an altitude of 302 km was selected. The selected nominal attitude of this mission include two main modes: an inertial sun pointing mode, used to ensure an optimal solar array sun incidence angle, and image acquisition in which the platform perform a roll maneuver up to 11 degree in order to collect a set of images of the target area.

The considered radar Earth observation mission, instead, a typical SAR configuration was considered. Indeed a dawn-dusk SSO at 620 km was chosen with a nominal attitude that include a sun pointing mode, which is close to a nadir pointing condition taking into account the orbital regime, and an attitude maneuver mode during which the payload can be exploited for all the considered image acquisition modes.

Beside to the earth observation the telecommunication missions have been also taken into account. The first telecommunication mission that have been assessed is a mission for low latency internet services provision with high payload rates. The full pay system has a very high power consumption so the considered platform, described hereafter, has a solar array configuration equipped with SADA to ensure a good sun incidence during the nominal operations. Being the service achieved with S/C constellation the orbit planes shall be separated in RAAN and the orbit selected for the control law performance assessment has a LTAN of 10:30. All the modelled circular orbits belonging to the constellation have an altitude of 1113.4 km and an inclination of 100.1°. The nominal attitude profile of the considered platform is based on a typical yaw steering law.

At last a telecommunication mission with precise pointing maneuver have been considered to further investigate the control law performances in operative conditions which are quite demanding for the considered platforms. Indeed in this mission the ground telecommunication is performed exploiting the capabilities of a quantum payload. Considering the precise pointing required in this application field, the platform shall be able to perform a ground motion compensation maneuver to maintain the ground data link for the entire pass duration. The considered orbit is a circular one with an altitude of about 540 km, and an inclination of 97.51 degrees. The nominal attitude profile of this mission is mainly based on two attitude modes: pointing maneuver and inertial sun pointing. This choice was led by the need of a good sun incidence, out of the pointing maneuvers, for a battery charge purpose. The orbital parameters of the described missions are summarized in the table below.

Table 1. Orbital parameters of the considered missions

Parameter	Optical	Radar	Telecom	Prec.point
a [km]	6681.19	6997.69	7491.57	6917.14
e	<0.01	<0.01	<0.01	<0.01
i [deg]	96.68	97.96	100.08	97.511
RAAN [deg]	129.559	67.775	136.589	331.609
True anomaly [deg]	180.065	179.964	180.075	359.911

The considered missions have been simulated with different platforms and the table below shows the match between mission and used platform

Table 2. Mission-Platform relation matrix

Platform	Optical	Radar	Telecom	Prec.point
Mini	X	X	X	X
Micro				X

Considering that the precise telecommunication mission is the most demanding in terms of pointing accuracy, in the Results section we will focus only on this mission.

4. Tube-Based Model Predictive Control

Tube-based Model Predictive Control is a class of robust controllers, i.e. controllers which are able to cope external disturbances and uncertainties which affect the system. The concept of *tube* [5,6] has been introduced in classical MPC in order to improve robustness of such controllers. TRMPC consists in forcing the perturbed system dynamics to converge to the center of a *tube* which is generated by propagating the unperturbed system dynamics. The outer bounding tube is generated in order to take into account all possible realization of the disturbances, which are supposed to belong to the set $\mathbb{W}$. A fundamental assumption is that $w \in \mathbb{W}$, where w is the disturbance. To realize a TRMPC controller, a classical MPC problem is implemented with respect to the nominal unperturbed system dynamics subject to *tighten constraints*. Constraints tightening ensure robustness of the TRMPC to external disturbances w.

Let's consider the perturbed system x and unperturbed dynamics z

$$\begin{aligned} x^+ &= Ax + Bu + w \\ z^+ &= Az + Bv \end{aligned} \qquad (1)$$

and the constraints $x \in \mathbb{X}, z \in \mathbb{Z}, u \in \mathbb{U}, v \in \mathbb{Z}$, $w \in \mathbb{W}$. The deviation of the perturbed state from the nominal state is then

$$e^+ = A(x - z) + w = Ae + w \Rightarrow$$
$$e(i) = A^i e(0) + \sum_{j=0}^{i-1} A^j w(j),$$

which implies that $e(i) \in S(i)$ and the set $S(i)$ is defined as

$$S(i) = \sum_{j=0}^{i-1} A^j \mathbb{W} = \mathbb{W} \oplus A\mathbb{W} \oplus A^2\mathbb{W} \oplus \dots \oplus A^{i-1}\mathbb{W}$$

where the set addition has been considered. If A is stable, then $i \to \infty$, $S(i) \to S(\infty)$ and it is positive invariant and then $Ax + w \in S(\infty)$ if $w \in \mathbb{W}$. $S(\infty)$ is the minimal robust invariant set for the perturbed system of Eq. 0.

It is also possible to define the tube

$$\hat{\boldsymbol{X}}(x, \boldsymbol{u}) = \{\hat{X}(0), \hat{X}(1; x, \boldsymbol{u}), \dots, \hat{X}(N; x, \boldsymbol{u})\}$$
$$\hat{X}(0) = \{x\} \quad \hat{X}(i; x, \boldsymbol{u}) = \{z(i)\} \oplus S \quad .$$

In general, the set $S(i)$ may not be bounded, i.e. it can become larger as i increases. To limit the size of the set $S(i)$, it is introduced the feedback policy

$$u = v + K(x - z), \qquad (3)$$

where x is the current state of the perturbed system, z is the current state of the unperturbed system subject to the control v and K is a feedback control gain matrix. Substituting the control u in the system of Eq. 0, the deviation dynamics is obtained as

$$e^+ = Ax + Bv + BKe + w = A_K e + w, \; A_K = A + BK. \qquad (4)$$

The matrix K can be designed in order to make A_K stable, and then the corresponding uncertainties set is defined as

$$S_K(i) = \sum_{j=0}^{i-1} A_K{}^j \mathbb{W},$$

which can be expected to be smaller than $S(i)$.

If A_K is stable, $S_K(\infty)$ exists and it is positive invariant for system of Eq. 0. This follows that if $e \in S_K(\infty)$ implies $e^+ \in S_K(\infty)$ for all $e^+ \in \{A_K e\} \oplus \mathbb{W}$, and hence if $e(0) \in S_K(\infty)$, then $e(i) \in S_K(\infty)$ for all i.

The purpose of TRMPC is to ensure that $z(i) \to 0$ with $i \to \infty$, in order to make $x(i) \in \{z(i)\} \oplus S_K(i)$, which converges to the set $S_K(\infty)$. Since the computation of $S_K(\infty)$ may be difficult, it is sufficient the knowledge of an outer approximation S of $S_K(\infty)$. So, if $x(i) \in \{z(i)\} \oplus S_K(\infty)$, it will lie in $x(i) \in \{z(i)\} \oplus S$ for sure, and if $\{z(i)\} \oplus S \subseteq \mathbb{X}$, $x(i)$ will satisfy the state constraints for all i and $w \in \mathbb{W}$. In a similar way it is also ensured that $\{u(i)\}$ lies in the tube $\{\{v(i)\} \oplus S\}$. It follows that if $\{z(i)\}$ and $\{v(i)\}$ are chosen to satisfy $\{z(i)\} \oplus S \subseteq \mathbb{X}$ and $\{v(i)\} \oplus KS \subseteq \mathbb{U}$, then $x(i) \in \mathbb{X}$ and $u(i) \in \mathbb{U}$ satisfy the constraints. Consequently, $\{z(i)\}$ and $\{v(i)\}$ shall be chosen to satisfy the tighten constraint

$$\mathbb{Z} = X \ominus S$$
$$\mathbb{V} = U \ominus KS$$

where X and U are intended as set. To implement a TRMPC controller, it is sufficient the knowledge of the inner approximation of the tighten constraints, which is

$$\hat{\mathbb{Z}} = \mathbb{X} \ominus S_K(\infty)$$
$$\hat{\mathbb{V}} = \mathbb{U} \ominus KS_K(\infty).$$

For the implementation of the TRMPC controller it is necessary to linearize the equation of motion in order to obtain a linear time invariant system in the classical form

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + Du \end{aligned} \qquad (4)$$

The state and control are assumed to be

$$\begin{aligned} x &= [q_{e1} \quad q_{e2} \quad q_{e3} \quad \omega_{e1} \quad \omega_{e2} \quad \omega_{e3}]^T \\ u &= [u_1 \quad u_2 \quad u_3]^T \end{aligned},$$

where q_{e1}, q_{e2} and q_{e3} are the vectorial components of the quaternion error $q_e = q \otimes q_{des}^{-1}$, where the symbol $\otimes$ is referred to the quaternion multiplication, while ω_i is the body angular velocity. See Appendix A for the definition of the matrices.

For the tuning of the matrix K, an LMI procedure is proposed. Advantages of using an LMI approach is that the feedback matrix K can be designed in order to stabilize the system affected by uncertainties in the state and control matrices. The LMI problem is set as

$$\begin{cases} Q + K^T RK + (A_d^+ + B_d^+ K)^T \tilde{P} (A_d^+ + B_d^+ K) - \tilde{P} \\ Q + K^T RK + (A_d^+ + B_d^- K)^T \tilde{P} (A_d^+ + B_d^- K) - \tilde{P} \\ Q + K^T RK + (A_d^- + B_d^+ K)^T \tilde{P} (A_d^- + B_d^+ K) - \tilde{P} \\ Q + K^T RK + (A_d^- + B_d^- K)^T \tilde{P} (A_d^- + B_d^- K) - \tilde{P} \end{cases}$$

where $A_d^{+,-}$ and $B_d^{+,-}$ are the state and control matrices computed taking into account positive and negative uncertainties, i.e.

$$A_d^{+,-} = A_d(q^{+,-}) \quad B_d^{+,-} = B_d(q^{+,-})$$

Solving the LMI is possible to compute the matrix K which stabilize the uncertain system and the terminal weighting matrix P which ensure satisfaction of the terminal constraints. This is a derivation of the Edge theorem, as discussed in [7]. If no uncertainties are considered, the gain K can be computed using LQR design techniques.

5. Results

As stated above the proposed TRMPC control law has been validated for different mission scenarios and small satellite platforms. In detail, the considered missions have been selected to assess the performances of TRMPC in different orbital regimes and satellite attitude agility and stability. Indeed the proposed algorithm performances have been assessed for the missions described hereafter. The selected missions are two earth observation missions, i.e. radar and optical based, and two telecommunication missions, which have different attitude nominal behavior.

The Precise Pointing Telecommunication mission is executed with two platform with different inertia properties, as specified in Section 2.

As it will be discussed in the following, this mission has been observed to be the most critical form the point of view of the pointing transients. Considered constraints are the same as for the Earth Observation mission for the Mini platform, while for the Micro platform they are considered a maximum torque of 0.015 Nm and a maximum angular momentum of 0.34 Nms. Pointing and stability constraints are the same as for the Earth Observation mission.

The system dynamics has an update frequency of 100 Hz, instead the TRMPC is updating with a frequency of 1 Hz and 10 Hz. A NASA configuration of the RWs is analysed.

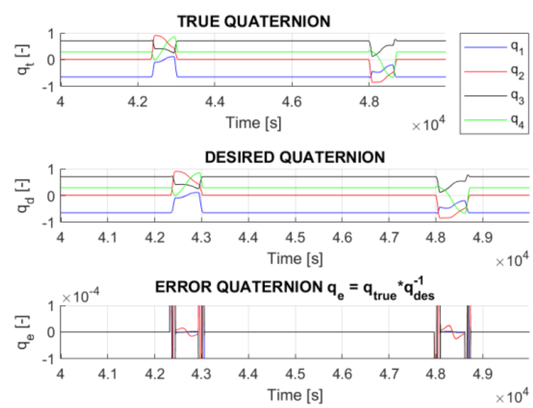

Figure 1 Desired and true quaternion dynamics at 1 Hz

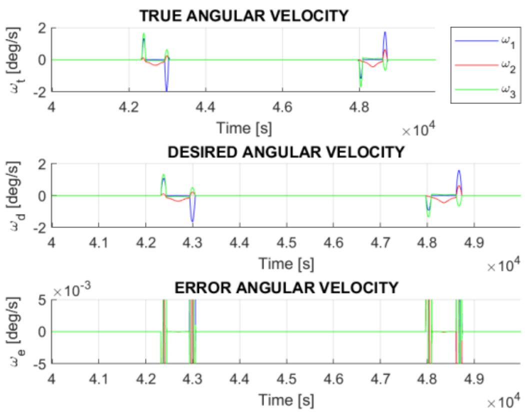

Figure 2 Desired and true angular velocity at 1 Hz

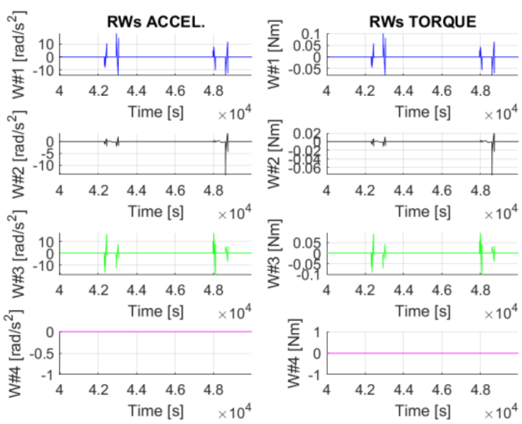

Figure 3 RW acceleration and torque at 1 Hz

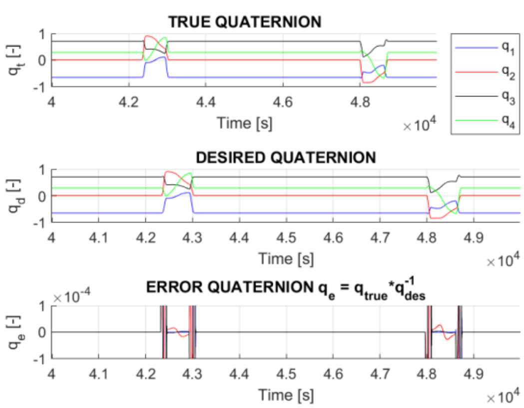

Figure 4 Desired and true quaternion dynamics at 10 Hz

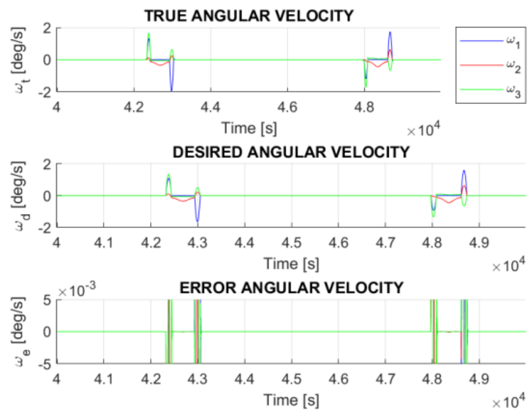

Figure 5 Desired and true angular velocity at 10 Hz

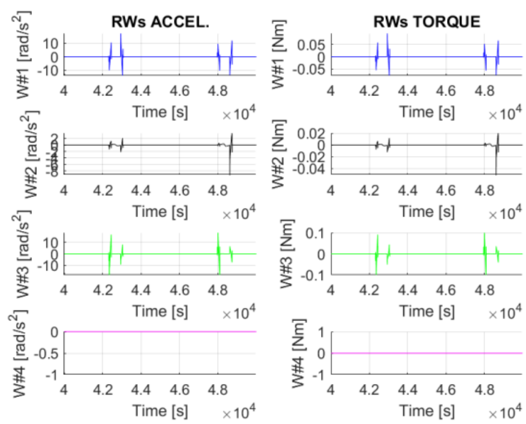

Figure 6 RW acceleration and torque at 10 Hz

The Precise Pointing Telecommunication mission requires a great effort to be satisfactory executed with the TRMPC controller as well. The maximum quaternion error is higher than 0.01 for both the controllers (up to 0.02 for the 1 Hz controller) and the maximum angular velocity error is higher than 0.5 deg/s (up to 0.8 deg/s) for both controllers. During the two pointing phases of the mission, instead, the quaternion error is lower than $1 \cdot 10^{-4}$ for both controllers and the angular velocity error is lower than 0.005 deg/s for both controllers as well, which means that the requirements during the two pointing phases are satisfied for almost all the phase. Even though the TRMPC controller shall admit an higher maximum angular velocity error during the transient phases with respect to the STW-SMC controller, overall performance can be considered to be higher than the previous controller. Differently, constrains of maximum torque and maximum angular momentum of reaction wheels are never exceeded in both controller frequencies, even if Wheel#3 reaches the maximum torque for the 10 Hz controller command.

6. Conclusions

The proposed TRMPC has the main advantage that the control robustness is ensured introducing a simple static feedback gain, which is the implementation of the tube. In addition, compared with classical Proportional-Integral-Derivative (PID) control laws, a TRMPC controller is able to inherently manage system constraints, without using complex tuning techniques.

The proposed TRMPC control law has been tested and validated for different mission scenarios considering LEO orbits and considering different small spacecraft platforms.

Extensive simulations have been executed, supported by a detailed orbital simulator which implements LEO orbital disturbances, such as magnetic dipole, residual aerodynamic torque, gravitational torque and solar radiation pressure torque.

Appendix A: Linear Spacecraft Model Matrices

The linear time invariant system in the classical form

$$\dot{x} = Ax + Bu + w$$
$$y = Cx + Du$$

includes the following matrices

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$A_{11} = \begin{bmatrix} 0 & \omega_{d3} & -\omega_{d2} \\ -\omega_{d3} & 0 & \omega_{d1} \\ \omega_{d2} & -\omega_{d1} & 0 \end{bmatrix}$$

$$A_{12} = \frac{1}{2} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_{21} = \oslash^{3x3}$$

$$A_{22} = \begin{bmatrix} 0 & k_1\omega_3 & k_1\omega_2 \\ k_2\omega_3 & 0 & k_2\omega_1 \\ k_3\omega_2 & k_3\omega_1 & 0 \end{bmatrix}$$

in which all the quantities are computed in the equilibrium point x_0.

Coefficients k_i are computed as

$$k_1 = \frac{J_2 - J_3}{J_1} \quad k_2 = \frac{J_3 - J_1}{J_2} \quad k_3 = \frac{J_1 - J_2}{J_3}$$

where the terms J_i are the diagonal terms of the inertia matrix of the system, which is supposed to be diagonal as well. Eventually, matrices B,C and D are computed as

$$B = \begin{bmatrix} \oslash^{3x3} \\ J^{-1} \end{bmatrix} \quad C = \mathbb{I}^{6x6} \quad D = \oslash^{6x3}$$

where $\mathbb{I}$ and $\oslash$ are respectively the identity and null matrices of proper dimensions. Since MPC is supposed to be a discrete-time controller, the resulting LTI system is then

$$x^+ = A_d x + B_d u + w$$
$$y = C_d x + D_d u$$

with

$$A_d = e^{AT} \quad B_d = \int_t^{t+T} e^{A\tau} B \, d\tau \quad C_d = C \quad D_d =$$

The discretized matrices of system are used to propagate the nominal unperturbed dynamics z, of the TRMPC controller.

References

[1] Sandau, R. (2010). Status and trends of small satellite missions for Earth observation. Acta Astronautica, 66(1), 1-12.

[2] Golkar, A., & i Cruz, I. L. (2015). The Federated Satellite Systems paradigm: Concept and business case evaluation. Acta Astronautica, 111, 230-248.

[3] Ge, M., Chiu, M. S., & Wang, Q. G. (2002). Robust PID controller design via LMI approach. Journal of process control, 12(1), 3-13.

[4] COOK, G. E. Satellite drag coefficients. Planetary and Space Science, 1965, 13.10: 929-946.

[5] RAWLINGS, James Blake; MAYNE, David Q. Model predictive control: Theory and design. Nob Hill Pub., 2009.

[6] S.V. Rakovic, E.C. Kerrigan, K. Kouramas, D.Q. Mayne, Approximation of the minimal robustly positively invariant set for discrete-time LTI systems with persistent state disturbances, in: 42nd IEEE Conference on Decision and Con-trol, Maui, Hawaii, 2003.

[7] B.R. Barmish, New Tools for Robustness of Linear Systems, Macmillan Publish-ing Company, New York, 1994.