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Sliding Mode Techniques for Precise Attitude Control

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Abstract

Small satellites have begun to play an important role in the space researches, especially about new technology development and attitude control. The main idea of this research is related to the design of Attitude Determination and Control System (ADCS) for small satellites. These ADCS systems should be compliant with strict pointing requirements and should be able to guarantee robustness against uncertainties and external disturbances. To accomplish the desired mission task and to design a robust flight software, two second-order Sliding Mode Controllers (SMCs) are taken into account: (i) a super twisting and (ii) a variable gain continuous twisting (CT) SMC. These approaches are able to provide finite-time theoretically exact convergence of the system states to zero. Moreover, the STW algorithm steers to zero in finite time its first time derivative in the presence of smooth matched disturbances with known bound and it contains a term that is obtained as the integral of a discontinuous component. The adaptive CT SMC is a homogeneous control algorithm for uncertain second order plants, for which the chattering phenomenon is strongly attenuated.

1. Introduction

The attitude tracking of rigid bodies, i.i. spacecraft systems, is an active research area,[?,?] since every system works in a harsh environment far from a direct human control. In detail, a space system requires a subsystem that can handle and control autonomously the attitude dynamics. This subsystem has the main purpose of control the orientation of the spacecraft with respect to an inertial reference frame. This subsystem includes sensors and actuators to measure the orientation and to apply the torques needed to change the orientation.

This research will focus on small satellites, i.e. object with mass lower than 500 kg. Common features of these objects are the small volume and mass, these features allow small satellites to be launched as cargo and later being deployed by an other spacecraft or, as payload, to have multiple objects put in orbit, even different ones, with a single launch vehicle. Since the cost of launch is heavily affected by the payload mass, small satellites offer a relatively low-cost solution to space access. However, the reduced dimensions brought new difficulties, since small satellites are more sensitive to disturbances and perturbations than larger satellites. The attitude control problem of a spacecraft in the presence of disturbance and/or uncertainties has been extensively studied. Many different control strategies including adaptive controllers,[?,?] robust control methods[?,?] or H_2/H_∞ controller.[?] The importance of robust controllers for attitude tracking and of the definition of the mathematical model is pointed out by Dasdemir,[?] in which a quaternion-based control is proposed. Even if external disturbances are included, only sinusoidal variations of them are considered and zero-tracking error is proposed. In[?] adaptive gains of a sliding mode controller are designed to counteract the presence of failures. Moreover, in[?] actuator limitations and dynamical constraints are also included.

Due to the presence of uncertainties and dynamical constraints, the main objective of this research is to design and compare different *robust* control system for attitude tracking. The two proposed control methodologies are based on model predictive control theory[?,?] and on variable structure theory.[?,?] A first definition of robustness, although not so rigorous, can be the capability of the control system to work well under sets of parameters different from the nominal one. For example, these parameters can be uncertainties within the system, not known but bounded.

Both of the proposed methods have advantages and drawbacks, briefly described in the following. As explained in,[?] a model predictive control approach is able to effectively handle constraints on torque magnitude, attitude angles and can be more effective than other classical methods. However, a high computational effort is required to solve online the optimization problem. Focusing on robust approach, in this chapter, a Tube-based Robust MPC (TRMPC) is proposed, which focuses on two main goals: (i) the robustness to additive disturbances and (ii) the computational efficiency of a classical MPC, due to an offline evaluation of the constraints. Thanks to this control strategy, the uncertain future trajectories lie in sequence of sets, known as *tubes*, and the online MPC scheme is applied *only* to the nominal trajectories, representing the center of the tube itself as in.[?]

The second proposed methodology is based on variable structure strategy,[?] in which the control law is a function of the system state and changes among the possible structures according to some suitably defined rules. In particular, in Sliding Mode Controller (SMC) systems a switching function is designed, which implicitly defines a sliding surface corresponding to the points in the state space in which the switching function is zero. At any time, the structure applied by the control law depends on the position of the state with respect to the sliding surface. When a sliding motion is established, the closed-loop system is in sliding mode and its trajectories are constrained on the sliding surface. Sliding mode methods provide controllers which can counteract uncertainties and disturbances, if the perturbations affecting the system are matched and bounded (first order SMC)[?] or smooth matched disturbances with bounded gradient (second order SMC).[?,?] One of the main drawback of SMC methods is the chattering phenomenon, that excites the high frequency unmodeled dynamics in practical applications. Moreover, the performance of the control system is affected by the quality of the measurements and of the computation frequency of the system.[?] For this reason, our idea is to design a Continuous Twisting controller (CTSMC) with adaptive gains.[?,?] The peculiarities of this controller are: (i) adaptation of the gains, (ii) continuous control inputs and (iii) the external disturbances are included in the definition of the control gains.

For both the proposed controller on-board hardware limitations are included in the design and implementation. As clearly explained in,[?] limited computational resources and reduced sampling time can be a problem in the design of robust controller. Moreover, the sample frequency reduction in the SMC design implies a residual chattering, which can be reduced with the introduction of an hyperbolic tangent.[?] A comparison with a Proportional Integrative Derivative (PD) controller, in which the gains are defined from loop shaping theory,[?] is also proposed.

2. Model Description

In this section, both the attitude dynamics and the model of external disturbances, affecting the spacecraft, are described. A detailed description of model disturbances in LEO missions is proposed.

2.1 Attitude Dynamics

Attitude dynamics is propagated by

$$\dot{\omega}_B = J^{-1}\left[M_B - \omega_B \times (J\omega_B + J_{\mathrm{RW}}\Omega_{\mathrm{RW}})\right], \qquad [1]$$

where $\omega \in \mathbb{R}^3$ is the spacecraft angular velocity vector, J is the moment of inertia of the spacecraft, $\Omega_{\mathrm{RW}} \in \mathbb{R}^3$ is the reaction wheels angular velocity vector, J_{RW} is the moment of inertia of the reaction wheels system and $M_B \in \mathbb{R}^3$ is the sum of the torques acting of the spacecraft, specifically

$$M_B = \Delta M_{\mathrm{ex}} + M_{\mathrm{RW}}, \qquad [2]$$

with $\Delta M_{\mathrm{ex}} \in \mathbb{R}^3$ is the disturbance torque vector and $M_{\mathrm{RW}} \in \mathbb{R}^3$ is the torque vector provided by the reaction wheels system.
The spacecraft attitude is expressed in quaternion form

$$q = [q_v, q_4]^T = [q_1, q_2, q_3, q_4]^T. \qquad [3]$$

$q_v \in \mathbb{R}^3$ and $q_4 \in \mathbb{R}$ are respectively the vectorial and the scalar components of the quaternion q. Attitude propagation is obtained by the kinematic equation in quaternion form

$$\begin{aligned}\dot{q}_v &= \frac{1}{2}(q_4\mathbb{I} + Q^{\mathrm{x}})\omega \\ \dot{q}_4 &= -\frac{1}{2}q_v^T\omega\end{aligned}, \qquad [4]$$

with $\mathbb{I} \in \mathbb{R}^{(3,3)}$ is the identity matrix. $Q^{\mathrm{x}} \in \mathbb{R}^{(3,3)}$ is the skew-symmetric matrix obtained by the vector q_v and it is defined as follows

$$Q^{\mathrm{x}} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \qquad [5]$$

.

2.2 External Disturbances

In this paper, the external disturbances model is based on the main torque disturbances, which affect the spacecraft attitude orbiting in LEO, such as gravity gradient torque, solar radiation pressure and aerodynamic torque. The residual magnetic dipole disturbance is neglected in the analyzed application, since its magnitude is usually lower than the other sources of disturbance in LEO.

The aerodynamic torque is due to the residual atmospheric gases present at high altitude. This disturbance torque has been modeled as a random three-axial disturbance of fixed magnitude, since its exact evaluation is very difficult due to the high uncertainty in computing the spacecraft center of pressure, while evaluation of the drag force can be easily evaluated.[1] The aerodynamic torque is thus modeled as

$$M_{\text{aer}} = r_{\text{cp}} \times T_{\text{aer}}, \quad [6]$$

where $T_{\text{aer}} = \frac{1}{2}\rho V_{SC}^2 S_{front} C_D$ is the magnitude of the aerodynamic torque, where ρ is the atmospheric drag, V_{SC} is the spacecraft velocity, S_{front} is the spacecraft frontal area, and C_D is the drag coefficient, and $r_{\text{cp}} \in \mathbb{R}^3$ is the distance between the center of pressure and the center of mass of the spacecraft.

The gravity gradient torque is modeled as

$$M_{\text{gg}} = 3\frac{\mu}{R^3}\hat{o}_3 \times J\hat{o}_3. \quad [7]$$

The parameters are defined as follows, μ is the Earth gravitational parameter, R is the orbit radius and $\hat{o}_3$ is the third column of the direction cosine matrix between body and local orbital frames.

Finally, the total disturbance torque acting on the spacecraft is computed as

$$\Delta M_{\text{ex}} = M_{\text{aer}} + M_{\text{gg}}. \quad [8]$$

3. Sliding Mode Controllers

Sliding Mode controllers belong to the class of discontinuous controllers. Extensive discussion about the fundamentals of classical, second-order and high-order SMC can be found in[3] and.[2] The main principle of a SMC is to drive the state of the system to converge to the sliding surface $\sigma = 0$, in finite time, by applying discontinuous control. Advantages of using SMC is that they are robust, by construction, to the so called *matched* uncertainties, i.e. uncertainties which affect the state of the system in the control channel. Considering the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + f(x_1, x_2, t) \end{aligned} \quad [9]$$

$f(x_1, x_2, t)$ expresses the matched uncertainties (nonlinearities, parasitic dynamics, disturbances). It is assumed that the matched uncertainties are bounded, such as $|f(x_1, x_2, t)| \leq L > 0$. In this section, a description of the developed Sliding Mode Controllers: classical first-order SMC, smoothed first-order SMC and second-order super twisting controller.

3.1 Super Twisitng Sliding Mode Controller

Second order sliding mode controllers, or 2-SMC, are designed to improve performance of classical first order SMC, since the aim is to drive to zero both σ and $\dot{\sigma}$, simultaneously. In addition, 2-SMC control system provides effective tools for chattering attenuation without affecting performance.[2] In this paper, a Super-twisting Sliding Mode Controller (STW-SMC) is proposed. STW-SMC is applied to a system of relative degree one, i.e. the control term u appears in the first derivative of σ. The sliding variable σ considered for the STSMC is defined as $\sigma = \omega_e + Kq_e$, where $\omega_e = \omega - \omega_{des}$ is the angular velocity error and q_e is the quaternion error defined by

$$\begin{aligned} q_{ev} &= q_{d4}q_v - Q_d^{\times}q_v - q_4 q_{dv} \\ q_{e4} &= q_{dv}^T q_v + q_4 q_{dv} \end{aligned}, \quad [10]$$

The first time-derivative of σ is then:

$$\begin{aligned} \dot{\sigma} = &\frac{1}{2}\Phi(q_e)\dot{\omega}_e - \frac{1}{2}q_{ev}^T\omega_e\omega_e + \frac{1}{4}Q^{\times}\Phi(q_e)\omega_e + \\ &+ \frac{1}{2}K\Phi(q_e)\omega_e \end{aligned}. \quad [11]$$

$\Phi(q_e) = (q_{4e}\mathbb{I} + Q^{\times})$ and $\dot{\omega}_e$ is computed as

$$\begin{aligned} \dot{\omega}_e = &J^{-1}\left[-(\omega_e + \omega_d) \times J(\omega_e + \omega_d) + u + d\right] + \\ &+ \dot{\omega}_d \end{aligned}. \quad [12]$$

Substituting Eq. (12) in Eq. (11) the sliding variable can be rewritten in the form

$$\dot{\sigma} = h(q_e, \omega_e) + g(q_e, \omega_e)\, u, \quad [13]$$

with $h(q_e, \omega_e)$ and $g(q_e, \omega_e)$ are respectively

$$\begin{aligned} h(q_e, \omega_e) = &\frac{1}{2}\Phi(q_e)J^{-1}[-(\omega_e + \omega_d) \times J(\omega_e + \omega_d) + \\ &+ d + J\dot{\omega}_d] + \\ &- \frac{1}{2}q_{ev}^T\omega_e\omega_e + \frac{1}{4}Q^{\times}\Phi(q_e)\omega_e + \\ &+ \frac{1}{2}K\Phi(q_e)\omega_e \end{aligned} \quad [14]$$

$$g(q_e, \omega_e) = \frac{1}{2}\Phi(q_e)J^{-1} \qquad [15]$$

and it is assumed that for some positive constants, C, K_M, $K_m\,U_M$, q, the following assumptions are valid:[2]

$$|\dot{h}| + U_M|\dot{g}| \leq C \qquad [16]$$
$$0 \leq K_m \leq g \leq K_M \qquad [17]$$
$$|h/g| < qU_M \qquad [18]$$
$$0 < q < 1 \qquad [19]$$

The STW-SMC algorithm is defined as

$$\begin{aligned} u &= -\lambda|\sigma|^{\frac{1}{2}}\mathrm{sign}(\sigma) + u_1, \\ \dot{u}_1 &= -\alpha\mathrm{sign}(\sigma). \end{aligned} \qquad [20]$$

Assuming $K_m\alpha > C$ and λ computed as suggested by[2]

$$\lambda > \left(\frac{2}{K_m\alpha - C}\right)^{\frac{1}{2}} \frac{(K_m\alpha + C)K_M(1+q)}{K_m^2(1-q)} \qquad [21]$$

the STSMC algorithm ensure the appearance of a second-order sliding mode in finite time, such as $\sigma = \dot{\sigma} = 0$, and the control u will be included in the interval $[-U_M, U_M]$.

3.2 Continuous Twisting Sliding Mode Controller

In the present work, the CTSMC will be further investigated. The system 9 represents a linearized attitude dynamics, in which they can be considered $x_1 = q_e$ and $x_2 = \omega_e$ as the quaternion error and the angular velocity error respectively, u is the control torque and $f(x_1, x_2, t)$ is the external disturbance. The external disturbance is assumed to be *Lipschitz*, i.e. it is differentiable with bounded derivative: $\dot{f}(x_1, x_2, t) \leq \mu$. According to system (9), a Continuous Twisting SMC can be designed as

$$\begin{cases} u &= -k_1|q_e|^{\frac{1}{3}}\mathrm{sign}(q_e) - k_2|\omega_e|^{\frac{1}{2}}\mathrm{sign}(\omega_e) + \eta \\ \dot{\eta} &= -k_3|q_e|\mathrm{sign}(q_e) - k_4|\omega_e|\mathrm{sign}(\omega_e) \end{cases} \qquad [22]$$

According to this control design, it results that $\sigma = q_e$ and $\dot{\sigma} = \omega_e$. Control gains k_i can be designed as[?]aggiunto bibitem al fondo:

$$k_1 = 7 \qquad k_2 = 5 \qquad k_3 = 2.3 \qquad k_4 = 1.1 \qquad [23]$$

Such gains ensure robustness against Lipschitz disturbances $f(x_1, x_2, t)$ bounded by $\mu = 1$. Since in practical cases the Lipschitz disturbances is usually bounded by $\dot{f}(x_1, x_2, t) \leq \mu^* = L\mu \neq 1$, control gains can be scaled as:

$$k_{p1} = k_1 L^{\frac{2}{3}} \quad k_{p2} = k_2 L^{\frac{1}{2}} \quad k_{p3} = k_3 L \quad k_{p4} = k_4 L \qquad [24]$$

The controller of Eq. (22) can be re-designed by substituting gains k_i with gains k_{pi}.

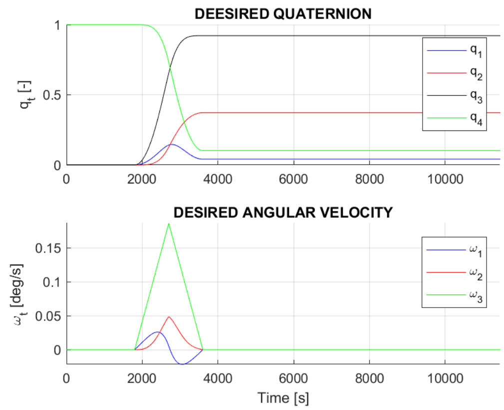

Fig. 1: Reference inertial quaternion and angular velocity.

4. Simulation Results

4.1 Mission Scenario

The mission scenario selected as test case for the two sliding mode controllers is the scientific observation of the Crab Nebula. A scientific observation scenario has been selected since it usually requires satisfaction of very strict pointing requirements and high pointing stability also in case uncertain external disturbances are applied to the system. Crab Nebula has been selected since its coordinates with respect to the inertial reference frame can be easily determined. The spacecraft is located in a circular orbit at 700 km of altitude with an inclination of about 28 deg. The reference quaternion and angular velocity profiles are depicted in Fig. 1. The spacecraft is supposed to be equipped with reaction wheels by Honewell with 0.1 Nm torque and angualr momentum of 25 Nms. Controller frequency is set as 10 Hz.

4.2 Super-Twisting SMC

In Fig. 2 it is depicted the angular error obtained with the STW-SMC control algorithm. During the transient of the pointing maneuver the maximum pointing error is lower than $3 \cdot 10^{-5}$ deg, while during the pointing phase the pointing is maintained with an error of about 10^{-7}deg. For what concern the control

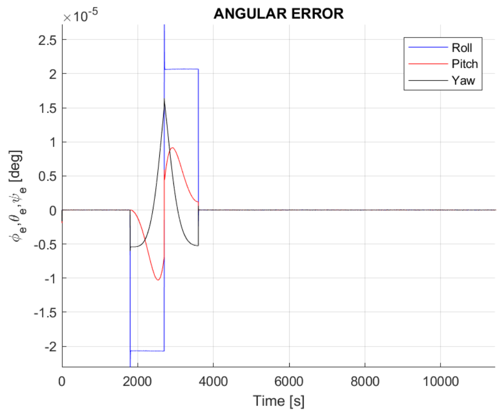

Fig. 2: STW-SMC angular error.

Fig. 4: STW-SMC Reaction Wheels momentum.

Fig. 3: STW-SMC control and Reaction Wheels torque.

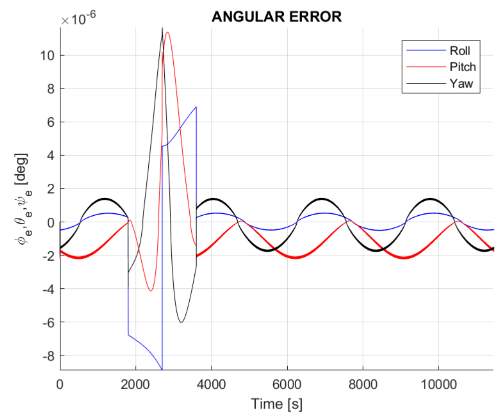

Fig. 5: CTA-SMC angular error.

and reaction wheels torque, depicted in Fig. 3, it can be observed that the commanded torque is always lower than the maximum torque which can be provided by the reaction wheels system. In addition, also the maximum angular momentum is never exceeded during the execution of this maneuver. As it can be observed in Fig. 4, reaction wheels are accumulating momentum due to coping external disturbances.

4.3 Continuous Twisting SMC

Simulation results related to the Continuous Twisting control law are depicted in Figs. 5-7. The angular error, Fig. 5, shows a maximum peak error less than $1.2 \cdot 10^{-5}$, which is almost the half of the peak error obtained with the STW-SMC, while the steady state pointing error is almost one order of magnitude higher than the one obtained with the STW-SMC control law. For what concern the control and reaction wheel torque and wheels angular momentum, it can be observed that results obtained with the Continuous Twisting control law are comparable with results obtained with the Super Twisting algorithm.

5. Conclusions

Two Sliding Mode Control laws have been developed for the implementation in a space observation mission scenario with strict pointing accuracy requirements. Both control laws, according to their mathematical definition, are able to cope against Lipschitz external disturbances. Pointing accuracy is satisfactory for both controllers, though the Super Twisting SMC shows a better steady state pointing accuracy. Instead, overall reaction wheels control performance are comparable. Both control laws shows their potential application in real spacecraft implementation. Even though STW-SMC shows better pointing accuracy, its tuning methodology is more

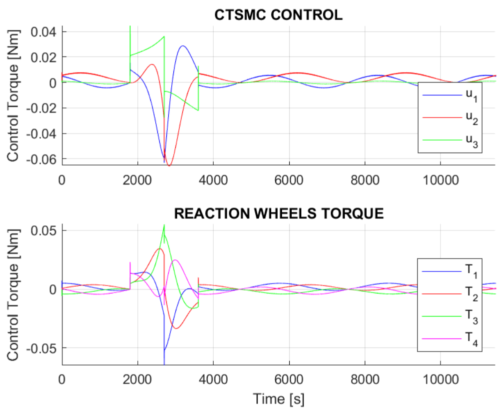

Fig. 6: CTA-SMC control and Reaction Wheels torque.

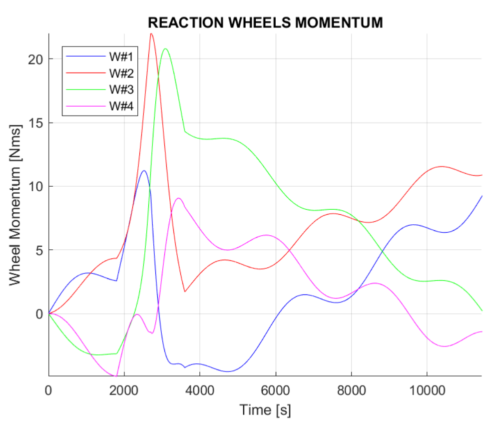

Fig. 7: CTA-SMC Reaction Wheels momentum.

complex than the Continuous Twisting Controller, since it requires the boundary of functions g, $\dot{g}$, h and $\dot{h}$. Differently, the Continuous Twisting Controller requires only the knowledge of the derivative of the external disturbances, and this could results in an advantage in the choice of a robust controller.

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