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Summary

Highly flexible composite thin-walled booms and plates are constantly employed in spacecraft science; applications include, but are not limited to, deployable satellites' instrumentation, antennas, and solar arrays. Generally subjected to large displacements and rotations, these composite structures are prone to suffering vibration and instability phenomena as a consequence of external excitations and operational loadings [1, 2]. Thus, predicting accurately the in-service nonlinear response and the modal characteristics around non-trivial equilibrium states of these thin-walled composite flexible structures is of great importance for design and verification.

In this work, the governing nonlinear equations of lower- to higher-order 1D (beam) and 2D (plate) structural theories for composite laminates are derived as degenerated cases of the three-dimensional elasticity equilibrium via an appropriate index notation and by employing the Carrera unified formulation (CUF), see Ref. [3]. According to CUF, 1D beam theories, for example, can be formulated from the three-dimensional displacement field ($\mathbf{u}$) as an arbitrary expansion of the generalized unknowns ($\mathbf{u}_\tau$); i.e.,

$$\mathbf{u}(x,y,z) = F_\tau^{1\text{D}}(x,z)\,\mathbf{u}_\tau(y), \qquad \tau = 1,2,....,M \tag{1}$$

where F_τ are generic functions on the beam cross-section domain, M is the number of expansion terms, and τ denotes summation. In contrast, the generalized displacements are functions of the (x,y) in-plane coordinates in the case of plate models and F_τ represent thickness functions to give:

$$\mathbf{u}(x,y,z) = F_\tau^{2\text{D}}(z)\,\mathbf{u}_\tau(x,y), \qquad \tau = 1,2,....,M \tag{2}$$

Depending on the choice of F_τ and the number of expansion terms M, different classes of beam and plate structural theories can be formulated and, thus, implemented in a straightforward manner [4].

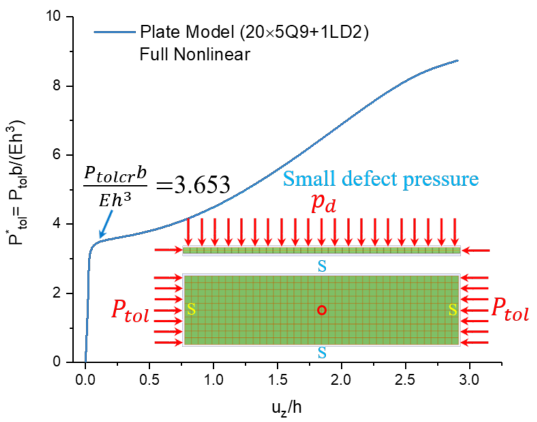
Regardless of the use of 1D or 2D formulations, quasi-static and eventually nonlinear equilibrium states of elastic structures can be found by using the principle of virtual work, which states that the sum of the virtual variation of the internal strain energy and the virtual variation of the work of external loadings is null. As a consequence, small-amplitude vibration of beams and plates subjected to initial pre-stress states and undergoing large displacements and rotations can be analysed by linearizing the virtual variation of the internal work, which – by using CUF, standard finite elements (FEs), the constitutive relations, and the Green-Lagrange strains – reads [5]

$$\begin{aligned} \delta(\delta L_{\text{int}}) &= \; < \delta(\delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}) > \\ &= \; < \delta\boldsymbol{\varepsilon}^T \delta\boldsymbol{\sigma} > + < \delta(\delta\boldsymbol{\varepsilon}^T)\boldsymbol{\sigma} > \\ &= \delta\mathbf{u}_{\tau i}^T(\mathbf{K}_0^{ij\tau s} + \mathbf{K}_{T_1}^{ij\tau s} + \mathbf{K}_\sigma^{ij\tau s})\delta\mathbf{u}_{sj} \\ &= \delta\mathbf{u}_{\tau i}^T \mathbf{K}_T^{ij\tau s} \delta\mathbf{u}_{sj} \end{aligned} \tag{3}$$

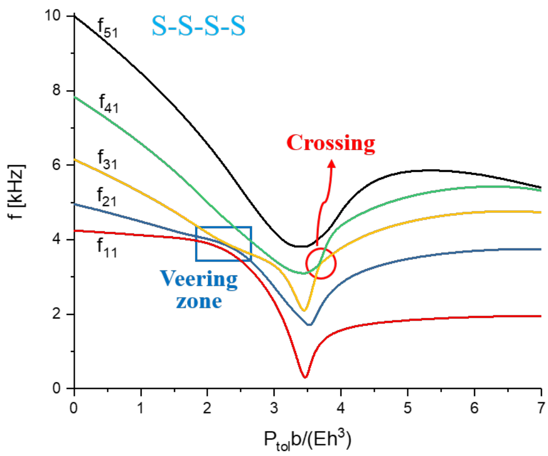
In Eq. (3), $<(\cdot)>=\int_V(\cdot)\,dV$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are respectively the strain and stress vectors, $\mathbf{u}_{\tau i}$ is the vector of the FE nodal unknowns, and $\mathbf{K}_T^{ij\tau s}$ is the tangent stiffness matrix in the form of 3×3 fundamental nucleus. Note that $\mathbf{K}_T^{ij\tau s}$ is the sum of $\mathbf{K}_0^{ij\tau s}$, which represent the linear stiffness matrix, $\mathbf{K}_{T_1}^{ij\tau s}$, which is the nonlinear contribution due to the linearization of the Hooke's law, and $\mathbf{K}_\sigma$, which comes from the linearization of the nonlinear form of the strain-displacement equations and is often called the *geometric stiffness*. Given the theory approximation order, these fundamental nuclei can be opportunely expanded in order to obtain the element tangent stiffness matrices of any arbitrarily refined beam and plate models. In other words, by opportunely choosing the theory kinematics (Eqs. (1) and (2)), classical to higher-order FE stiffness arrays can be implemented in an automatic manner by exploiting the index notation of CUF. The explicit derivation of the tangent stiffness matrix is not provided here for the sake of brevity, but it can be found in [6]. Once the global tangent stiffness matrix $\mathbf{K}_T$ is known, the natural frequencies and mode shapes of the structure can be evaluated by solving the usual eigenvalue problem, which holds:

$$\left(\mathbf{K}_T-\omega^2\mathbf{M}\right)\bar{\mathbf{u}}=0 \tag{4}$$

where $\mathbf{M}$ is the FE mass matrix, assumed linear in the present study.



(a) Equilibrium curve



(b) Effect of in-plane loading on natural frequencies

Figure 1: Vibration of simply-supported plate under uni-axial compression.

For representative purposes, Fig. 1 shows the mode aberration of a metallic simply-supported rectangular plate subjected to uni-axial in-plane compression. In particular, Fig. 1(a) shows the static equilibrium curve of the elastic structure under consideration. Note that a small defect pressure is applied to avoid singularities close to the buckling load. Important natural frequencies are thus analysed all along the equilibrium path and shown in Fig. 1(b). It is clear that instabilities, veering phenomena as well as crossing frequencies may arise as a consequence of the operational loadings. Some further considerations can be done:

- The accuracy of the proposed methodology, of course, depends on the capability of the structural theory to describe nonlinear analysis in an accurate manner, which is the case of the present CUF methodology.
- In fact, internal stress distributions are accurate and large-displacement states are described by 3D Green-Lagrange relations.
- The nonlinear vibrations have low amplitudes, so the linearization around discrete states of the equilibrium path and the assumption of harmonic oscillations are legit.

- Inertial work is neglected in the evaluation of the equilibrium path. In other words, vibrations are evaluated around quasi-static equilibrium states.
- The proposed method is able to identify bifurcations, elastic instabilities or buckling phenomena as those conditions which render the tangent stiffness matrix singular.

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