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Original
Efficient analysis of geometrically nonlinear deployable thin shell structures using Carrera unified formulation / Pagani, Alfonso; Carrera, Erasmo; de Miguel, Alberto G.; Hasanyan, Armanj; Pellegrino, Sergio; Reddy, Narravula H.; Zappino, Enrico. - ELETTRONICO. - (2019). (Intervento presentato al convegno 70th International Astronautical Congress (IAC) tenutosi a Washington, D.C. nel 21-25 October 2019).

Publisher:
International Astronautical Federation (IAF)

Published
DOI:

IAC-19-C2.2.3

Efficient Analysis of Geometrically Nonlinear Deployable Thin Shell Structures Using Carrera Unified Formulation

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Abstract

This work is for an exploratory study into the use of advanced beam models for the analysis of nonlinear deployable thin shell structures. For this purpose, the governing nonlinear equations of higher-order 1D structural theories for highly flexible booms are expressed in a total Lagrangian sense as degenerated cases of the three-dimensional elasticity equilibrium via an appropriate index notation and by employing the Carrera unified formulation. The Newton–Raphson linearization scheme along with a path-following method based on the arc-length constraint is exploited to find the equilibrium path of a tape spring subjected to coiling bending. Dedicated experimental tests reveal the validity of the proposed finite element approach and give confidence for further research in this direction and for the extension to composite laminated deployable structures.

Keywords: deployable structures; thin shells; tape springs; Carrera unified formulation; finite element method.

Nomenclature

F_τ	Cross-sectional kinematics
$\boldsymbol{K}_T^{ij\tau s}$	Fundamental nucleus of the tangent stiffness matrix
L_{int}	Internal strain energy
M	Number of terms for the given theory approximation order
N_i	Shape functions
$\boldsymbol{u}$	Three-dimensional displacement field
$\boldsymbol{u}_\tau$	Generalized displacements
$\boldsymbol{u}_{\tau i}$	Nodal parameters

Acronyms/Abbreviations

CUF	Carrera Unified Formulation
FEM	Finite Element Method

1. Introduction

Highly flexible booms are constantly employed in a number of engineering applications, including deployable satellites’ instrumentation, antennas and advanced aircraft structures [1]. Generally subjected to large displacements and rotations, these booms – eventually made of composite material – are prone to suffering instability phenomena and failure. Thus, predicting accurately stiffness and stress distribution in both linear and nonlinear regimes is of great importance for design and verification.

In this study, the Carrera Unified Formulation (CUF) [2] is used as an alternative finite element approach to modelling long, deployable, thin shell structures. The advantage of the CUF approach is in the approximation of the span-wise coordinate axes of a long structure using one-dimensional shape functions. Under this consideration, CUF provides an efficient approach to simulating long deployable shells compared to 3D continuum, 2D shell, or continuum shell based finite element approaches. As a matter of fact, due to the slenderness and the length of thin shell structures, both 3D continuum and shell models can be limited in simulating their behaviour. In addition, these models can be latent with numerical locking phenomena.

To evaluate the accuracy and the efficiency of CUF, a tape spring shell structure is examined in this study. The analysis is performed in the geometrically nonlinear regime, where the structure is bent in both opposite sense and equal sense to simulate the formation of localized folds during the process of coiling. For this purpose, the total Lagrangian finite element approach based on the CUF assumptions is developed. An implicit Newthon-Raphson method is considered along with a dedicated arc-length governing constraint in order to capture the unstable equilibrium path of the tape spring associated with geometric nonlinearities. The attention is focussed on the accurate description of the post-buckling mechanics, the evaluation of the limit points, and the prediction of the steady-state moments in far post-buckling.

The paper compares the predictions and the efficiency of CUF to experimental results obtained at the California Institute of Technology. By comparing the experimental and numerical results for the tape spring structure, the goal is to develop and verify the utility of CUF for application towards the broader class of deployable, long thin shell structures.

2. Mathematical description of the thin boom in the framework of CUF

Figure 1 shows a representative thin shell oriented in a Cartesian reference system.

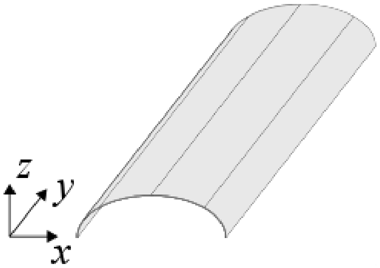

Fig. 1. Representative thin boom

Within the framework of the Carrera Unified Formulation (CUF), the three-dimensional displacement field $\boldsymbol{u}(x,y,z)$ can be expressed as a general expansion of the primary unknowns. In the case of one-dimensional beam theories, one has:

$$\boldsymbol{u}(x,y,z) = F_\tau(x,z)\boldsymbol{u}_\tau(y), \quad \tau = 1,2,\ldots,M \qquad (1)$$

where F_τ are the expansion functions of the coordinates x and z on the cross-section, $\boldsymbol{u}_\tau$ is the vector of the generalized displacements which lay along the beam axis, M stands for the number of terms used in the expansion, and the repeated subscript τ indicates summation. The choice of F_τ determines the class of the 1D CUF model that is required and subsequently to be adopted. On the other hand, if the Finite Element Method (FEM) is adopted, the generalized displacements are approximated as $\boldsymbol{u}_\tau(y) = N_i(y)\boldsymbol{u}_{\tau i}$, where i represents the index summation from 1 to $(p+1)$, p is the polynomial order of the given 1D shape functions N_i, and $\boldsymbol{u}_{\tau i}$ is the vector of nodal parameters.

2.1 Fundamental nuclei for the resolution of nonlinear problems

The nonlinear governing equations corresponding to tape springs subjected to large displacements are obtained in this paper by using the principle of virtual displacements as described by Pagani and Carrera [3]. Here, the Newton-Raphson method was employed along with a dedicated arc-length constraint equation, which makes use of the linearized consistent solution.

It can be easily demonstrated that, by using CUF and a recursive notation for the expansion of the kinematics, one can write the equilibrium equations in terms of fundamental nuclei, which are invariant of the theory approximation order. In this manner, in a FEM framework, stiffness arrays can be obtained in an automatic manner with ease and the accuracy of the analysis can be tuned opportunely.

For completeness reasons, we report here the explicit expression of the tangent stiffness matrix. It is obtained by linearizing the virtual variation of the internal strain energy, which holds:

$$\begin{aligned} d(\delta L_{int}) &= < d(\delta \boldsymbol{\epsilon}^T \boldsymbol{\sigma}) > \\ &= < \delta \boldsymbol{\epsilon}^T d\boldsymbol{\sigma} > + < d(\delta \boldsymbol{\epsilon}^T)\boldsymbol{\sigma} > \end{aligned} \qquad (2)$$

where $< (\cdot) >$ stands for volume integral, $\boldsymbol{\epsilon}$ and $\boldsymbol{\sigma}$ are the Green-Lagrange strain and the second Piola-Kirchhoff stress vectors respectively, and δ is the virtual variation. If CUF is used along with FE approximation, and by using the constitutive equations (elastic materials are considered in this work), Eq. (2) becomes:

$$\begin{aligned} d(\delta L_{int}) &= \delta \boldsymbol{u}_{sj}^T \left(\boldsymbol{K}_0^{ij\tau s} + \boldsymbol{K}_{T1}^{ij\tau s} + \boldsymbol{K}_\sigma^{ij\tau s}\right) d\boldsymbol{u}_{\tau i} \\ &= \delta \boldsymbol{u}_{sj}^T \boldsymbol{K}_T^{ij\tau s} d\boldsymbol{u}_{\tau i} \end{aligned} \qquad (3)$$

where $\boldsymbol{K}_T^{ij\tau s}$ is the fundamental nucleus of the tangent stiffness matrix. According to CUF, this 3x3 matrix, given the cross-section functions F_τ and the 1D shape functions N_i, can be expanded using the indexes $\tau, s = 1,\ldots,M$ and $i,j = 1,\ldots,p+1$ to obtain the element tangent stiffness matrix of any arbitrarily refined beam model. Note that $\boldsymbol{K}_0^{ij\tau s}$ is the fundamental nucleus of the linear stiffness matrix, $\boldsymbol{K}_{T1}^{ij\tau s}$ is the nonlinear contribution of the fundamental nucleus of the tangent stiffness matrix due to the linearization of the nonlinear constitutive relation, and $\boldsymbol{K}_\sigma^{ij\tau s}$ is the contribution of the geometric stiffness (nonlinear pre-stresses).

The full derivation of the nonlinear problem in the framework of CUF is not given here for the sake of brevity. However, interested readers are referred to the recent paper of Pagani and Carrera [3].

Fig. 2. Commercial tape spring; 170 mm sample

3. Experimental setup

Figure 2 shows the 170 mm sample that is the subject of this study. With no loss of generality, the sample was opportunely obtained from a commercial tape spring composed of isotropic steel and with a thin layer of coating. Because shells are very sensitive to small imperfections, accurate measurements of the tape spring cross-section were made using a FaroArm scanner and are shown in Fig. 3. In the figure, the radius of curvature

R measures 12.3 mm, $\alpha = 62^o$, $l = 6.1$ mm, and the thickness is $t = 0.114 \pm 0.007$ mm.

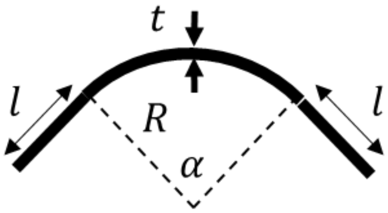

Fig. 3. Cross-section geometry

Testing was done using the Fischer's machine [4], which makes use of two rotation control dials to apply moment at the free ends of the samples, see Fig. 4. Note that one side of the machine is fixed, while the other end is allowed to translate. Two strain gauges are attached on adaptors, one on each side of the machine. The strain gauges were calibrated to find the factor between the data from the amplifier (strain reading) and the moment. For the sake of completeness, Fig. 5 shows equilibrium states of the tape spring subjected to different opposite sense and equal sense bending moments.

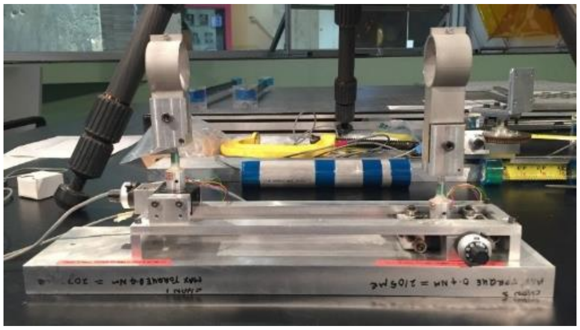

Fig. 4. Fischer's testing machine

4. Results

The results from the experiments are compared to numerical simulations based on CUF finite elements in this section. For this purpose, higher-order beam models are used by expressing the theory kinematics by means of Lagrange Expansions (LE). LE models have been demonstrated to provide efficient solutions for thin-walled beams in both linear and geometrical nonlinear regimes [5]. As a matter of fact, by using piece-wise higher-order LE kinematics, one can describe severe cross-section deformations although 1D models are used.

The CUF model adopted in the proposed analysis employs 32 L9 (nine-node quadratic) polynomial subdomains on the beam cross-section. The equilibrium curve from this model is depicted in Fig. 6, where the experimental results are also given. It should be noted that linear stiffness (the elastic modulus) is slightly overestimated by the simulation. However, the steady-state moment in far post-buckling is perfectly predicted. Also, more important, limit loads are demonstrated to be accurately foreseen by the proposed simulation. Nevertheless, there is difference between finite element model and experiments in the case of the equal sense post-buckling mechanics.

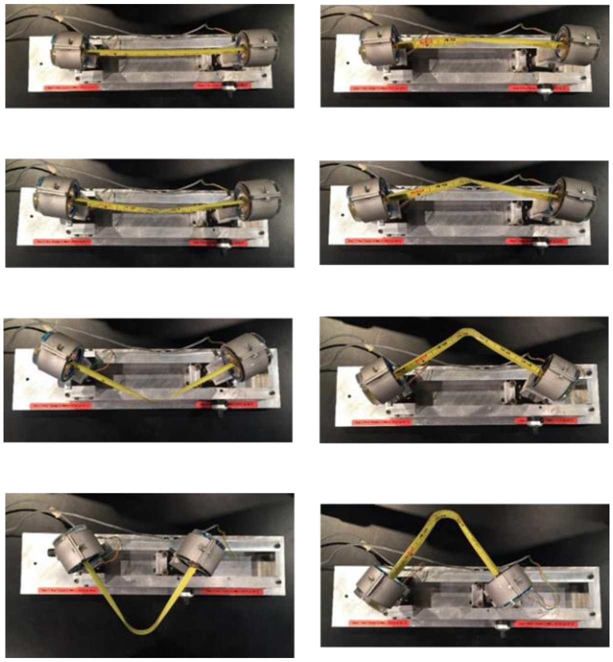

Fig. 5. Deformed configurations of the tape spring subjected to opposite sense (left) and equal sense (right) bending moments

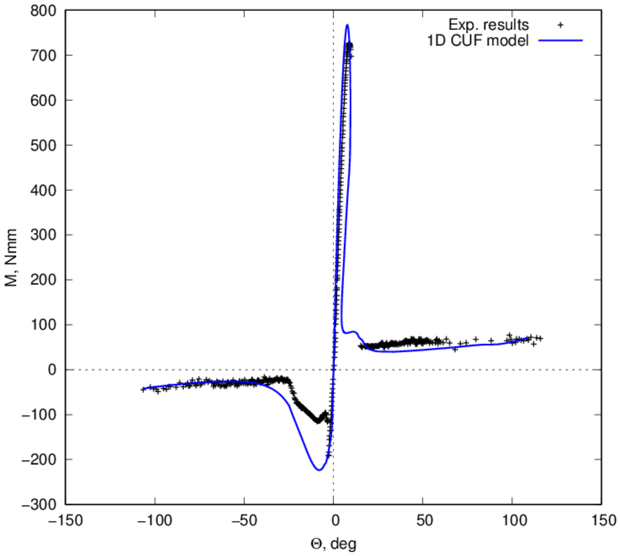

Fig. 6. Equilibrium curves from experiments and CUF-based finite element model

To discuss this aspect, Figs. 7 and 8 show the post-buckling states of the tape spring by CUF when subjected to opposite and equal sense moments, respectively. By comparing with Fig. 5, it is clear that the equal sense post-buckling presents a few differences with experiments. This is due to slight asymmetries (defects) of the tested structure that cause a torque mechanics, which cannot be predicted by the numerical model. Future research will investigate the defect sensitivity in the simulation analysis for tape spring under equal sense moments.

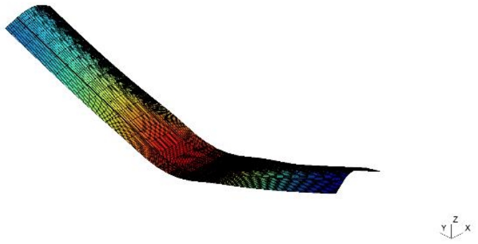

Fig. 7. Opposite sense post-buckling by CUF

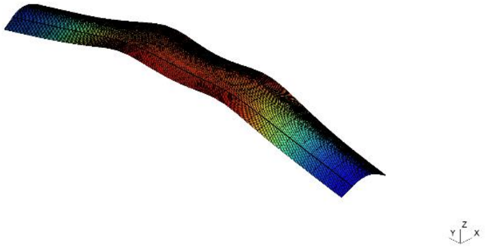

Fig. 8. Equal sense post-buckling by CUF

6. Conclusions

This work has explored the use of the Carrera Unified Formulation (CUF) for simulating the formation of localized folds during the process of coiling of deployable thin shells. CUF, in fact, is a hierarchical methodology, which expresses the governing equations in terms of fundamental nuclei. Because the nuclei are invariant of the theory approximation order, one can tune the efficiency and the accuracy of the analysis opportunely.

The efficacy of the proposed method in describing near and far post-buckling mechanics of a representative tape spring has been demonstrated via dedicated experiments. The model is able to foresee limit loads and steady-state moments. On the other hand, it has been shown that the structure is very sensitive to geometry defects in the case of equal sense moment. From this standpoint, a dedicated sensitivity analysis by CUF will be performed.

Future work will also include the extension to composite deployable booms with the aim of studying accurately 3D nonlinear stress fields and related failure mechanics, including delamination and free-edge effects.

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