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Some Groupoids and their Representations by Means of Integer Sequences

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Abstract: In some previous works, we have discussed the groupoids related to the integer sequences of Mersenne, Fermat, Cullen, Woodall and other numbers. These groupoids possess different binary operators. As we can easily see, other integer sequences can have the same binary operators, and therefore can be used to represent the related groupoids. Using the On-Line Encyclopedia of Integer Sequences (OEIS), we are able to identify the properties of these representations of groupoids. At the same time, we can also find integer sequences not given in OEIS and probably not yet studied.

Keywords: Groupoid Representations, Integer Sequences, Binary Operators, Generalized Sums, Generalized Entropies, Tsallis Entropy, Q-Calculus, Abelian Groups, Fermat Numbers, Mersenne Numbers, Triangular Numbers, Repunits, Oblong Numbers

Introduction

A groupoid is an algebraic structure made by a set with a binary operator [1]. The only restriction on the operator is closure. This property means that, applying the binary operator to two elements of a given set S, we obtain a value which is itself a member of S. If this binary operation is associative and we have a neutral element and opposite elements into the set, the groupoid becomes a group.

Groupoids are interesting also for the study of integer numbers. As shown in some previous works [2-7], the integer sequences of Mersenne, Fermat, Cullen, Woodall and other numbers are groupoid possessing different binary operators. Here we show that other integer sequences can have the same binary operators, and therefore can be used to represent the related groupoids. That is, we can obtain different integer sequences by means of the recurrence relations generated by the considered binary operations.

In [7], we started the search for different representations for the groupoid of Triangular Numbers. Here we generalize this search, using the binary operators obtained in the previous analyses. In particular, we will see the representations linked to Mersenne, Fermat, Cullen, Woodall, Carol and Kynea, and Oblong numbers. The binary operators of these numbers have been already discussed in previous works. The results concerning the Triangular numbers are also reported.

Using the On-Line Encyclopedia of Integer Sequences (OEIS), we are able to identify the several representations of groupoids. At the same time, we can also find integer sequences not given in OEIS and probably not yet studied.

Mersenne numbers

We discussed the binary operator of the set of Mersenne numbers in [8,9]. The numbers are given as$M_n = 2^n - 1$. The binary operator is:

$$M_{n+m} = M_n \oplus M_m = M_n + M_m + M_n M_m \quad (1)$$

As shown in [9], this binary operation is a specific case of the binary operator of q-integers, which can be linked to the generalized sum of Tsallis entropy [10,11].

The binary operator can be used to have a recurrence relation:

$$M_{n+1} = M_n \oplus M_1 \quad (2)$$

Here in the following, let us show the sequences that we can generate from (1) and (2).

We use OEIS, the On-Line Encyclopedia of Integer Sequences, to give more details on them.

$M_1 = 0$, sequence 0, 0, 0, 0, 0, 0, … .

$M_1 = 1$, sequence 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535, 131071, 262143, 524287, 1048575, 2097151, and so on. The Mersenne numbers $2^n - 1$. This sequence is OEIS A000225. (OEIS tells that these numbers are sometimes called Mersenne numbers, "although that name is usually reserved for A001348").

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$M_1 = 2$, sequence 8, 26, 80, 242, 728, 2186, 6560, 19682, 59048, 177146, 531440, 1594322, 4782968, 14348906, 43046720, 129140162, 387420488, and so on (OEIS A024023, $a_n = 3^n - 1$).

$M_1 = 3$, sequence 15, 63, 255, 1023, 4095, 16383, 65535, 262143, 1048575, 4194303, 16777215, 67108863, 268435455, and so on (OEIS A046092, $a_n = 4^n - 1$).

And we can continue: $M_1 = 4$, OEIS A024049, $a_n = 5^n - 1$; $M_1 = 5$, OEIS A024062, $a_n = 6^n - 1$; $M_1 = 6$ OEIS A024075, $a_n = 7^n - 1$, and so on. An interesting sequence is $M_1 = 9$, A002283, $a_n = 10^n - 1$. Dividing this sequence by 9, we have the repunits A002275, $a_n = (10^n - 1)/9$. The generalized sum of the repunits is given in [12].

Fermat numbers

The group of Fermat numbers has been discussed in [13]. As explained in [14], there are two definitions of the Fermat numbers. "The less common is a number of the form $2^n + 1$ obtained by setting x=1 in a Fermat polynomial, the first few of which are 3, 5, 9, 17, 33, ... (OEIS A000051)" [14]. We used this definition.

$$F_n = 2^n + 1$$

$$F_{n+m} = F_n \oplus F_m = (1 - F_n) + (1 - F_m) + F_n F_m \quad (3)$$

The binary operator can be used to have a recurrence relation: $F_{n+1} = F_n \oplus F_1$ (4)

Sequences can generate from (3) and (4).

$F_1 = 0$, sequence 2, 0, 2, 0, 2, 0, ... ;

$F_1 = 1$, sequence 1, 1, 1, 1, 1, 1, ...

$F_1 = 2$, sequence 2, 2, 2, 2, 2, 2, ...

$F_1 = 3$, sequence 5, 9, 17, 33, 65, 129, 257, 513, 1025, 2049, 4097, 8193, 16385, 32769, 65537, 131073, 262145, 524289, 1048577, 2097153, and so on, the Fermat numbers. (OEIS A000051, $a_n = 2^n + 1$.

$F_1 = 4$, sequence A034472, $a_n = 3^n + 1$; for $F_1 = 5$, sequence A052539, $a_n = 4^n + 1$. Continuing with 6, we have A034474, $a_n = 5^n + 1$. For 7, we have A062394, $a_n = 6^n + 1$. And so on.

Cullen and Woodall numbers

These numbers had been studied in [15].
Let us consider the Cullen numbers, $C_n = n2^n + 1$. We have the binary operator:

$$C_{n+m} = C_n \oplus C_m = \left(\frac{1}{n} + \frac{1}{m}\right)(C_n - 1)(C_m - 1) + 1 \quad (5)$$

$$C_{n+1} = C_n \oplus C_1 \quad (6)$$

$C_1 = 1$, sequence 1, 1, 1, 1, 1, 1, 1, and so on.

$C_1 = 2$, sequence 3, 4, 5, 6, 7, 8, 9, and so on.

$C_1 = 3$, sequence 9, 25, 65, 161, 385, 897, 2049, 4609, 10241, 22529, 49153, 106497, 229377, 491521, 1048577, 2228225, 4718593, 9961473, 20971521, 44040193, and so on. OEIS A002064, Cullen numbers: $a_n = n2^n + 1$.

$C_1 = 4$, sequence 19, 82, 325, 1216, 4375, 15310, 52489, 177148, 590491, 1948618, 6377293, 20726200, 66961567, 215233606, 688747537, and so on. OEIS A050914, $a_n = n3^n + 1$. For $C_1 = 5$ sequence A050915, $a_n = n4^n + 1$. And so on. Let us mention the case $C_1 = 11$which is giving A064748, $a_n = n10^n + 1$. That is: 201, 3001, 40001, 500001, 6000001, 70000001, 800000001, and so on.

Woodall numbers are $W_n = n2^n - 1$, and the binary operator is:

$$W_{n+m} = W_n \oplus W_m = \left(\frac{1}{n} + \frac{1}{m}\right)(W_n + 1)(W_m + 1) - 1 \quad (7)$$

$$W_{n+1} = W_n \oplus W_1 \quad (8)$$

$W_1 = 0$, sequence 1, 2, 3, 4, 5, 6, 7, and so on.

$W_1 = 1$, sequence 7, 23, 63, 159, 383, 895, 2047, 4607, 10239, 22527, 49151, 106495, 229375, 491519, 1048575, 2228223, 4718591, 9961471, 20971519, 44040191, and so on. A003261, Woodall (or Riesel) numbers:$a_n = n2^n - 1$.

$W_1 = 2$, sequence A060352, $a_n = n3^n - 1$. For $W_1 = 3$, we have sequence A060416, $a_n = n4^n - 1$. And so on. Let us mention the case $W_1 = 9$, which is giving A064756, $a_n = n10^n - 1$, that is, 199, 2999, 39999, 499999, 5999999, 69999999, 799999999, and so on.

Carol and Kynea Numbers

These numbers have been studied in [3]. Carol number is: $C_n = (2^n - 1)^2 - 2$. The binary operator $C_n \oplus C_m$is given in [3]:

$$C_m \oplus C_n = 6 + C_m C_n + 3C_m + 3C_n + a + b + c$$

where $a = 2(C_m + 2)(C_n + 2)^{1/2}$,$b = 2(C_m + 2)^{1/2}(C_n + 2)$,$c = 2(C_m + 2)^{1/2}(C_n + 2)^{1/2}$.
We can use again $C_{n+1} = C_n \oplus C_1$. Since the binary operator contains square roots, we can obtain integer

sequences only in some cases.

$C_1 = -1$, sequence A093112, $a_n = (2^n - 1)^2 - 2$, that is 7, 47, 223, 959, 3967, 16127, 65023, 261119, 1046527, 4190207, … As told in [16], Cletus Emmanuel called these numbers as "Carol numbers".

$C_1 = 2$, sequence 62, 674, 6398, 58562, 529982, 4778594, 43033598, 387381122, 3486666302, 31380705314, and so on. Not given in OEIS.

$C_1 = 7$, sequence 223, 3967, 65023, 1046527, 16769023, 268402687, 4294836223, 68718952447, 1099509530623, 17592177655807, and so on. Not given in OEIS.

Let us consider the Kynea numbers.

$$K_n = (2^n + 1)^2 - 2$$

The binary operator $K_n \oplus K_m$ is given in [3]. We use again $K_{n+1} = K_n \oplus K_1$. Again, we have square roots, so we can obtain integer sequences only in some cases.

$K_1 = -1$, sequence -1, -1, -1, -1, -1, and so on.

$K_1 = 2$, sequence 2, 2, 2, 2, 2, 2, and so on.

$K_1 = 7$, sequence A093069 , $a_n = (2^n + 1)^2 - 2$, that is 7, 23, 79, 287, 1087, 4223, 16639, 66047, 263167, 1050623, 4198399, and so on. As told in [17], Cletus Emmanuel calls these "Kynea numbers" [17].

$K_1 = 14$, sequence 98, 782, 6722, 59534, 532898, 4787342, 43059842, 387459854, 3486902498, 31381413902, and so on. Not given in OEIS.

Oblong numbers
These numbers are discussed in [4]. The oblong number is defined as: $O_n = n(n+1)$. It is given by OEIS A002378. An oblong number is also known as a promic, pronic, or heteromecic number. OEIS gives the list: 2, 6, 12, 20, 30, 42, 56, 72, 90, 110, 132, 156, 182, 210, 240, 272, 306, 342, 380, 420, 462, 506, 552, 600, 650, 702, 756, 812, 870, 930, 992, 1056, and so on.
The binary operator $O_n \oplus O_m$ is:

$$O_m \oplus O_n = \frac{1}{2} + O_m + O_n + a + b$$

where $a = 2(O_m + 1/4)^{1/2}(O_n + 1/4)^{1/2}$, $b = -(O_m + 1/4)^{1/2} - (O_n + 1/4)^2$. Again, as we did before we have:

$O_1 = 0$, sequence 0, 0, 0, 0, 0, and so on.

$O_1 = 2$, sequence OEIS A002378, as given above.

$O_1 = 6$, sequence, A002943, $a_n = 2n(2n+1)$.

$O_1 = 12$, sequence A045945, Hexagonal matchstick numbers: $a_n = 3n(3n+1)$.

$O_1 = 20$, sequence 72, 156, 272, 420, 600, 812, 1056, 1332, 1640, 1980, and so on. Not given in OEIS. It is $a_n = 4n(4n+1)$. And we can continue.

Of course, we can repeat the same approach for the odd squares (A016754) numbers. Their binary operator is given in [4]. Also for the centered square numbers and the star numbers, we have the binary operators [5,6], so we can find the related representations by means of integer sequences too. As previously told, among the generated sequences, news sequences are produced that can be interesting for further investigation of integer sequences.

Triangular numbers
These numbers are really interesting. The numbers are of the form (OEIS A000217):

$$T_n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

I have discussed them in [7]. For these numbers we can give two binary operators. For the convenience of the reader, I show the results that we can obtain.
The first binary operator is [7]:

$$T_n \oplus T_m = T_n + T_m + \tfrac{1}{4}\left[1 - (1+8T_n)^{1/2} - (1+8T_m)^{1/2} + (1+8T_n)^{1/2}(1+8T_m)^{1/2}\right]$$

Again we consider $T_{n+1} = T_n \oplus T_1$, and change the value of T_1. Here in the following the sequences that we generate.

$T_1 = 0$, sequence 0, 0, 0, 0, 0, 0, … .

$T_1 = 1$, sequence 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, and so on. And this is OEIS A000217, the sequence of triangular numbers.

$T_1 = 3$, sequence 10, 21, 36, 55, 78, 105, 136, 171, 210, 253, 300, 351, 406, 465, 528, 595, 666, 741, 820, 903, … . Searching this sequence in OEIS, we can easily find that it is A014105, that is, the Second Hexagonal Numbers: $H_n = n(2n+1)$.

$T_1 = 4$, sequence 12, 24, 40, 60, 84, 112, 144, 180, 220, 264, 312, 364, 420, 480, 544, 612, 684, 760, 840, 924, … OEIS A046092 (four times triangular numbers).

$T_1 = 6$, sequence 21, 45, 78, 120, 171, 231, 300, 378, 465, 561, 666, 780, 903, 1035, 1176, 1326, 1485, 1653, 1830, 2016, … OEIS A081266 (Staggered diagonal of triangular spiral in A051682).

$T_1 = 7$, sequence 23, 48, 82, 125, 177, 238, 308, 387, 475, 572, 678, 793, 917, 1050, 1192, 1343, 1503, 1672, 1850, 2037, … OEIS A062725.

$T_1 = 10$, sequence 36, 78, 136, 210, 300, 406, 528, 666, 820, 990, 1176, 1378, 1596, 1830, 2080, 2346, 2628, 2926, 3240, 3570, … OEIS A033585, that is, numbers: $2n(4n+1)$.

$T_1 = 11$, sequence 38, 81, 140, 215, 306, 413, 536, 675, 830, 1001, 1188, 1391, 1610, 1845, 2096, 2363, 2646, 2945, 3260, 3591, … OEIS A139276, that is, numbers $n(8n+3)$.

Of course, we can continue and obtain further sequences.
As previously told, we have a second binary operator for the triangular numbers [7]. It is the following:

$$T_n \oplus T_m = T_n + T_m + \frac{1}{4}\left[1 + (1+8T_n)^{1/2} + (1+8T_m)^{1/2} + (1+8T_n)^{1/2}(1+8T_m)^{1/2}\right]$$

Again, let us consider $T_{n+1} = T_n \oplus T_1$ as we did before.

$T_1 = 0$, sequence 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, and so on. OEIS A000217, the sequence of triangular numbers.

$T_1 = 1$, sequence 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435, 496, 561, 630, 703, 780, 861, … OEIS A000384, Hexagonal numbers $H_n = n(2n-1)$.

$T_1 = 3$, sequence 15, 36, 66, 105, 153, 210, 276, 351, 435, 528, 630, 741, 861, 990, 1128, 1275, 1431, 1596, 1770, 1953, … OEIS A062741, three times pentagonal numbers $3n\,(3n-1)/2$.

$T_1 = 4$, sequence 17, 39, 70, 110, 159, 217, 284, 360, 445, 539, 642, 754, 875, 1005, 1144, 1292, 1449, 1615, 1790, 1974, … OEIS A022266, numbers $n\,(9n-1)/2$.

$T_1 = 6$, sequence 28, 66, 120, 190, 276, 378, 496, 630, 780, 946, 1128, 1326, 1540, 1770, 2016, 2278, 2556, 2850, 3160, 3486, … OEIS A014635, numbers $2n(4n-1)$.

$T_1 = 7$, sequence 30, 69, 124, 195, 282, 385, 504, 639, 790, 957, 1140, 1339, 1554, 1785, 2032, 2295, 2574, 2869, 3180, 3507, … OEIS A139274, numbers $n(8n-1)$.

$T_1 = 10$, sequence 45, 105, 190, 300, 435, 595, 780, 990, 1225, 1485, 1770, 2080, 2415, 2775, 3160, 3570, 4005, 4465, 4950, 5460 …. This sequence is not present in OEIS.

$T_1 = 11$, sequence 47, 108, 194, 305, 441, 602, 788, 999, 1235, 1496, 1782, 2093, 2429, 2790, 3176, 3587, 4023, 4484, 4970, 5481, …. OEIS A178572, numbers with ordered partitions that have periods of length 5.

Using the On-Line Encyclopedia of Integer Sequences (OEIS), we have seen that quite different sequences can have the same binary operators. We have also found integer sequences not given in OEIS and that need to be studied.

Conclusion

Groupoids are related to the integer sequences. These groupoid possess different binary operators. As we have shown, other integer sequences can have the same binary operators, and therefore can be used to represent the related groupoids.

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