

# Preliminary Study of a Cylindrical Microstrip Metasurface Using the State Space Method

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**Abstract**—In this work a preliminary study of the state space method applied to the electromagnetic cloaking problem is presented. A dielectric coated circular cylinder cloaked with a metasurface based on a sinusoidally modulated unit cell is considered. The state space method is used to examine the object's scattered field and to analyze its composition in terms of specular, trapped and creeping wave modes around the cloaking working frequency.

## I. INTRODUCTION

Electromagnetic metamaterials are well-known for their exotic characteristics (such as the possibility to show a negative permittivity at microwave frequencies [1]), which derive from the periodic repetition of a specific unit cell, usually composed of metallic inclusions on a dielectric substrate. By modifying the shape of this unit cell and its dimensions, it is possible to control the material characteristics and thus change its electromagnetic (EM) scattering response.

One application of metamaterials in electromagnetics is to cloak a target object, i.e., to minimize its scattering in order to make the object invisible to an external observer.

This effect could be realized in different ways. In [2] an anisotropic 3D metamaterial is proposed such that the EM wave is guided around the target object and the field is reconstructed behind it. In [3] a different technique based on cancellation of the principal harmonic of the scattered field has been proposed. This could be realized by changing the input impedance of the object by covering it with an appropriate coating such that the impedance of the object plus that of the cloaking layer equals the intrinsic impedance of the background medium [4].

Isolation of relevant scattering mechanisms is crucial to defining a data-driven approach for the efficient design of an EM cloak. In this paper, the state space method (SSM) is used to examine cloaking by a sinusoidally modulated metasurface on an infinite coated cylinder, leading to the extraction of waves species of the scattered field in terms of backscattered creeping waves and trapped modes.

The novelty of this paper is to utilize a state-space approach [5] to formulate the linear system matrix of the underlying scattering problem, whose complex eigenvalues and eigenvectors allow for the isolation of specific modal responses and their contributions to backscattering.

## II. STRUCTURE GEOMETRY

In the following, a coated metallic cylinder is considered. The structure is composed of an inner perfect electrically conducting (PEC) cylinder of radius  $a = 20$  mm, coated with a dielectric layer of thickness  $t = 2.9$  mm and relative permittivity  $\epsilon_r = 2.3$ . On the top of the dielectric is placed a modulated microstrip line metasurface based on a sinusoidally shaped unit cell, which is described by:

$$W(u) = W_{min} + (W_{max} - W_{min}) \left( \sin \frac{\pi \cdot u}{D_u} \right)^\alpha \quad (1)$$

where  $W_{max}$  and  $W_{min}$  are the maximum and the minimum line width, respectively,  $D_u$  is the cell length, and  $\alpha$  is the constant which determines the cell modulation.

In [6] the cloaking behavior of this structure has been demonstrated, showing how by acting on the geometrical characteristics of the unit cell, in particular on  $W_{max}$ , it is possible to tune the working frequency of the structure.

In the following,  $W_{max} = 27.3$  mm,  $W_{min} = 0.2$  mm,  $D_u = 4$  mm and  $\alpha = 0.5$  is considered. The number of cells in the azimuthal and longitudinal direction are  $N_u = 36$  and  $N_v = 3$ , respectively (see the inset of Fig. 1 for the geometry). In order to reduce the effect of the edge diffraction two absorbers are placed on the top and bottom of the cylinder. The coated structure is illuminated by a linearly polarized plane wave with the electric field vector parallel to the cylinder axis.

The behavior of the surface waves propagating along the object's boundary has been studied using the SSM, briefly discussed in the next section. The dielectric coated cylinder with and without the presence of the metasurface layer is considered for a comparative study of the wave decomposition.

## III. SPACE STATE METHOD

The EM response of any scattering object may be represented as a sum of complex sinusoids, whose amplitude and phase are related closely to the EM parameters of interest (e.g., complex propagation constants and their modal amplitudes). Let the radar cross section (RCS) be given by the data sequence  $y(k)$  comprising  $N$  uniformly spaced frequency samples, each represented as a sum of  $M$  complex sinusoids (or point scatterers) corrupted by white Gaussian noise  $w(k)$ .

Over a given bandwidth, the signal measurements at these  $N$  frequencies can be modeled as:

$$y(k) = \sum_{i=1}^M a_i p_i^k + w(k); \quad k = 1, \dots, N \quad (2)$$

where  $a_i$  refers to the amplitude of the  $i^{\text{th}}$  scattering center, and the complex pole  $p_i$  is given by:

$$p_i = \exp \left[ -\Delta f \left( \alpha_i + j \frac{4\pi}{c} R_i \right) \right] \quad (3)$$

In (3),  $\alpha_i$  and  $R_i$  denote the spectral decay/growth constant and range, respectively, associated with the  $i^{\text{th}}$  scattering center. The notation  $c$  refers to the speed of light, and  $\Delta f$  is the sampling frequency. The primary interest is to estimate the parameters  $\alpha_i$  and  $R_i$  embedded in the data sequence  $y(k)$ . The state space method, originally presented in [7] and extended to EM problems in [5], provides an efficient computation of the decay/growth constant and range from the eigenvalues of an open-loop matrix. Once these parameters are estimated, the amplitudes  $a_i$  can readily be derived using least-squares fit of the data to the frequency response of a sum of poles in the form of (2). The reader is referred to [5] for details.

#### IV. RESULTS

The EM field, hence the RCS, has been computed, using CST Microwave Studio, for the cylindrical metasurface shown in Fig.1 inset. Then SSM is used to extract specular and creeping wave components from the monostatic RCS computed at broadside. The creeping waves propagate circumferentially along the surface of a curved object and shed energy tangentially into the shadow region.

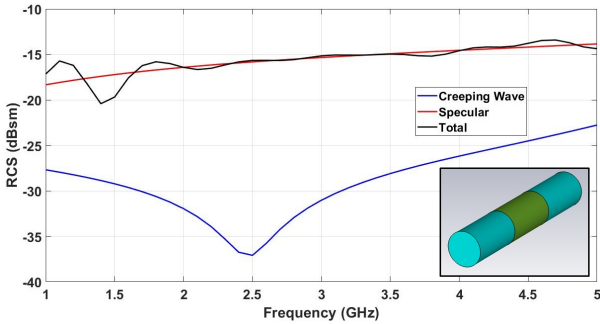


Fig. 1. SSM-extracted wave species for the coated cylinder.

Fig. 1 displays the extracted wave constituents for a coated dielectric cylinder without the metasurface. The scattered field consists of two components: the specular reflection off the front of the cylinder and the surface wave that traverses half the circumference in the shadow region and detaches along parallel rays off the cylinder to reach the observer. In SSM each wave is uniquely characterized by two distinct poles corresponding to the range traversed by each wave. As the frequency increases, the creeping wave path becomes

electrically large and the scattering approaches the limit of a planar slab (trapped mode) [8]. The transition from creeping wave to trapped mode occurs at 2.5 GHz. Trapped modes are predominantly guided waves and radiate very little around this cutoff frequency.

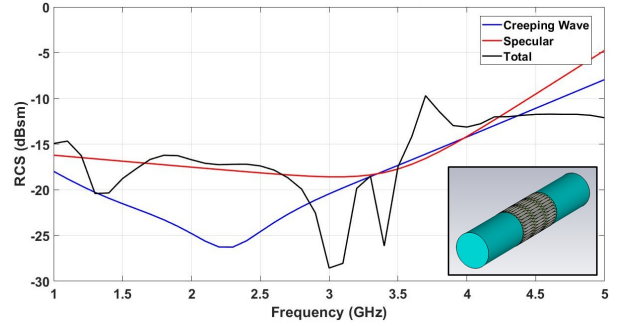


Fig. 2. SSM-extracted wave species for the coated cylinder with the microstrip metasurface.

Fig. 2 displays the extracted specular and creeping wave constituents for a coated dielectric cylinder *with* the metasurface. The difference in amplitude between the two wave types is much smaller than that for the coated cylinder, and the amplitudes become comparable for  $f > 3$  GHz. The creeping wave transitions to trapped mode around 2.3 GHz. Below this cutoff, the attenuation of the creeping wave is relatively large and the scattered response is dominated by the specular wave.

#### V. CONCLUSION

The application of the SSM to a metasurface cloaking structure is discussed. The SSM is used to examine the specular and the creeping wave nature of the scattered field a cylinder with and without the metasurface coating. This analysis could be a starting point for a further investigation on the cloaking phenomena and the metasurface design.

#### REFERENCES

- [1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity," *Physical Review Letters*, vol. 84, no. 18, pp. 4184-4187, 2000.
- [2] D. Schurig et al., "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, vol. 314, pp. 977-980, 2006.
- [3] Y. R. Padooru, A. B. Yakovlev, P. Chen, A. Alù, "Analytical modeling of conformal mantle cloaks for cylindrical objects using subwavelength printed and slotted arrays," *Journal of Applied Physics* 112, 034907, 2012.
- [4] G. Labate, S.K. Podilchak, L. Matekovits, "Closed-form harmonic contrast control with surface impedance coatings for conductive objects," *Applied Optics*, vol. 56, Issue 36, pp. 10055-10059, 2017.
- [5] K. Naishadham and J. E. Piou, "A Robust State Space Model for the Characterization of Extended Returns in Radar Target Signatures," *IEEE Trans. Antennas and Propag.*, vol. 56, no. 6, pp. 1742-1751, June 2008.
- [6] B. Cappello, L. Matekovits, "Effect of Geometrical Parameters of a Width Modulated Microstrip Line based Mantle-Cloak," *Proceedings of the 2018 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting*, 8-13 July 2018.
- [7] S. Y. Kung, K. S. Arun, and D. V. Bhaskar Rao, "State-Space and SVD-Based Approximation Methods for the Harmonic Retrieval Problem," *J. Opt. Soc. Am.*, vol. 73, pp. 1799-1811, Dec. 1983.
- [8] K. Naishadham and L. B. Felsen, "Dispersion of Waves Guided Along a Cylindrically Stratified Medium," *IEEE Trans. Antennas and Propag.*, vol. 41, no. 3, pp. 304-313, March 1993.