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## 25th International Conference on Fracture and Structural Integrity

# Fatigue crack onset by Finite Fracture Mechanics

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### Abstract

The paper investigates the fatigue crack/notch sensitivity by the coupled criterion of Finite Fracture Mechanics (FFM). The approach involves two parameters: the plain specimen fatigue limit, and the threshold value of the stress intensity factor range for fatigue crack growth. Useful analytical relationships are presented. The accuracy of FFM is verified by considering experimental data available in the literature, showing the potentiality of the coupled approach to predict size effects.

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**Keywords:** Finite Fracture Mechanics; fatigue limit; cracks; notches; size effects

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### 1. Introduction

The failure mechanism under cyclic loadings of elements containing cracks or notches was traditionally faced by two different approaches. On one hand, the fatigue strength of cracked structures was addressed by Linear Elastic Fracture Mechanics (LEFM), involving the well-known concept of stress intensity factor (SIF)  $K_I$ . On the other hand, the strength reduction related to notches was treated by some stress-based approaches involving the stress concentration factor  $K_t$ .

In this framework, the Theory of Critical Distances (TCD) by Taylor (1999) (see also El Haddad et al. (1979); Tanaka (1983)) allowed to encompass the two distinct areas of cracks and notches removing some drawbacks, and accounting at the same time for size effects in damaged structures (Atzori et al., 2001).

The most simple criterion in the framework of TCD is the Point Method (PM) by Taylor (1999) (see also Tanaka (1983)). According to it the fatigue limit conditions are achieved when the range of the maximum principal stress at a distance  $l_c = 1/2\pi(\Delta K_{th}/\Delta\sigma_0)^2$  from the notch tip equals the plain fatigue limit  $\Delta\sigma_0$ ,  $\Delta K_{th}$  being the range of the threshold value of the SIF. By referring to the frame of reference in Fig. 1 the PM criterion can be expressed as:

$$\Delta\sigma_y(x = a + l_c) = \Delta\sigma_0 \quad (1)$$

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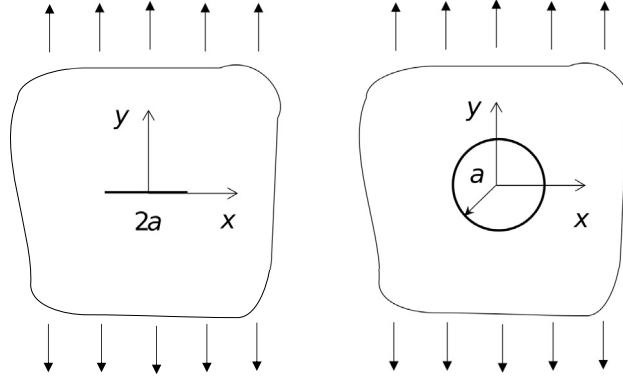


Fig. 1. Infinite tensile plate: center crack of length  $2a$  and a circular hole with radius  $a$ .

The other common stress criterion is the line stress one, also termed as Line Method (LM);

$$\frac{1}{l_c} \int_a^{a+l_c} \Delta\sigma_y(x) dx = \Delta\sigma_0 \quad (2)$$

Several studies have been carried out to establish which criterion between (1) and (2) provides the most accurate predictions (Atzori et al., 2001; Taylor, 2007; Susmel, 2008; Susmel and Taylor, 2011). Indeed, the situation varies from case to case, and the best approach depends on the particular geometry (Livieri and Tovo, 2004; Da Silva et al., 2012; Beber et al., 2019). On the other hand, stress approaches have some drawbacks, related to the fact that the crack advance  $l_c$  results a material constant: for very low sizes approaching  $l_c$ , the criteria fail in providing reasonable estimates. The FFM criterion by Cornetti et al. (2006) assumes a contemporaneous fulfilment of two conditions. The former is the stress condition expressed by Eq. (2). The latter one provides the relationship between the SIF range and the threshold value in the following terms:

$$\sqrt{\frac{1}{l_c} \int_0^{l_c} \Delta K_I^2(c) dc} = \Delta K_{th} \quad (3)$$

where  $c$  is the length of a crack stemming from the feature tip. Note that since  $\Delta K_I^2 \sim \Delta J$  (Anderson, 2017), the  $J$ -integral coinciding with the crack driving force under linear elastic conditions, Eq.(3) can be seen in terms of an energy requirement, similarly to the static case (Carpinteri et al., 2008).

At fatigue limit, the approach is thus expressed by a system of two equations, (2) and (3), in two unknowns: the critical crack advance  $l_c$  (which is no longer a mere material function) and the fatigue strength  $\sigma_f$ . FFM has recently been proved to provide nearly identical predictions to the cohesive zone model for different geometries (Cornetti et al., 2016). Finally, although not considered in the present work, it is worthwhile to mention the Strain Energy Density (SED) approach by Lazzarin and Zambardi (2001), which assumes as a critical parameter the strain energy in a small region around the notch tip, and which has been proved to provide accurate results in different fatigue contexts (Berto and Lazzarin, 2011; Meneghetti et al., 2016).

## 2. Crack and notch effects

Let us start by considering the case of a Griffith crack of length  $2a$  in an infinite plate subjected to a remote uniaxial tension (Fig. 1).

The stress field ahead of the crack tip can be expressed as:

$$\Delta\sigma_y(x) = \frac{x}{\sqrt{x^2 - a^2}} \Delta\sigma \quad (4)$$

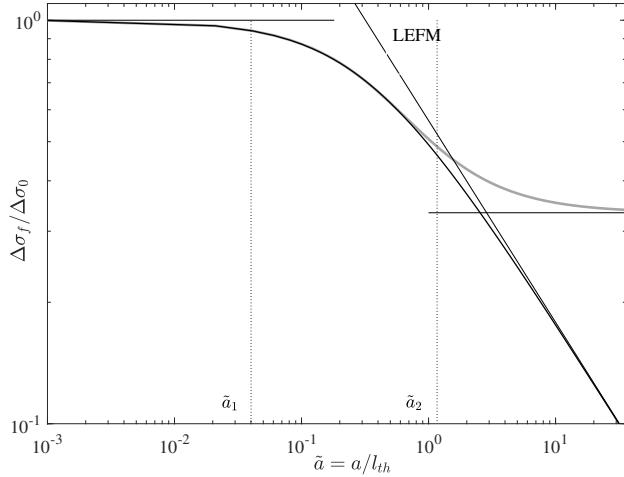


Fig. 2. FFM size effects on the fatigue limit related to cracks (thick black line) and notch-like-holes (gray line). The two horizontal asymptotes for small and large sizes, and LEFM predictions are also depicted.

whereas the SIF range writes:

$$\Delta K_I(a) = \Delta\sigma \sqrt{\pi a} \quad (5)$$

Substituting Eqs.(4) and (5) into Eqs. (2) and (3), respectively, yields the same result:

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \frac{1}{\sqrt{1 + \pi(a/l_{th})}} \quad (6)$$

where  $l_{th} = (\Delta K_{th}/\Delta\sigma_0)^2$ . Thus in this case, there is not any difference from the LM. Results on the failure stress are reported in Fig. 2, together with LEFM predictions. As it can be seen, the FFM allows to catch the transition from long to short cracks, avoiding the LEFM drawback.

As concerns the case of a circular hole of radius  $a$  in an infinite plate subjected to a remote uniaxial tension (Fig. 1), the stress field is equal to:

$$\Delta\sigma_y(x) = \frac{\Delta\sigma}{2} \left( 2 + \frac{a^2}{x^2} + 3 \frac{a^4}{x^4} \right) \quad (7)$$

whereas the SIF can be expressed as

$$\Delta K_I(c) = \Delta\sigma \sqrt{\pi c} F(s) \quad (8)$$

$c$  being the length of a crack stemming from the hole edge. By considering a symmetrical crack propagation (Sapora et al., 2018), the following relationship was proposed by Bowie (1956)

$$F = 0.5(3 - s)[1 + 1.243(1 - s)^3], \quad s = \frac{c}{c + a} \quad (9)$$

After substituting the expression for the stress field (7) into the stress condition (2) and the SIF (8) into Eq. (3), it is possible to get FFM predictions in a semi-analytical way. Results are presented in Fig. 2. As can be seen FFM satisfies the two asymptotic limits of a vanishing radius, and a well-developed hole when the strength is governed by the concentration factor  $K_t = 3$ . Furthermore, by introducing the notation  $\tilde{a} = a/l_{th}$ , a more detailed comparison between cracks and notches can be developed. Three ranges can be identified:

- $\tilde{a} \geq \tilde{a}_2$ : in this case, the structure is *feature sensitive*: the differences of a notch from a crack are consistent, and they increase as the size increases.

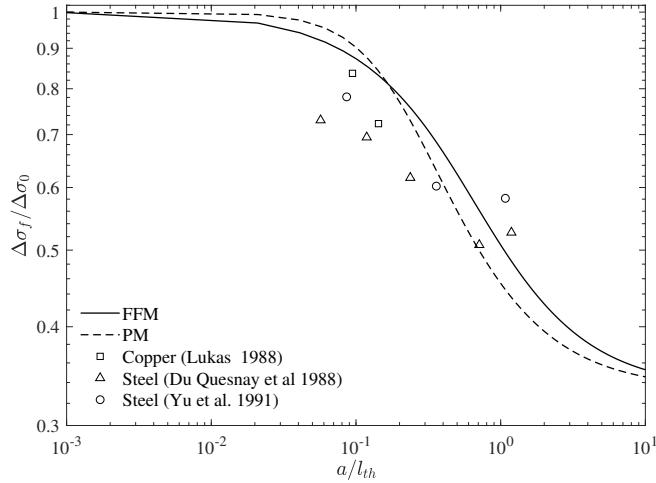


Fig. 3. Fatigue limit for elements containing a circular notch: predictions by FFM, PM, and experimental data.

- $\tilde{a}_1 < \tilde{a} < \tilde{a}_2$ : in this case, the structure can be supposed to be *feature shape insensitive*: the strength is affected by the presence of a flaw, but regardless its type.
- $\tilde{a} \leq \tilde{a}_1$ : in this case, the structure is *feature insensitive*: the fatigue limit is not affected by the presence of a feature.

By considering the case in exam, i.e. a sharp crack and a circular hole, and fixing an engineering tolerance of 5%, the following estimations can be provided:  $\tilde{a}_1 \simeq 0.04$  and  $\tilde{a}_2 \simeq 1.19$ . The diagram displayed in Fig. 2 can be thought as the FFM interpretation of the Atzori-Lazzarin diagram (Atzori and Lazzarin, 2001; Atzori et al., 2003), which was introduced to extend the Kitagawa-Takahashi diagram (Kitagawa and Takahashi, 1976) describing the size effects of cracks inside structures.

### 3. Comparison with experimental data

In order to verify the applicability of FFM in the fatigue framework a comparison with experimental data is invoked. The mechanical properties of the considered materials and necessary for the FFM implementation are reported in the corresponding references. Note that the loading ratio  $R$  affects the values of both  $\Delta K_{th}$  and  $\Delta\sigma_0$ , thus implicitly influencing the FFM analysis.

Data related to circular notches are firstly implemented by considering the experimental tests carried out by Du Quesnay et al. (1988); Lukás et al. (1989); Yu et al. (1991). By looking at the geometry of the samples, the size of the notch with respect to that of the sample is such to exploit the relationships presented in the previous Section. Results are depicted in Fig.3, together with predictions by the PM (Eq.(2)). The FFM accuracy is relatively good (indeed, some data show a not negligible uncertainty, as observed by Taylor (1999)), and the criterion is in tune with the PM: again it is hard to say which criterion is the most accurate, depending the answer on the particular size  $a/l_{th}$ . By comparing Figs. 1 and 3 it should be observed however that almost all the data fall in the range  $\tilde{a}_1 < \tilde{a} < \tilde{a}_2$ , where the behavior at failure is insensitive to the notch shape. A more interesting analysis would be that to consider data related to the range  $\tilde{a} > \tilde{a}_2$ , as recently performed in the static case by testing PMMA samples (Sapora et al., 2018).

As concerns the case of center trough thickness cracks, the experimental data reported in El Haddad et al. (1979) are taken into account and the comparison is shown in Fig. 4. In this case, as already observed, FFM predictions coincide with those by the LM and they reveal to be extremely accurate, whereas the PM generally tends to be overestimate the results.

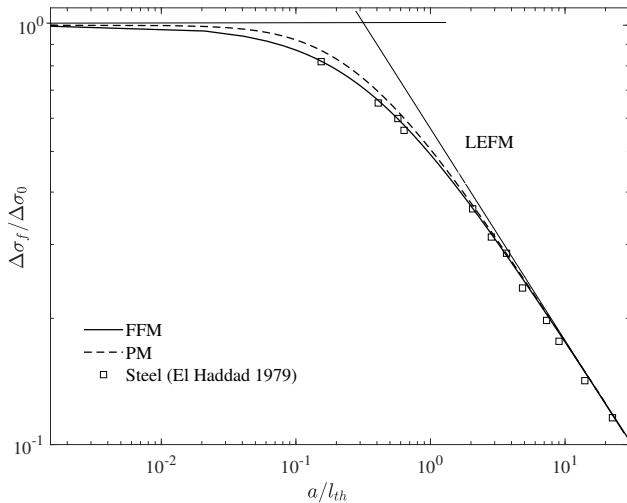


Fig. 4. Fatigue limit for elements containing a center through thickness crack: predictions by FFM, PM, and experimental data.

#### 4. Conclusions

The coupled criterion of FFM, already established in the framework of static fracture, was applied to assess the fatigue limit of structures presenting different features and subjected to mode I loading conditions. The analysis was focused on cracks and circular holes, the effects of which decrease with the size. The shape has no effect on the plain fatigue limit below a certain size  $\tilde{a}_2$ , but it is just below a lower value  $\tilde{a}_1$  that the structure becomes feature insensitive. FFM estimations on  $\tilde{a}_1$  and  $\tilde{a}_2$  are provided.

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