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Reduced Order Modeling of Thermal Transients in High-Performance Engineering Systems

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Abstract. Reduced order FEM modeling of thermal transients in high-performance engineering systems is proposed. The approach is based on the reduction of the fully 3D configurations by means of an averaging process along a preferred direction. The method takes into account for anisotropic material, variable thermal features and time-dependent boundary conditions. The numerical scheme is outlined and the application to two practical problems is illustrated. The simulations are compared with numerical and experimental results.

INTRODUCTION

The working conditions of many high-performance engineering systems are often characterized by the presence of strong energy exchanges involving steep thermal transients. For instance, in jet engine components such as the combustor, turbines and afterburner the hot gases can lead to temperatures higher than what the material can sustain. These temperatures are allowed either for limited time intervals or by means of aggressive cooling technologies [1–3]. In systems as the high-performance carbon-carbon brakes, optimal performances are obtained in a certain range of temperatures only, so that the unsteady evolution of the system must be maintained inside that range [4–7]. In both cases, the thermal design process relies on a series of analyses of the unsteady performances [1, 8–10]. Three-dimensional FEM thermal analyses of the full system are CPU and time-consuming. Their use can be motivated in case of an aggressive redesign. In the preliminary design phase or for optimization purposes, lower order models and computationally less intensive approaches are preferred. Optimal design does not require accurate system analyses but reliable, trend-following models of the system tha allow for the investigation of the full design space. In the following a reduced order FEM analysis of complex system is proposed. The full optimization process consists in: (i) performing an order reduction of the system to a two-dimensional or axis-symmetric representation; (ii) deducing appropriate boundary conditions and thermal features; (iii) carrying out an extensive investigation of the design space by using the Reduced Order Model (ROM); (iv) checking the optimal solution by the 3D analysis of the optimal configuration found. In the next section the mathematical model is explained and thermal analyses of two practical application are presented.

MATHEMATICAL MODEL

Governing equations

The mathematical model is based on the unsteady heat equation written in the form

$$\nabla \cdot (\nabla_\kappa T) + g(\boldsymbol{x}, T) = \rho(T)\, c_p(T)\, \frac{\partial T}{\partial t} \tag{1}$$

where T is temperature, c_p and ρ are the specific heat and the density of the medium, respectively. Both c_p and ρ are functions of the material and temperature. The source term $g(\boldsymbol{x}, T)$ expresses a thermal energy generation per unit

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volume within the medium. In cylindrical coordinates, the ∇ and ∇_κ operators are defined as

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{\partial}{\partial z}\hat{z}, \qquad \nabla_\kappa = k_r(T)\frac{\partial}{\partial r}\hat{r} + k_\theta(T)\frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + k_z(T)\frac{\partial}{\partial z}\hat{z} \tag{2}$$

$\hat{r}, \theta, \hat{z}$ are the radial, tangential and axial directions respectively. The functions k_r, k_θ and k_z defines the variation of thermal conductivity in the three directions. In the general case, Eq. (1) is a nonlinear PDE. The nonlinearity can rely on the dependence of c_p and κ from the temperature and on the expression of the source term $g(\boldsymbol{x}, T)$. The latter has been introduced for modeling purposes, but it is often $g = 0$, moreover the system is reduced to a 2D or axis-symmetric case ($\partial/\partial\varphi = 0$). In the latter case we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_r\frac{\partial T}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial z}\left(rk_z\frac{\partial T}{\partial z}\right) = \rho c_p\frac{\partial T}{\partial t} \tag{3}$$

An appropriate representation of the initial and boundary conditions must be specified. Typical BC's are of Robin type

$$-k\nabla T \cdot \mathbf{n} = \Phi(\boldsymbol{x}, t) + h(T - T_\infty), \qquad t \geq 0 \tag{4}$$

where h is the heat transfer coefficient, T_∞ is the temperature of the surrounding environment and $\Phi(\boldsymbol{x}, t)$ is an externally prescribed heat flux, e.g. the frictional effects of a pad on a disc brake. For a numerical treatment of the problem with the finite element method, we reformulate it in a weak form. By Galerkin's method, system (3) is written as

$$\int_\Omega \rho c_p \frac{\partial T}{\partial t} w \, d\Omega = \int_\Omega \nabla \cdot (\nabla_\kappa T) w \, d\Omega \tag{5}$$

where $w = w(r, z)$ is a virtual variation of the temperature field $T(r, z)$. By applying the Gauss theorem

$$\int_\Omega \rho c_p \frac{\partial T}{\partial t} w \, d\Omega + \int_\Omega \nabla_\kappa T \cdot \nabla w \, d\Omega = \int_\Sigma \nabla_\kappa T \cdot \hat{n} \, w \, d\Sigma \tag{6}$$

Finally, by using a finite difference approximation of the time derivative in the first integral

$$\int_\Omega \rho c_p \frac{T^n - T^{n-1}}{\Delta t} w \, d\Omega + \int_\Omega \nabla_\kappa T^n \cdot \nabla w \, d\Omega = \int_\Sigma \nabla_\kappa T^n \cdot \hat{n} \, w \, d\Sigma \tag{7}$$

we are lead to a system that can be solved according to the FEM technique describe in Refs. [11, 12].

Concerning the boundary conditions, a critical point is the evaluation of the heat transfer coefficient $h(T, t)$ distribution. In the present approach the evaluation of $h(T, t)$ is based on empirical correlations, e.g deduced from experimental testing or from 3D simulations. Generally, the results of experimental heat transfer problems are represented by functional relationships between non-dimensional variables and parameters such as Nusselt number Nu, the Prandtl number Pr and the Reynolds number Re. For many convection problems, the correlations are of the form

$$Nu = c \, Re^{\alpha} \, Pr^{\beta} \tag{8}$$

where the parameters c, α and β are evaluated experimentally. The heat transfer coefficient is then computed as $h_{conv} = Nu \, k_a/L$, where L the reference length, and k_a is the thermal conductivity of the fluid, e.g. air.

NUMERICAL RESULTS

Subscale aircraft brake test-case

In this section the numerical procedure is applied to the simulation of the thermal transient on a subscale aircraft brake in normal landing conditions, and compared to the corresponding experimental testcase[13]. The brake-calliper system is solved according to the FEM modeling proposed. The adopted thermal diffusivity $k_r(T)$ and $k_\theta(T)$ are different and variable with temperature as shown in Figure 1(f). The braking action of the system is simulated by a time varying heat flux $\Phi(t)$ on the brake-pad interface. The heat transfer coefficient is computed by the correlation for lightweight discs [4, 14]

$$Nu = 0.70 \, Re^{0.55} \cdot (Re < 2.4 \cdot 10^5) + 0.04 \, Re^{0.8} \cdot (Re \geq 2.4 \cdot 10^5) \tag{9}$$

where the Reynolds number is defined as $Re = UD/\nu_a$, with $U = \Omega D/2$. D is the disc diameter, ν_a is the kinematic viscosity of air. In the logical expressions $false = 0$ and $true = 1$. Some snapshots of the thermal transient are shown in Figure 1(b)-(e). Experimental unsteady temperature measures are compared with the numerical results in Figure 1(g). Temperatures below 300 K were dropped out from the experimental data according to sensor precision[13].

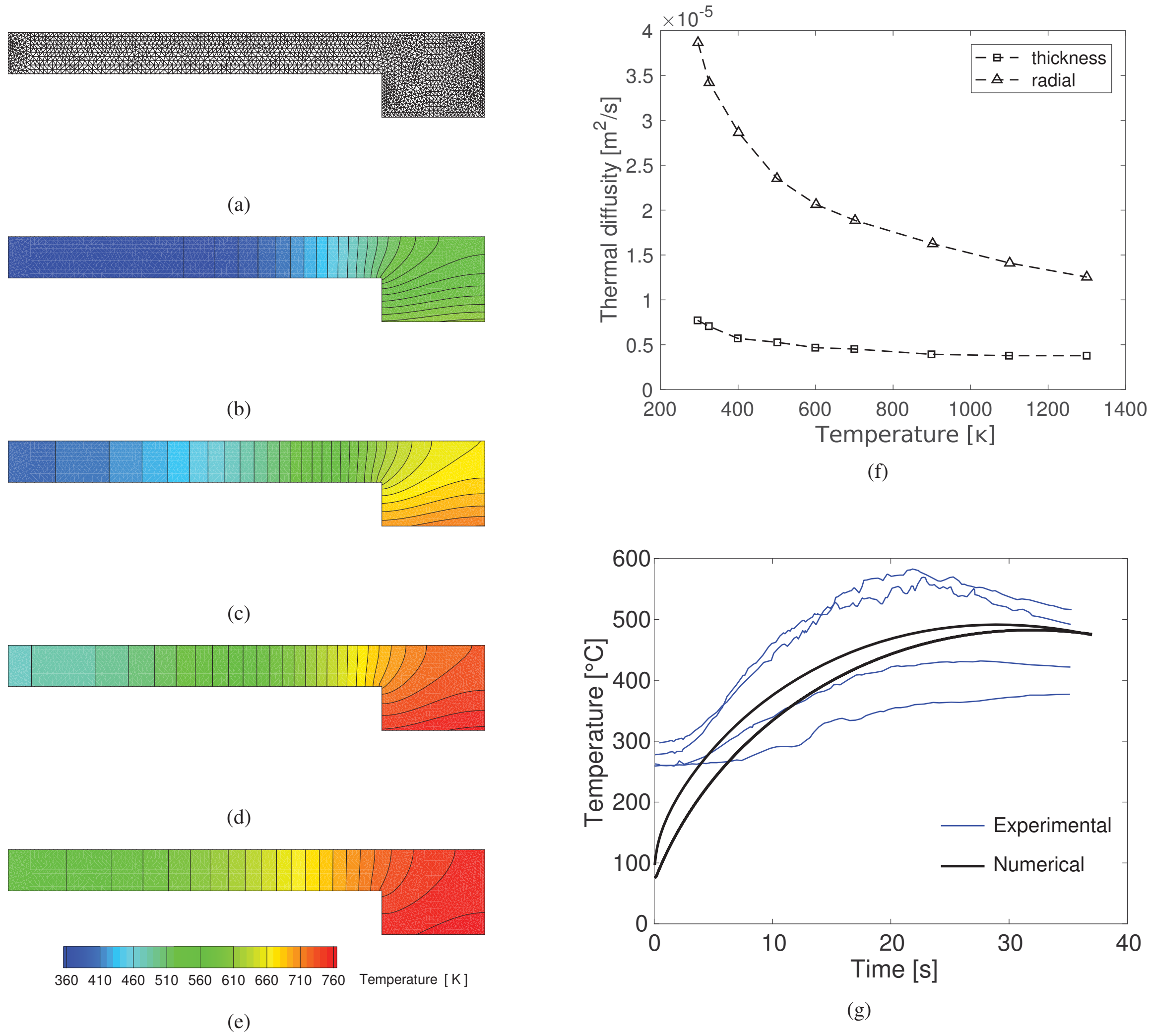


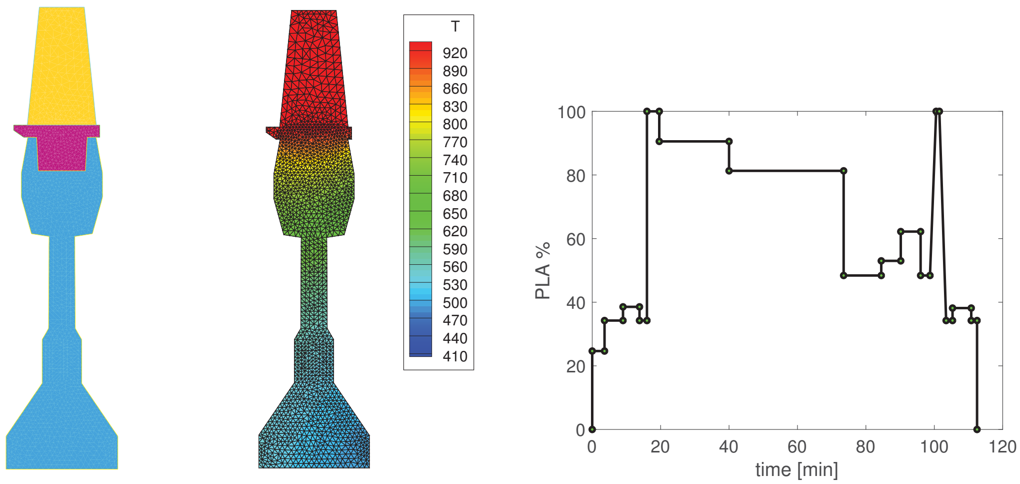
FIGURE 1. Subscale aircraft brake in normal landing conditions. (a) FEM grid. (b)-(c)-(d)-(e) Snapshots of the temperature feld, (f) thermal diffusivities k_θ and k_r. (f) Computed *(black)* and experimental *(blue)* interfacial temperature during the transient.

HP Turbine disc thermal analysis

In this section, an axis-symmetric FEM reduced order model of an HP turbine bladed rotor disc assembly is described. The example refers to the second HPT turbine rotor of the GE-CF56-50 turbofan engine [1]. We study the thermal effects that the hot-gases-flowpath generates on the turbine rotor disc. The computational domain has been divided in three parts having different material characteristics and mass distributions as shown in Figure 2(a). The FEM grid and the temperature field at the takeoff design conditions are shown in Figure 2(b). The reference conditions for the thermal analysis were deduced from the conventional engine flight cycle depicted in Figure 2(c) . Then, from the engine experimental testing or from numerical simulations [15] the most critical transients are identified and the related unsteady thermal stresses are evaluated.

CONCLUSIONS

An approach for the fast analysis of thermal transients in high-performance engineering systems has been presented. It is based on a reduction process of the 3D system to 2D or axis-symmetric configurations, then discretized by FEM.



(a) (b) (c)

FIGURE 2. HP turbine bladed disc. (a) Sketch of the computationa domains, (b) FEM grid and temperature field, (c) typical CF56-50 engine flight cycle with Power Level Angle (PLA) settings.

The method can deal with anisotropic materials and variable thermal features. The BCs may vary in time in order to simulate the interactions with the external ambient or the variation of the working conditions The method allows for higher details than the classical lumped system thermal analysis but maintains an affordable computational costs. As practical examples the simulation of thermal transients in a subscale aircraft brake and the thermal analysis of a HP turbine bladed disc have been presented.

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REFERENCES

[1] F. Barack and P. Domas, NASA CR-135033 (1976).
[2] E. Degregori and M. Ferlauto, Paper AIAA 2018-1143, AIAA Aerospace Sciences Meeting (2018).
[3] J.-G Yang, H. Wu, P. Yang, and K. Mao, Tuijin Jishu/Journal of Propulsion Technology **34**, 161–167 (2013).
[4] R. Limpert, *Brake Design And Safety. 2nd Ed.)* (Society of Automotive Engineers, Washington, DC, 1999).
[5] A. Adamowicz and P. Grzes, Applied Thermal Engineering **31**, 1003–1012 (2011).
[6] B. Boersma, R. Pecnik, H. Nemati, and J. Peeters, AIP Conference Proceedings **1648**, p. 030006 (2015).
[7] J. K. Grabski and J. A. Kolodziej, AIP Conference Proceedings **1738**, p. 030015 (2016).
[8] A. Alnaqi, D. Barton, and P. Brooks, Applied Thermal Engineering **75**, 658–668 (2015).
[9] Q. Jian and Y. Shui, International Journal of Thermal Sciences **122**, 115–123 (2017).
[10] D. Meresse, S. Harmand, M. Siroux, M. Watremez, and L. Dubar, Appl. Thermal Eng. **48**, 202–210 (2012).
[11] M. Ferlauto, Inverse Problems in Science and Engineering **21**, 269–282 (2013).
[12] M. Ferlauto, IMechE Part A: Journal of Power and Energy **228**, 328–339 (2014).
[13] D. Marx, T. Policandriotes, S. Zhang, J. Scott, and H. Wang, Journal of Physics D **34**, 976–984 (2001).
[14] W. Hucho, *Aerodynamics of Road Vehicles. 4th edition* (SAE International, 1998).
[15] M. Ferlauto and R. Marsilio, Advances in Aircraft and Spacecraft Science **5**, 349–362 (2018).