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# Coupled Bloch-Wave Analysis of Active PhC Waveguides and Cavities

Marco Saldutti,<sup>1</sup> Jesper Mørk<sup>2</sup>, Paolo Bardella<sup>1</sup>, Ivo Montrosset<sup>1</sup> and Mariangela Gioannini<sup>1</sup>

<sup>1</sup> Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino, Corso Duca degli Abruzzi 24, Torino, Italy

<sup>2</sup> DTU Fotonik, Department of Photonics Engineering, Technical University of Denmark, Ørstedss Plads, Building 343, DK-2800 Lyngby, Denmark  
marco.saldutti@polito.it

**Abstract**— A coupled Bloch-wave approach is employed to analyze active photonic-crystal (PhC) waveguides and cavities. Gain couples the otherwise independent counter-propagating Bloch modes. This coupling is shown to limit the maximum attainable slow-light enhancement of gain itself and to strongly affect the mode selection in PhC lasers.

**Keywords**—PhC lasers, Coupled-mode theory, Bloch waves, Slow-light.

## INTRODUCTION

The slow-light (SL) enhancement of gain in photonic-crystal waveguides allows for the fabrication of shorter devices when realizing active structures. In particular, PhC lasers based on line-defect waveguides are ideal candidates for energy efficient light sources in high density PhC integrated circuits [1,2]. Solving Maxwell equations by a finite-difference-time-domain (FDTD) technique is a rigorous, but extremely time- and memory-consuming approach to analyze PhC devices [3]. Furthermore, FDTD simulations are not always useful to shed light on the physics of the investigated structures. Conversely, coupled-mode theory has been widely used to investigate the impact of SL effects in both passive [4,5] and active [6] PhC waveguides. In particular, the complex optical susceptibility arising by the interaction of the field with the active medium is treated in [6] as a weak perturbation of the passive structure, which induces a coupling between the otherwise independent counter-propagating Bloch modes. The fundamental limitations to the SL gain-enhancement imposed by the gain itself have been investigated in [7] by a rigorous, non-perturbative approach. In this work we use the perturbative approach of [6] to study an active PhC waveguide; we analyze the implications of the gain perturbation on the group index and then we study a PhC laser modelled as a cavity consisting of an active PhC waveguide and two mirrors. Interestingly, it is shown that our model predicts, consistently with [7], a reduction of the maximal group index caused by increasing the gain and it can be used to understand the impact of the gain-induced coupling on the selection of PhC laser lasing mode.

## I. NUMERICAL MODEL

The forward- (+) and backward-propagating (−) guided electric field of the passive waveguide in the frequency-domain are denoted by  $\mathbf{E}_{0,\pm}(\mathbf{r}, \omega) = \mathbf{e}_{0,\pm}(\mathbf{r}, \omega)e^{\pm ik_z(\omega)z}$ , where  $z$  is the propagation direction and  $\mathbf{e}_{0,\pm}(x, y, z) = \mathbf{e}_{0,\pm}(x, y, z + a)$  are the Bloch waves, with  $k_z$  propagation constant and  $a$  the PhC lattice constant. The electric field of the active waveguide is expanded as  $\mathbf{E} = \psi_+(z, \omega)\mathbf{E}_{0,+} + \psi_-(z, \omega)\mathbf{E}_{0,-}$ , where  $\psi_{\pm}(z, \omega)$  are slowly-varying amplitudes. By neglecting nonlinear effects, two coupled differential equations for  $\psi_{\pm}(z, \omega)$  are derived [6]:

$$\begin{cases} \frac{\partial \psi_+(z, \omega)}{\partial z} = i\kappa_{11}(z, \omega)\psi_+(z, \omega) + i\kappa_{12}(z, \omega)e^{-i2k_z(\omega)z}\psi_-(z, \omega) \\ -\frac{\partial \psi_-(z, \omega)}{\partial z} = i\kappa_{21}(z, \omega)e^{i2k_z(\omega)z}\psi_+(z, \omega) + i\kappa_{22}(z, \omega)\psi_-(z, \omega) \end{cases} \quad (1)$$

The self- and cross-coupling coefficients induced by the active material gain  $g_0(\omega)$  are indicated as  $\kappa_{11;12;21}(z, \omega) \simeq -\frac{i}{2}g_0(\omega) [n_g(\omega)/n_s] \Gamma_{xy,11;12;21}(z, \omega)$ , where  $n_s$  and  $n_g$  are the slab material refractive index and the passive waveguide group index. Confinement factors  $\Gamma_{xy,11;12;21}(z, \omega)$  are given by

$$\begin{aligned} \Gamma_{xy,11}(z, \omega) &= \frac{a \int_S \epsilon_0 n_s^2 |\mathbf{e}_0(\mathbf{r}, \omega)|^2 F(\mathbf{r}) dS}{\int_V \epsilon_0 n_b^2(\mathbf{r}) |\mathbf{e}_0(\mathbf{r}, \omega)|^2 dV} \\ \Gamma_{xy,12}(z, \omega) &= \frac{a \int_S \epsilon_0 n_s^2 [\mathbf{e}_{0,-}(\mathbf{r}, \omega) \cdot \mathbf{e}_{0,+}^*(\mathbf{r}, \omega)] F(\mathbf{r}) dS}{\int_V \epsilon_0 n_b^2(\mathbf{r}) |\mathbf{e}_0(\mathbf{r}, \omega)|^2 dV} \end{aligned}$$

with  $\Gamma_{xy,21} = \Gamma_{xy,12}^*$ ;  $V$  is the volume of a PhC supercell,  $S$  the transverse plane at position  $z$  and  $n_b(\mathbf{r})$  the background refractive index, whereas  $F(\mathbf{r}) = 1$  ( $= 0$ ) in the slab (holes). Due to the  $z$ -periodicity of  $\mathbf{e}_{0,\pm}$  and  $F(\mathbf{r})$ , the coupling coefficients are periodic with  $z$ . If the single unit cell is discretized with a sufficiently small space step  $\Delta_z$ , the coupling coefficients can be assumed constant within it. By defining  $c_{\pm} = \psi_{\pm} e^{\pm ik_z z}$ , Eq. (1) is turned, in each  $\Delta_z$ , into an initial-value problem, whose solution in matrix form is

$$\begin{bmatrix} c_+(z_0 + \Delta_z) \\ c_-(z_0 + \Delta_z) \end{bmatrix} = \begin{bmatrix} T_{\Delta_z,11} & T_{\Delta_z,12} \\ T_{\Delta_z,21} & T_{\Delta_z,22} \end{bmatrix} \begin{bmatrix} c_+(z_0) \\ c_-(z_0) \end{bmatrix} \quad (2)$$

with

$$\begin{aligned} T_{\Delta_z,11;22} &= \cosh[\gamma(z_0)\Delta_z] \pm i \frac{\kappa_{11}(z_0) + k_z}{\gamma(z_0)} \sinh[\gamma(z_0)\Delta_z], \\ T_{\Delta_z,12;21} &= \pm i \frac{\kappa_{12;21}(z_0)}{\gamma(z_0)} \sinh[\gamma(z_0)\Delta_z], \text{ and} \\ \gamma(z_0) &= \sqrt{\kappa_{12}(z_0)\kappa_{21}(z_0) - [\kappa_{11}(z_0) + k_z]^2}. \end{aligned}$$

By successive application of Eq. (2), the single unit cell transmission matrix  $\mathbf{T}_a$  is obtained and the transmission matrix of  $N$  cascaded cells is given by  $\mathbf{T}_a^N$ . From Frobenius theorem,  $\mathbf{T}_a^N$  can be written as  $\mathbf{T}_a^N = \mathbf{M}\boldsymbol{\lambda}^N\mathbf{M}^{-1}$ , where  $\mathbf{M}$  contains the eigenvectors of  $\mathbf{T}_a$  arranged by columns and  $\boldsymbol{\lambda}$  is a diagonal matrix with the eigenvalues of  $\mathbf{T}_a$  on the main diagonal. Multiplying by  $\mathbf{M}^{-1}$  both sides of

$$\begin{bmatrix} c_+(Na) \\ c_-(Na) \end{bmatrix} = \mathbf{M}\boldsymbol{\lambda}^N\mathbf{M}^{-1} \begin{bmatrix} c_+(0) \\ c_-(0) \end{bmatrix} \quad (3)$$

the Bloch waves of the active waveguide at input and output are obtained, i.e.  $\mathbf{c}_B(Na) = \boldsymbol{\lambda}^N \mathbf{c}_B(0)$ . From here, it is apparent that  $\boldsymbol{\lambda}^N$  is the evolution matrix in the basis of the Bloch waves of the active waveguide. If the eigenvalues of  $\mathbf{T}_a$  are denoted by  $\lambda_{1,2} = e^{\pm i\phi}$ ,  $\phi = \cos^{-1}[\text{Tr}(\mathbf{T}_a)/2]$  is the dispersion relation of the active waveguide and  $n_{g,Pert}(\omega, g_0) = c \text{Re}\{\partial\phi(\omega, g_0)/\partial\omega\}$  the associated group index, with  $c$  vacuum light speed. Within this approach, a PhC laser consists in the cascade of an active PhC waveguide and two mirrors, which, for simplicity, are modelled as standard reflectors. The complex round-trip-gain (RTG) of the cavity is computed as the product, at a given reference plane, of the left and right field reflectivity. Longitudinal resonant modes are those for which  $\angle\text{RTG}$  is an integer multiple of  $2\pi$ . For each longitudinal mode, threshold gain is found as the smallest  $g_0$  value which ensures  $|\text{RTG}| = 1$  [8].

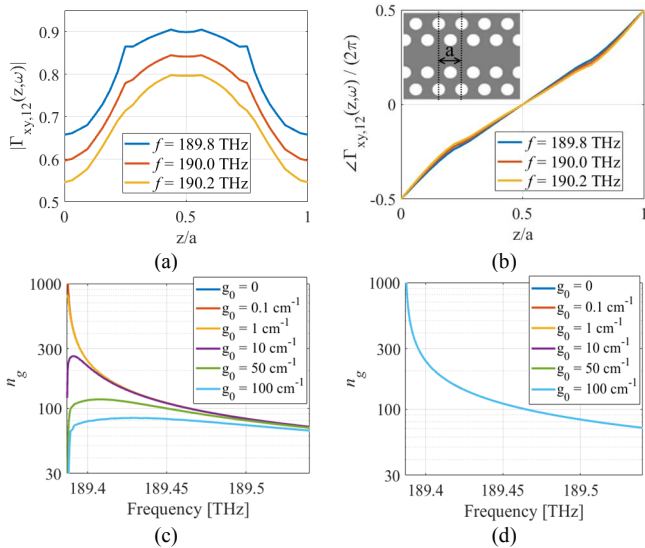


Fig. 1. (a) Magnitude and (b) phase of  $\Gamma_{xy,12}$ , at different frequencies, for the same line-waveguide of PhC lasers in [2]; inset in (b): unit cell reference planes. Group index with (c) and without (d) gain-induced coupling.

## II. SIMULATION RESULTS

The reference structure is the line-defect waveguide on which the PhC lasers realized in [2] are based. Dispersion relation and Bloch modes of the passive waveguide are computed by the free software package MIT Photonic-Bands (MPB) [9]. Fig. 1a and 1b display magnitude and phase of  $\Gamma_{xy,12}(z, \omega)$  at different frequencies. Since the  $z$ -variation of  $\angle \Gamma_{xy,12}(z, \omega)$  on a unit cell is approximately linear with a slope equal to  $2\pi/a$ , the first-order Fourier component of  $\kappa_{12}(z, \omega)$ , which synchronously couples  $\mathbf{E}_{0,+}$  and  $\mathbf{E}_{0,-}$ , is proportional to  $g_0(\omega) [n_g(\omega)/n_s] < |\Gamma_{xy,12}(z, \omega)| >$ . Since  $< \Gamma_{xy,11}(z, \omega) >$  and  $< |\Gamma_{xy,12}(z, \omega)| >$  have comparable values, the magnitude of the cross-coupling coefficients is comparable with the self-coupling coefficient. This peculiar characteristic of the active PhC waveguides arises from the  $2\pi$  phase shift of the non-negligible  $z$ -component of  $\mathbf{e}_{0,\pm}$  along the propagation direction. Fig. 1c reports the group index  $n_{g,Pert}$  of the active waveguide as a function of frequency at different  $g_0$  values. At small gain values, the dispersion relation of the active waveguide is not significantly perturbed, and the group index diverges as the frequency approaches the band-edge of the passive waveguide, i.e.  $k_z a / 2\pi = 0.5$  with a frequency  $f \approx 189.387$  THz. On the contrary, at larger gain values the group index is reduced and it even starts to decrease in the close proximity of the band-edge. Remarkably, this behaviour is consistent with that reported in [7] and obtained with a non-perturbative treatment. Furthermore, if the gain-induced coupling is neglected (i.e.,  $\kappa_{12;21} = 0$ ), the group index monotonically diverges with the frequency approaching the band-edge (Fig.1d). This proves the key role played by cross-coupling in limiting the maximum attainable SL enhancement of gain. With this coupled Bloch-wave approach we have then modelled the PhC lasers presented in [2]. The mirrors reflectivity is set to  $r^2 = 0.98$  [10] and  $g_0$  is assumed to be frequency-independent. The inset of Fig. 2b displays a scheme of principle of the cavity, with the field reflectivity from the left facet towards the cavity denoted by  $r_{eq,R}$ . Fig. 2a and 2b focus on a cavity with length  $L = 5a$ , showing magni-

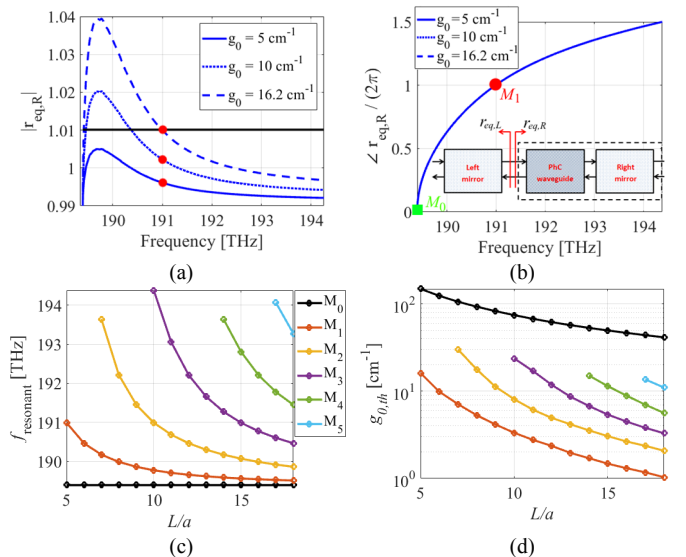


Fig. 2. Magnitude of  $r_{eq,R}$ , at different  $g_0$ , for  $L = 5a$ ; black line is level  $1/r$ . (b) Phase of  $r_{eq,R}$ ; inset: scheme of principle of the cavity (c) Mode frequencies versus cavity length.  $M_0$  is the mode at the band-edge. (d) Threshold gain for the onset of lasing of the various modes.

tude and phase of  $r_{eq,R}$  versus frequency at increasing  $g_0$  values. The threshold condition corresponds to the level  $1/r$ , corresponding to the horizontal line in Fig. 2a. The red spots track the longitudinal resonant mode  $M_1$ , with frequency  $f = 191$  THz, as it approaches the lasing onset at  $g_0 = 16.2$   $\text{cm}^{-1}$ . The mode located exactly at the band-edge ( $M_0$ , shown in Fig. 2b) requires higher gain for achieving threshold, because the maximum attainable  $|r_{eq,R}|$  around the band-edge is limited by the gain induced cross-coupling. This is a consequence of the fact that the field backscattered by the waveguide and the field backscattered by the right mirror facet are out of phase at the band-edge. Fig. 2c,d report, at each cavity length, all the longitudinal modes and corresponding threshold gain. The frequency shift of mode  $M_1$  towards the SL region observed by increasing cavity length (Fig. 2c) well reproduces the experimental [2] and numerical [3] trends. Conversely, without the gain-induced distributed feedback, the group index and the effective gain resulting from SL enhancement would monotonically increase towards the band-edge; consequently, the cavity would behave as a SL enhanced FP laser and, independently of the cavity length, the mode  $M_0$  would be the sole lasing one.

## III. CONCLUSIONS

In conclusion, the gain-induced coupling between counter-propagating Bloch modes has been found to be responsible for the degradation of the SL enhancement of gain discussed in [7]. Moreover, this coupling strongly affects the lasing mode threshold gain properties of PhC lasers.

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