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A recursive formula for Thabit numbers

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Abstract Here we discuss the Thabit numbers. An operation of addition of these numbers is proposed. A recursive relation is given accordingly.

Keywords Thabit numbers

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In recent papers we have discussed some properties of the Mersenne numbers [1,2] and of the Fermat numbers [3], using an approach based on generalized operations of addition [4-8]. In [9], we have discussed the Cullen and Woodall numbers too (for references on these numbers, see [10-13]). Here we consider the Thabit numbers [14]. These numbers are given as $T_n = 3 * 2^n - 1$, where the asterisk represents the ordinary multiplication.

Let us consider the following operation:

$$T_{m+n} = T_m \oplus T_n$$

Therefore

$$T_{m+n} = 3 * 2^{m+n} - 1 = 3 * 2^{m+n} - 1 + 2^n - 2^n = 2^n (3 * 2^m - 1) - 1 + 2^n = 2^n T_m + 2^n - 1$$

$$T_{m+n} = 2^n \frac{3}{3} T_m + 2^n - 1 - \frac{T_m}{3} + \frac{T_m}{3} = \frac{1}{3} T_m T_n + \frac{3}{3} 2^n - \frac{1}{3} + \frac{1}{3} - 1 + \frac{T_m}{3}$$

So we have:

$$(1) \quad T_m \oplus T_n = \frac{1}{3} T_m T_n + \frac{1}{3} T_m + \frac{1}{3} T_n - \frac{2}{3} = \frac{1}{3} (T_m + T_n + T_m T_n - 2)$$

Using (1), we can see that the neutral element is $T_0 = 2$, so that:

$$T_m \oplus T_0 = \frac{1}{3} (T_m + T_0 + T_m T_0 - 2) = \frac{1}{3} (3 T_m) = T_m$$

The recursive relation is given accordingly to (1), starting from $T_1 = 5$:

$$T_{n+1} = T_n \oplus T_1 = \frac{1}{3}(T_n + T_1 + T_n T_1 - 2) = \frac{1}{3}(T_n + 5 + 5T_n - 2) = \frac{1}{3}(6T_n + 3) = 2T_n + 1$$

With a Fortran program (double precision), we have **5, 11, 23, 47, 95, 191, 383, 767, 1535, 3071, 6143, 12287, 24575, 49151, 98303, 196607, 393215, 786431, 1572863, 3145727, 6291455, 12582911, 25165823, 50331647, 100663295, 201326591, 402653183, 805306367, 1610612735, 3221225471, 6442450943, 12884901887, 25769803775, 51539607551, 103079215103, 206158430207, 412316860415, 824633720831, 1649267441663, 3298534883327, 6597069766655, 13194139533311, 26388279066623, 52776558133247, 105553116266495, 211106232532991, 422212465065983, 844424930131967, 1688849860263935, 3377699720527871, 6755399441055743**. In bold characters, the prime numbers as from <http://oeis.org/A007505>.

References

- [1] Sparavigna, A. C. (2018). The q-integers and the Mersenne numbers. Zenodo. <http://doi.org/10.5281/zenodo.1251833>
- [2] Sparavigna, A. C. (2018). On a generalized sum of the Mersenne Numbers. Zenodo. <http://doi.org/10.5281/zenodo.1250048>
- [3] Sparavigna, A. C. (2018). The group of the Fermat Numbers. Zenodo. <http://doi.org/10.5281/zenodo.1252422>
- [4] Sparavigna, A. C. (2018). Generalized Sums Based on Transcendental Functions. Zenodo . DOI: 10.5281/zenodo.1250020
- [5] Sparavigna, A. C. (2018). On the generalized sum of the symmetric q-integers. Zenodo. DOI: 10.5281/zenodo.1248959
- [6] Sparavigna, A. C. (2018). ON THE ADDITIVE GROUP OF q-INTEGERS. Zenodo . DOI: 10.5281/zenodo.1245849
- [7] Sicuro, G., & Tempesta, P. (2016). Groups, information theory, and Einstein's likelihood principle. Phys. Rev. E 93, 040101(R).
- [8] Curado, E. M., Tempesta, P., & Tsallis, C. (2016). A new entropy based on a group-theoretical structure. Annals of Physics, 366, 22-31.
- [9] Sparavigna, A. C. (2019). On the generalized sums of Mersenne, Fermat, Cullen and Woodall Numbers. Zenodo. <http://doi.org/10.5281/zenodo.2634312>

[10] Weisstein, Eric W. "Mersenne Number." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/MersenneNumber.html>

[11] Weisstein, Eric W. "Fermat Number." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/FermatNumber.html>

[12] Weisstein, Eric W. "Cullen Number." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/CullenNumber.html>

[13] Weisstein, Eric W. "Woodall Number." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/WoodallNumber.html>

[14] Weisstein, Eric W. "Thâbit ibn Kurrah Number." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/ThabitibnKurrahNumber.html>