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## A recursive formula for Thabit numbers

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**Abstract** Here we discuss the Thabit numbers. An operation of addition of these numbers is proposed. A recursive relation is given accordingly.

**Keywords** Thabit numbers

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In recent papers we have discussed some properties of the Mersenne numbers [1,2] and of the Fermat numbers [3], using an approach based on generalized operations of addition [4-8]. In [9], we have discussed the Cullen and Woodall numbers too (for references on these numbers, see [10-13]). Here we consider the Thabit numbers [14]. These numbers are given as  $T_n = 3 * 2^n - 1$ , where the asterisk represents the ordinary multiplication.

Let us consider the following operation:

$$T_{m+n} = T_m \oplus T_n$$

Therefore

$$T_{m+n} = 3 * 2^{m+n} - 1 = 3 * 2^{m+n} - 1 + 2^n - 2^n = 2^n (3 * 2^m - 1) - 1 + 2^n = 2^n T_m + 2^n - 1$$

$$T_{m+n} = 2^n \frac{3}{3} T_m + 2^n - 1 - \frac{T_m}{3} + \frac{T_m}{3} = \frac{1}{3} T_m T_n + \frac{3}{3} 2^n - \frac{1}{3} + \frac{1}{3} - 1 + \frac{T_m}{3}$$

So we have:

$$(1) \quad T_m \oplus T_n = \frac{1}{3} T_m T_n + \frac{1}{3} T_m + \frac{1}{3} T_n - \frac{2}{3} = \frac{1}{3} (T_m + T_n + T_m T_n - 2)$$

Using (1), we can see that the neutral element is  $T_0 = 2$ , so that:

$$T_m \oplus T_0 = \frac{1}{3} (T_m + T_0 + T_m T_0 - 2) = \frac{1}{3} (3 T_m) = T_m$$

The recursive relation is given accordingly to (1), starting from  $T_1 = 5$  :

$$T_{n+1} = T_n \oplus T_1 = \frac{1}{3}(T_n + T_1 + T_n T_1 - 2) = \frac{1}{3}(T_n + 5 + 5T_n - 2) = \frac{1}{3}(6T_n + 3) = 2T_n + 1$$

With a Fortran program (double precision), we have **5, 11, 23, 47**, 95, **191, 383**, 767, 1535, 3071, **6143**, 12287, 24575, 49151, 98303, 196607, 393215, **786431**, 1572863, 3145727, 6291455, 12582911, 25165823, 50331647, 100663295, 201326591, 402653183, 805306367, 1610612735, 3221225471, 6442450943, 12884901887, 25769803775, **51539607551**, 103079215103, 206158430207, 412316860415, **824633720831**, 1649267441663, 3298534883327, 6597069766655, 13194139533311, **26388279066623**, 52776558133247, 105553116266495, 211106232532991, 422212465065983, 844424930131967, 1688849860263935, 3377699720527871, 6755399441055743. In bold characters, the prime numbers as from <http://oeis.org/A007505>.

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