

FITTING THE LUMINOSITY DATA FROM TYPE Ia SUPERNOVAE IN THE FRAME OF THE COSMIC DEFECT THEORY

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5 **FITTING THE LUMINOSITY DATA FROM TYPE Ia  
 SUPERNOVAE IN THE FRAME OF THE COSMIC  
 DEFECT THEORY**

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15 The cosmic defect (CD) theory is reviewed and used to fit the data for the acceler-  
 17 ated expansion of the universe, obtained from the apparent luminosity of 192 SnIa's.  
 The fit from the CD theory is compared with the one obtained by means of  $\Lambda$ CDM.  
 19 The results from the two theories are in good agreement and the fits are satisfactory.  
 The correspondence between the two approaches is discussed and interpreted.

*Keywords:*

21 **1. Introduction**

23 As is well known, an extremely important finding of the last decade has been the  
 accelerated expansion of the universe. This was rather a surprise, mainly based on  
 25 the observation of luminosity distance of type Ia supernovae (SnIa).<sup>1,2</sup> Nowadays,  
 the picture which seems to emerge from the data is that of a universe which has  
 undergone a transition from a decelerated to an accelerated phase, with a relatively  
 27 recent turning point located at  $z_{\text{tr}} \simeq 0.46$ .<sup>3</sup> This framework seems to be confirmed  
 by cross-comparison with other pieces of evidence.<sup>4–7</sup> The discovery gave rise to  
 29 an active search for an explanation on the theoretical side, within and outside  
 general relativity (GR). An immediate effect was to revive the old cosmological  
 31 constant,  $\Lambda$ .<sup>8</sup> Afterward, a number of evolutionary sons of  $\Lambda$  or new exotic fields were  
 elaborated, mostly based on the idea of “dark energy.”<sup>9–17</sup> Also various possibilities  
 33 of alternative, modified or extended versions of GR have actively been explored.<sup>18–22</sup>

35 Here, our purpose is to review the existing observational data and some proposed  
 fits, comparing them one with another and with the results of a recently introduced  
 four-vector theory, which we shall call the “cosmic defect” theory, CD for short.<sup>23,24</sup>

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1 The CD theory, which also has correspondences in the group of the so-called vector  
 “ether” theories,<sup>25</sup> will also be revised and recast in the following.

3 Whenever a theory is contrasted with the data from experience (here, from  
 observation), one has to face a number of different problems. First of all, there is  
 5 the reliability and cleanness of the data: we shall not elaborate on this, assuming the  
 discussion to have been effectively conducted in the literature.<sup>26–28</sup> A second, subtle  
 7 issue is that, even in presenting apparently raw data, underlying assumptions often  
 exist, originating in one or another theoretical view: as far as possible, we shall try  
 9 to express the existing information in a model-independent way. Finally, any theory  
 usually has (a number of) free parameters to adjust, in order to fit the experiment;  
 11 of course, the more parameters you have, the more you will be able to reproduce a  
 given empirical trend, but any choice must be checked for consistency in as many  
 13 different physical situations as possible.

As we shall see, the CD theory gives a reasonably good fit for the SnIa data,  
 15 making use of a limited number of parameters and, at the same time, offers an  
 interpretation paradigm based on correspondences with known physical phenomena  
 17 without calling for new dark entities.

## 2. Luminosity Distance, Magnitude and Redshift

19 In the framework of the supernova observations, a key role is played by the concept  
 of luminosity distance,  $d_l$ , which is defined as

$$21 \quad d_l \doteq \sqrt{\frac{L_{\text{obs}}}{4\pi\Phi}}, \quad (1)$$

where  $L_{\text{obs}}$  is the absolute luminosity of the source (released energy per unit time)  
 23 corresponding to the  $z$  value measured by the observer, and  $\Phi$  is the energy flux  
 density (energy per unit time and surface) measured at the observer’s site. In an  
 25 expanding universe both energy and time are affected by the expansion so that  
 the effective luminosity for the observer, in terms of the absolute luminosity at the  
 27 source, is<sup>29</sup>

$$L_{\text{obs}} = \frac{L_S}{(1+z)^2}.$$

29 In a universe endowed with the typical Robertson–Walker (RW) symmetries, (1)  
 becomes

$$31 \quad d_l = a_0 r_S (1+z),$$

where  $a_0$  is the scale parameter at the observer, and  $r_S$  is the coordinate distance of  
 33 the source from the observer. The latter, written in terms of the distance traveled  
 by a light ray, is in turn

$$35 \quad r_S = c \int_{t_S}^{t_0} \frac{dt}{a(t)},$$

1 where of course  $t$  is the cosmic time. In terms of the redshift and the scale factor  
we may also write

$$3 \quad cdt = c \frac{da}{\dot{a}} = - \frac{cdz}{(1+z)H(z)}, \quad (2)$$

5 where the dot denotes the derivative with respect to  $t$ , and  $H = \dot{a}/a$  is the Hubble  
parameter.

It is then easily seen that the luminosity distance is

$$7 \quad d_l = c(1+z) \int_0^z (1+\zeta) \frac{da(\zeta)}{\dot{a}(\zeta)} = c(1+z) \int_0^z \frac{d\zeta}{H(\zeta)}.$$

Usually, astronomical objects are classified in terms of their magnitude  $m$ , rather  
than their luminosity. By definition, the bolometric magnitude (integrated over all  
frequencies) depends logarithmically on the luminosity distance, according to the  
formula

$$\begin{aligned} m - M_S &= 25 + 5 \log d_l = 25 + 5 \log \left( a_0 c (1+z) \int_{t_S}^{t_0} \frac{dt}{a} \right) \\ &= 25 + 5 \log \left( c(1+z) \int_0^z \frac{d\zeta}{H(\zeta)} \right) \\ &= 25 + 5 \log \left( c \frac{(1+z)}{H_0} \int_0^z \frac{d\zeta}{E(\zeta)} \right), \end{aligned} \quad (3)$$

9 where distances are expressed in Mpc and it is  $H_0 = H(0)$  and  $E(z) = H(z)/H_0$ ;  
 $m - M_S$  is usually called the “distance modulus.”

11 The integral in (3) depends of course on the model which one uses to describe  
the cosmic expansion. For a dust-filled universe in a typical Friedman–Robertson–  
Walker (FRW) scenario, it is indeed

$$13 \quad a(t) = a_0 \sqrt[3]{6\pi G \rho_{m0} t^2}, \quad (4)$$

$\rho_{m0}$  being the present matter energy density and  $G$  the gravitation constant.

15 As a consequence one expects that

$$(m - M_S)_{\text{FRW}} = 25 + 5 \log \left[ \frac{3c}{\sqrt{6\pi G \rho_{m0}}} (1+z - \sqrt{1+z}) \right]. \quad (5)$$

17 If one considers a  $\Lambda$ -cold-dark-matter universe ( $\Lambda$ CDM), i.e. an FRW universe  
with a cosmological constant  $\Lambda$ , it is

$$19 \quad E(z) = \sqrt{\Omega_m (1+z)^3 + 1 - \Omega_m}, \quad (6)$$

21 where  $\Omega_m = \rho_m/\rho_c$  represents the ratio between the matter density and the critical  
density (ensuring the flatness of space). The difference  $\Omega_\Lambda = 1 - \Omega_m$  allows for the  
effect of the cosmological constant.

23 The formula (6) is a special case of the more general

$$E(z) = \sqrt{\sum_i \Omega_i (1+z)^{3(1+w_i)}},$$

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1 allowing for any number of components of the content of the universe, with different  
equations of state.

### 3 **3. The Cosmic Defect Theory**

5 The CD theory is based on the presence of a cosmic (four-)vector field in the  
universe. This vector field is interpreted as the strain flux density in a continuum  
7 with a pointlike defect.<sup>a</sup> Actually we start from a universe which, at the large scale,  
is considered to be isotropic, homogeneous and globally expanding. Our idea, which  
9 is explained in more detail in Refs. 23 and 24, is that the global symmetry of the  
universe, including the expansion, is a consequence of the presence of a texture  
11 defect in the four-dimensional space–time, and of its symmetry. A defect like this,  
as is the case for any material continuum, is not, per se, a dynamical feature (it is  
13 there or not); however it induces a strained state in the medium which shows up as  
a non flat intrinsic metric tensor. Of course, we are referring to the full space–time  
15 curvature and not to the simple space curvature; in what follows we shall indeed  
consider a spatially flat RW universe. A strained state in a continuum may indeed  
17 be represented by means of a vectorial displacement field. The defect is described  
as a singular event (or a singular spacelike hypersurface); if now the strain tensor is  
19 projected, at each event, onto a direction orthogonal to the spacelike defect (single  
event or hyperplane, in the case of a flat-space global RW symmetry), a vector field  
21 is obtained, whose flow lines materially diverge only at the defect and nowhere else:  
any intersection would act as a “source,” i.e. as an additional defect, but, since the  
23 observation of the universe at large suggests so, we assume that there is only one  
defect at the origin of cosmic time. From this picture two features of the vector  
25 field naturally emerge: (a) it is timelike; (b) it is divergenceless everywhere except  
at the defect. These constraints and conditions, in an RW universe, lead to a unique  
27 solution for the norm of the vector,  $\gamma$  (coinciding with the absolute value of its time  
component), namely

$$\gamma = \frac{Q^3}{a^3}, \quad (7)$$

29 where  $Q$  is a constant and  $a$  is the scale factor of the RW metric. We stress the  
fact that  $\gamma$  is not a dynamical quantity, which means that, as for any defect in a  
31 solid, its form is not the consequence of the application of a variational principle: it  
depends on extrinsic conditions. In a solid we would have, for example, impurities or  
33 dislocations along the domain walls formed at the moment of some phase transition,  
or other kinds of defects. Of course, the dynamical properties of the material will  
35 depend on the presence of the defects, but the latter will not be the result of any  
internal extremization of anything. In practice, as we shall see, we will not vary  
37 with respect to  $\gamma$ : (7) is a consequence of the symmetry of the defect.

<sup>a</sup>Actually the defect could correspond to any singular hypersurface.

1 The other relevant feature of the CD theory is in the choice of the Lagrangian for  
 2 the space–time containing the defect. This choice is inspired by the correspondence  
 3 between the (bidimensional) phase space of an RW universe and that of a point  
 4 particle moving through a viscous fluid.<sup>23,24</sup> Including the presence of matter (i.e.  
 5 whatever is not accounted for by space–time), the action integral is

$$S = \int (\kappa e^{-g_{\mu\nu}\gamma^\mu\gamma^\nu} R + \mathcal{L}_{\text{matter}}) \sqrt{-g} d^4x, \quad (8)$$

7 with  $\kappa \equiv c^4/16\pi G$  and  $d^4x = dt dr d\theta d\varphi$ . Explicitly introducing the RW symmetry  
 8 and considering matter in terms of scalar functions, the Lagrangian read out of  
 9 (8) is

$$\mathcal{L}_0 = -\mathcal{V}_k [6\kappa e^{-\gamma^2} (a^2\ddot{a} + a\dot{a}^2) + \kappa_0 f a^3 \dot{a}^2 + \varpi h a^3], \quad (9)$$

11 where  $\mathcal{V}_k$  is the part of the Lagrangian which is not affected by any variation with  
 12 respect to the metric, and, in the flat  $k = 0$  case (polar coordinates), equals  $r^2 \sin \theta$ .  
 13 The presence of matter is represented by two scalar functions,  $f$  and  $h$ , coupling with  
 14 space–time through the constants  $\kappa_0$  and  $\varpi$ . The function  $h$  represents a matter-  
 15 energy density of a perfect fluid: the  $f$  function accounts for a possible coupling  
 16 with the rate of expansion of the universe,  $\dot{a}$ , representing a kind of “drag” by the  
 17 expanding space–time. Actually  $f$  has been included just for the sake of generality,  
 18 but it will indeed be dropped soon.

19 The second derivative of  $a$  with respect to  $t$ , appearing in (9), is easily elimi-  
 20 nated, once the action is integrated by parts, thus giving a final effective Lagrangian  
 21 of the universe:

$$\mathcal{L} = -\mathcal{V}_k \left[ -6\kappa e^{-\gamma^2} \left( \frac{6}{a^5} + a \right) \dot{a}^2 + \kappa_0 f a^3 \dot{a}^2 + \varpi h a^3 \right]. \quad (10)$$

23 From (10) the Hamiltonian function is readily obtained:

$$\mathcal{H} \doteq \dot{a} \frac{\partial \mathcal{L}}{\partial \dot{a}} - \mathcal{L} = -\mathcal{V}_k \left\{ \left[ \kappa_0 f a^3 - 6\kappa e^{-\gamma^2} \left( \frac{6}{a^5} + a \right) \right] \dot{a}^2 - \varpi h a^3 \right\}. \quad (11)$$

25 As usual,  $\mathcal{H}$  can be interpreted as the energy content of the system described by  
 26 the effective Lagrangian (10), so that in our case it represents the energy content of  
 27 the universe. The Hamiltonian of an isolated system is a conserved quantity, since  
 28 it is identically

$$\frac{d\mathcal{H}}{dt} = \ddot{a} \frac{\partial \mathcal{L}}{\partial \dot{a}} + \dot{a} \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{a}} - \frac{\partial \mathcal{L}}{\partial a} \dot{a} - \frac{\partial \mathcal{L}}{\partial \dot{a}} \ddot{a} \equiv 0. \quad (12)$$

From now on use will be made of  $\alpha = a/Q$ , so we write

$$\left[ \kappa_0 f \alpha^3 - 6\kappa e^{-\gamma^2} \left( \frac{6}{\alpha^5} + \alpha \right) \right] \dot{\alpha}^2 - \varpi h \alpha^3 = \mathcal{W} = \text{const.} \quad (13)$$

From (13) one directly gets the expansion rate equation:

$$\dot{\alpha}^2 = \frac{\mathcal{W} + \varpi h \alpha^3}{\kappa_0 f \alpha^3 - 6\kappa e^{-\gamma^2} \left( \frac{6}{\alpha^5} + \alpha \right)}. \quad (14)$$

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1 Actually, if we want to recover the usual meaning of the matter term in a  
 2 comoving reference frame, we must choose

$$3 \quad \kappa_0 = 0,$$

so that the expansion rate can be rewritten as

$$4 \quad \dot{\alpha}^2 = -\frac{\mathcal{W} + \varpi h \alpha^3}{6\kappa e^{-1/\alpha^6} \left( \frac{6}{\alpha^5} + \alpha \right)}. \quad (14)$$

5 In the absence of a defect we should recover the classical FRW model; for this  
 6 reason, it should be

$$7 \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho = \frac{1}{6\kappa} \rho c^4, \quad (15)$$

8 where  $\rho c^2$  is the energy density of matter. However, under the same condition  
 9 ( $Q = \gamma = 0$ ) Eq. (14) gives

$$10 \quad \frac{\dot{a}^2}{a^2} = -\frac{\mathcal{W}Q^3 + \varpi h \alpha^3}{\kappa \alpha^3} = -\frac{\varpi}{\kappa} h. \quad (16)$$

Consistency between (15) and (16) then requires that

$$11 \quad \begin{aligned} \varpi &= -\frac{1}{6}, \\ h &= \rho c^4. \end{aligned}$$

The final formula for the expansion rate of the universe is

$$12 \quad \dot{\alpha}^2 = -\frac{\mathcal{W} - \rho c^4 \alpha^3}{6\kappa e^{-1/\alpha^6} \left( \frac{6}{\alpha^5} + \alpha \right)}. \quad (17)$$

13 Let us now suppose that the cosmic fluid is made up of a number of different  
 14 noninteracting components, each with its equation of state in the form

$$15 \quad p_i = w_i \rho_i c^2,$$

16 where  $w_i$  are real positive numbers ( $w_i \geq 0$ ) and  $p_i$  is the partial pressure of the  
 17  $i$ th component.

18 The conservation laws imply that

$$19 \quad \rho_i = \rho_{i0} \frac{\alpha_0^{3(1+w_i)}}{\alpha^{3(1+w_i)}}.$$

20 Introducing this relation into (17) we have

$$21 \quad \dot{\alpha}^2 = -\frac{\mathcal{W} - c^4 \sum_i \rho_{i0} \frac{\alpha_0^{3(1+w_i)}}{\alpha^{3w_i}}}{6\kappa e^{-1/\alpha^6} \left( \frac{6}{\alpha^5} + \alpha \right)}. \quad (18)$$

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The corresponding Hubble parameter is

$$\begin{aligned}
 H &= \frac{\dot{a}}{a} = \frac{\dot{\alpha}}{\alpha} = \frac{1}{\alpha} \sqrt{\frac{c^4 \sum_i \rho_{i0} \frac{\alpha_0^{3(1+w_i)}}{\alpha^{3w_i}} - \mathcal{W}}{6\kappa e^{-1/\alpha^6} \left( \frac{6}{\alpha^5} + \alpha \right)}} \\
 &= \frac{c^2}{\sqrt{6\kappa}} (1+z)^{3/2} \sqrt{\frac{\sum_i \rho_{i0} (1+z)^{3w_i} - \mathfrak{w}}{e^{-(1+z)^6/\alpha_0^6} [1 + 6(1+z)^6/\alpha_0^6]}}}, \quad (19)
 \end{aligned}$$

1 with  $\mathfrak{w} = \mathcal{W}/(c^4 \alpha_0^3)$ .

In the case of dust ( $w = 0$ ) and radiation ( $w = 1/3$ ) it is

$$3 \quad H(z) = (1+z)^{3/2} \sqrt{\frac{c^4}{6\kappa} \rho_{m0}} \sqrt{\frac{1 + \varepsilon_0 (1+z) - b}{e^{-(1+z)^6/\alpha_0^6} [1 + 6(1+z)^6/\alpha_0^6]}}. \quad (20)$$

5 The adimensional quantity  $\varepsilon_0 = \rho_{r0}/\rho_{m0}$  is the present ratio between the radiation and the matter energy density in the universe.  $b$  is  $\mathfrak{w}/\rho_{m0}$ .

#### 4. Observations Versus Theory

7 In order to compare theory and observation we make reference to the same set  
 8 of data used recently by Davis *et al.*<sup>30</sup> and incorporating supernovae analyzed in  
 9 four different groups: 60 from the ESSENCE (Equation of State: SuperNova trace  
 10 Cosmic Expansion) project,<sup>31,32</sup> 57 from SNLS (SuperNova Legacy Survey),<sup>33</sup> 45  
 11 nearby supernovae, and 30 detected by the Hubble Space Telescope and qualified  
 12 as “golden” supernovae by Riess *et al.*<sup>34</sup> As mentioned in the introduction, we shall  
 13 not enter into a discussion on the elaboration of the data, but assume them exactly  
 14 the way they are published or anyway accessible, considering them as the best  
 15 available at the moment.

17 Altogether we use the luminosity data from 192 SnIa,<sup>35–37</sup> which we try to fit  
 18 with theoretical models. For the optimization as well as for the determination of  
 19 the uncertainty of the values of the parameters, we use a multidimensional nonlinear  
 20 minimization by means of the MINUIT engine.<sup>38,b</sup> The optimization is made  
 21 minimizing the reduced  $\chi^2$  of the fit, where

$$21 \quad \chi^2(\mathbf{p}) = \frac{\sum_i^N (f(x_i, \mathbf{p}) - e_i)^2}{\sigma_i^2}.$$

<sup>b</sup>The open source routine we used (due to G. Allodi of the University of Parma), named `fminuit`, is called from within MATLAB, and may be retrieved from `ftp://ftp.fis.unipr.it/pub/matlab/fminuit.mex`



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1 Here  $\mathbf{p}$  is the vector of free parameters being fitted (in our case they are usually  
2 two in number and  $\sigma_i^2$  are the uncertainties in the individual measurements  $e_i$ ; of  
3 course,  $f$  is the function to be fitted and  $x_i$  is the redshift parameter for the  $i$ th  
4 supernova. Therefore, the reduced  $\chi^2$  is defined as  $\chi^2/\text{d.o.f.}$ , where d.o.f. is the  
5 number of data minus the number of parameters we want to fit.

6 In what follows, the values of the parameters we get from the fit are given with  
7 a one-standard-deviation error, which corresponds to a 68.3% confidence level.

8 First, we use (5) and obtain the result shown in Fig. 1. Direct inspection of the  
9 graph shows that the data correspond to systematically lower luminosities than the  
10 ones given by the FRW model, whence the accelerated expansion interpretation  
11 comes.

12 The next step will be to test on the data the  $\Lambda$ CDM model in its simplest  
13 version. For that purpose we use (3) and (6). In practice

$$m - M_S = \mu + 5 \log(1 + z) + 5 \log \int_0^z \frac{d\zeta}{\sqrt{\Omega_m (1 + \zeta)^3 + 1 - \Omega_m}}. \quad (21)$$

14 The result, as is well known, is better than before, since the reduced  $\chi^2$  is now  $\chi^2 =$   
15  $1.0295$  with a best-fitting  $\mu = 43.30 \pm 0.03$ , which corresponds to  $H_0 = 65.6 \pm 0.9$   
16  $\text{km/s} \times \text{Mpc}$ , and  $\Omega_m = 0.27 \pm 0.03$ , i.e. 27% of ordinary and dark matter plus 73%  
17 of dark energy (cosmological constant) in a spatially flat universe.

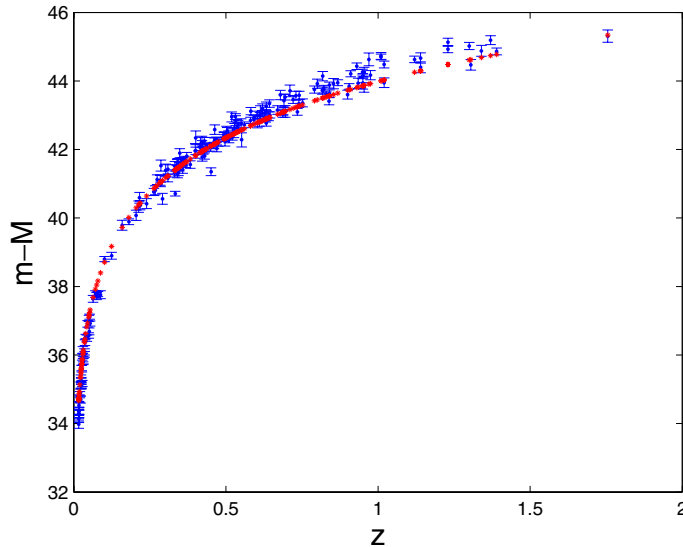


Fig. 1. Fit of distance modulus observations using a standard dust-filled Friedman–Robertson–Walker universe. The data are from 192 SnIa’s, as explained in the text. Vertical bars represent the experimental uncertainties ( $2\sigma$ ). The uncertainty on the redshift parameter  $z$  would be imperceptible at the scale of the graph. The reduced  $\chi^2$  is 2.1276.

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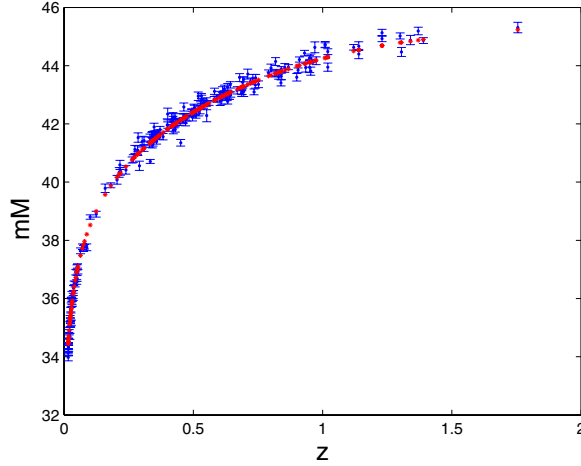


Fig. 2. Fit of distance modulus observations using the CD theory with two free parameters. Radiation is overlooked with respect to dust. The symbols are as in (1). The reduced chi square is  $\chi^2 = 1.092$ .

1 Finally, we test the CD theory. Use is made of (3) and (20) considering dust  
and radiation, so that

$$3 \quad m - M_S = \mu + 5 \log(1+z) + 5 \log \int_0^z \sqrt{\frac{e^{-(1+\zeta)^6/\alpha_0^6} [1 + 6(1+\zeta)^6/\alpha_0^6]}{(1+\zeta)^3 (1 + \varepsilon_0(1+\zeta) - b)}} d\zeta. \quad (22)$$

5 We could treat  $\mu$ ,  $\alpha_0$  and  $b$  as optimization parameters; however, the value to be  
introduced for  $\varepsilon_0$  is the one currently agreed upon, excluding any dark contribution:  
7  $\varepsilon_0 \sim 10^{-4}$ . Of course, as long as  $z$  is in the order of a few units (as is the case for  
SnIa's), the radiation term in the denominator of the integrand is negligible, so  
9 that the contribution of  $b$  may also be embedded in  $\mu$  and the free parameters  
remain  $\mu$  and  $\alpha_0$  only. The result of the optimization process is  $\mu = 43.26 \pm 0.03$   
11 and  $\alpha_0 = 1.79 \pm 0.04$ ; the reduced  $\chi^2$  is  $\chi^2 = 1.092$ , almost as good as for  $\Lambda$ CDM.  
The graph is shown in Fig. 2.

We summarize the results of the three cases (FRW,  $\Lambda$ CDM, CD) in the following  
table:

Model	$\chi^2/\text{d.o.f.}$	Parameters
FRW	2.1276	$\mu = 25 + 5 \log \frac{3c}{\sqrt{6\pi G \rho_0}} = 45.14 \pm 0.01$
LCDM	1.0295	$\mu = 25 + 5 \log \frac{c}{H_0} = 43.30 \pm 0.03$ $\Omega_m = 0.27 \pm 0.03$
CD	1.092	$\mu = 43.26 \pm 0.03$ $\alpha_0 = 1.79 \pm 0.04$

The parameters are the ones which are found and described  
in the text.

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## 1 5. The Hubble Parameter and the Age of the Universe

3 Reconsidering now the explicit spelling-out of the parameters appearing in the CD theory used to draw Fig. 2, we see that it is

$$\mu = 25 - 5 \log c + \frac{5}{2} \log \frac{6\kappa}{\rho_{m0}(1-b)},$$

where

$$\begin{aligned} \rho_{m0}(1-b) &= \frac{6\kappa}{c^2} 10^{-\frac{2}{5}(\mu-25)} \\ &= (8.5 \mp 0.2) \times 10^{-27} \text{kg/m}^3. \end{aligned} \quad (23)$$

5 The “visible” matter density in the universe is commonly assumed to be around  $\sim 10^{-27}$ – $10^{-28}$  kg/m<sup>3</sup>, which means that  $b$  must be  $\sim -10$ .

Then, introducing (23) into (20) and evaluating for  $z = 0$ , we obtain

$$\begin{aligned} H_0 &= \sqrt{\frac{c^4}{6\kappa} \frac{\rho_{m0}(1+\varepsilon_0-b)}{e^{-1/\alpha_0^6}(1+6/\alpha_0^6)}} \\ &= 62.8 \mp 1.7 \text{ km/(s} \times \text{Mpc)}, \end{aligned}$$

7 which is an acceptable result ( $\varepsilon_0$  has been neglected with respect to  $b$ ). The corresponding Hubble time is 15.6 Gy.

Of course, we should determine the age of the universe using the CD model; this can be done by means of (18), through integration:

$$\begin{aligned} t_0 &= \frac{1}{c^2} \sqrt{\frac{6\kappa}{\rho_{m0}\alpha_0^3}} \int_0^{\alpha_0} \sqrt{\frac{(6+\xi^6)e^{-\frac{1}{\xi^6}}}{\xi^4[(1-b)\xi + \varepsilon_0\alpha_0]}} d\xi \\ &= 9.0 \pm 0.2 \text{ Gy}. \end{aligned}$$

9 The final numerical result has been obtained neglecting  $\varepsilon_0\alpha_0$  with respect to the other terms in the denominator of the integrand. The value falls rather short as  
11 compared to the age of globular clusters, which fact may probably be interpreted as an inadequacy of the model at very early cosmic times.

## 13 6. Conclusion and Discussion

15 We have fitted the apparent luminosity data from SnIa’s with the values predicted by the  $\Lambda$ CDM and the CD theories, comparing both with a traditional FRW universe. The result is of course partly known, but we see now that CD also improves  
17 with respect to FRW and gives a fit comparable with that of  $\Lambda$ CDM. Using the same data and the same number of parameters, we obtained similar values of the reduced  
19  $\chi^2$ ’s, suggesting the idea that CD is also a viable theory. It is, however, true that the apparently small difference in the reduced  $\chi^2$ ’s of the fits corresponds to a rather  
21 big difference in the full  $\chi^2$  which, when analyzed in the light of statistical information criteria, such as the Akaike information criterion (AIC)<sup>39</sup> and the Bayesian  
23 information criterion (BIC),<sup>40</sup> enhances the distance between the two theories in

1 favor of  $\Lambda$ CDM. At the same time it is also true that both reduced values of  $\chi^2$  are  
 3 bigger than 1; furthermore, the  $H_0$  values obtained from observation using different  
 5 methods are systematically higher than those of the two-parameter best fits above.  
 7 The most recent data from WMAP<sup>41</sup> yield  $H_0 = 73.2_{-3.2}^{+3.1}$  km/s $\times$  Mpc, which is  
 9 consistent with a number of other results produced by different methods and indica-  
 11 tors (like SnI, SnII, Cepheids in nearby galaxies, Sunyaev–Zeldovitch effect, X-rays  
 13 from clusters, and gravitationally lensed systems), all quoted in Ref. 41. The cen-  
 15 tral values from these different observations range from 72 to 76 km/s $\times$ Mpc and  
 in general the historical evolution of the estimated values of the Hubble constant  
 seems to progressively converge toward something around 75 km/s $\times$ Mpc,<sup>c</sup> which  
 is  $\sim 15\%$  more than the results got by means of the fits in this paper. If the “exper-  
 imental” value of  $H_0$  were used in the fits (so reduced to one-parameter ones), the  
 agreement with the data would consistently worsen both for  $\Lambda$ CDM and for CD.

In practice there is something missing beyond the details of the theories and their  
 interpretation, which deserves investigation and insight.

$\Lambda$ CDM is indeed different from CD: the former assumes in the universe the  
 presence of a cosmological constant corresponding to a sort of uniformly and homo-  
 geneously distributed dark energy; the latter interprets space–time as a continuum  
 with a cosmic defect inducing a strained state containing both the symmetry and  
 the nonuniform expansion rate. Besides this, we know that  $\Lambda$ CDM requires also that  
 the matter content in the universe be one order of magnitude bigger than what is  
 expected from baryonic particles only. In the case of CD, instead, we saw that the  
 ordinary matter density is combined with the effect induced by the defect via the  $b$   
 parameter [see (23)], so that, in a sense, it gives rise to an effective matter/energy  
 density one order of magnitude bigger than the actual one. Adding the fact that  
 one can interpret the strained state induced by the cosmic defect as being the  
 equivalent of a nonuniform (in time) dark energy, we see that in fact the principal  
 difference between  $\Lambda$ CDM and CD could not be that deep. However, CD produces,  
 somehow unexpectedly, one additional result, which is an inflationary phase in the  
 initial life of the universe, with no need for an ad hoc field.<sup>23,24</sup> This is not the case  
 with  $\Lambda$ CDM. The latter is of course mathematically simple and practically work-  
 ing, but it is not that simple on the side of the interpretation of what  $\Lambda$  actually  
 is; furthermore it apparently implies a never-ending acceleration of the expansion.  
 Our theory, instead, leads back to a final decelerated phase, which we think is a  
 good feature. On the formal side we may also remark that CD has already proved  
 to correspond to vector theories developed with different motivations and within a  
 different scenario.<sup>25</sup> Of course, there are many observational facts against which to  
 test the theory. We have started with the most well-known and considered one, i.e.  
 SnIa luminosity, with no pretence that this is the end of the story. In this test the  
 range of  $z$  values is limited, and the poor result obtained for the age of the universe

<sup>c</sup>Look for instance at <http://cfa-www.harvard.edu/~huchra/hubble>

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1 seems to indicate an inadequacy of the theory at high redshift values, where prob-  
 2 ably a better treatment of the matter content is in order. However, result with the  
 3 type Ia supernovae, summed with the other features of the theory, is encouraging.

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