Detecting the Angular Momentum of the Galactic Dark Halo

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Detecting the Angular Momentum of the Galactic Dark Halo

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Abstract

General relativity predicts the presence of a gravito-magnetic component of the gravitational interaction induced by a rotating mass. It is currently assumed that our galaxy (as well as the others) is immersed in a dark halo. If so, it is reasonable to expect the halo to rotate, then to produce a gravito-magnetic field. Here we present a proposal to measure such effect exploiting the fully relativistic version of the Sagnac effect. When an electromagnetic signal is led to travel along a spacely closed path immersed in a gravito-magnetic field the time of flight for a complete turn depends on the direction of rotation. The proposed physical loop would be based on the Lagrange points of the Sun-Earth pair. An evaluation of the sensitivity of such a measurement, together with a discussion of more opportunities that the experiment would offer is also presented.

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1. Introduction

General Relativity (GR) is a global paradigm for physics encompassing the gravitational interaction and providing the conceptual envelop for all interactions at the classical level. Among the weak manifestations of gravity, GR predicts also a contribution from the relative motion of the source of the interaction and the observer, assumed not to perturb the field (Tartaglia , 2005). This feature is, at first sight, similar to an aspect of classical electromagnetism (EM), where moving charges interact not only via their electric field,

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but also through the magnetic field originated by their currents (fluxes). In fact, when linearizing Einstein's equations (i.e. assuming that the interaction is sufficiently weak), they take a form much similar to Maxwell's equations for the EM field (Misner, Thorne, Wheeler , 1973, chapter 18). The similarity is approximated and not complete, so it has to be handled with care, but it works reasonably well for the purposes of this paper.

Leaving GR in the background, let us come to phenomena on the scale of galaxies or at least globular clusters. Since the 30's of the 20^{th} century and even before, it is known that in order to explain the behaviour of matter at those scales much more mass than the visible one is needed (Bertone & Hooper , 2016). The additional stuff has been called "dark matter"; initially the name simply referred to non-bright, i.e. low temperature, matter, but now people surmise that it is "something" non-interacting electromagnetically, but carrying mass, then interacting gravitationally. Recent global evaluations in the framework of the so called standard model tell us that dark matter (DM) is approximately 27% of the cosmic cocktail, whereas ordinary matter is just 5% and what is left is dark energy (Planck collaboration , 2018).

Here we would like to call the attention on our galaxy, the Milky Way, as well as on other similar spiral galaxies. DM plays a role in all types of galaxies and galaxy clusters, but spirals are easier to study. Commonly accepted wisdom assumes that, in order to explain the rotation velocity curves of the stars in the Milky Way, our galaxy (as well as its kins) must be immersed in a dark halo bigger than the whole visible extent of the stellar pinwheel (Battaglia et al. , 2005). The remark is: should such huge halo exist, could it be non-rotating? Should it not be rotating more or less at the same rate as the visible galaxy? If the galactic dark halo(s) were non-rotating, it would be a sort of miracle worth special investigations in order to understand why: everything in the universe rotates with the exception of puzzling spin zero particles. The next comment is that, if the massive dark halo rotates, it also possesses an important angular momentum and, according to GR, it must produce non-negligible gravito-magnetic (GM) effects.

The final question is: how could we evidence the GM field of the dark halo of the Milky Way? The proposal presented in this paper is to use a circuit based on the Lagrange points of the Sun-Earth pair and exploit the fully relativistic Sagnac effect, i.e. the anisotropic propagation of EM signals induced both by the kinematical rotation of the observer and by the presence of a GM field.

In the following sections we shall summarize the essentials of gravito-magnetism, then the way Sagnac effect works, applying it to a Lagrangian circuit. Such explanations will be followed by an evaluation of the expected signal; the whole proposed experiment is nicknamed LAGRANGE. Practical problems together with other opportunities offered by the LAGRANGE configuration will also be discussed before drawing a conclusion.

2. Gravito-magnetism

GR is essentially geometry of a four-dimensional Riemannian manifold, curved by the presence of matter/energy. What we usually call gravitational field is indeed expressed by the curvature of the manifold. Geometric properties are synthesized in the line element of the manifold; in arbitrary coordinates x^μ [1] we write:

$$ds^2 = g_{00}c^2dt^2 + 2g_{0i}cdtdx^i + g_{ij}dx^i dx^j \tag{1}$$

Latin indices run from 1 to 3 and are assigned to space variables. The Einstein convention in the notation implies that, whenever the same index appears both as subscript and as superscript a summation over the full range of values of the indices is intended.

Variable s is proportional to the proper time of an observer at rest in the (arbitrarily chosen) reference frame. The $g_{\mu\nu}$'s are the elements of the symmetric metric tensor, incorporating gravity into geometry. The mixed time-space components g_{0i} account for relative motion of the source of gravity with respect to the observer.

In some cases (uniform relative motion along a straight line) the mixed terms may be eliminated by a global reference change, thus showing that one is facing a coordinate effect. However in other situations it is not possible to eliminate the g_{0i}'s by a *global* coordinate transformation: it is the case of a massive source endowed with an angular momentum. The latter situation is the one of interest here. Again, in weak field approximation, for a stationary configuration, the mixed terms assume a role recalling the components of the three-vector potential, $\overline{h}$, of electromagnetism

$$g_{0i} \equiv h_i \tag{2}$$

[1] The index μ runs from 0 to 3; conventionally index 0 is allotted to time.

and the corresponding GM field $\overline{B}_{GM}$ is $\overline{B}_{GM} = \overline{\nabla} \wedge \overline{h}$.

Following on along this path we may verify, working out the approximated geodesics, that a test mass is subject to a (three-)acceleration $\overline{a}$ writable in the form:

$$\overline{a} \simeq -c^2 \overline{\nabla} g_{00} + 2c\overline{v} \wedge \overline{B}_{GM} \tag{3}$$

The first term on the right is the usual gravitational attraction (g_{00} is the adimentional gravitational potential) and, according to the EM analogy, may be called "gravito-electric". The second term looks like the EM Lorentz force and depends on the (three-)velocity $\overline{v}$ of the test body: this is the gravito-magnetic acceleration.

3. Light propagation

Let us now concentrate on the propagation of light, or in general of an EM signal, in the space time of a moving, and in particular a rotating, mass, i.e. including a gravito-magnetic interaction. The line element of an EM wave is null, which means that we may use Eq. 1 to solve for dt, i.e. the coordinated time interval for a space displacement vector $\{dx^i\}$ along a ray. The result is:

$$dt = -\frac{g_{0i} dx^i}{c g_{00}} + \frac{\sqrt{(g_{0i} dx^i)^2 - g_{00} g_{ij} dx^i dx^j}}{c g_{00}} \tag{4}$$

The sign in front of the square root has been chosen so that the propagation always be to the future, irrespective of the sign of the dx^i's, i.e. of the direction of propagation along the space path.

For a finite length travel from event A to event B and using the three-dimensional space line element dl along the trajectory, the coordinated time of flight will be:

$$t_{AB} = -\int_A^B \frac{g_{0i}}{c g_{00}} \frac{dx^i}{dl} dl + \int_A^B \frac{\sqrt{g_{0i}^2 (dx^i/dl)^2 - g_{00} g_{ij} \frac{dx^i}{dl} \frac{dx^j}{dl}}}{c g_{00}} dl \tag{5}$$

If we reverse the propagation direction (from B to A, rather than from A to B) the first integral in Eq. 5 changes sign, which is not the case of the second. Going one step further we may imagine an observer O at rest in the chosen reference frame, who sends EM signals along a closed path so that they come back to him: this of course requires some appropriate devices (mirrors, optical fibers...). The above remark on the sign of the contributions to the time of flight, tells us that the right-handed time will be different from the left-handed one. The difference is obtained from Eq. 5 subtracting the times of flight along the same closed path, in opposite directions. Including the fact that every measurement will be made referring to the observer's proper time, whose element is $d\tau = \sqrt{g_{00}}dt$, we get:

$$\delta\tau|_{obs} = 2\frac{\sqrt{g_{00}}_{obs}}{c}|\oint\frac{g_{0i}dx^i}{g_{00}}| \tag{6}$$

The absolute value of the difference has been considered.

The above is the essence of the generalized Sagnac effect, which was initially, in its kinematical version, proposed as a means to corroborate the role of the ether *against* the Einsteinian special relativity (Sagnac, 1913), but actually turned out to be a fully (general) relativistic phenomenon (Tartaglia & Ruggiero , 2015).

An advantage of measurement strategies based on the Sagnac effect is that the relevant quantity is indeed a locally measured proper time interval along the world-line of the observer; the ends of the interval are two events corresponding to the intersections with the null world-lines of EM signals travelling in opposite directions on a trajectory whose space projection in the observer's frame is the same. Such proper time span is a rank-0 tensor, i.e. a scalar quantity and as such it is "objective", not requiring any external landmark. That is not the case for experiments concerning the mechanical drag on gyroscopes, such as Gravity Probe-B (Everitt et al. , 2011), where a faraway physical reference is needed in a region where space-time is assumed to be flat, in order to ascertain the reality of the induced precession motion. Of course in the Sagnac signal the practical problem, which is considered in sec. (4), is to separate the kinematical rotations corresponding to the orbital motion of the observer, from the GM effect.

4. LAGRANGE

The Sagnac effect, in its simplest version, is currently exploited in angular rotation sensors for airplanes and other aerial and maritime vehicles; the most sensitive devices are able to reveal the diurnal rotation of the Earth and even the wobbling of the axis of the planet (Schreiber & Wells , 2013). In its generalized form, synthesized in Eq. 5, it is under implementation for the GINGER experiment (Bosi et al. , 2011; Tartaglia et al. , 2017); here we

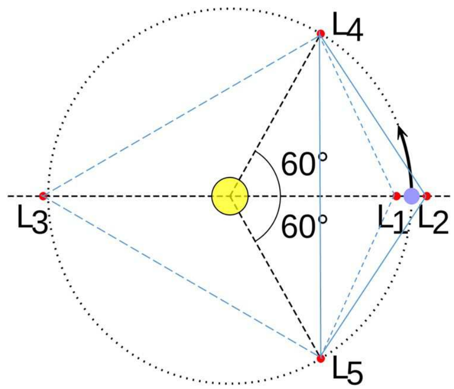

Figure 1: This figure shows the configuration of the Lagrange points (dark dots) in the plane of the orbit of the Earth (grey disk). Numbering is the conventional one. Straight continuous lines mark a possible triangular path for electromagnetic signals.

apply the experimental approach to the Lagrange points of the Sun-Earth pair.

The set of the Lagrange points (L-points) of a pair of mutually orbiting masses comprises the positions where gravitational attraction and centrifugal force on a test particle balance each other. This description is entirely classical; when a GR approach is introduced the situation becomes more complicated and it is not a priori guaranteed that such positions always exist: it depends on the ratio of the masses of the main bodies. In the case of the Sun-Earth pair, GR introduces a small correction in the positions of the L-points (Battista et al. , 2017; Tartaglia, Esposito et al. , 2018) which is not relevant for the purpose of the present paper. What matters here is that the L-points rotate "rigidly" together with Earth and Sun co-orbiting the center of mass of the pair (which actually is located inside the volume of the Sun). This statement is not literally true, since the orbit of the Earth is not exactly circular and the other bodies of the solar system introduce perturbations, but the essence remains: the L-points may be thought as the vertices of a polygon (actually various polygons) following the Earth while keeping their configuration fixed. The situation is represented in Fig. 1.

The idea now is to position emitters and/or transponders in at least three L-points, then use them to obtain spacely closed paths for EM signals: an asymmetry in the times of flight (TOF) is expected according to the direction of propagation. The difference in the closed circuit TOFs would be due: a) to the rotation of the L-points and the Earth with respect to the "fixed" stars (distant quasars); b) to the angular momentum of the Sun; c) (this is the relevant point here) to the rotation both of the baryonic matter and of the dark halo of the Milky Way (if it exists). Considering the size of the configuration and the distances, we assume that the contributions from the angular momenta of the Earth and of other bodies of the solar system are negligible.

Let us concentrate on the $L_4 - L_5 - L_2$ triangle: one of the combinations visible on Fig. 1. Of course other choices would equally be viable; the most favorable, due to the scale, would be $L_3 - L_4 - L_5$, but the drawback there is that L_3 lies on the opposite side of the Sun with respect to the Earth, thus being invisible from the ground. The total TOF along the perimeter of the chosen triangle is ~ 2000 s.

The solar contribution to the gravito-magnetic propagation asymmetry (which is proportional to the angular momentum of the Sun) has been evaluated in Tartaglia, Lorenzini et al. (2018). The starting point is the approximated line element in the plane of the ecliptic of a spinning mass (the Sun), viewed in a non rotating reference frame centered in the main body:

$$ds^2 \simeq (1 - 2\frac{\mathfrak{M}}{r})c^2dt^2 - (1 - 2\frac{\mathfrak{M}}{r})^{-1}dr^2 - r^2d\phi^2 + 4\frac{j}{r^2}crdtd\phi \tag{7}$$

Polar coordinates in space have been used and it is:

$$\mathfrak{M} = \frac{G}{c^2}M \quad \text{and} \quad j = \frac{G}{c^3}J \tag{8}$$

M is the mass of the Sun and J its angular momentum. Numerical values are:

$$\mathfrak{M} \simeq 1475\, m \quad \text{and} \quad j \simeq 4.71 \times 10^6\, m^2 \tag{9}$$

The small terms for the approximation are $\mathfrak{M}/r$ and j/r^2 assuming that r always be of the order of the radius of the orbit of the Earth $R \simeq 150 \times 10^9$m: $r \sim R$. The smallest term kept will be of the order of $\sim j/r^2$.

We are interested in the viewpoint of an observer at rest with the L-points, i.e. co-rotating with the Earth at its orbital angular velocity Ω. Staying within the approximation approach, let us assume that the orbit of the Earth be circular, so that Ω is fixed.[2] Its value, as it is suggested by Eq.

[2]In terms of principle the exterior exact metric of a massive singularity endowed with

3 depends both on the mass and on the angular momentum of the central body (the Sun), however the chosen approximation level permits to simply use the Keplerian form:

$$\Omega \simeq \frac{c}{R}\sqrt{\frac{\mathfrak{M}}{R}} \tag{10}$$

Put it all together (Tartaglia, Lorenzini et al. , 2018), the solar contribution to the TOF asymmetry turns out to be:

$$\delta\tau_{Sun} \simeq -4\oint \frac{j}{r} d\phi \sim 4.3 \times 10^{-13}\ s \tag{11}$$

The term due to the kinematical Sagnac effect is much bigger:

$$\delta\tau_{Sagnac} \simeq \frac{2}{c}\sqrt{\frac{\mathfrak{M}}{R}}\oint (\frac{r}{R} + 2\frac{\mathfrak{M}^2}{R^2} + \frac{\mathfrak{M}^2}{R^2}\frac{r^3}{R^3} - \frac{3}{2}\frac{\mathfrak{M}r}{R^2}) r d\phi \tag{12}$$

The explicit calculation can be made using the same method as in Tartaglia, Lorenzini et al. (2018, Sect 3.1). The dominant term yields:

$$\delta\tau_{Sagnac} \sim 0.36\ s \tag{13}$$

Actually in order to discriminate the solar GM signal from the rest, the kinematic TOF asymmetry should be expressed up to 10^{-15} s and of course this is not a matter for calculation but of accuracy in the measurement: I leave this for the final discussion.

4.1. GM field of the galactic dark halo

Let us now focus on the possible contribution to the generalized Sagnac effect due to the rotation of masses, be they visible or dark, in the Milky Way. If the dark halo exists, we know very little about it; essentially one considers the DM mass distribution. The mass distribution, on its turn, is usually induced from the peripheral velocities curve of ordinary matter in the Milky Way (Battaglia et al. , 2005). No unique answer is available but various models are in use, starting from the simplest spherical uniform distribution up to more refined choices.

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angular momentum, i.e. the Kerr metric, would not allow for closed orbits contained in a plane.

Here I will not try to privilege any specific model, but simply discuss what we could expect in any case, just assuming that the massive dark halo is rotating. A first remark is that our Lagrangian triangle (or any other Lagrangian polygon we would like to use) will be orbiting the Sun together with the Earth, hence, during the year, it will change its position with respect to the center of the Milky Way. This periodic motion implies the "device" will meet a changing galactic GM field, $\overline{B}_{GM}$. The maximum size of the displacement is of the order of the diameter of the Earth's orbit, i.e. $\delta\rho \sim 3 \times 10^{11}$ m; on the other side, the distance of the Sun from the galactic center is roughly $\rho \sim 2.5 \times 10^{20}$ m (Honma , 2012). It is reasonable to expect that the relative change of the intensity of $\overline{B}_{GM}$ be

$$\frac{\delta B_{GM}}{B_{GM}} \sim \frac{\delta\rho}{\rho} \sim 10^{-9} \tag{14}$$

Actually, if we are interested in detecting the possible GM field of our galaxy with accuracies at best in the order of, say, 1% we conclude that B_{GM} is practically constant over the interior solar system. Of course $\overline{B}_{GM}$ is a vector, but we may easily extend the above conclusion to its direction also: the GM field of an axially symmetric space-time is expected to have a dipolar structure (Bosi et al. , 2011), so the field is expected to be perpendicular to the galactic plane all over that plane. The Sun is thought not to exactly lye on the galactic plane, furthermore the plane of the ecliptic is at an angle $\alpha \sim 60^o$ with respect to the galactic plane. Summing up, and staying within the borders of reasonable approximations, we may assume that, the whole year round, the GM field of the Milky Way is a constant vector at an angle α with the ecliptic north pole.

This situation simplifies the evaluation of the possibly detectable signal in LAGRANGE. Recalling Eq. 2 and Eq. 6 we may use Stokes' theorem and write:

$$\delta\tau_{GM} = \frac{2}{c}\int \overline{B}_{GM} \cdot \hat{u}_n dS \tag{15}$$

The flux integral is over the area, S_L, contoured by the Lagrangian triangle (or other polygon). The practical local uniformity of $\overline{B}_{GM}$ immediately gives the result of the integral in Eq. 15:

$$\delta\tau_{GM} \simeq \frac{2}{c} B_{GM} S_L \cos\alpha \tag{16}$$

We may reverse Eq. 16 to get B_{GM}:

$$B_{GM} \simeq \frac{c\delta\tau_{GM}}{2S_L \cos\alpha} \tag{17}$$

The area of the $L_4 - L_5 - L_2$ triangle is $\sim 9.9 \times 10^{21}\,\mathrm{m}^2$. Let us suppose the smallest detectable TOF asymmetry be $\sim 10^{-15}$ s. In such conditions we should be able to sense GM fields of the Milky Way at the position of the solar system

$$B_{GM} \geq\sim 10^{-29}\,\mathrm{m}^{-1} \tag{18}$$

Just for comparison, the GM field of the angular momentum of the Earth, at the surface of the planet is $\sim 10^{-30}\,\mathrm{m}^{-1}$.

Of course, what we would measure with LAGRANGE would indeed combine the effect of the visible matter and of the dark halo. Just to give an idea of the relative importance of the two contributions let us recall that GM fields originating from rotations are in general proportional to the angular momentum of the source. A rough guess can then be obtained comparing the angular momentum of an homogeneous disk (symbolizing baryonic matter) with that of an homogeneous sphere (the dark halo) rigidly rotating at the same angular speed. The ratio of the latter to the former is just, modulo a factor of order 1, the ratio between the two masses, which, in the case of the Milky Way could be in the order of 10. In other words, the contribution of the dark halo would clearly prevail on the one of ordinary matter.

5. Discussion and Conclusion

I have presented a concept strategy for revealing the possible presence of a GM drag due to the rotation of the dark halo of our galaxy. I have not entered technological problems concerning the implementation of the proposal. Measuring time differences in the order of 10^{-15} s over total TOFs as big as ~ 2000 s is indeed not an easy task, especially in space. Furthermore the looked for effect would be superposed to a dominant kinematical Sagnac effect (Eq. 12) and could be comparable with the GM field of the Sun (Eq. 11). It has also been assumed that the different GM fields originated by the Sun, the baryonic matter in the Milky Way and the dark halo be linearly superposable, which seems to be a reasonable hypothesis, due to their weakness.

Despite these difficulties I think the challenge should not be unsurmountable. On Earth similar measurements, like for GINGER (Tartaglia et al. , 2017) or G-Wettzell (Schreiber & Wells , 2013), correspond to TOF asymmetries (1% accuracy) $\sim 10^{-16}$ s. Terrestrial experiments are based on the use of light, while in space tests radio frequencies could be easier to use; however it is worth mentioning that the LISA mission for detecting gravitational waves is based on the use of light for interferometry over distances of the order of 2.5×10^9 m and with the purpose to reveal relative motions in the order of 1 part in $\sim 10^{20}$ or less. Apparently this is a task even harder than the one proposed here for LAGRANGE. LISA is under science development (see for instance Amaro-Seoane et al. (2018)) and the most relevant related technologies have been tested by the LISA pathfinder mission (ESA, 2018).

Reviewing the problems to be tackled for a mission like LAGRANGE we should recall that of course the calculations have been done as though transponders and emitters were fixed in the positions of the L-point. Of course this would not be the case: the spacecraft carrying the devices would orbit the L-points either along weakly stable quasi-periodic orbits (L_4 and L_5) or weakly unstable Lissajous orbits (L_1 and L_2): the issue has been discussed in Tartaglia, Lorenzini et al. (2018, Sect. 6). The situation would be reasonably manageable if an appropriate control is exerted on the initial conditions for the orbits and sufficiently long duration strategy for the measurement is adopted.

An important remark is that placing emitters or transponders in the L-points would offer interesting side opportunities for more experiments and for positioning and navigation in the solar system (see Tartaglia, Lorenzini et al. (2018)). The Sun-Earth Lagrange points are and have been often exploited locations for a number of past, present and future space missions, which means that a multi-opportunity approach is practicable, thus comparting the relevant costs of the mission over many projects and reducing individual economic burden.

Summing up, we may conclude that the LAGRANGE mission is in the range of technical feasibility and could be worth doing for its main goals (retrieving information about the galactic dark halo and performing an independent measurement of the angular momentum of the Sun) and for a number of additional opportunities it would offer.

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Aknowledgements The idea proposed in this paper rests on the work done for paper Tartaglia, Lorenzini et al. (2018), so I would like to thank

my coauthors there, David Lucchesi, Enrico C. Lorenzini, Giuseppe Pucacco, Matteo Luca Ruggiero and Pavol Valko, for inspiring me the developments presented in the present article.

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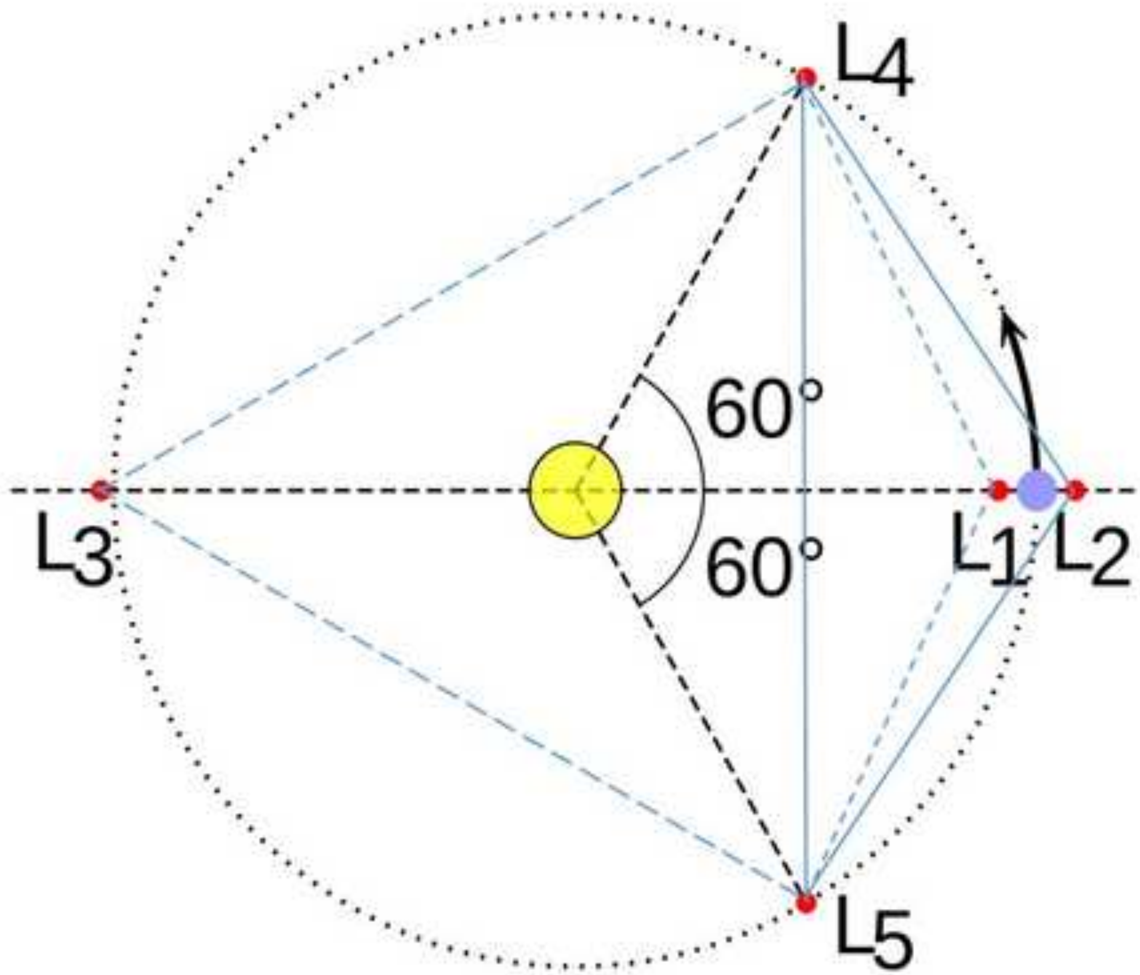
Figure



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I thank the referee for pointing out misprints and unclear sentences and formulae in the text. I have taken care of all remarks. The list of amendments follows.

List of amendments

- On pages 6 and 7 the figure reference has been corrected: now it is properly to Fig. 1
- On page 5. The misprint has been corrected: the right spelling is now "Einsteinian"
- Page 6, first line. The referee's remark is indeed appropriate; I have replaced "generalize" with a plain "apply".
- Formula (5). It is true that it looks rather cumbersome, so I have changed a bit the two lines before the formula, introducing the space line element *dl*. The new text is:
  "For a finite length travel from event *A* to event *B* and using the three-dimensional space line element *dl* along the trajectory, the coordinated time of flight will be:"
  Then I have amended the formula consequently.
- For clarity, having used "*l*" for the line element along the space trajectory of light, I have replaced *l* in formula (14) and the four preceding lines with ρ

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