Doctoral Dissertation

Linear and weakly nonlinear analyses of morphological instabilities

By

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Abstract

The interactions between a fluid, such as water or air, and a deformable surface, such as rock, ice or sand, create marvelous natural patterns. The desert dunes generated by the winds, the cave speleothems shaped by thin water films and the sediment patterns on a river bed are just some examples. The branch of fluid mechanics devoted to the study of these patterns is the so-called *morphodynamics*.

In this thesis, some morphological patterns from different natural environments are analytically modelled through stability analysis. This mathematical approach, commonly used to study every morphological pattern, addresses the stability of the spatially uniform solution to a small spatial perturbation. In unstable conditions, the perturbation grows in time to eventually form a finite-amplitude pattern. This thesis focuses on alternate bars, which are macro-scale river patterns, and on some small-scale ice and karst patterns.

In first place, bar formation in rivers with considerable amount of suspended load is investigated. Closed forms relations for the wavelength (linear stability analysis) and the finite amplitude (weakly nonlinear analysis through Center Manifold Projection) are achieved. Results show that suspension plays a destabilizing role in bar instability and it affects both the bar wavelength and amplitude. The theoretical outcomes are validated with field observations. In second place, the conditions for vegetation spreading on finite-amplitude bars are mathematically framed in a model that also includes flow stochasticity. Flow unsteadiness is identified as the main factor discouraging vegetation growth, up to the point that, above a certain threshold, plant spread is completely inhibited. Comparison with field data demonstrates that the model captures the physical conditions heralding the transition between bare and vegetated fluvial states.

Small-scale patterns in karst and glacial environments are usually generated by a falling liquid film. In some cases, the resulting patterns are so similar that a unified approach for the two environments is possible. This happens in the case of the longitudinally oriented organ-pipe-like structures, called flutings, which are widespread in caves and in ice-falls. In this thesis, an analytical model for fluting formation is proposed and closed form relations for the wavelength and the finite-amplitude (linear and weakly nonlinear analyses) are provided. The theoretical results are confirmed by numerical simulations of the fully nonlinear equations.

The last part of this thesis deals with the problem of icicle formation. Recent experiments have revealed that small amounts of dissolved impurities are required for radial ripples to appear on icicle surface. This is contrary to existing theories, which would predict ripples on icicles formed by pure water. The theoretical model here proposed shows that icicles made by pure water do not develop ripples. Moreover, some considerations on the reason why dissolved impurities drive the ripple instability are presented.