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On Urban Morphology and Mathematics

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Cities are ultimately the results of human behaviour whose understanding might be reductive if solely framed within a strictly 'mathematical' confine[1]. Human behaviour "is so complex and influenced by such a wide range of factors that any claim to provide precise, deterministic prediction is unrealistic" (Inglehart 2018, p. 10).
But there is another side of the coin: quantification and mathematical modelling are means enabling us to discover partially predictable macro paths of our behaviours otherwise unreadable. Even if "not deterministic […,] some trajectories are more probable than others" (Inglehart 2018, p. 11): the mathematical language helps both in seeing these trajectories and in quantifying these probabilities.

1. Morphology, Urban and Mathematics

The expression 'mathematics of urban morphology', has three terms: mathematics, urban, and morphology; whose individual meanings are:
'Morphology' is built from the suffix 'logy', from ancient Greek λογία, meaning 'the study of', and *morphé* meaning 'form', 'shape'. It intrinsically involves a quantitative component, as it is defined "the scientific study of the structure and form of…"[2]; where "scientific" evokes the *scientific method* introduced in the 17th century in the natural sciences "consisting in systematic observation, measurement, and experiment […]"[3].
'Urban' is related to the Latin word *urbs* used to indicate the physicality of the city, as an ensemble of built structures[4]. According to Pomponio[5], who recalls the authority of Alfenus Varus, *urbs* comes from *urbum*, the handle of the plough, used in the Roman tradition to define the border of a new city, and *urbare* means the action of "defining with the plough". Varrone[6] prefers to associate the etymology of *urbs* to *orbis*; the *orbis urbis principium* of the Romans was the founding element of the edification of the city. The *orbis* constitutes the *principium urbis* and is the strip of land around the *urbs* which is generated by the plough (De Sanctis 2007).
'Mathematics', from Greek μάθημα máthēma, originally meant 'knowledge, study, learning'. Among many definitions[7], is nowadays often defined as the science of number, quantity, and space, as abstract concepts or as applied to other disciplines[8].
The meanings arising when grouping together these terms – mathematics, urban, and morphology – become:

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[1] Among many, we just recall as example William S. Jevons's ambition to mathematically formulate human behaviour as the ultimate in scientific soundness.
[2] https://dictionary.cambridge.org/dictionary/english/morphology (Cambridge University Press 2018).
[3] https://en.oxforddictionaries.com/definition/scientific_method (Oxford University Press 2018).
[4] The Latin used another term, *civitas*, to refer to the city as an ensemble of people, which said in Rousseau's words becomes: "les maisons font la ville, mais […] le citoyens font la cite" (Rousseau 1762, reprint 2011, Livre 1, chap. 6, note p.57) (houses make the 'ville' [urbs], but citizens make the 'cite' [civitas]). While the Greeks used the word *polis* to indicate the city as the place where to exercise politics.
[5] "Urbs ab urbo appellata est: urbare est aratro definire. Et Varus ait urbum appellari curvaturam aratri, quod in urbe condenda adhiberi solet" Pomponio, *Digesta*. Libro 16, 239.
[6] Varrone, *De lingua Latina*. V, 143.
[7] A poetical one from Nicola Bellomo (Highly cited in mathematics, from 2014 by WEB of SCIENCE), is "Mathematics: a virtual bridge that starts from the phenomenon in search of the noumenon" (Nicola Bellomo personal communication, 3 May 2018).
[8] https://en.oxforddictionaries.com/definition/mathematics (Oxford University Press 2018).

'Urban morphology' is *the study of urban form*. It concerns the size, shape, and physical structure of urban settlements. The American Planning Association sees urban morphology as the understanding of "the spatial structure and character of an urban area by examining its patterns and the process of its development" [9]; Kropf (2017, p. 9) as "concerned with the form and structure of cities, towns and villages, the way that they grow and change and their characteristics […]; and Batty as "patterns of urban structure based on the way activities are ordered with respect to their locations" [10] stressing how the urban morphology, as [*physical*] patterns of urban structure, is linked with *activities*.
'Mathematics of urban morphology' injects a double dose of 'quantitative' in the study of urban form: one from 'mathematics', one from the *scientific* study intrinsic in the term morphology we saw at the beginning of this introduction.

2. Urban Morphology

Urban morphology is a relatively young field which is becoming, as Whitehand (2016) underlines, more and more significant[11] and multidisciplinary[12] and its aim is to "contribute to our understanding of the built environment as a complex physical object, a cultural artefact and quasi-natural phenomenon […]" (Kropf 2017 p. 9).
A better understanding of urban forms helps the availability of measures, models, analysis which in turn helps to make effective previsions (that eventually lead to decisions) on urban form issues, which is among the most urgent needs and with the longest lasting effects, in the sustainable development arena of a world whose current urbanization and population growth rates are unique in history.
Rooted in the morphological tradition of European geographers interested in the urban layout forms in-between the nineteenth and twentieth centuries (human geographers such as Schlüter and Geisler, whose work Conzen later brought to the British school), urban morphological studies in urban design and architecture, began in Italy (Muratori and his student Caniggia) and France (Castex, Panerai, Depaule and later in the 1980s Borie, Micheloni, Pinon and Fortier) in the 1950s and 1960s as an attempt to explain the structural continuity of traditional cities, after the failure of the modern movement; while in north America still during the second half of the twentieth century urban layout studies focused on aesthetics or economics, but historians of architecture (Fitch, Garvan, Lewis, Mayer, Whitehill, Kennedy), made some early progress in the study of urban forms (Rashid 2017, p. 23-25).
We might cluster urban morphology lines into four: the (mostly British) human geography field (influenced by Conzenian's description and classification approach, recently carried on by Whitehand and his Birmingham School[13]); the (mostly Italian) architecture/planning field (as the Caniggian school underlining urban structure's components); the utopian approach (in search for ideal cities, e.g. Le Corbusier, Howard and Wright); and the anthropological perspective (Lynch, Jacobs, Alexander) whose interest was to observe "what actually works in real cities. This sowed the seeds of a mathematical approach for quantifying urban morphology" (Xiao 2017, p. 42), where Alexander was among the first to attempt the introduction of formal mathematics to the subject of design.

3. Mathematical treatment of urban forms

Since then, the mathematical treatment of the urban forms has become more and more sophisticated, and, generally speaking, mathematics has proved to be an efficient language to deal with the aim of the scientific tradition in urban studies (rooted in neoclassical economics and embracing urban and regional economics, urban and economic

[9] https://www.planning.org/tuesdaysatapa/2010/aug.htm
[10] http://discovery.ucl.ac.uk/15183/1/15183.pdf
[11] "[…] the number of indexed documents containing the term 'urban morphology' increased from 26 in 1991–95 to 363 in 2011–15, more than doubling in successive 5-year periods from 1996–2000 to 2011–15 (Web of Science, All Databases, accessed 30 January 2016). This is almost certainly indicative of a significant increase in the actual amount of work undertaken […]" (Whitehand 2016 http://www.urbanform.org/online_public/2016_1_editorial.shtml).
[12] "[…] journals carrying documents that contain the term 'urban morphology' […] In 1991–95 just 21 […] 2015 roughly triple" (Whitehand 2016 http://www.urbanform.org/online_public/2016_1_editorial.shtml)
[13] "Founded in 1974, the Urban Morphology Research Group (UMRG), in the School of Geography, Earth and Environmental Sciences at the University of Birmingham, is the major centre in the United Kingdom for the study of the geographical aspects of urban form." https://www.birmingham.ac.uk/research/activity/urban-morphology . See also Oliveira, V. (Ed) (2019) *J.W.R. Whitehand and the Historico-geographical Approach to Urban Morphology*. Springer.

geography), which is to generate a theory of urban form which is scientific in nature: namely formal, deductive, and based on postulates of human behaviour.
The *scientisation* process[14] of the study of urban forms in the planning sector dates back at least to Ildefonso Cerda (1867), often regarded as the father or Urbanism, who promoted a rational urban method with the declared aim to define a kind of urban science, even if his top-down view is sometimes conceptually distant from the bottom-up emergence paradigm which mostly characterizes the new urban science of today framed within the recent science of complexity[15].
This 'rational' attitude in planning urban forms, would travel even much further back into the past if we think about the strictly geometrical proportional rules used in Ancient China to design the size of ex novo cities and their growth; the Harappan civilization planned towns, the earliest known gridiron urban form planning (e.g. Harappa, ~2150 BC, Kalibangan, Lothal, Mohenjo-daro); the ancient Egyptian gridiron planning (e.g. Tel-el Amarna, ~1346 BC, and Kahun, 1853 BC) as the Hippodamian's of the Ancient Greeks (started with the rebuilding plan of Miletus, 479 BC); the *centuratio* of the Romans; or more recently the grid of European colonies, the Renaissance symmetries, order and idealistic rationality, and the Baroque lavish geometries (Morris 1994, Benevolo 1980).
However, the mathematical approach to urban forms of contemporary times is not much oriented toward the above examples – which we may better define as top-down planning of forms rather than the science of cities we intend today. The latter is more oriented towards the analytical (e.g. Geoffrey West's research) finding for universal laws, and in the phenomenological (e.g. Michael Batty's research) understanding of processes dealing with the emergence of various urban phenomena as forms, which are treated as a physical expression of our behaviors to be understood within the complexity science framework with the help of statistical physics, computation and formal models. Recent availability of disaggregated data, together with the betterment of computer capacity and mathematical tools, allow a proper introduction of the complexity paradigm in the study of cities, by, among many examples, multi-agent based modelling and behavioural economic elements acting as theoretical/technical tools to simulate interactions whose positive/negative feedbacks induce emergent phenomena dynamically translated into urban forms. This enormous availability of data presents a great opportunity for the science of cities, or, to use the words of Marina Alberti, a challenge to "turn the unprecedented floods of data into new knowledge about urban systems and novel insights for their effective management" (Alberti 2017, p.2).
If we enlarge the overview of the use of mathematics more generally to urban and regional modelling (which still influence urban forms), the modern history of these models began with the contribution of von Thünen (1826), the Prussian economist, agriculturalist and social theorist who wished to understand in more depth human society and economics thanks to mathematical formulation of socio-economic principles.
A century later, two theorists writing in the tradition of German location theory, the geographer Walter Christaller (1933) and the economist August Lösch (1940), proposed, independently, formal theories of the spatial locations of towns and cities which also indirectly relate to urban sizes, see also Isard (1956).
Mathematical language carried on being successfully used in urban phenomena with, to mention only a few, Alfred Weber (1929) in his optimal location models; Ira Lowry (1964) in his mathematical model for spatial organization of urban activities; William Alonso (1964) and Richard Muth (1969) in their urban economic reinterpretation of the agricultural von Thünen model, which in turn relates to the Ricardian rent theory (Ricardo 1821), more recently re-elaborated in much more sophisticated mathematics in Fujita (1989) and Fujita, Krugman, Venables (1999).
The modelling approach to cities, "emphasizes the development of robust theoretical constructs […] it seeks to fashion precise analytic representations of the world […] creating formalized, empirically testable, descriptions of the social world […] They may also be presented as mathematical formula […] closely related to […] physical sciences" (Koch, Latham 2017, p.9).
To use the expression of John Landis, "if there is a golden age of urban models, it is surely now." (Landis p. 323). The list of authors of mathematical formulations for models (static or dynamic, aggregated or disaggregated) in some way related to urban morphology to mention in order to provide a fairly complete picture is enormous, particularly in the last decades. We can however briefly describe three main groups within the research community adopting mathematical models for spatial structures, which started fragmenting by the 1980's: one group oriented itself toward

[14] Here used in the common sense, without the literal meaning of 'scientific approach' and without meaning the use of mathematics.
[15] One of the most known catch-phrase of complexity science is the antireductionism affirmation that "the whole is more than the sum of its parts", ironically implicit in Hofstadter book: "Reductionism is the most natural thing in the world to grasp. It's simply the belief that 'a whole can be understood completely if you understand its parts, and the nature of their 'sum'". No on in her left brain could reject reductionism. (Hofstadter 1979).

inferential statistics rather than formal theory and models; another toward mathematical models rather than empirical; another abandoned both statistical and mathematical theoretical modelling preferring postmodern approaches (White, Engelen, Uljee 2015, p. 53).
A fine way to express the importance of bridging theorists (close to the first group above mentioned) and empiricists (second group) is described from Melanie Mitchell in her eminently readable book: "the more established the theory or principles, the more sceptical you have to be of any contradicting facts, and conversely the more convincing the contradicting facts are, the more sceptical you need to be of your supposed principles. This is the nature of science – an endless cycle of proud proposing and disdainful doubting" (Mitchell 2011, p. 295).
More critical to the scientific lack in urban studies is Geoffrey West, former President of the Santa Fe Institute, in the beginning of the chapter "Toward a science of cities" of his book: "Almost all theories of the city are largely qualitative, developed primarily from focused studies on specific cities or groups of cities supplemented by narratives, anecdotes, and intuition" (West 2017, p. 269).
To carry on later on the same book with the necessity to mix qualitative and quantitative approaches in the study of cities: "much of what we experience in cities […] is embodied in this quantitative framework. In this respect it should be viewed as complementary to traditional social science and economic theories whose character is typically more qualitative, more localized, more based on narrative, less analytic, and less mechanistic." (West 2017, p. 325).
As the Géographie Vautrin Lud prize[16] winner Michael Batty of the Centre for Advanced Spatial Analysis (CASA) tells us (2018), the idea of a science of cities is not new; what is new is the availability of data and technologies.
Using the words of Koonin and Holland (2014), the new opportunity is "to collect and analyze data that will allow us to characterize and quantify the 'pulse of the city'. We are not alone in believing that a new science of cities is emerging".
Batty bases this new urban science on three principles: relations, scale, and prediction; Goodwin (2004) indicates science to identify quality in cities through analytic-cognitive tools; and Ball (2004) claims how patterns of movement and activity can provide a more vigorous approach to urban planning and he went so far as to define a "Physics of Societies".
Hillier believes in the enormous potential of scientific analysis of movement's patterns, and recommends that urban designers have to internalise this knowledge and, according to complexity science, to use the self organizational behaviour of cities. He (Hillier 2004) noted that the making of cities is both fully an art and fully a science: "The art of urban design, as I firmly believe it to be, does rest on the foundation of the science of space".
Hanson's ideas (2004) follows Alexander's magnum opus, *The Nature of Order* which is based on organised complexity principles – recalling pioneers like Jacobs – arguing that order does not mean top-down design, but should emerge from the actions of countless individuals.
All the above quarrels are sympathetic to Batty's approach, and more generally, quoting Wolfram's opera (2002), to our *New Kind of Science*.
In one of the classics of urban science from Patrick Geddes written over a century ago, we read "[…] appear the methods of a Science of Cities – that our cities should be individually surveyed, scientifically compared: as their architecture long has been – cathedral with cathedral, style with style" (Geddes 1915).
More recently also West claims the urgency to approach urban studies scientifically, quantitatively, when he firmly says: "we desperately need a serious scientific theory of cities" (West 2011).
As biology shows, despite life being the most complex and varied system in the universe, an astonishing simplicity is expressed by relationships among variables and the same universality was found in cities when numerous variables are plotted against city sizes (Batty 2013, Bettencourt 2013); "all the data shows it's the same, despite the fact that these cities have evolved independently. Something universal is going on. The universality is *us* – *we* are the city" (West 2011).

4. Cities between Hard and Soft sciences

Perhaps it is inside the "us" the reason why most theories treat cities as a *social* system rather than spatial, or why we often see *urban studies* departments/institutes/groups under macro categories labelled *Social Science* (such as, among many, the University of Oxford, the University of Amsterdam, and the University of Sheffield), *Arts and Humanities* (as the University of Cambridge), *Humanities* (as the University of Manchester), *Social and Political Sciences* (as the

[16] See https://en.wikipedia.org/wiki/Vautrin_Lud_Prize

University of Glasgow), *Arts, Humanities and Social Sciences* (as Cardiff University[17]), or under multidisciplinary-interuniversity departments between *Hard* and *Soft* sciences (as Politecnico di Torino[18]).
The abovementioned statement of West ("The universality is *us* – *we* are the city") encloses the polyhedric nature of urban studies and urban forms, and is, in some senses, somewhat close to the Latin terms to define cities, *civitas*, or the Aristotle[19] conception of city, more radically rooted in the political nature of human beings, whose common interest notion, *to koinon sympheron*, anticipates the Cicerone's *utilitas rei publicae*[20] and *utilitas communis*[21]. In this sense the "us" may be interpreted as an inextricable bundle of forces intrinsically (universally?) built in us, which, even if with significant variations due to culture, climate, geography and historical individual paths, acts in a perpetual search for an "equilibrium" among often contradictory forces; one of these forces is the Aristotelian common interest.
As soon as we adventure on the learning of (whatever) forms, we discern that they are always due to the action of some sort of force. D'Arcy Thompson describes the form of an object as "a 'diagram of forces' in this sense, at least, that from it we can judge or deduce the forces that are acting or have acted upon it"; forms come from an "interaction or balance of forces" (D'Arcy Thompson 2014, p. 11).
Forces shaping cities and their forms come from economics reasoning of individuals, firms, companies and governments, behaviours and instincts of humans, cultural processes, historic paths, geographic-environmental influences/constrictions, technology, utopia, politics, alliances, markets, flows, trades and communications, physical and socio-economics networks, private/public interests, striving for wellbeing (objective and subjective, personal and collective),… all acting contemporarily at different physical and temporal scales; all the above is "us", a multiscale multitemporal summation of our behaviors and decisions, conscious or not, rational or not, instinctive or not.
These forces constantly change their reciprocal 'equilibrium' and so the related urban forms, in a continuous dynamic evolution. This urban form metamorphosis is poetically expressed by Lefebvre "la ville est un emploi du temps" (Lefebvre 2001, p. 224), and Baudelaire[22] "Le vieux Paris n'est plus (la forme d'une ville change plus vite, hélas! que le cœur d'un mortel)", and mathematics is an efficient language for its understanding.

5. In search of universal laws: from Calvino to Santa Fe

Modern urban scientists, supported from unprecedented amounts of data on almost every aspect of cities, their availability and mathematical-technical capacity of elaborations, are starting to systematically analyse and model cities, "in the hope of uncovering underlying laws governing the dynamics and the evolution of these systems" (Barthelemy 2016, p.xii).
Statistical regularities systematically emerge despite immense variety among cities. We may expect two reasons for this universality: that systems with large numbers of components lead to collective behaviors with statistical regularities; and the existence of fundamental processes shared to all cities such as spatial organization of activities and residences, mobility, etc… (Barthelemy 2016, p.1).
Of a different opinion is the architectural historian Berkeleyan professor Spiro Kostof: "cities are too particular as phenomena – specific to moments in time and to the vicissitudes of site and culture – to be pinned down by absolute taxonomies" (Kostof 2014a, p. 8). Thought shared from West himself: "It may be that the sort of quantitative "physics-

[17] Although this university also categorizes urban studies under Physical Sciences and Engineering, inside the School of Architecture.
[18] DIST: the Interuniversity Department of Regional and Urban Studies and Planning between Politecnico of Torino and University of Torino, excitingly bringing in an effervescent team of 235 people (75 professors, 59 researchers, 66 visiting professors-researchers, and 34 technicians-administrators) Economists, Historians, Anthropologists, Engineers, Architects, Urbanists, Geographers, Naturalists, Computer Scientists, Sociologists, Mathematicians, Agronomist and Ecologists. It has been recently qualified as one of the best Italian Departments within the already prestigious list of Departments of Excellence in Research.
[19] "[…] man is by nature a political animal; and so even when men have no need of assistance from each other they none the less desire to live together. At the same time they are also brought together by common interest, so far as each achieves a share of the good life. The good life then is the chief aim of society, both collectively for all its members and individually" (Aristotle, *Politics*, III, 1278b). "It is clear that all partnerships aim at some good, and that the partnership that is most authoritative of all and embraces all the others does so particularly, and aims at the most authoritative good of all. This is what is called the city or the political partnership" (1252a3). "[T]he city belongs among the things that exist by nature, and…man is by nature a political animal" (1252b30-1253a3).
[20] Cicerone M.T. (85 b.C. circa). *De inventione*. I, 40.
[21] Cicerone M.T. (44 b.C. circa). *De Officiis*. III.
[22] "Le Cygne" in the section "Tableaux parisiens" of "Les Fleurs du Mal".

inspired" theory of cities that I am advocating is simply not conceivable. Cities and the process of urbanization may be just "too complex" to be subjected to laws and rules that transcend their individuality in a useful way" (West 2017, p. 269), even if he is optimistically oriented toward a truly scientific approach to cities and its, at least up to a certain extent, universality principles: "science at his best is the search for commonalities, regularities, principles, and universalities that transcend and underlie the structure and behaviour of any particular individual constituent, whether it be a quark, a galaxy, an electron, a cell, an airplane, a computer, a person, or a city. And it is at its very best when it can do that in a quantitative, mathematically computational, predictive framework […]" (West 2017, p. 269).
In search of a unifying law, almost three decades ago, also the distinguished mathematician Mitchell Feigenbaum asked (and replied to) himself: "Are there intrinsic geometries that describe various chaotic motions, that serve as a unifying way of viewing these disparate nonlinear problems, as kindred? I ask the question because I know the answer to be affirmative in certain broad circumstances. The moment this is accepted, then strongly nonlinear problems appear no longer as each one its own case, but rather coordinated and suitable for theorizing upon as their own abstract entity"; he later continues toward a both qualitative and quantitative universality of behaviours "an even stronger notion than this generality of shared qualitative geometry is the notion of universality […] this shared geometry is not only one of a qualitative similarity but also one of true quantitative identicality"[23] (Feigenbaum 1992, p.4).
Also the Nobel Prize Niels Bohr wrote about the use of mathematics in the help of unread phenomena: "for more and more phenomena, their governing laws were wrung from Nature and their rules were recognized. Simultaneously, mathematics developed hand in hand with the natural sciences, and thus an understanding of the nature of a phenomenon soon came to also include the discovery of an appropriate mathematization of it" (Bohr 1992, p. 11). We should underline the other way around too: the progress and use of mathematics helps the discovering of processes behind phenomena.
Mathematics is essential in this regard, for analysing data, founding scientific theories, creating quantified models, where 'quantified model' doesn't *necessarily* mean a model fitting to real data but "a theoretically consistent model whose parameters are based on some mix of data and assumptions, so that realistic simulation exercises can be carried out" (Fujita, Krugman, Venables 2001, p. 347).
In a Santa Fe Institute lecture[24], Melanie Mitchell interviews Luis Bettencourt, who replies: "we need to create a statistical theory of what cities are. We have a very poor idea of their statistical character. For example, scaling laws only give you an idea of how a system behaves on the average, given its size, etc…, but when you look at a particular city it's never quite that number. What creates these deviations?".
Bettencourt's reply and the quantitative approach among urban scientists toward universal laws, slightly remind me Kublai Kan's reply to Marco Polo in their imaginary dialogue in Calvino's romance: "I have constructed a model of city in my mind from which deduct every possible city. It comprehends everything corresponding to the norm. Since the cities that exist diverge in varying degree from the norm, it is enough to predict the exceptions to the norm and calculate the most probable combinations."
In a within city context, syntactic analysis of street networks of large numbers of cities highlighted another kind of universal law: "spatially speaking, and at a deep enough level, cities seem to be the same kind of thing […] there is at a deep enough level a *generic city*, that is, a structure that makes a city a city in the first place […]" (Hillier 2016, p. 200).
Hillier's *generic city*, Bettencourt's 'statistical city', and Kublai's comprehensive city in the Calvino's romance, tease our thoughts about the goal of a science of cities, that, according to Barthelemy, "will be reached when, considering a specific case, we can basically say what will happen and which ingredients it is necessary to introduce in a model in order to get more detailed information and predictions" (Barthelemy 2016, p.3).
In Popper's idea, science is "the art of systematic over-simplification – the art of discerning what we may with advantage omit" (Popper 1992), such as the exceptions we cannot predict from general laws.
Sceptical about quantitative universal meanings, if without being accompanied from proper qualitative historical/cultural analysis, is again the historian architect Kostof: "city form is neutral until it is impressed with specific cultural intent. So there is no point in noticing the formal similarities […] unless we can elaborate on the nature of the content that was to be housed within each, and the social premises of the designers", in fact we cannot be really able to read cities until we turn "to the archives, the history books, the old maps – until we assemble all the evidence, some

[23] He was referring to the universality (today known as the Feigenbaum constants) in the transition to chaos that several systems share, regardless their *practical* nature, as long as are mathematizible in a certain way.
[24] Introduction to Complexity, 2018. Video-lecture 10.4. https://www.complexityexplorer.org/courses/89-introduction-to-complexity

of it often contradictory, that will help explain how a particular downtown got the look it now has." (Kostof 2014b, p. 10, 11).
Other historians, as the Professor of Ancient History Arjan Zuiderhoek, well show their consciousness of the importance of the understanding of some kind of universality in urban form elements; he writes in his recent book about the observation of "striking similarities between cities across space and time, particularly in terms of layout and the general structure of urban landscapes" (Zuiderhoek 2017, p. 6) which, using the words of the archaeologist Monica Smith, suggests "that the capacities for human interaction in concentrated locations [villages, towns, cities] are exercised within a limited set of parameters" (Smith 2003, p- 3-8), which may be seen in evolutionary terms by using the words of the anthropologist Glenn Storey: "human nucleation behaviour into cities might be a form of group selection strategy that has proved eminently adaptable for humans and has fostered strong interspecific ties of cooperation" (Storey 2006, p. 23). Zuiderhoek continues promoting an interdisciplinary universal urbanism: "Along such broad interdisciplinary lines, combining insights from human geography, ecology and evolutionary biology, we may eventually be able to arrive at some universal understanding of urbanism"; and, similarly to the Kublai's city model in the Calvino's romance, and the Bettencourt's ambition for a statistical theory of city, Zuiferhoek defends the importance that the understanding of such hidden urban spatial patterns revealing universal mechanisms at work has also for those historians and archaeologists qualitatively interested in specific urban cultures rather than quantitatively deduced macro-universal-laws: "the broad comparative study of world urbanism does supply us with a rough cross-cultural template that can be used to sketch the outlines of a particular type of urbanism, in order to bring out, as sharply as possible, its cultural specificities" (Zuiderhoek 2017, p. 7).
There are signs for believing that these universal behaviours urban scientists are trying to uncover are linked with the self-organization property of complex systems as cities are.

6. Complex self-organizing systems urban approaches

Ilya Prigogine's original investigation on self-organization systems, in the 1950's and 1960's, focused on far-from-equilibrium chemical systems and the emergence of macro-scale spatial structures in chemical reactions[25].
Prigogine's group and many others, extended their research about self-organization to other fields as biology and urban systems, continuing to focus on the spatial structure, the physical forms taken, or better saying, *emerging* from these systems.
The ultimate message is that, whatever complex system is being analysed – economic, social, urban, biological, chemical, physical – characteristic spatial structures emerge.
Three main schools posed the basements to complex self-organizing systems urban approach: Brussels school, Santa Fe school, and CASA school.
The Brussels school is associated with the research of Prigogine and investigates how *real* systems (whether chemical, physical, social…) behave (under energy input).
The Santa Fe school, from the Santa Fe Institute, focuses on the algorithmic logic behind complex systems; it is to some extent more related to *abstract* systems virtually explored by computer modelling, seeking to capture universal, general ways to explain how self-organization and adaptation of complex systems happen. It emphasizes the *analytic-mathematical* traditions of physics toward underlying laws.
The CASA school is rooted in the work of Michael Batty whose *phenomenological* accent is linked with the social, physical and geographical sciences traditions, by passing through *abstract* simulations too. It lies in some sense in between the first two schools.
The language to understand and model these emergences of complex systems, regardless if governed by simple or complicated rules, is logical-mathematical.
All these systems "come into existence by virtue of processes that create a spatial structuring of their constituent elements" (White, Engelen, Uljee 2015, p. 4): for the urban systems, these processes are the "us" of West which are studied by the related disciplines (sociology, demography, economics, psychology, organization theory, sociobiology, anthropology…).

[25] The classical example is the Belousov-Zhabotinsky reaction which takes place in a shallow dish where the concentration of chemicals is kept far from equilibrium, and visible patterns may appear, such as spirals, concentric circles, multi-armed spirals… (Nicolis and Prigogine 1989).

Complexity and self-organizing systems theories' application to urban studies is "an attempt to generalize and formalize the qualitative understandings developed within the framework of the humanities and social sciences" (White, Engelen, Uljee 2015, p. 14).

7. Laplace's demon in cities

The aim of this scientific approach is to disclose the in-built complexity nature of the city and its morphology. Despite passing through formal methodologies and rigorous mathematical language, the scientific approach applied to cities has, somehow, the softness of social sciences and humanities in the necessary link with histories, contexts and (recalling the before mentioned Bettencourt and Calvino), especially, in their limited predictive power. In this sense, its quantitative nature should be used to scientifically extrapolate laws and formally write models, whose predictions should be *qualitatively*, rather than (as Laplace's demon would like) quantitatively, read, for the impossibility of predicting with certitude the output or the state that the system under investigation we will assume.
Within the inherent previsions incertitude of socioeconomic systems as *civitas* (ensembles of people) and *urbs* (their physical manifestations, like urban forms ultimately are), mathematical language remains an essential tool to decipher their internal mechanisms.
It often helps better measurement and analyses, which, when they go together with more data availability and computational capacity, "our ability to predict and understand and explain would be greater. […] With these advances, the thinking goes, it should be possible to get closer to Laplace's demon […]" (Feldman 2014, p. 71, 72).
It is a positive feedback: "As we gain knowledge, we create more sophisticated tools and these tools enable us to ask and answer new questions" (Wilensky, Rand 2015, p. 6).

8. A science but not a Science?

Sir D'Arcy Thompson – a truly Leonardian man[26] whose Italian Renaissance polyhedric spirit would be so precious, in any epoch, for the highly multi-interdisciplinary nature of urban studies – starts his seminal book[27] *On Growth and Form*, stating that "the criterion of truth science lay in its relation to mathematics". D'Arcy was conveying the Kantian quote '*eine Wissenschaft, aber nicth Wissenschaft*' (referring to chemistry, Kant[28] wrote that 'it is a science but not a science') translating it into 'it is a science but not a Science'.
West (2017, p.86), citing D'Arcy's interpretation of the above Kant quote, reflects on the provocative (but for him still, with caution, valid in the spirit) argument that a science (West refers to biology) would become a Science when its principles can be mathematizable, without however misevaluating in any way sciences that per nature are (or were till now) predominantly qualitative.
If we recall what defines a science a science (systematic study of structures and behaviours through observation and experiment[29]), it is not a *condicio sine qua non* to have mathematical language explaining its structures to be classified 'science', even if it might be efficient and enlarge horizons in some occasions, or undoubtedly essential in others.
Among many authors promoting mathematical approach to urban studies were Brian Berry, who brought quantitative spatial analysis and scientific method into the study of cities, building a science of cities "with the potential to elevate urban studies and geography to the status, influence, and certainty of physics and the other 'hard' sciences" (Wily 2017, p. 39); and Janet Anu-Lughoud, an earlier adopter of statistical model and computer-based data processing, she also likes to explore integrations of qualitative and quantitative methods in social research.

[26] "an aristocrat of learning whose intellectual endowments are not likely ever again to be combined within one man". That was his description by the Nobel Prize Sir Peter Medawar.
[27] Which the Nobel Prize Sir Peter Medawar described as "the finest work of literature in all the annals of science".
[28] Not the Kant of the *Critics of Judgment* but the one (later called by Stephen J. Gould 'physicalist') infatuated with Laplace, Euclid, Newton, of the *Critics of Pure Reason*.
[29] https://en.oxforddictionaries.com/definition/science (Oxford University Press 2018)

9. The language of Mathematics

"Mathematics is a language, and an exceedingly beautiful one, and the applications of that language are vast and extensive" (Adam 2012, p. xiv). I would go even further in reminding us that mathematics, as music and arts (yet these two being culturally biased), is a *universal* language *among* humans and, perhaps more importantly, *between* humans and nature (there is a stimulating dispute if it is a language *of* humans, or *of* nature which humans happen to learn) that helps to parsimoniously describe, understand and extend complicated phenomena, principles and reasoning that would have been impossible otherwise. It is both a support and an extension of our minds.

Steven Strogatz, the MIT's prized and Schurman Professor of Applied Mathematics at Cornell University, lyrically describes his being "captivated by the mathematics of nature" and the hidden "beautiful world that can be seen only through mathematics" in his discursive book (2003, p. 4), who his 'only' partially reminds the Pythagorian (everything is number[30]) and Galiliean (mathematics is the language Nature wrote her book) views.

Similarly, the well-known mathematician and astronomer John Barrow from the University of Cambridge, is not hesitant in stating that the reason we were so good in releasing the mystery of the universe is because we discovered the language in which it was written, that, as three hundred years ago Galileo believed, is mathematics. According to Barrow, science itself exists because the natural world seems algorithmically squeezable. Mathematical formulas we call Natural Laws are efficient reductions of enormous sequences of data about changes of the state of the system (Barrow 1992, p. 5, 93).

Mathematical equations, Marshall believed (1890, p.10), are beneficial in "helping a person to write down quickly, shortly and exactly, some of his thoughts"; thought shared from Samuelson in his first major opus (1947) where indicate the role of equations in sharpening muddled thinking replacing it with methodical exactness.

The same happens when we deal with urban systems and their physical forms which are the ultimate result of a complex sequence of interconnected multiscale multitemporal socioeconomic forces constrained by local contexts, driven inside historical distinctive paths.

History, not only mathematics, as Glaeser wisely starts one of his influential book, is essential for a complete urban study: "understanding cities requires more than current statistics. Many urban areas are extraordinarily ancient and their structures reflect events far in the past. We also need to know history if we are to understand our cities." (Glaeser 2007, p. 2). And he carries on pointing for a gainful trade between social scientists working in anthropology or sociology, and economists.

10. The Art and Science of Cities[31]

The view of our world has regularly shifted between these two pendulum: scientific and humanistic. According to Snow (1959), one of the major barriers to solving our world's problems resides in the absence of dialogue between these 'two cultures'.

During the first half of the 20th century, both sides were present such as the humanistic perspective of Lewis Mumford (1938) and the quantitative's of Christaller (1933), Losch (1962), Reilly (1931) and others. During that period, the system theory approach was preeminent and, during the 50s, induced researchers to see systems as centrally ordered, and as a hierarchical sum of subsystems dominated by negative feedback.

Until the middle of the 20th century, a standard theory of cities prevailed as an economic and transportation model, and based predominantly on the monocentric city. Ideas and models were built on statistical aggregations of units, exemplified in models based on macroeconomics, such as econometric, population and Keynesian models. description

In the 1950s the quantitative revolution criticized the scientific validity of the humanistic trend, which defined descriptive approaches.

In turn, in the early 1970s scholars adopting urban social theories, through Structuralist Marxist and phenomenological idealistic perspectives, in the qualitative revolution, criticized the positivistic-quantitative approach.

From that decade on also the underlying conceptual ideas changed: cities were observed as governed by positive feedback rather negatives, and from the bottom-up rather than top-down, giving space to the new complexity science. According to Portugali (2012), the urban complexity science can be seen as a second scientific culture of cities or as a junction between the scientific and humanistic culture.

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[30] Yet mathematics is not much interested in numbers on themselves but in their interrelations.

[31] Some parts from (D'Acci 2016).

Also Batty, even if pushes the pendulum to the quantitative side, would like to think that is humanistic too in some sense, according to his way of thinking about science; in the urban context you also need intuition: quantifications without intuition may be a black box[32].
The quantitative-scientific approach finds universal rules aiming to see cities as part of the domain of nature and to be studied by the scientific method. The humanistic approach claims a difference between the human domain and the natural domain, so that studying cities and their phenomena as quantitative may lead to reductionism, and it finds soft hermeneutic methods more suitable.
This opposition may be only apparent, as it could be transformed into a profitable complementarity; namely referring to the scientific methods for what concerns urban phenomena that are objective and universal, and to the humanistic approach for phenomena that are not. We are also able, not always, to quantify qualitative phenomena and to qualitatively interpret quantified data.
Art is viewed as the opposing counterpart to science. Batty (2013) explains terms of *science* and *art* as: "by science, we mean an organized body of knowledge produced using commonly agreed tools and methods that can be reproduced over and over again by different individuals. [...] this is quite different from ideas and knowledge we consider art, since the production of art is individual and formed by an intuition that is personal".
According to Croce and Read (1941), for art we do not intend "beauty" – which is a "very fluctuating phenomenon, with manifestation in the course of history that are very uncertain and often baffling" – but *intuition*. "As for a work of art, what we expect in a city is a personal element. Each city reveals original, unique elements; each of them is *special* and, even more, is *differently* special for each of us" (D'Acci 2015).
The above call for wise "intuition" claimed from Batty, and the systematic "deviations" from the modelled trajectory augmented from Bettencourt, lies in what mathematical models and quantifications in the social and behavioural sciences – the ultimate foundation of cities – leave invisible. A concept well expressed in the words of the Director of the MIT System Dynamics Group, (Sterman 2002, p. 513): "the most important assumptions of a model are not in the equations, but what's not in them; […] not in the variables on the computer screen, but in the blank spaces around them".
Science sees the many in the one; art the one in the many. However, this new science based on the complexity paradigm[33], is a science that induces art, identified as personal uniqueness: each city emerges from unique contexts, from where the randomness of the micro-fluctuations, the unpredictability of the agent's behaviours' positive feedbacks, and the contextual historic successions, generate – although within universal macro patterns – unique scenarios; and each of them is personally read.

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[32] Batty's personal conversation, 27 June 2018, University of Cambridge.
[33] In a seminal paper about Science and Complexity, Weaver (1948) recognised how most of new humanity challenges (economic, social, political, ecological, biological) are organized complexity in their nature and "require science to make a third great advance". The latter started in the 1970s with Complexity Science which study how the behaviour of a system is shaped by the relationships among its parts.

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