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High-fidelity vibration analysis of tapered swept tailored composite wing boxes / Viglietti, Andrea; Zappino, Enrico; Carrera, Erasmo. - In: JOURNAL OF AIRCRAFT. - ISSN 0021-8669. - 56:2(2019), pp. 842-846.

*Availability:*

This version is available at: 11583/2731491 since: 2019-04-24T10:06:46Z

*Publisher:*

American Institute of Aeronautics and Astronautics Inc.

*Published*

DOI:10.2514/1.C034795

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# High-Fidelity Vibration Analysis of Tapered Swept Tailored Composite Wing Boxes

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## I. Introduction

Composite laminates are primarily used in those engineering fields where high specific mechanical properties are required maintaining a low weight. Layers with different orientations can be overlapped through a process called *tailoring*, in order to obtain a lamination with the desired properties [1]. Tailoring is used extensively to address aero-elastic stability problems, where the coupling between torsion and bending plays an important role [3]. Shirk *et al.* [2] presented an overview of the theories used in this research field such as the beam model proposed by Librescu [4]. The development of theories able to describe the mechanical behavior of laminated structures, e.g. the development of Layer-Wise models, is of primary importance to provide computationally efficient tools that can be used in the structural design.

Carrera *et al.* [5] presented an advanced numerical tool, named Carrera Unified formulation (CUF), which allows refined structural models to be derived in a compact form. By using the Lagrange Expansion (LE Model) to approximate the behaviour of the beam cross-section, Carrera *et al.* [6] have performed *tailoring* analyses of a simple prismatic laminated thin-walled wing box. Recently, Zappino *et al.* [7] have extended the model to the analyses of complex structures with tapered shapes. Since the capabilities of this model allow complex geometrical couplings to be taken into account, the model has been extended in this work to the tailoring analyses of taper swept reinforced structures characterized by a multi-component nature, evaluating the alterations

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on the free-vibration response.

## II. Refined one-dimensional models formulation

The current FE one-dimensional model considers the displacement field  $\mathbf{u}$  as the product of two contributions, one over the cross-section and one along the beam axis:

$$\mathbf{u}(x, y, z) = F_\tau(x, z)N_i(y) \quad \tau = 1, 2 \dots M, \quad i = 1, 2 \dots N \quad (1)$$

where  $\mathbf{u}_\tau$  is the displacement vector,  $F_\tau$  represents a function expansion used to approximate the kinematic of the beam cross-section, and  $M$  is the number of the expansion terms.  $N_i$  is the shape function of the beam element with  $N$  nodes. The present work uses the Lagrange polynomials to describe any shape geometry of the cross-section through high-order elements which use an isoparametric formulation. Several sets of Lagrange polynomial exist, the nine-point (L9) functions have been adopted in this work. The use of Lagrange functions leads to models with only displacements as degrees of freedom, as shown reported by Carrera and Petrolo [9]. In this way, this feature makes it possible to connect different models just imposing the congruence of the displacements at the shared nodes [10]. Zappino *et al.* [7] exploited these models to study complex structures including tapered beams and wing structures.

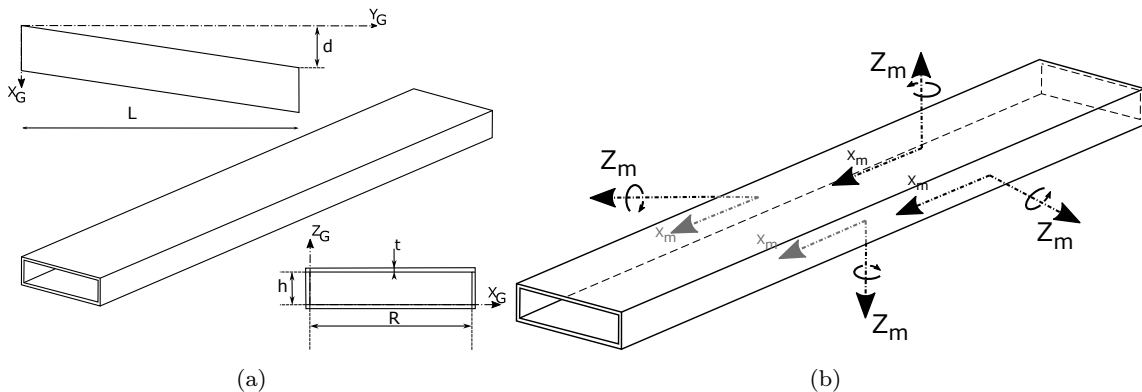
A detailed explanation of the CUF and the derivation of the stiffness and mass matrix are not reported here for sake of brevity, but they can be found in the work by Viglietti *et al.*[8] and in the book by Carrera *et al.* [5].

## III. Results

Two thin-walled boxes with wing-like geometry are presented in this work. The structures are made of a 2-ply laminate of *CFRP: Carbon Fiber Reinforced Polymer*. Each layer of the laminate has the following properties:  $E_{LL} = 50e9 \text{ Pa}$ ,  $E_{TT} = E_{ZZ} = 10e9 \text{ Pa}$ ,  $G = 5e9 \text{ Pa}$ , Poissons's ratio  $\nu = 0.25$  and density of  $1700 \text{ kg/m}^3$ .  $E_{LL}$  is referred to the fiber direction. The results presented in the following sections have been compared with those from the commercial code NASTRAN using CQUAD elements to validate the accuracy of the model. Different lamination cases have

	1	2	3	4	5	6	7	8	9	10	11	12	13
$\theta_1$	$90^\circ$	$75^\circ$	$60^\circ$	$45^\circ$	$30^\circ$	$15^\circ$	$0^\circ$	$-15^\circ$	$-30^\circ$	$-45^\circ$	$-60^\circ$	$-75^\circ$	$-90^\circ$
$\theta_2$	$0^\circ$	$-15^\circ$	$-30^\circ$	$-45^\circ$	$-60^\circ$	$-75^\circ$	$-90^\circ$	$-105^\circ$	$-120^\circ$	$-135^\circ$	$-150^\circ$	$-165^\circ$	$-180^\circ$

**Table 1 Lamination Cases.**  $\theta_1$  is referred to the inner layer.  $\theta_2$  is related to the outer one.



**Fig. 1 Geometry of the rectangular box with sweep angle.**

been considered, Table 1 shows the values of the lamination angles for each case.  $\theta_1$  and  $\theta_2$  refer to the inner and the outer layer respectively. As an example, lamination case 5 has  $\theta_1 = 30^\circ$  and  $\theta_2 = -60^\circ$ .

### A. Rectangular sweep box

A simple rectangular box has been taken into account. A sweep angle has been introduced as shown in Figure 1a. The structure has the following dimensions:  $L = 2\text{ m}$ ,  $h = 0.08\text{ m}$ ,  $R = 0.3\text{ m}$ ,  $d = 0.3\text{ m}$  and thickness equal to  $1\text{ cm}$ . The direction of the fibers rotation is shown in Figure 1b. The panels are described using one beam element placed along the thickness of each layer. Figure 2 shows a comparison of the results obtained with the present model with the first four frequencies evaluated using a Nastran 2D model. The present approach can describe the tailoring effects with a reasonable approximation even if it uses a much lower number of degrees of freedom DOF, in fact, the current model has 11844 DOFs while the Nastran model requires 130000 DOFs. The third frequency, which corresponds to the second bending mode, has a minimum when the lamination is equal to  $\pm 45^\circ$ . In contrast, the first torsional mode, which corresponds to the fourth frequency, has a maximum for this lamination, see case 4 and 10. As expected the choice of the lamination can

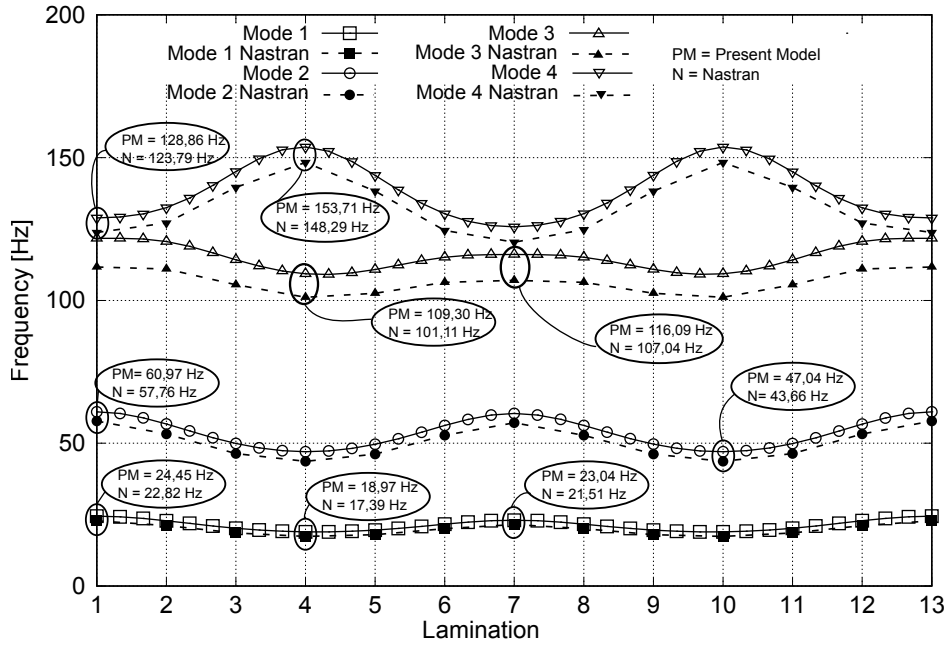


Fig. 2 First 4 modes of the swept rectangular box. Comparison between the results obtained from the present model (11844 Dofs) and from the Shell model (130000 Dofs).

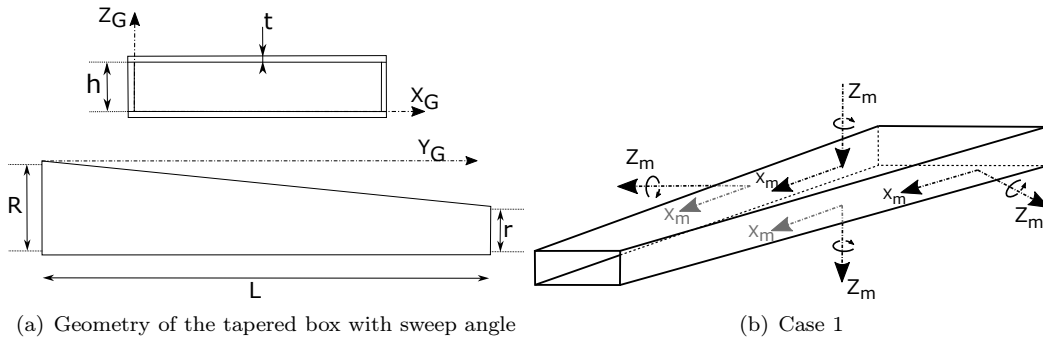


Fig. 3 Geometry and reference systems for different tailoring approaches.

have a significant effect on the dynamic response of the structure, and this can be exploited in the aeroelastic design of wing structure.

### B. Tapered sweep box

In this section, a tapered box characterized by a swept angle, shown in figure 3a, is considered. The structure has been described using the beam elements through the thickness of the panels. The geometrical characteristics are the following:  $L = 2\text{ m}$ ,  $R = 0.4\text{ m}$ ,  $r = 0.2\text{ m}$ . The height and the thickness are constant over the whole wingspan and they are equal to  $h = 0.08\text{ m}$  and  $t = 0.01\text{ m}$ . Several cases with different rotation directions of the fibers have been investigated. The lamination

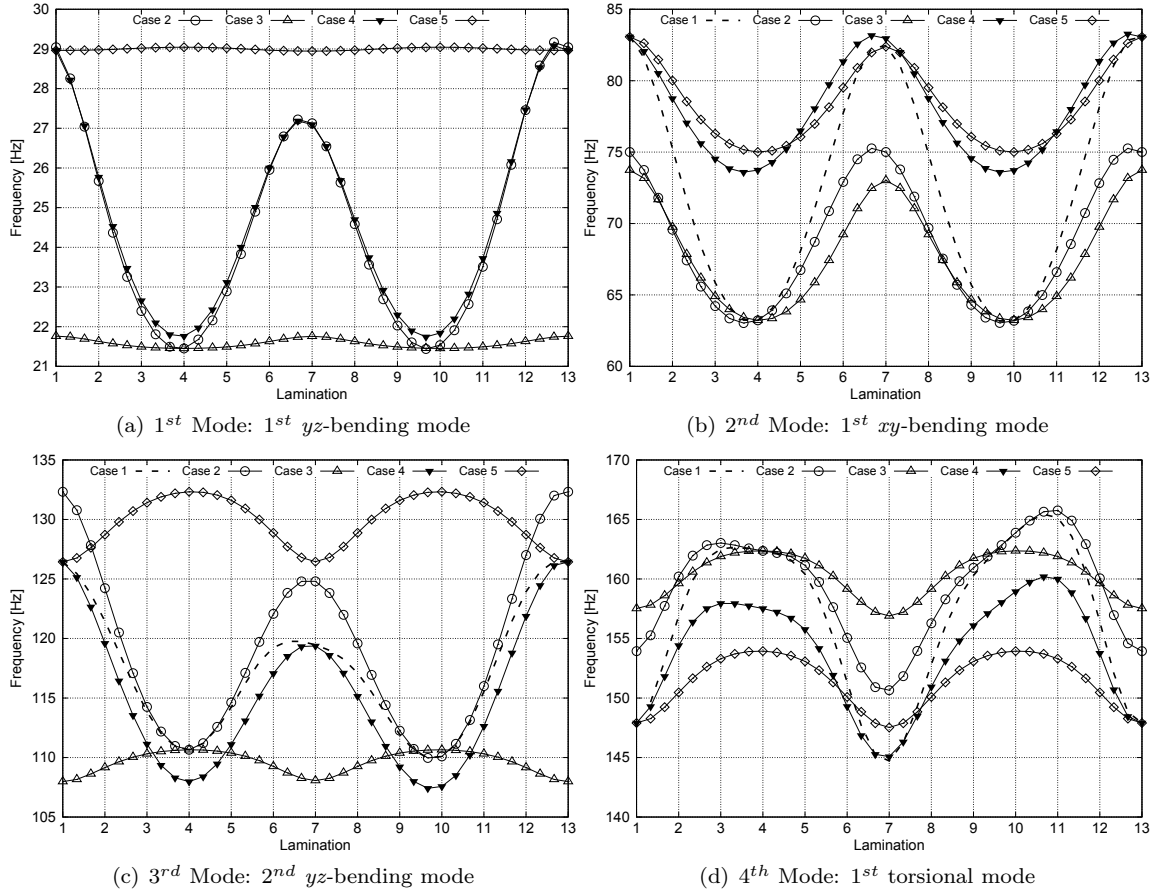
	Panels		Webs	
	Inner Layer	Outer Layer	Inner Layer	Outer Layer
Case 1	$\theta_1$	$\theta_2$	$\theta_1$	$\theta_2$
Case 2	$\theta_1$	$\theta_2$	$45^\circ$	$-45^\circ$
Case 3	$45^\circ$	$-45^\circ$	$\theta_1$	$\theta_2$
Case 4	$\theta_1$	$\theta_2$	$90^\circ$	$0^\circ$
Case 5	$90^\circ$	$0^\circ$	$\theta_1$	$\theta_2$

**Table 2 Lamination cases details**

angles in each panel are defined according with the normals shown in 3*b*. Five lamination setups have been considered. Case 1 has a variable lamination in all the components. Cases 2 and 4 consider a fixed lamination angle in the webs and investigate the effects of the tailoring of the upper and lower panels while Cases 3 and 5 consider a variation of the web lamination angle given a fixed layout of the upper and lower skins, as reported in Table 2. Figures 4a-d show the variation of the natural frequencies when Case 1 to 5 are considered. When the results from cases 3 and 5 are considered, the first frequency is not influenced by the lay-up of the webs and maintains an almost constant value for both cases. Cases 2 and 4 consider a variable angle in the upper and lower skins. They show a large variation of the first frequency highlighting that the stiffness of the panels governs the first mode. A cross-ply lamination ensures the higher frequency value if compared with that of the angle-ply lamination. The effects on the second mode are shown in Figure 4b. In this case, both panels and webs play an essential role in the dynamic response. In fact, the higher frequency values are obtained for Cases 4 and 5. Figure 4c shows the variation of the third frequency. The results show that the frequency can be strongly modified with the tailoring of the panels while the web lamination has not such a big effect. The higher frequency value is obtained when the panels have a cross-ply lamination, and the webs have an angle-ply set-up. Finally, Figure 4d shows the results related to the torsional mode. In this case, an angle-ply lamination of both panels and webs produces the higher frequency value, see Case 2 lamination 11.

### C. Tapered wing-box structure

The last case concerns the evaluation of the effects due to a tailoring process on a complex structure, shown in Figure 5, that involves several structural components such as unidirectional ele-



**Fig. 4** Variation of the first four frequencies at different laminations for cases 2<sup>nd</sup>, 3<sup>th</sup>, 4<sup>th</sup> and 5<sup>th</sup>.

ments (stiffeners), transverse reinforcements (rib), top and bottom skin and two spar webs (vertical panels). The global dimensions are the following:  $L = 2 \text{ m}$ ,  $R = 0.5 \text{ m}$ ,  $r = 0.3 \text{ m}$ ,  $h = 0.1 \text{ m}$ . The skin and the ribs have a thickness equal to  $t = 5 \text{ mm}$ . The cross-section of the stiffeners have the following dimensions expressed in meters:  $A = 0.03$ ,  $B = 0.015$  and  $a = b = 0.005$ . The stiffeners are described using ten three-node beam elements (B3) over the length of each reinforcement. Panels and webs maintain the same description of the previous cases: one B3 element through the thickness of each layer. To conclude, five B3 elements, placed parallel to  $Z_G$ , are used to describe the ribs. The frequencies are compared with those obtained using a refined NASTRAN solid model. Table 3 reports the first five global frequencies considering a cross-ply lamination of the skin and the webs, unidirectional reinforcements and traversal ribs with a lamination of  $45^\circ / -45^\circ$ . The tailoring effects on this structure have been investigated referring to the lamination scenarios reported in Table 2. The traversal ribs have been considered with a fixed stacking sequence equal to  $45^\circ / -45^\circ$ , and the

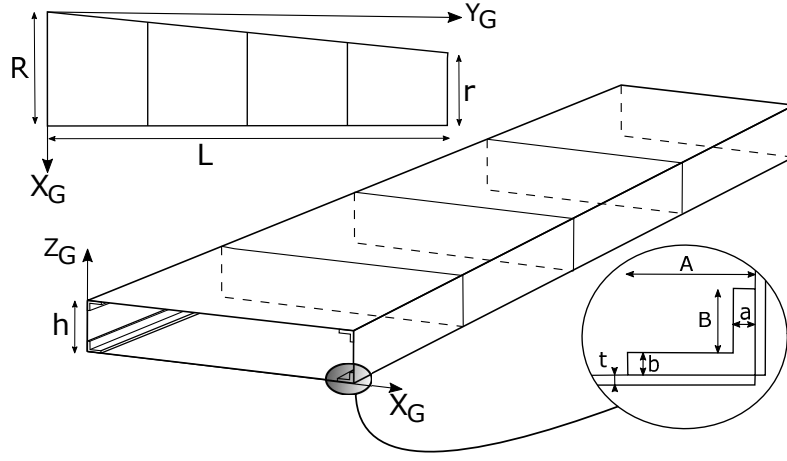


Fig. 5 Geometry of the tapered wing box with stiffeners and traversal webs.

DOF	Present Model Solid Nastran	
	26952	181740
1 <sup>st</sup> yz-Bending	28.99	29.12
1 <sup>st</sup> xy-Bending	104.65	105.51
1 <sup>st</sup> Torsional	124.30	123.61
3 <sup>rd</sup> yz-Bending	334.01	317.29
2 <sup>nd</sup> xy-Bending	362.51	362.82

Table 3 Global frequencies of tapered reinforced wing box:  $90^\circ/0^\circ$  lamination.

stiffeners are unidirectional. The fiber rotations have been performed accordingly with the material reference systems shown in Figure 3b. Figure 6 shows the variation of the first three modes considering different lamination setup. The first bending mode in the plane  $yz$  has the highest frequencies with a lamination of the panel blocked at  $90^\circ/0^\circ$ . These results confirm the importance of the panel respect the webs in this mode. A variation in the web lamination introduces weak effects on the frequency. The second mode has the higher frequency when a lamination of  $90^\circ/0^\circ$  for the spar webs are used. The highest frequency peaks are achieved by introducing the same lamination in the panels. As expected, the panels characterized by a lamination of  $45^\circ/-45^\circ$  increase the torsional frequency and the more significant peaks of this value is obtained when the same lamination is used in the webs.



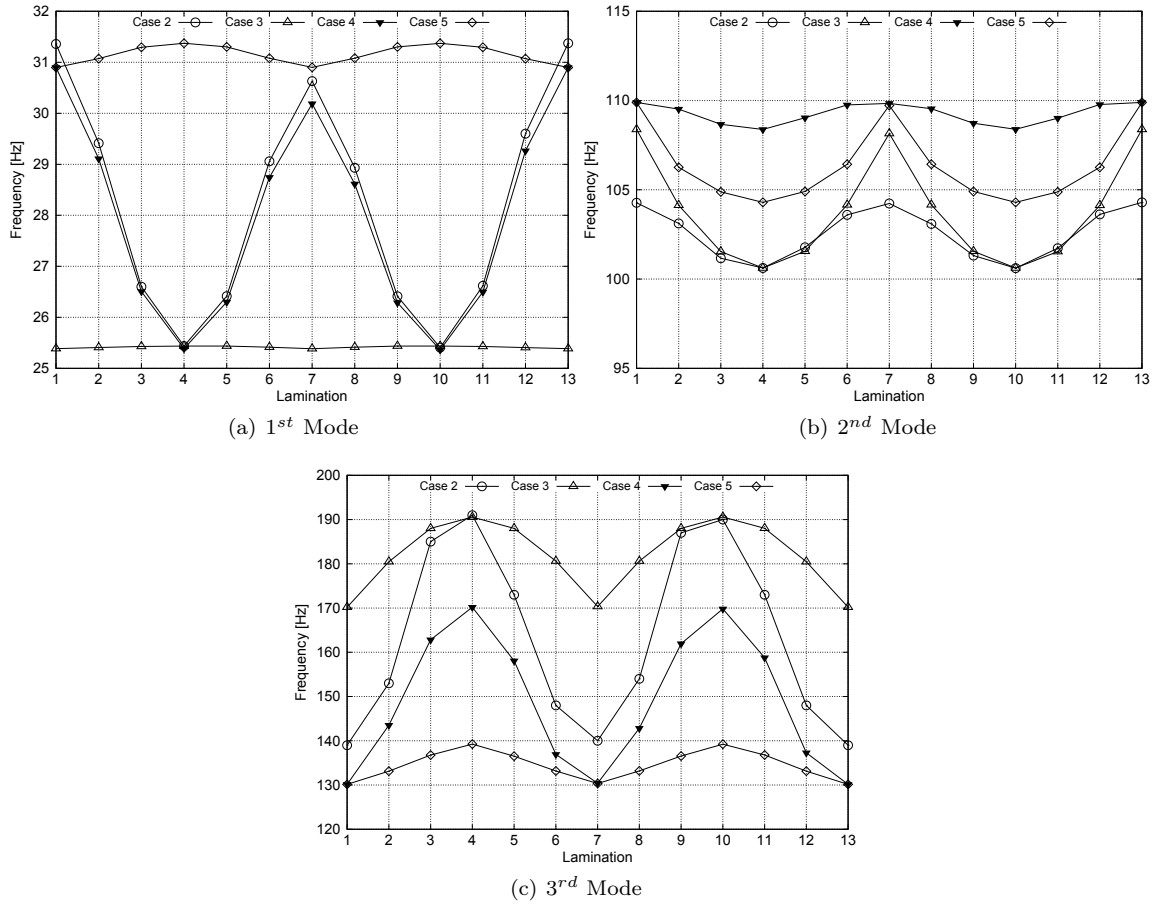


Fig. 6 Tailoring effects on the natural frequencies.

#### IV. Conclusions

A refined one-dimensional model, based on the Carrera Unified Formulation, has been adopted to describe the kinematic field of structures with non-prismatic shapes. In particular, the current model can investigate complex structures composed of several components, each one represented by an *ad-hoc* beam formulation. In this way, the effects of the tailoring on each structural component of these structures have been considered in order to emphasize the role played by each part in the dynamics of the structure. The model has provided accurate results compared with those from commercial codes that are more expensive in term of computational costs. As confirmed by the results, the tailoring of the composite materials has a significant influence on the frequencies.. The results show that the frequencies can be shifted to the suitable values, without modifying the geometry, using appropriate laminations. Furthermore, direct control of the geometrical coupling between the bending and torsional effects is possible. The results confirm the potentialities of

the present model and make it a reliable tool for the tailoring analysis during preliminary design processes of complex structures.

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