A Hybrid Integral Equation Approach to Solve the Anisotropic Forward Problem in Electroencephalography

A Hybrid Integral Equation Approach to Solve the Anisotropic Forward Problem in Electroencephalography

Maxime Y. Monin†*, Lyes Rahmouni‡, Francesco P. Andriulli†
†Department of Electronics and Telecommunications, Politecnico di Torino, Turin, Italy
‡Computational Electromagnetic Research Laboratory, IMT Atlantique, Brest, France

***Abstract*—The boundary element method provides a computationally efficient solution to the Electroencephalography (EEG) forward problem on piece-wise homogeneous head models by using surface integral equations. However, realistic modeling of the head medium requires a proper account of the anisotropic electric nature of the human skull, which cannot be handled by standard surface integral equations. This work addresses this issue by presenting a new formulation which perturbs the standard Lippmann-Schwinger approach with volume elements within the skull. The resulting surface/volume integral equation can handle computations of fully realistic modeling for both skull and white matter. Numerical results will confirm the validity of the approach as well as its applicability to real case scenarios.**

I. Introduction

Electroencephalography (EEG) is a standard method of recording the brain activity used for a wide range of medical and research applications. In particular, the accurate solution of the so-called forward EEG problem is a crucial step to obtain insight on the source activity which generates the electric potential measured on scalp electrodes. A standard numerical technique to solve the forward problem is the Boundary element method (BEM) [1]. Boundary integral techniques feature several advantages over differential formulations, among which the convenient need to discretize only the boundary of the studied medium. A major drawback of standard BEM is the requirement for the conductivity to be isotropic and homogeneous. Thus, it cannot account for anisotropic human tissues in the head. The skull for instance, is known to be ten times more conductive in the tangential direction to its surface than in the normal direction [2]. To bypass this constraint, the authors of [3] have derived an adequate formulation which handles anisotropy and inhomogeneity of the conductivity in the EEG problem by adapting a volume integral technique previously used in high frequency electromagnetic solvers [4]. Consequently however, this formulation requires the discretization of the whole head model and the computational efficiency of the BEM is partially lost. This costly requirement can nonetheless be restricted. In fact, we show that the volume discretization can be reduced to only the anisotropic and inhomogeneous regions, whereas surface discretization is sufficient for regions with constant scalar conductivity. This is achieved by suitable definition of a conductivity contrast which reduces the inhomogeneous problem to an equivalent piecewise homogeneous problem. Accordingly, we introduce a new formulation to solve the forward problem which can efficiently handle an arbitrary number of isotropic and non-isotropic layers. The validity of the approach is confirmed by numerical results which show the practical relevance of the newly developed formulation.

II. Background and Notation

The head medium Ω is modelled as a superposition of nested regions Ω_i, $i = 1, ..., N$ with boundary Γ_i. In each region, the 3×3 conductivity tensor $\bar{\bar{\sigma}}(\boldsymbol{r})$ is either homogeneous and isotropic ($\bar{\bar{\sigma}}(\boldsymbol{r}) = \sigma_i I$) or inhomogeneous and anisotropic. In the latter case we still define an arbitrary background scalar conductivity $\sigma_i = \sigma_b$. The forward EEG problem then consists in the determination of the unknown electric potential $\phi(\boldsymbol{r})$, $\boldsymbol{r} \in \Gamma_N$ in the presence of a source dipole located in $\boldsymbol{r}_0 \in \Omega_1$ inducing a primary current $\boldsymbol{J}_p(\boldsymbol{r})$.

In the quasi-static regime, Maxwell's system reduces to Poisson's equation

$$\nabla \cdot (\bar{\bar{\sigma}}(\boldsymbol{r}) \nabla \phi(\boldsymbol{r})) = \nabla \cdot \boldsymbol{J}_p(\boldsymbol{r}) \quad \boldsymbol{r} \in \Omega \tag{1}$$

with the boundary conditions

$$\phi(r)|_i^- = \phi(\boldsymbol{r})|_i^+ \quad \boldsymbol{r} \in \Gamma_i \tag{2a}$$

$$\hat{\boldsymbol{n}}(\boldsymbol{r})\bar{\bar{\sigma}}(\boldsymbol{r})\nabla\phi(r)|_i^- = \hat{\boldsymbol{n}}(\boldsymbol{r})\bar{\bar{\sigma}}(\boldsymbol{r})\nabla\phi(\boldsymbol{r})|_i^+, \quad \boldsymbol{r} \in \Gamma_{i<N} \tag{2b}$$

$$\hat{\boldsymbol{n}}(\boldsymbol{r})\bar{\bar{\sigma}}(\boldsymbol{r})\nabla\phi(\boldsymbol{r}) = 0, \quad \boldsymbol{r} \in \Gamma_N \tag{2c}$$

where $\hat{\boldsymbol{n}}(\boldsymbol{r})$ is a unit vector normal to Γ_i. Define the following surface and volume operators

$$\mathcal{S}(f_s)(\boldsymbol{r}) = \int_\Gamma G(\boldsymbol{r}, \boldsymbol{r}') f_s(\boldsymbol{r}') d\Gamma' \tag{3}$$

$$\mathcal{D}^*(f_s)(\boldsymbol{r}) = \int_\Gamma \hat{\boldsymbol{n}}(\boldsymbol{r}) \cdot \nabla G(\boldsymbol{r}, \boldsymbol{r}') f_s(\boldsymbol{r}') d\Gamma' \tag{4}$$

$$\mathcal{S}_v^*(\boldsymbol{f}_v)(\boldsymbol{r}) = \int_\Omega G(\boldsymbol{r}, \boldsymbol{r}') \nabla' \cdot \boldsymbol{f}_v(\boldsymbol{r}') d\Omega' \tag{5}$$

$$\mathcal{D}_v^*(\boldsymbol{f}_v)(\boldsymbol{r}) = \int_\Omega \hat{\boldsymbol{n}}(\boldsymbol{r}) \cdot \nabla G(\boldsymbol{r}, \boldsymbol{r}') \nabla' \cdot \boldsymbol{f}_v(\boldsymbol{r}') d\Omega' \tag{6}$$

where

$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|} \tag{7}$$

is the static Green's function.

III. A Hybrid Formulation

We introduce the conductivity contrast $\chi(\boldsymbol{r})$, piecewise-defined as:

$$\chi(\boldsymbol{r}) = (\sigma_i I - \bar{\bar{\sigma}}(\boldsymbol{r}))\bar{\bar{\sigma}}^{-1}(\boldsymbol{r}) \quad \boldsymbol{r} \in \Omega_i. \tag{8}$$

In particular, we have $\chi(\boldsymbol{r}) = 0$ in homogeneous isotropic layers. Equation (1) is then rewritten as:

$$\sigma_i \Delta\phi(\boldsymbol{r}) = \nabla \cdot (\boldsymbol{J}_p(\boldsymbol{r}) + \chi(\boldsymbol{r})\bar{\bar{\sigma}}(\boldsymbol{r})\nabla\phi(\boldsymbol{r})) \quad \boldsymbol{r} \in \Omega_i. \tag{9}$$

Thus the second term on the right-hand side $\chi(\boldsymbol{r})\bar{\bar{\sigma}}(\boldsymbol{r})\nabla\phi(\boldsymbol{r})) = \boldsymbol{J}_{eq}(\boldsymbol{r})$ can be seen as an equivalent volume current source originated from the anisotropic behaviour of the conductivity. Because of our definition of the contrast, the volume unknown exists only in anisotropic regions. In these conditions, we can then follow a single-layer approach [1] to obtain surface equations for each unknown single-layer potential $\xi_{\Gamma_i}(\boldsymbol{r})$

$$\frac{\sigma_i + \sigma_{i+1}}{2(\sigma_{i+1} - \sigma_i)}\xi_{\Gamma_i}(\boldsymbol{r}) - \sum_k \mathcal{D}^*(\xi_{\Gamma_k})(\boldsymbol{r}) + \frac{1}{\sigma_b}\mathcal{D}_v^*(\boldsymbol{J}_{eq})(\boldsymbol{r})$$
$$= -\frac{1}{\sigma_1}\mathcal{D}_v^*(\boldsymbol{J}_p)(\boldsymbol{r}), \quad \boldsymbol{r} \in \Gamma_i. \tag{10}$$

The potential can then be computed as

$$\phi(\boldsymbol{r}) = \sum_k \mathcal{S}(\xi_{\Gamma_k})(\boldsymbol{r}) - \frac{1}{\sigma_b}\mathcal{S}_v^*(\boldsymbol{J}_{eq})(\boldsymbol{r}) - \frac{1}{\sigma_1}\mathcal{S}_v^*(\boldsymbol{J}_p)(\boldsymbol{r}) \tag{11}$$

but to retrieve it, another volume equation is required which we can get by taking the gradient of (11) in the anisotropic regions, obtaining

$$\sum_k \nabla\mathcal{S}(\xi_{\Gamma_k})(\boldsymbol{r}) - (\sigma_b I - \bar{\bar{\sigma}}(\boldsymbol{r}))^{-1}\boldsymbol{J}_{eq}(\boldsymbol{r}) - \frac{1}{\sigma_b}\nabla\mathcal{S}_v^*(\boldsymbol{J}_{eq})(\boldsymbol{r})$$
$$= -\frac{1}{\sigma_1}\nabla\mathcal{S}_v^*(\boldsymbol{J}_p)(\boldsymbol{r}), \quad \boldsymbol{r} \in \Omega,\ \bar{\bar{\sigma}}(\boldsymbol{r}) \neq \sigma_i. \tag{12}$$

In order to obtain a discrete matrix system, the surface unknowns are discretized with pyramidal basis functions and the volume unknown with SWG functions [5]. Following a Galerkin approach, the equations are tested with the corresponding basis functions. It must be noted that with this scheme, only the anisotropic regions ($\bar{\bar{\sigma}}(\boldsymbol{r}) \neq \sigma_i$) need full volume discretization, while homogeneous ones still lead to efficient surface contributions.

IV. Numerical Results

A first set of test has been focusing on multi-layer spherical geometries since analytic solutions are available as a benchmark in this case. The model used is then a 3-layer sphere accounting for brain, skull and scalp media. The brain and scalp have a scalar constant conductivity, whereas the skull has inhomogeneous and anisotropic conductivity [2]. Fig. 1 shows the relative error with respect to the analytic solution as a function of the average mesh size h. From the figure it is clear that our scheme is converging to the analytic reference. To show the applicability of our new scheme to a real-case scenario, we have used a Magnetic-Resonance-Imaging generated brain-head model and both scalp potential and skull electric currents have been obtained as shown in Fig. 2.

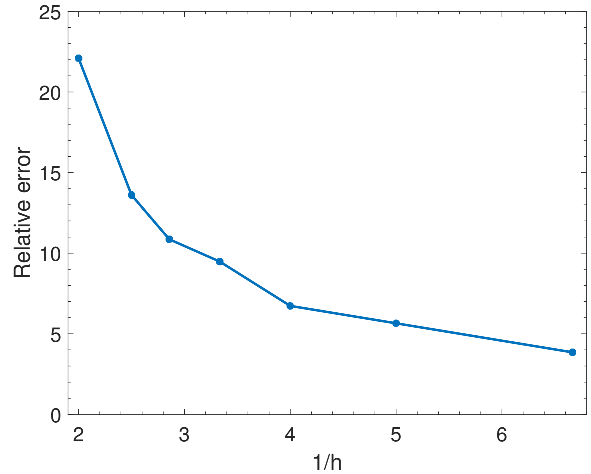

Fig. 1. Relative error as a function of the mesh refinement

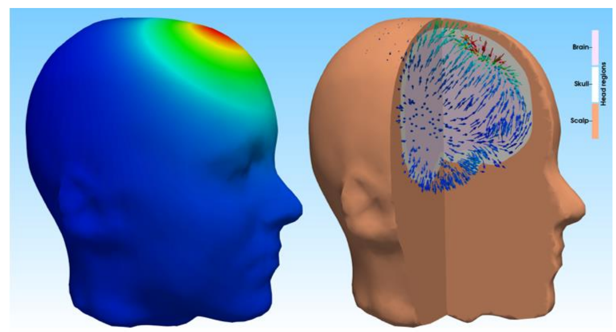

Fig. 2. Scalp potential (left) and volume currents (right) on a realistic model

This work was supported by the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation program (ERC project 321, grant No. 724846).

References

[1] J. Kybic, M. Clerc, T. Abboud, O. Faugeras, R. Keriven, and T. Papadopoulo, "A common formalism for the integral formulations of the forward eeg problem," *IEEE transactions on medical imaging*, vol. 24, no. 1 pages=12–28, year=2005, publisher=IEEE.

[2] H. Hallez, B. Vanrumste, R. Grech, J. Muscat, W. De Clercq, A. Vergult, Y. D'Asseler, K. P. Camilleri, S. G. Fabri, S. Van Huffel *et al.*, "Review on solving the forward problem in eeg source analysis," *Journal of neuroengineering and rehabilitation*, vol. 4, no. 1, p. 46, 2007.

[3] L. Rahmouni, R. Mitharwal, and F. P. Andriulli, "Two volume integral equations for the inhomogeneous and anisotropic forward problem in electroencephalography," *Journal of Computational Physics*, vol. 348, pp. 732–743, 2017.

[4] G. Kobidze and B. Shanker, "Integral equation based analysis of scattering from 3-d inhomogeneous anisotropic bodies," *IEEE transactions on antennas and propagation*, vol. 52, no. 10, pp. 2650–2658, 2004.

[5] D. Schaubert, D. Wilton, and A. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 1, pp. 77–85, 1984.