Update: On the synthesis of quantum Hall array resistance standards (2015 Metrologia 52 31)

Update: On the synthesis of quantum Hall array resistance standards (2015 Metrologia 52 31)

Massimo Ortolano[1,2] and Luca Callegaro[2],

[1] Dipartimento di Elettronica e Telecomunicazioni, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino, Italy
[2]INRIM - Istituto Nazionale di Ricerca Metrologica
Strada delle Cacce, 91 - 10135 Torino, Italy

E-mail: massimo.ortolano@polito.it

Abstract. This work provides an update, according to the revised SI, to table 1 of M. Ortolano et al., "On the synthesis of quantum Hall array resistance standards", *Metrologia*, 52, p. 31–39, 2015. The table reports fractions of the quantized Hall resistance approximating decadic values and the associated deviations. In several cases, the deviations have become smaller in the revised SI.

Submitted to: *Metrologia*

Quantum Hall array resistance standards (QHARS) are integrated circuits of interconnected quantum Hall elements. They are designed to provide a quantized resistance R approximating a nominal resistance value R_0 of practical interest, typically a decadic value (e.g., $10\,\mathrm{k}\Omega$). R is a fraction $R = (p/q)R_\mathrm{H}$ of the quantized Hall resistance R_H of the individual elements composing the QHARS, and p and q are integers. The quantized Hall resistance is a submultiple of the von Klitzing constant, $R_\mathrm{H} = R_\mathrm{K}/i$, i being a small integer (here we assume $i = 2$, as is usually the case).

In [1], we proposed an algorithm that, given the target resistance R_0, allows to calculate the fractions p/q of interest, and to design the corresponding QHARS networks. Table 1 of [1] provides several examples of practical interest for the decadic values $100\,\Omega, 1\,\mathrm{k}\Omega, 10\,\mathrm{k}\Omega, 100\,\mathrm{k}\Omega$ and $1\,\mathrm{M}\Omega$. The approximations reported in [1], and the corresponding deviations of R from R_0, were calculated by assuming the conventional value of the von Klitzing constant $R_\text{K-90} = 25\,812.807\,\Omega$ [2, Appendix 1].

The redefinition of the International System of Units [3] fixes the value of the von Klitzing constant R_K according to the last CODATA 2017 recommendation [4, table 3], $R_\mathrm{K} = h/e^2 = 25\,812.807\,459\,304\,5\,\Omega$ (calculated to 15 significant figures, as given in [5]). This value relatively differs from $R_\text{K-90}$ by 1.78×10^{-8}.

The present work provides in table 1 below an update to table 1 of [1] according to the revised SI. A few sign errors in the original δ_{90} values have also been corrected. It is of interest that several approximations provide a lower relative error in the revised SI.

The Mathematica [6] notebook implementing the search of approximating fractions as described in [1] and employed to generate table 1 is available at `https://github.com/INRIMQuantumElectricalMetrology/SternBrocot`.

Table 1. Summary of the cases analyzed in [1]: $\rho_\text{0-90} = R_0/R_\text{H-90}$ is the target resistance value normalized to the Hall resistance $R_\text{H-90} = R_\text{K-90}/2$; $\rho_0 = R_0/R_\mathrm{H}$ is the target resistance value normalized to the Hall resistance $R_\mathrm{H} = R_\mathrm{K}/2$, calculated with the revised value of R_K; $\rho = p/q$ is a rational approximation for ρ_0 and $\rho_\text{0-90}$; $\delta_{90} = (\rho - \rho_\text{0-90})/\rho_\text{0-90}$ and $\delta = (\rho - \rho_0)/\rho_0$ are the relative errors of the approximation. Values are rounded at the 10^{-10} level. Table 1 of [1] reported δ_{90}.

ρ	δ_{90}	δ
	$R_0 = 100\,\Omega$	
47/6066	$+1.5900 \times 10^{-6}$	$+1.6078 \times 10^{-6}$
78/10067	-5.235×10^{-7}	-5.057×10^{-7}
125/16133	$+2.712 \times 10^{-7}$	$+2.890 \times 10^{-7}$
203/26200	-3.42×10^{-8}	-1.64×10^{-8}
	$R_0 = 1\,\mathrm{k}\Omega$	
203/2620	-3.42×10^{-8}	-1.64×10^{-8}
235/3033	$+1.5900 \times 10^{-6}$	$+1.6078 \times 10^{-6}$
	$R_0 = 10\,\mathrm{k}\Omega$	
203/262	-3.42×10^{-8}	-1.64×10^{-8}
	$R_0 = 100\,\mathrm{k}\Omega$	
1015/131	-3.42×10^{-8}	-1.64×10^{-8}
	$R_0 = 1\,\mathrm{M}\Omega$	
4029/52	-1.9288×10^{-6}	-1.9110×10^{-6}
6121/79	$+1.2130 \times 10^{-6}$	$+1.2307 \times 10^{-6}$
10150/131	-3.42×10^{-8}	-1.64×10^{-8}

References

[1] M. Ortolano, M. Abrate, and L. Callegaro, “On the synthesis of quantum Hall array resistance standards,” *Metrologia*, vol. 52, p. 31, 2015. [Online]. Available: https://doi.org/10.1088/0026-1394/52/1/31

[2] Bureau International des Poids et Mesures, “The International System of Units (SI),” 2006, 8th edition. [Online]. Available: www.bipm.org

[3] ——, “Draft of the 9th SI brochure,” Feb. 2018. [Online]. Available: www.bipm.org

[4] D. B. Newell, F. Cabiati, J. Fischer, K. Fujii, S. G. Karshenboim, H. S. Margolis, E. de Mirandés, P. J. Mohr, F. Nez, K. Pachucki, T. J. Quinn, B. N. Taylor, M. Wang, B. M. Wood, and Z. Zhang, “The CODATA 2017 values of h, e, k, and N_A for the revision of the SI,” *Metrologia*, vol. 55, p. L13, 2018. [Online]. Available: http://stacks.iop.org/0026-1394/55/i=1/a=L13

[5] Consultative Committee for Electricity and Magnetism, “*Mise en pratique* for the definition of the ampere and other electric units in the SI,” Dec. 2017, draft for Appendix 2 of the SI Brochure for the “Revised SI”. [Online]. Available: www.bipm.org

[6] Wolfram, “Mathematica 11.” [Online]. Available: http://www.wolfram.com/mathematica/