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# A new proposal to improve the customer competitive benchmarking in QFD

Fiorenzo Franceschini[a] and Domenico Maisano[b]

[a]*fiorenzo.franceschini@polito.it* [b]*domenico.maisano@polito.it*
Politecnico di Torino, Department of Management and Production Engineering (DIGEP)
Corso Duca degli Abruzzi 24, 10129, Torino (Italy)

## Abstract

*Quality Function Deployment* (QFD) is a structured tool that supports the design of new products/services, translating customer requirements into technical and process characteristics. The so-called *Customer Competitive Benchmarking* is a module of the QFD's *House of Quality*, in which a sample of (potential) customers express their perceptions on a set of competing products/services; this information is then elaborated by a cross-functional team of experts and used to define improvement and strategic goals. Despite the importance of this kind of benchmarking for the whole QFD process, the scientific literature reveals limited research.

This paper critically analyzes the traditional procedure of customer-competitive benchmarking, highlighting its major weaknesses and problematic aspects. Additionally, it proposes an alternative procedure to overcome (at least partly) those weaknesses, without undermining the simplicity in data collection and processing. The alternative procedure utilizes the Thurstone's *Law of Comparative Judgment*, which allows to transform subjective judgments by multiple respondents into a collective *cardinal* scaling. The description is supported by several pedagogical and real-life examples.

**Keywords:** QFD, Customer requirements, Customer competitive benchmarking, Preference ordering, Law of Comparative Judgment, Thurstone's scaling, Ratio scale, Indicator aggregation.

## Introduction

About 50 years after its conception (Akao, 1990), *Quality Function Deployment* (QFD) continues to be a very popular and diffused tool to support the design and development process of products/services, helping companies make the key trade-offs between what the customer really wants and what the company can afford to build.

Typical benefits are fewer and earlier design changes, improved cross-functional communication, improved product/service quality, and reduced development time and cost (Hauser and Clausing, 1988; Griffin and Hauser, 1993; Tran and Sherif, 1995; Franceschini, 2002). One of the greatest merits of QFD is to rely on the so-called *Voice of the Customer* (VoC) and to translate the *customer requirements* (CRs) into appropriate technical characteristics, for each stage of the development process of the product/service. In addition, QFD can incorporate benchmarking information of the products/services by representative competitors, from the dual perspective of customer satisfaction and technical characteristics. This information may help QFD users to make strategic decisions, both from the marketing and technical viewpoint (Shen et al., 2000).

QFD generally utilizes four sets of matrices, which respectively translate (1) CRs into engineering characteristics and, in turn, into (2) parts characteristics, (3) process characteristics, and (4) process/quality-control parameters, according to a sequential approach. For details, we refer the reader to the vast literature and extensive reviews (Chan and Wu, 2002; Franceschini, 2002; Sharma et al., 2008).

The first matrix, defined as *Product Planning House of Quality* or simply *House of Quality* (HoQ), is probably the most important one, since it regards the collection and analysis of the VoC. Not surprisingly, the majority of the (literally) thousands of scientific articles on QFD-related topics are focused exclusively on the HoQ, neglecting the other three matrices (Vinayak and Kodali, 2013). In addition, many proposed variants/improvements of the traditional QFD process often remain pure theory, since the real QFD users (e.g., product/service companies) hardly venture out of it. This apparent dichotomy is probably due to two reasons:

1. In spite of being a very practical tool, QFD has several "methodological weaknesses" that have been stimulating the development of a number of variants/improvements in the Quality Engineering/Management research field;
2. A fundamental attribute of the traditional QFD process is its inherent simplicity of use for the parties involved, i.e., (i) a sample of (potential) customers, with reasonable knowledge of the product/service to be designed, and (ii) a cross-functional team of experts (hereafter abbreviated as "QFD team"), consisting of members from marketing, design, quality, finance and production. Since most of the proposed variants/improvements tend to undermine this simplicity, they are actually unused.

The focus of this paper is on the HoQ, whose construction process can be summarized into ten phases, as represented in Figure 1 (Gonzales et al., 2003; Franceschini and Maisano, 2015). In particular, we will deal with the phases concerning the CR-data analysis, namely: Phase 2, "Relative Importance Ratings", Phase 3, "Customer Competitive Benchmarking", and Phase 4, "Final Importance Ratings". Since this set of phases are closely related to each other and the most relevant one is Phase 3, the expression "Customer Competitive Benchmarking" will hereafter refer to the whole set.

In the traditional HoQ-construction procedure, the Customer Competitive Benchmarking includes several activities of data collection and aggregation, which involve (potential) customers and QFD experts. Most of the data are ordinal subjective/attitudinal measurements that are operationalized via *ordinal* response scales (Iqbal et al., 2017). A typical abuse is to improperly promote these scales to *cardinal* ones, i.e., *interval* or even *ratio* scales (Stevens, 1946; Roberts, 1979; Burke et al., 2002). Another issue is that ordinal response scales tend to be used subjectively, as there is no absolute reference shared by all respondents (Franceschini et al., 2015a). Yet another issue is the conceptually questionable aggregation model of (sub)indicators (Franceschini et al., 2007).

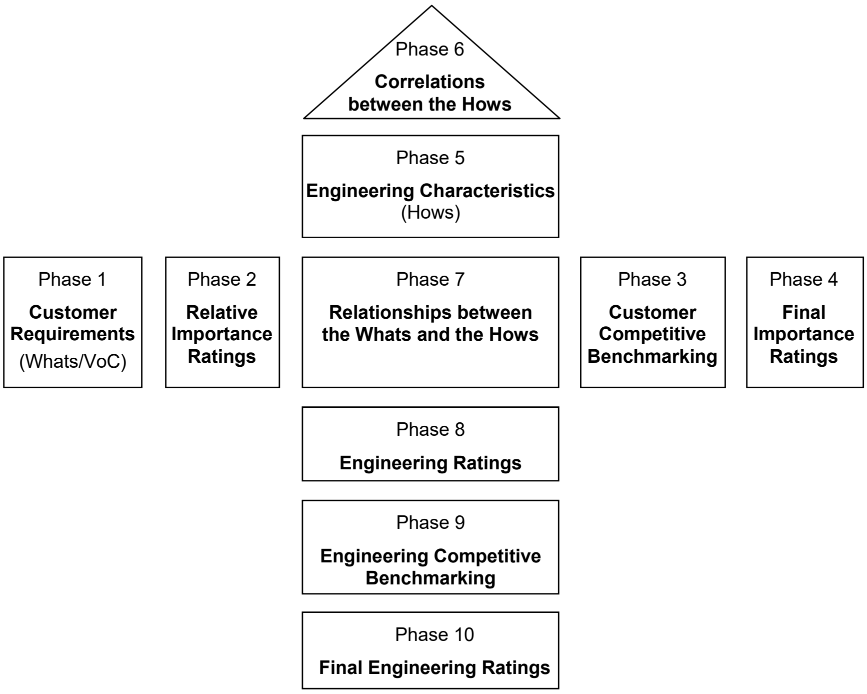
Phase 6
Correlations between the Hows
Phase 5
Engineering Characteristics
(Hows)
Phase 1
Customer Requirements
(Whats/VoC)
Phase 2
Relative Importance Ratings
Phase 7
Relationships between the Whats and the Hows
Phase 3
Customer Competitive Benchmarking
Phase 4
Final Importance Ratings
Phase 8
Engineering Ratings
Phase 9
Engineering Competitive Benchmarking
Phase 10
Final Engineering Ratings

**Figure 1. Phases of the *Product-Planning House of Quality* (HoQ) construction process.**

The aim of this paper is to critically analyze the Phases 2, 3 and 4 of the traditional HoQ-construction procedure, and then propose an alternative procedure to overcome its major weaknesses, without undermining simplicity in data collection and processing. The new procedure will make the customer competitive benchmarking easier to manage, as well as more correct from a conceptual point of view. A key element is the repeated application of the *Thurstone's Law of Comparative Judgment* (LCJ), which allows to aggregate multi-respondent subjective judgments into a cardinal scaling (Thurstone, 1927; Edwards, 1957; Franceschini and Maisano, 2015). The LCJ will be integrated with a practical response mode to collect the subjective judgments, through the formulation of preference orderings (Yager, 2001; Chen et al., 2012; Zheng et al., 2016).

The rest of this paper is organized into five sections. The section "Critical Description of Customer Competitive Benchmarking" illustrates the traditional procedure for Phases 2, 3 and 4 of the HoQ, identifying the major weaknesses and problematic aspects. The section "Basics of the Thurstone's LCJ" recalls the Thurstone's LCJ, while the section "A new response mode" introduces a practical response mode that facilitates its integration into the HoQ. The section "New Procedure for Customer Competitive Benchmarking" illustrates the proposed procedure in detail, with the aid of several pedagogical examples. Then, the section "Real-life Application" shows a real-life application example concerned with the design of an aircraft seat for passengers. The concluding section summarizes the original contributions of the article, focusing on the benefits and limitations of the proposed procedure and possible future research. Further information is contained in the "Appendix" section.

## Critical Description of Customer Competitive Benchmarking

This section presents a description of the traditional procedure for Phases 2, 3 and 4 of the HoQ. The description emphasizes the weaknesses of this procedure, which will be overcome by the proposed procedure.

For the purpose of completeness, we recall that the HoQ's Phase 1 ("Customer Requirements") results into the formulation of a list of CRs that represent what the customer truly expects from the product/service of interest. This phase is carried out by selecting and interviewing a representative sample of (potential) customers, with a reasonable knowledge of the product/service to be designed (Urban and Hauser, 1993). Subsequently, the QFD experts have to review, reorganize and include the CRs into the HoQ.

### *Phase 2: Relative Importance Rating*

#### *Description*

This phase concerns the prioritization of CRs. The expression "Relative Importance Ratings" indicates that this prioritization is aimed at discriminating a CR based on its importance over the others. A sample of respondents – generally the same (potential) customers involved in the VoC collection process – have to express their judgements, using a five-level ordinal response scale (1=Very low importance, 2=Low importance, 3=Medium importance, 4=High importance, and 5=Very high importance). The multi-respondent judgments related to each CR are then aggregated through central tendency indicators, such as the median.

Let us consider the pedagogical example in Table 1, about the construction of the HoQ for a service that is familiar to scholars: an *international scientific conference*. A sample of five respondents ($R_1$ to $R_5$) are interviewed to collect the VoC and prioritize the relevant CRs. The resulting importance judgments are then aggregated through the median ($I^{(1)}$).

| List of CRs | | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $I^{(1)}$ |
|---|---|---|---|---|---|---|---|
| $CR_1$ | International reputation | 3 | 3 | 2 | 3 | 2 | 3 |
| $CR_2$ | Suitable mix of research topics | 1 | 3 | 2 | 1 | 3 | 2 |
| $CR_3$ | Public relations | 4 | 5 | 5 | 4 | 5 | 5 |
| $CR_4$ | Suitable location | 3 | 5 | 5 | 3 | 3 | 3 |

**Table 1. Pedagogical example of Phase 2 for the HoQ related to an international scientific conference. Five respondents ($R_1$ to $R_5$) express their judgements on the relative importance of four CRs ($CR_1$ to $CR_4$).**

#### *Critical Analysis*

Although being simple and intuitive for respondents, the traditional CR-prioritization procedure has (at least) three weaknesses:

1. Since the CR importance is expressed on an ordinal scale, it only allows comparisons like "$CR_1$ and $CR_2$ have equal importance", "$CR_1$ is more important than $CR_2$", etc.. Unfortunately, a typical abuse is "promoting" this scale to an *interval* or even *ratio* scale, in which the interval/distance between scale categories is meaningful (Stevens, 1946; Burke et al., 2002; Franceschini et al., 2007). At the risk of oversimplifying, we recall that a ratio scale has the interval property and an *absolute* (or non-arbitrary) *zero*, corresponding to the absence of the attribute of interest; e.g., for a mass scale, the zero position is unique and corresponds to the absence of mass, independently from the unit in use (grams, pounds, stones, etc.).
2. The five-level ordinal scale tends to be used subjectively, as there is no absolute reference shared by all respondents. For example, "indulgent" respondents will tend to assign higher levels of importance, whereas "severe" respondents will tend to assign lower ones. For this reason, it is questionable to aggregate judgments by different respondents through indicators of central tendency.
3. The resolution of the scale (i.e., five levels) may conflict with the real discriminatory power of respondents; e.g., it can be a limitation for respondents able to distinguish among a greater number of hierarchical levels, or it can be overdetailed for respondents unable to distinguish among more than two/three hierarchical levels.

Apart from the traditional procedure, the scientific literature encompasses several alternative techniques for the CR prioritization.

- Techniques based on the *analytic hierarchy process* (AHP) and *analytic network process* (ANP) methods, which require CR judgments expressed in the form of paired-comparison data and defined on a ratio scale; e.g. "$CR_1$ is twice as important as $CR_2$" (Chuang, 2001; Franceschini, 2002; Kwong and Bai, 2002; Li et al., 2009). Techniques based on the ANP method are generally more comprehensive than those based on the AHP one, as they can also be used for Phase 8, which focuses on the prioritization of Engineering Characteristics.
- Techniques that allow to model the uncertainty in CRs, taking into account the uncertainty in the relevant customer judgments (Nahm et al., 2013).
- Techniques of scaling, such as the *generalized Yager's algorithm* (Franceschini et al., 2015a) and the *Thurstone's LCJ* (Franceschini and Maisano, 2015). Although being based on different response modes, these techniques can be used to aggregate multi-respondent judgments into a CR scaling.

The latter technique represents the starting point of the present research as – with some adaptations – its use will be extended from Phase 2 to Phase 3, with the aim of overcoming the three weaknesses mentioned above.

*Phase 3: Customer Competitive Benchmarking*

*Description*

The QFD team, with the predominant contribution of marketing experts, identifies a number of existing products/services to be benchmarked: usually our product/service[1] and two or three others by relevant competitors. One of the purposes of this phase is to know how competing products/services match up to the CRs, compared to our product/service.

Returning to the pedagogical example, let us suppose to compare our existing conference ($P_A$), with two other competing conferences ($P_B$ and $P_C$ respectively). Of course, the choice of the products/services to be benchmarked should be consistent with the so-called *market segment* of interest: e.g., international scientific conferences should not be confused with regional/national conferences, workshops, exhibitions, etc..

Then, each respondent expresses his/her degree of satisfaction on the CRs of the products/services benchmarked, using another five-level ordinal scale (1=Very low satisfaction, 2=Low satisfaction, 3=Medium satisfaction, 4=High satisfaction, and 5=Very high satisfaction). Judgments by different respondents are aggregated through a central tendency indicator, such as the median, and included into the HoQ. For the purpose of example, Table 2 shows the judgements of five respondents ($R_1$ to $R_5$) on the satisfaction level of each CR, for the three conferences benchmarked; these judgements are aggregated through the median (see the $S_A$, $S_B$ and $S_C$ values in Table 2).

On the basis of competitive benchmarking and strategic consideration, the QFD team then defines the *target* ($T$) satisfaction level of each CR, on the same afore-described five-level ordinal scale.

The *improvement rate* ($I^{(2)}$) of each CR of the product/service to be designed is then calculated as:

$$I^{(2)} = T / S_A, \tag{1}$$

$S_A$ being the satisfaction level of that CR, for our existing product/service ($P_A$). For the purpose of example, see the calculation of $I^{(2)}$ in the last column of Table 2.

Next, the focus is on the CRs that are expected to impact more on sales, i.e., those that tend to give the company a competitive advantage over competitors, differentiating its products/services (Van De Poel, 2007). For example, a car manufacturer with a consolidated brand image in terms of *comfort* or *low fuel consumption* will reasonably preserve these features in the design of new car models. For each CR, the so-called *sales point* ($I^{(3)}$) usually takes the value 1.5=Real, 1.2=Potential (e.g., the company plans to invest in it in the future), or 1=Uninfluential (Franceschini, 2002). Returning to the example, it can be assumed that $CR_2$ is a real sales point, $CR_3$ is a potential one, while the remaining ones are uninfluential.

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[1] The adjective "our" denotes the existing product/service of the company implementing the QFD process. Similarly, the expression "our company" will be used to denote the company itself.

| | Satisfaction on $P_A$ | | | | | | Satisfaction on $P_B$ | | | | | | Satisfaction on $P_C$ | | | | | | $T$ | $I^{(2)}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $\rightarrow S_A$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $\rightarrow S_B$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $\rightarrow S_C$ | | |
| $CR_1$ | 2 | 2 | 1 | 2 | 3 | **2** | 4 | 4 | 2 | 3 | 5 | **4** | 3 | 2 | 3 | 3 | 4 | **3** | 4 | **2.0** |
| $CR_2$ | 1 | 4 | 3 | 2 | 3 | **3** | 1 | 2 | 2 | 1 | 3 | **2** | 4 | 4 | 3 | 2 | 4 | **4** | 3 | **1.0** |
| $CR_3$ | 1 | 3 | 2 | 1 | 2 | **2** | 3 | 5 | 1 | 2 | 3 | **3** | 2 | 1 | 2 | 2 | 3 | **2** | 2 | **1.0** |
| $CR_4$ | 1 | 2 | 1 | 1 | 2 | **1** | 2 | 3 | 2 | 1 | 2 | **2** | 5 | 3 | 4 | 4 | 5 | **4** | 3 | **3.0** |

Notes:
$P_A$, $P_B$ and $P_C$ are respectively our conference and two other competing ones;
$S_A$, $S_B$ and $S_C$ are the median satisfaction levels of each CR, for $P_A$, $P_B$ and $P_C$ respectively;
$T$ is the target level of satisfaction of each CR, for the conference to be designed;
$I^{(2)}$ is the improvement rate of each CR.

**Table 2. Pedagogical example of Phase 3 of the HoQ related to an international scientific conference (see also Table 1).**

From the perspective of a generic CR, $I^{(2)}$ and $I^{(3)}$ represent two additional dimensions of importance, which complement the information provided by $I^{(1)}$.

*Critical Analysis*

The three weaknesses highlighted for the CR-importance judgments (Sect. “Relative Importance Rating”) can be extended to the CR-satisfaction judgements, i.e., (1) the five-level ordinal scale in use has no interval (or distance) property; (2) this scale tends to be used subjectively, depending on the severity/indulgence of respondents; (3) this scale imposes a certain resolution (i.e., 5 hierarchic levels) which could conflict with the real discriminatory power of respondents.

Another possible issue is that QFD experts are supposed to make a collective choice and determine one-and-only-one $T$ value for each CR; this is not so obvious, since they may have conflicting opinions (Van De Poel, 2007). In addition, there is no clear guideline on how to define $T$ values: several authors suggest to focus more on the CRs where our existing product/service is weaker than competitors (Carnevalli and Miguel, 2008); other authors claim that significant effort is required when there is similar performance between our product/service and the others, while, if our product/service outperforms or underperforms, significant effort may not be worth the potential gain (Iqbal et al., 2017); other authors yet suggest to set the degree of improvement, taking into account various aspects, such as *technical* or *financial difficulty*, although it is not perfectly clear how these aspects could be evaluated at this stage (Shen et al., 2000; Vinayak and Kodali, 2013).

In our opinion, the problem of defining $T$ values is a consequence of the intrinsically elusive and ambiguous definition of this indicator. The alternative procedure, illustrated later on, will overcome this problem.

Another weakness of Phase 3 concerns the calculation of $I^{(2)}$ through a ratio (see Eq. 1): this operation is conceptually prohibited as $T$ and $S_A$ are defined on ordinal scales (Stevens, 1946; Roberts, 1979; van de Poel, 2007).

Focusing on *sales points*, QFD experts may, again, find it difficult to make a collective choice on the $I^{(3)}$ score for each CR. Also, the fact that this indicator is expressed on a 3-level ordinal scale (1, 1.2 and 1.5) entails that it cannot be aggregated with other indicators through additive or multiplicative models (Roberts, 1979); this point will be clarified in the next subsection.

Even though several authors recommend that QFD practitioners should always integrate customer competitive benchmarking within the QFD process (Vaziri, 1992; Jeong and Oh, 1998; Chan and Wu, 2005), literature reveals limited research undertaken. Some relevant contributions are the following ones:

- Swanson (1993) proposed the *quality benchmark deployment* method, i.e., a variation of QFD to help organizations logically select critical areas to benchmark and understand the relationship between customers' expectations and performance drivers;
- Lu et al. (1994) developed an integrative approach for strategic marketing by using QFD, AHP and benchmarking;
- Iqbal et al. (2017) proposed a statistical method allowing to perform comparisons between a company's performance and that of competitors, resulting in improved decision making.

Unfortunately, most of these (and other) procedures tend to complicate the data collection process, without overcoming the weaknesses of the traditional procedure.

*Phase 4: Final Importance Ratings*

*Description*

The objective of this phase is to determine the so-called *final importance* ($I$) of each CR, aggregating the three importance dimensions:

$I^{(1)}$ *relative importance*, reflecting the importance of a certain CR over the others, regardless of specific products/services benchmarked;

$I^{(2)}$ *improvement rate*, reflecting the target level of improvement of the product/service to be designed with respect to the existing one, trying to bridge the gap with competitors;

$I^{(3)}$ *sales points*, reflecting the impact of a certain CR on sales.

The traditional aggregation is carried out through the following multiplicative model:

$$I = I^{(1)} \cdot I^{(2)} \cdot I^{(3)}. \tag{2}$$

In case one wants to add extra importance dimensions, the model can be easily adapted by adding multiplicative terms. The $I$ values associated with the CRs can also be expressed in percent form, dividing them by their sum (see the example in Table 3).

| | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ | $I$ | |
|---|---|---|---|---|---|
| $CR_1$ | 3 | 2.0 | 1.0 | 6.0 | (25.0%) |
| $CR_2$ | 2 | 1.0 | 1.5 | 3.0 | (12.5%) |
| $CR_3$ | 5 | 1.0 | 1.2 | 6.0 | (25.0%) |
| $CR_4$ | 3 | 3.0 | 1.0 | 9.0 | (37.5%) |

**Table 3. Pedagogical example of Phase 4 for the HoQ related to an international scientific conference (see also Table 1 and Table 2).**

*Critical Analysis*

The aggregation of $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$ through a multiplicative model is questionable for several reasons:

1. This operation would be acceptable for sub-indicators defined on ratio scales (Roberts, 1979; Franceschini and Maisano, 2010) but, unfortunately, $I^{(1)}$ and $I^{(3)}$ are defined on ordinal scales, while $I^{(2)}$ is obtained through the (conceptually prohibited) ratio of two quantities defined on ordinal scales. One of the possible effects of this aggregation is that transformations that shift the zero-point position or distort the unit of (at least) one sub-indicator may produce uncontrolled variations into $I$ (see the example in Table 4).

| | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ | $I$ |
|---|---|---|---|---|
| $CR_1$ | $1 \rightarrow 2$ | 1.5 | 1.5 | $2.3 \rightarrow 4.5$ |
| $CR_2$ | $2 \rightarrow 3$ | 1.3 | 1 | $2.6 \rightarrow 3.9$ |

**Table 4. Rank reversal in the *I* values related to two CRs (*$CR_1$* and *$CR_2$*), caused by a scale transformation of the sub-indicator $I^{(1)}$: the initial 1-to-5 scale is transformed into a 2-to-6 one. When adopting the first scale, the resulting *I* value of *$CR_2$* overcomes that of *$CR_1$*, while when adopting the second one, *vice versa*.**

2. The aggregation in Eq. 2 makes it difficult to assess the influence of $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$ on $I$ (JRC-EU, 2008; Iqbal et al., 2017). An aggregation procedure that weighs the contribution of these sub-indicators depending on the QFD-team's policy/strategy would probably be more effective.
3. The multiplicative aggregation model entails that the *substitution rate*[2] between sub-indicators is not constant (Franceschini and Maisano, 2010). Let us consider the example in Table 5, in which the variation in $I^{(1)}$ between the states 1 and 2 ($\Delta I^{(1)} = +0.83$) is compensated by a certain variation in $I^{(3)}$ ($\Delta I^{(3)} = -0.3$, $I = 5$ being unchanged) and the substitution rate is $\Delta I^{(1)}/\Delta I^{(3)} = -2.78$. In response to the same variation in $I^{(1)}$ (i.e., $\Delta I^{(1)} = +0.83$) between the states 2 and 3, the variation in $I^{(3)}$ is significantly different ($\Delta I^{(3)} = -0.2$) and the substitution rate is almost doubled (i.e., $\Delta I^{(1)}/\Delta I^{(3)} = -4.17$). In other words, the substitution rate is not constant over the $I^{(1)}$-$I^{(3)}$ plane, as it depends on the $I^{(1)}$ and $I^{(3)}$ values related to the CR of interest.
4. According to some authors, the aggregation through a multiplicative model can be excessively penalizing in the case (at least) one of the sub-indicators has a relatively low value (Iqbal et al., 2017).

(a) States

| State | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ | $I$ |
|---|---|---|---|---|
| 1 | 3.33 | 1.0 | 1.5 | 5 |
| 2 | 4.17 | 1.0 | 1.2 | 5 |
| 3 | 5.00 | 1.0 | 1.0 | 5 |

(b) Substitution-rate calculation

| State transition | $\Delta I^{(1)}$ | $\Delta I^{(3)}$ | $\Delta I^{(1)}/\Delta I^{(3)}$ |
|---|---|---|---|
| From 1 to 2 | +0.83 | -0.30 | -2.78 |
| From 2 to 3 | +0.83 | -0.20 | -4.17 |

**Table 5. Substitution rate between $I^{(1)}$ and $I^{(3)}$, in the transition from state 1 to 2 and in the transition from state 2 to 3.**

*Summary*

Table 6 summarizes the main weaknesses found in Phases 2, 3 and 4 of the traditional procedure, which have been discussed in detail in the previous subsections.

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[2] The *substitution rate* between two generic sub-indicators (e.g., $I^{(1)}$ and $I^{(3)}$) is defined as the rate at which the value of one sub-indicator (e.g., $I^{(1)}$) can be increased/decreased in exchange for a decrease/increase in the value of the other sub-indicator (e.g., $I^{(3)}$), maintaining the same value of the aggregated indicator (e.g., $I$).

| HoQ Phase | Goal | Weaknesses |
|---|---|---|
| 2 - Relative Importance Ratings | Prioritizing CRs in terms of relative importance ($I^{(1)}$) | • The traditional five-level scale for customer judgements is just ordinal.<br>• This scale can be used subjectively, depending in the level of severity/indulgence of (potential) customers.<br>• The resolution of the scale may conflict with the discriminatory power of (potential) customers. |
| 3 - Customer Competitive Benchmarking | Comparing different competing products/services and defining the strategic improvements for the new product/service, in terms of customer satisfaction | • The traditional five-level scale for customer judgements is just ordinal.<br>• This scale can be used subjectively, depending in the level of severity/indulgence of (potential) customers.<br>• The resolution of the scale may conflict with the discriminatory power of (potential) customers.<br>• The QFD team may find it difficult to converge collectively to some target values ($T$), which are concerned with the satisfaction of CRs, for the new product/service.<br>• The improvement rate ($I^{(2)}$) is conceptually incorrect, being based on the ratio between two sub-indicators that are defined on ordinal scales.<br>• The QFD team may find it difficult to converge to a collective choice of sales points ($I^{(3)}$), for the new product/service. |
| 4 - Final Importance Ratings | Aggregation of the importance sub-indicators ($I^{(1)}$, $I^{(2)}$ and $I^{(3)}$) into a final importance indicator ($I$). | • The aggregation by means of a multiplicative model assumes that sub-indicators are defined on ratio scales (not just ordinal ones).<br>• The QFD experts do not have any chance to weigh the contributions of the three sub-indicators.<br>• The substitution rate between the importance sub-indicators is not constant. |

**Table 6. Summary of the weaknesses in the HoQ's Phases 2, 3 and 4, according to the traditional procedure.**

## Basics of the Thurstone's LCJ

The LCJ is a mathematical model to estimate scale values based on binary choices between specific empirical *objects*[3] to be compared. Precisely, Thurstone (1927) postulated the existence of a *psychological continuum*, i.e., an abstract scale, in which objects are placed. The evaluation is based on the degree of a certain *attribute*, i.e., a specific characteristic of the objects, on the basis of which some *respondents* develop their subjective perceptions.

Unfortunately, envisaging the continuum and placing the objects in a reliable and repeatable manner is very difficult for respondents; on the other hand, they may find it easier to formulate comparative judgments by means of comparisons of pairs of objects. Following this idea, a set of ($m$) respondents express their preferences for each object ($O_i$) versus any other object ($O_j$), considering all possible pairs. Preferences are expressed through relations of *strict preference* (e.g., $O_1 > O_2$ or $O_1 < O_2$) or *indifference* (e.g., $O_1 \sim O_2$).

At the risk of oversimplifying, the LCJ can be seen as a "black box" that transforms a set of multi-respondent paired-comparison judgments into scale values with the *interval* property (i.e., with meaningful distance but arbitrary zero (Stevens, 1946)); see the schematic representation in Figure 2.

The LCJ is based on the following additional postulates/assumptions (Thurstone, 1927; Edwards, 1957):

- For the attribute of a generic $i$-th object ($O_i$), a preference will exist among respondents;

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[3] In the original formulation, Thurstone (1927) uses the term "*stimuli*", which is commonly used in the field of cognitive science.

- For the attribute of $O_i$, the preference will be distributed normally: $O_i \sim N(\mu_i, \sigma_i^2)$, where $\mu_i$ and $\sigma_i^2$ are the unknown mean value and variance of that object's attribute. This distribution has been postulated to reflect the intrinsic respondent-to-respondent variability of perceptions;
- For simplicity, the attribute's variances are supposed to be all equal ($\sigma_i^2 = \sigma_j^2 = \ldots = \sigma^2$);
- The intercorrelations (in the form of Pearson coefficients $\rho_{ij}$) between the attributes of pairs of objects ($O_i$, $O_j$) are supposed to be all equal ( $\rho_{ij} = \rho, \forall i, j$ ).

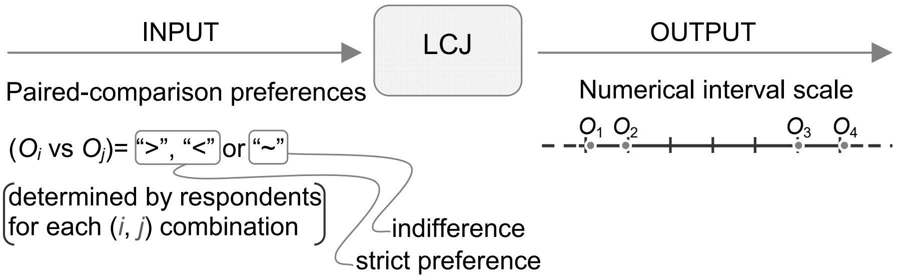
INPUT
LCJ
OUTPUT
Paired-comparison preferences
($O_i$ vs $O_j$)= ">", "<" or "~"
determined by respondents for each ($i$, $j$) combination
indifference
strict preference
Numerical interval scale
$O_1$ $O_2$ $O_3$ $O_4$

**Figure 2. Conceptual scheme of the Thurstone's LCJ.**

The application of the LCJ is based on five steps:

1. Each respondent expresses a preference for one object over another one. All possible $\binom{n}{2} = \frac{n \cdot (n-2)}{2}$ pairs are assessed, where $n$ is the number of objects of interest. Results may then be aggregated into a frequency matrix (***F***), whose general element $f_{ij}$ represents the number of times that $O_i$ was preferred to $O_j$ (i.e., absolute frequency of the preference $O_i > O_j$). Precisely, for each respondent who prefers $O_i$ to $O_j$, the indicator $f_{ij} \in [0, m]$ is incremented by one unit ($m$ being the total number of respondents). If two objects are considered indifferent (i.e., $O_i \sim O_j$), $f_{ij}$ and $f_{ji}$ are both conventionally incremented by 0.5. Of course, the complementarity relationship $f_{ij} = m - f_{ji}$ holds.

   Let us consider an example in which $m = 5$ respondents ($R_1$ to $R_5$) express their paired-comparison preferences on the specific attribute of $n = 4$ objects ($O_1$ to $O_4$). The matrix ***F*** (in Figure 3) contains the $f_{ij}$ values, which are calculated by aggregating the preferences; for example, considering the pairwise comparison between $O_1$ and $O_2$, three respondents (i.e., $R_1$, $R_2$ and $R_4$) prefer $O_1$ to $O_2$ (partial score 1+1+1 = 3), one respondent (i.e., $R_5$) prefers $O_2$ to $O_1$ (partial score 0), and one other respondent (i.e., $R_3$) considers them indifferent (partial score 0.5), therefore $f_{12}$=3.5.
2. Next, the $f_{ij}$ values are transformed into $p_{ij}$ values, through the relationship:

$$p_{ij} = \frac{f_{ij}}{m} \, , \tag{3}$$

   where $p_{ij}$ represents the observed proportion of times that $O_i$ was chosen over $O_j$. The matrix ***P*** in Figure 3 shows the $p_{ij}$ values obtained from the $f_{ij}$ values in the matrix ***F***. For example, regarding the comparison between $O_1$ and $O_2$, $p_{12} = 3.5/5 = 0.70$, denoting that the tendency of preferring $O_1$ to $O_2$ is around 70%. Of course, the relationship of complementarity $p_{ij} = 1 - p_{ji}$ holds.

3. Next, $p_{ij}$ values are transformed into $z_{ij}$ values, through the relationship:

$$z_{ij} = \Phi^{-1}(1 - p_{ij}), \tag{4}$$

$\Phi()$ being the cumulative distribution function of the standard normal distribution. The element $z_{ij}$ represents a unit normal deviate, which will be positive for all values of $(1 - p_{ij})$ over 0.50 and negative for all values of $(1 - p_{ij})$ under 0.50.

In general, objects are judged differently by respondents. However, if all respondents express the same preference for each outcome, the model is no more viable ($p_{ij}$ values of 1.00 and 0.00 would correspond to $z_{ij}$ values of $\pm\infty$). A simplified approach for tackling this problem is associating values of $p_{ij} \leq 0.023$ with $z_{ij} = \Phi^{-1}(1 - 0.023) = 1.995$ and values of $p_{ij} \geq 0.977$ with $z_{ij} = \Phi^{-1}(1 - 0.977) = -1.995$. More sophisticated solutions to deal with this issue have been proposed (Edwards, 1957; Krus and Kennedy, 1977).

The example in Figure 3 shows the $z_{ij}$ values related to the $p_{ij}$ values, reported in the preceding column; the items marked with "*" are those for which the afore-described simplification is applied.

(a) Respondent judgements

| Paired comparison | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $f_{ij}$ | $p_{ij}$ | $z_{ij}$ |
|---|---|---|---|---|---|---|---|---|
| $(O_1, O_2)$ | > | > | ~ | > | < | 3.5 | 0.70 | -0.524 |
| $(O_1, O_3)$ | < | < | < | < | < | 0 | 0.00* | 1.995 |
| $(O_1, O_4)$ | < | < | < | ~ | < | 0.5 | 0.10 | 1.282 |
| $(O_2, O_3)$ | < | < | < | < | < | 0 | 0.00* | 1.995 |
| $(O_2, O_4)$ | < | < | < | < | < | 0 | 0.00* | 1.995 |
| $(O_3, O_4)$ | ~ | ~ | < | > | > | 3 | 0.60 | -0.253 |

(b) matrix ***F***

|  | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|
| $O_1$ | 2.5 | 3.5 | 0 | 0.5 |
| $O_2$ | 1.5 | 2.5 | 0 | 0 |
| $O_3$ | 5 | 5 | 2.5 | 3 |
| $O_4$ | 4.5 | 5 | 2 | 2.5 |

(c) matrix ***P***

|  | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|
| $O_1$ | 0.50 | 0.70 | 0.00* | 0.10 |
| $O_2$ | 0.30 | 0.50 | 0.00* | 0.00* |
| $O_3$ | 1.00* | 1.00* | 0.50 | 0.60 |
| $O_4$ | 0.90 | 1.00* | 0.40 | 0.50 |

(d) matrix ***Z***

|  | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|
| $O_1$ | 0.000 | -0.524 | 1.995 | 1.282 |
| $O_2$ | 0.524 | 0.000 | 1.995 | 1.995 |
| $O_3$ | -1.995 | -1.995 | 0.000 | -0.253 |
| $O_4$ | -1.282 | -1.995 | 0.253 | 0.000 |

Notes:

$f_{ij}$ denotes the number of times that $O_i$ is preferred to $O_j$;

$p_{ij}$ denotes the proportion of times that $O_i$ is preferred to $O_j$;

$z_{ij} = \Phi^{-1}(1 - p_{ij})$ ;

(*) for $p_{ij} \leq 0.023$, $z_{ij}$ is conventionally set to 1.995, while for $p_{ij} \geq 0.977$, it is set to -1.995.

**Figure 3. Example of construction of the $f_{ij}$, $p_{ij}$ and $z_{ij}$ values (and relevant matrices *F*, *P* and *Z*) related to the paired-comparison preferences by five respondents ($R_1$ to $R_5$) on four objects ($O_1$ to $O_4$).**

4. Next, the $z_{ij}$ values related to the possible paired comparisons are reported into the matrix ***Z*** (see Figure 3). Being $z_{ij}$ and $z_{ji}$ unit normal deviates related to complementary cumulative probabilities (i.e., $p_{ji} = (1 - p_{ij})$), the relationship $z_{ji} = -z_{ij}$ holds.

5. Finally, the so-called *Thurstone's scaling* can be performed according to the following operations (see Figure 4):
   - summing the values into each column of the matrix **Z**;
   - dividing these sums by $n$, i.e., the number of objects ($n$ = 4 in this case).

   It can be demonstrated that the result obtained for each column corresponds to a linear transformation of the unknown average value ($\mu_j$) of the $j$-th object's attribute; in mathematical terms:

$$\mu_j' = \sum_i (z_{ij})/n = k_1 \cdot \mu_j + k_2 , \tag{5}$$

   being $k_1 = \dfrac{1}{\sqrt{2 \cdot \sigma^2 (1-\rho)}}$, $k_2 = -\dfrac{\left(\sum_i \mu_i\right)/n}{\sqrt{2 \cdot \sigma^2 (1-\rho)}}$.

   The Thurstone's scaling therefore allows to determine the $\mu_j$ values up to a positive (unknown) scale factor ($k_1$) and an additive constant ($k_2$). A formal proof is given in (Thurstone, 1927; Edwards, 1957; Franceschini and Maisano, 2015). Since the LCJ operates only on judgements of differences between objects, it cannot determine an absolute zero point. In other words, the resulting scale values are defined on an *interval* scale, i.e., a scale with meaningful interval/distance but arbitrary origin and unit (Stevens, 1946; Torgerson, 1958).

   Since LCJ is a statistical procedure, the larger the number of respondents is, the more reasonable and robust results tend to be. Empirical studies show that this condition is generally reached with a number of respondents around (at least) ten units (Thurstone, 1927; Edwards, 1957; Torgerson, 1958).

   Figure 4 exemplifies the Thurstone's scaling relating to the example in Figure 3.

|  | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|
| $O_1$ | 0.000 | -0.524 | 1.995 | 1.282 |
| $O_2$ | 0.524 | 0.000 | 1.995 | 1.995 |
| $O_3$ | -1.995 | -1.995 | 0.000 | -0.253 |
| $O_4$ | -1.282 | -1.995 | 0.253 | 0.000 |

|  |  |  |  |  |
|---|---|---|---|---|
| $\Sigma_j$ | -2.753 | -4.515 | 4.244 | 3.024 |
| $\mu_j' = \Sigma_j / n$ | **-0.688** | **-1.129** | **1.061** | **0.756** |

**Figure 4. *Z* matrix related to the $z_{ij}$ data reported in Figure 3 and corresponding Thurstone's scaling.**

## New response mode for the LCJ integration into the HoQ

Although the LCJ is a well-established, elegant and effective technique, there are three limitations to its use within the HoQ:

1. The response mode based on paired comparisons is laborious and tedious for respondents,

especially when the number of objects is large.

2. Often it is advantageous to express the positioning of the objects with reference to an *absolute* zero point rather than an *arbitrary* one[4]. The scientific literature includes several techniques to provide a rough estimate of an absolute zero, which is also denominated *rational zero point* (Lim, 2011; Thurstone and Jones, 1957; Torgerson, 1958). For example, Torgerson (1958) proposed a technique based on the correlation between the results of the LCJ and those of the so-called *Method of Single Stimuli* (Volkmann, 1932); this technique will be recalled later on in the paper.

   Unfortunately, these (and other) techniques tend to overcomplicate the response mode, requiring additional evaluations by respondents.
3. Even assuming that an absolute-zero point could be estimated, the LCJ results in an arbitrary unit, which does not allow to identify the absolute degree of a certain attribute, with respect to the extreme situations of (i) *absence* (i.e., absolute zero) and (ii) *maximum-imaginable*[5] degree. In other words, the interval scale resulting from the LCJ is not "anchored" with respect to the (unknown) psychological continuum, in which objects are positioned. This limitation makes the results of different Thurstone's scaling processes incomparable.

The two previous limitations will be addressed in the following subsections.

### *Preference orderings*

In the standard LCJ procedure, response data are expressed in the form of paired-comparison preference judgements. A significant drawback of this approach is that it can be tedious and complex to manage, especially when the number of objects is large and much repetitious information is required from respondents. However, paired-comparison preference judgments can be obtained indirectly, by using more practical response modes (Franceschini and Maisano, 2015). For example, judgements can be expressed using the classical five-level ordinal scale, then turned into *preference orderings* and, then again, into paired-comparison data (see the example in Figure 5).

Alternatively, preference orderings may be directly formulated by respondents and then turned into paired-comparison data. A practical way to do this is asking each respondent to position some *tags* (even immaterial ones, through some software interface) in order of preference: the more preferred ones should be positioned in the top positions while the less preferred ones in the lower ones; those positioned at the same hierarchical level are considered indifferent (see the example Figure 6).

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[4] This also applies to Phases 2, 3 and 4 of the HoQ, where aggregated judgments are treated as if they were defined on a (ratio) scale with an absolute-zero point (cf. Sect. "Critical Description of Customer Competitive Benchmarking").

[5] We have implicitly assumed that the (unknown) *psychological continuum* is included between an absolute-zero point, corresponding to the *absence* of the attribute, and a point corresponding to the *maximum-imaginable* degree of the attribute (Torgerson, 1958). This assumption, which is quite common for psychometric studies on subjective perceptions (Lim, 2011), will be discussed in more detail later on in the paper.

In this way, each respondent constructs a *linear* preference ordering, i.e., a chain of objects linked by *strict preference* ("$>$") and *indifference* ("$\sim$") relationships; the resulting number of hierarchical levels may change depending on the number of objects and their mutual relationships (Yager, 2001; Nederpelt and Kamareddine, 2004). We also note that this response mode forces respondents to be *transitive* (e.g., if $O_1 > O_2$ and $O_2 > O_3$, then $O_1 > O_3$).

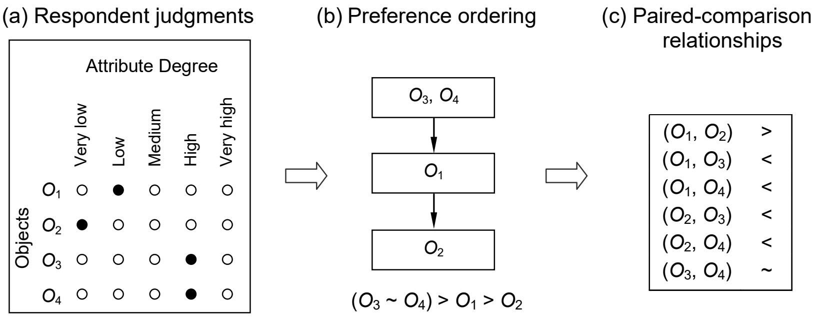
(a) Respondent judgments
Attribute Degree
Very low
Low
Medium
High
Very high
Objects
O1 ○ ● ○ ○ ○
O2 ● ○ ○ ○ ○
O3 ○ ○ ○ ● ○
O4 ○ ○ ○ ● ○
(b) Preference ordering
O3, O4
O1
O2
(O3 ~ O4) > O1 > O2
(c) Paired-comparison relationships
(O1, O2) >
(O1, O3) <
(O1, O4) <
(O2, O3) <
(O2, O4) <
(O3, O4) ~

**Figure 5. Example of (indirect) determination of paired-comparison relationships from judgments expressed using a five-level ordinal scale, then turned into a preference ordering.**

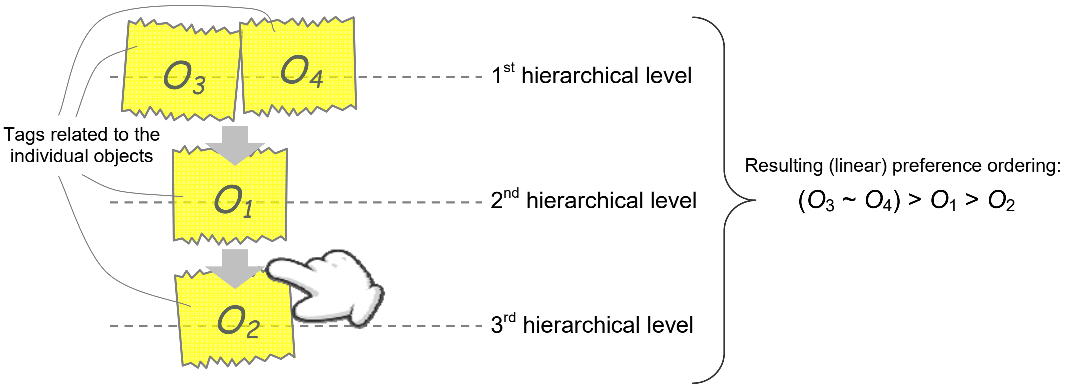
O3
O4
O1
O2
Tags related to the individual objects
1st hierarchical level
2nd hierarchical level
3rd hierarchical level
Resulting (linear) preference ordering:
(O3 ~ O4) > O1 > O2

**Figure 6. Practical technique for supporting the construction of preference orderings, using tags.**

Even though the direct formulation of preference orderings may sometimes be less practical then the use of five-level ordinal scales (e.g., in the case of telephone or street interviews) (Alwin and Krosnick, 1985), the fact remains that it is less prone to subjective interpretations and does not impose any discriminatory power to respondents (cf. critical considerations on HoQ's Phases 2 and 3).

### *Anchoring the Thurstone's Scaling: the ZM-technique*

Another obstacle to the integration of the LCJ within the HoQ is that the resulting (interval) scale is not "anchored", as it has an arbitrary zero point and an arbitrary unit. One may be tempted to overcome this obstacle promoting this scale to a ratio one with a conventional unit, introducing conceptually wrong transformations (see Sect. "Example of Improper Scale Promotion", in the appendix).

As mentioned above, the scientific literature includes several techniques to estimate the position of the absolute zero and/or anchor the LCJ's scale, even though they inevitably complicate the response mode. For example, the technique proposed by Torgerson (1958, page 196) requires that each judge directly assigns the scale values of the objects, in a range included between two anchor points: i.e., a (presumed) absolute-zero point (set to 0), corresponding to the absence of the attribute, and a point corresponding to the *maximum-imaginable* degree of the attribute, which is conventionally set to 5. While aware of the difficulty and potential unreliability of this direct-assignment process, Torgerson suggests to use it just for the purpose of anchoring the LCJ scale (Torgerson, 1958). An application example of this technique is reported in the section "Torgerson's anchoring" (in the appendix).

We have developed a new anchoring technique, denominated "ZM-technique", that is more consistent with the response mode based on preference orderings (in the section "Preference orderings"). Our proposal is to apply the LCJ including two *dummy* or *anchor* objects in addition to the regular ones:

($Z$) a dummy/anchor object corresponding to the absence of the attribute of interest ("$Z$" stands for "zero");

($M$) a dummy/anchor object corresponding to the maximum-imaginable degree of the attribute ("$M$" stands for "maximum-imaginable").

Likewise the regular objects, $Z$ and $M$ can be represented on the psychological continuum and follow a normal distribution, with unknown mean value and variance (see Sect. "Basics of the Thurstone's LCJ"). The procedure of collection of respondent judgments is then modified by considering the regular objects ($O_1$, $O_2$, etc.) and the dummy/anchor objects ($Z$ and $M$). Each respondent then formulates a preference ordering of these objects, with two important requirements (see an example of questionnaire in Figure A.4, in the appendix).

1. $Z$ should be positioned at the bottom of the preference ordering, i.e, there should not be any other object with preference lower than $Z$. In the case the attribute of another object is judged to be absent, that object will be considered indifferent to $Z$ and positioned at the same hierarchical level.
2. $M$ should be positioned at the top of the preference orderings, i.e., there should not be any other object with preference higher than $M$. In the case the attribute of another object is judged to be the maximum-imaginable, that object will be considered indifferent to $M$ and positioned at the same hierarchical level.

Next, the Thurstone's scaling is performed and the resulting (interval) scale is transformed into a new one, which is defined in the conventional range [0, 10]; the following linear transformation is used:

$$\frac{y-0}{10-0}=\frac{x-x_Z}{x_M-x_Z} \;\Rightarrow\; y=10\cdot\frac{x-x_Z}{x_M-x_Z}, \tag{6}$$

where

$x_Z$ and $x_M$ are the scale values of $Z$ and $M$, resulting from the LCJ;

$x$ is the scale value of a generic object, resulting from the LCJ;

$y$ is the relevant transformed scale value, in the conventional range [0, 10].

The introduction of $Z$ and $M$ allows to anchor the LCJ scale into a new scale ($y$) with a conventional unit and an absolute zero point (since it corresponds to the absence of the attribute); it is therefore not unreasonable to consider $y$ as a *ratio* scale. On the other hand, setting the value of $M$ to 10 is a conventional assumption to make the scale comparable[6] with those obtained from analogous LCJ processes.

Let us return to the example in Sect. "Basics of the Thurstone's LCJ", in which five respondents ($R_1$ to $R_5$) formulate their preference orderings of four objects ($O_1$ to $O_4$); consistently with what explained before, these orderings also include the two dummy/anchor objects $Z$ and $M$ (see Figure 7).

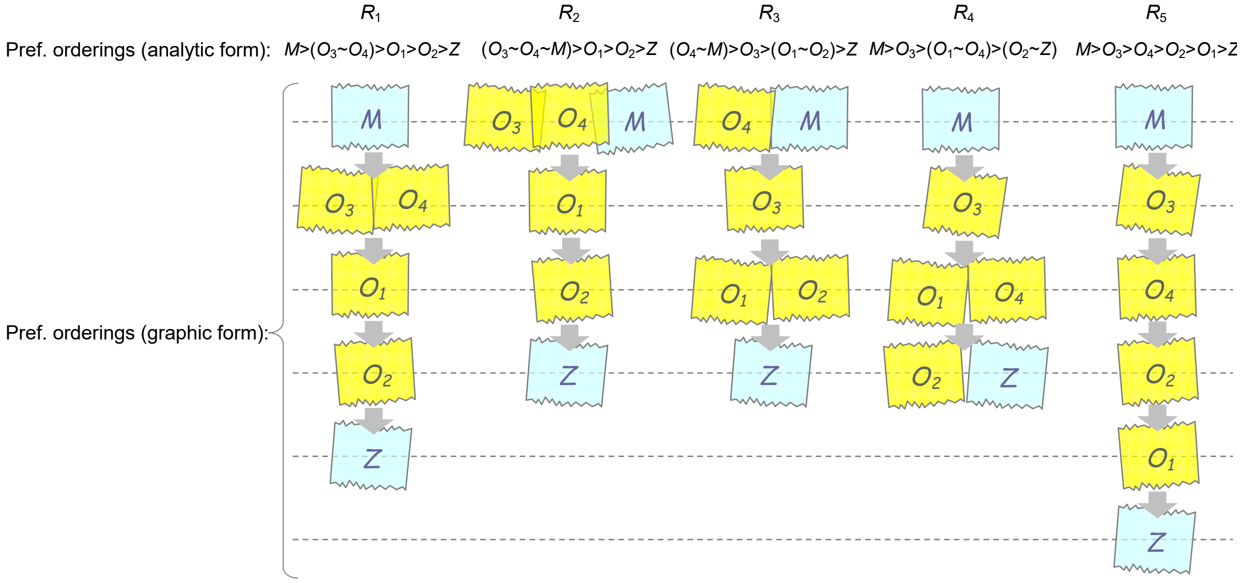

**Figure 7. Example of preference orderings formulated by five respondents (i.e., $R_1$ to $R_5$), including four regular objects ($O_1$ to $O_4$) and two dummy/anchor objects ($Z$ and $M$).**

It can be noticed that $R_4$ has positioned $Z$ and $O_2$ at the bottom of the preference ordering (absence of the attribute). On the other hand, $R_2$ and $R_3$ have positioned $M$ and other objects at the top of their orderings (maximum-imaginable degree of the attribute).

The preference orderings are then translated into paired-comparison relationships, as shown in

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[6] The adjective "comparable" means that the resulting scales should have a common unit; e.g., let us assume that the LCJ is used to evaluate the courtesy of some call-center operators, according to the judgments of a sample of customers, and this evaluation is repeated annually: without proper normalization, comparing the results of two processes would not be correct.

Figure 8(a). If one excludes the paired-comparison relationships with at least one of the dummy/anchor objects, the remaining ones are identical to those in the example in Figure 3; in other words, the problem in Figure 3 is in some ways "encapsulated" into the (more general) one in Figure 8.

(a) Respondent judgements, paired comparisons, and $f_{ij}$, $p_{ij}$, $z_{ij}$ values

| Paired comparison | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $f_{ij}$ | $p_{ij}$ | $z_{ij}$ |
|---|---|---|---|---|---|---|---|---|
| $(O_1, O_2)$ | > | > | ~ | > | < | 3.5 | 0.70 | -0.524 |
| $(O_1, O_3)$ | < | < | < | < | < | 0 | $0.00^*$ | 1.995 |
| $(O_1, O_4)$ | < | < | < | ~ | < | 0.5 | 0.10 | 1.282 |
| $(O_2, O_3)$ | < | < | < | < | < | 0 | $0.00^*$ | 1.995 |
| $(O_2, O_4)$ | < | < | < | < | < | 0 | $0.00^*$ | 1.995 |
| $(O_3, O_4)$ | ~ | ~ | < | > | > | 3 | 0.60 | -0.253 |
| $(O_1, Z)$ | > | > | > | > | > | 5 | $1.00^*$ | -1.995 |
| $(O_2, Z)$ | > | > | > | ~ | > | 4.5 | 0.90 | -1.282 |
| $(O_3, Z)$ | > | > | > | > | > | 5 | 1.00* | -1.995 |
| $(O_4, Z)$ | > | > | > | > | > | 5 | 1.00* | -1.995 |
| $(O_1, M)$ | < | < | < | < | < | 0 | $0.00^*$ | 1.995 |
| $(O_2, M)$ | < | < | < | < | < | 0 | $0.00^*$ | 1.995 |
| $(O_3, M)$ | < | ~ | < | < | < | 0.5 | 0.10 | 1.282 |
| $(O_4, M)$ | < | ~ | ~ | < | < | 1 | $1.00^*$ | -1.995 |

(b) matrix ***F***

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $O_1$ | 2.5 | 3.5 | 0.0 | 0.5 | 5.0 | 0.0 |
| $O_2$ | 1.5 | 2.5 | 0.0 | 0.0 | 4.5 | 0.0 |
| $O_3$ | 5.0 | 5.0 | 2.5 | 3.0 | 5.0 | 0.5 |
| $O_4$ | 4.5 | 5.0 | 2.0 | 2.5 | 5.0 | 1.0 |
| $Z$ | 0.0 | 0.5 | 0.0 | 0.0 | 2.5 | 0.0 |
| $M$ | 5 | 5 | 4.5 | 4 | 5 | 2.5 |

(c) matrix ***P***

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $O_1$ | 0.50 | 0.70 | 0.00* | 0.10 | 1.00* | 0.00* |
| $O_2$ | 0.30 | 0.50 | 0.00* | 0.00* | 0.90 | 0.00* |
| $O_3$ | 1.00* | 1.00* | 0.50 | 0.60 | 1.00* | 0.10 |
| $O_4$ | 0.90 | 1.00* | 0.40 | 0.50 | 1.00* | 0.20 |
| $Z$ | 0.00* | 0.10 | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 1.00* | 0.90 | 0.80 | 1.00* | 0.50 |

(d) matrix ***Z***

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $O_1$ | 0.00 | -0.52 | 2.00 | 1.28 | -2.00 | 2.00 |
| $O_2$ | 0.52 | 0.00 | 2.00 | 2.00 | -1.28 | 2.00 |
| $O_3$ | -2.00 | -2.00 | 0.00 | -0.25 | -2.00 | 1.28 |
| $O_4$ | -1.28 | -2.00 | 0.25 | 0.00 | -2.00 | 0.84 |
| $Z$ | 2.00 | 1.28 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -2.00 | -1.28 | -0.84 | -2.00 | 0.00 |
| $\Sigma_j$ | -2.75 | -5.23 | 4.96 | 4.18 | -9.26 | 8.11 |
| $\mu_j' = \Sigma_j / n$ | -0.46 | -0.87 | 0.83 | 0.70 | -1.54 | 1.35 |
| $\mu_j''$ [0,10] | **3.7** | **2.3** | **8.2** | **7.7** | **0** | **10** |

Notes:
$Z$ is a dummy/anchor object denoting the zero preference level;
$M$ is a dummy/anchor object denoting the maximum-possible preference level;
$n$=6 is the total number of objects, including $Z$ and $M$;
(*)values of $p_{ij} \leq 0.023$ and $\geq 0.977$ have been conventionally associated with $z_{ij}$ = 1.995 and -1.995 respectively;
$f_{ij}$ denotes the number of times that $O_i$ is preferred to $O_j$;
$p_{ij}$ denotes the proportion of times that $O_i$ is preferred to $O_j$;
$z_{ij} = \Phi^{-1}(1 - p_{ij})$.

**Figure 8. Example of LCJ application to the preference orderings in Figure 7: (a) paired-comparison relationships, (b) matrix *F*, (c) matrix *P*, (d) matrix *Z* and resulting Thurstone's scaling.**

Comparing the resulting Thurstone's scaling (in Figure 8(d)) with that in Figure 4 (in the absence of $Z$ and $M$), we note that the rankings of the regular objects ($O_1$ to $O_4$) are coincident (i.e., $O_2 < O_1 < O_4 < O_3$), even if the distances between the scale values are slightly changed. This reveals a certain robustness of the LCJ, although it can be demonstrated that this technique may violate the Arrow's axiom of *independence of irrelevant alternatives*[7] (IIR) (Arrow and Rayanaud, 1986; Dym et al., 2002; Van De Poel, 2007; Franceschini et al., 2015b).

Given that the introduction of $Z$ and $M$ increases the information content of preference orderings, it may also cause some variation in the results. For example, the information that the attribute of a specific object is absent or with the maximum-imaginable degree is richer than the information that the same attribute is just lower or higher than the remaining ones.

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[7] According to this axiom, the preference between two objects $O_i$ and $O_j$ should depend only on the individual preferences between $O_i$ and $O_j$ exclusively: if one object is removed, the algorithm scaling should result into the same ordering of the remaining objects.

The price to pay for this information enrichment is the increased effort of respondents, who should also consider the two dummy/anchor objects and envisage their “absolute” meaning. This certainly represents a new element of complexity (Paruolo et al. 2013).

We have experimentally verified that the proposed anchoring technique provides results in line with those obtained from other techniques in the literature. For example, it can be seen that the results in Figure 8 are strongly correlated with those obtained through the Torgerson’s technique (see the section “Torgerson’s anchoring”, in the appendix). Additionally, it was empirically observed that the correlation tends to increase for problems with a larger number of objects and/or respondents.

## New Procedure for Customer Competitive Benchmarking

The following sub-sections describe in detail the Phases 2, 3 and 4 of the new procedure for the Customer Competitive Benchmarking.

### *New phase 2*

Initially, the LCJ is used to determine the relative importance of CRs. Consistently with what proposed in Sect. “Critical Description of Customer Competitive Benchmarking”, a response mode based on preference orderings can be used. Specifically, each of the interviewed customers formulates a preference ordering of the CRs, based on their importance for the products/services in the market segment of interest. Preference orderings include the two dummy/anchor CRs: *Z*, with zero importance, and *M*, with the maximum-imaginable degree of importance.

For the purpose of example, Table 7 contains the preference orderings about the CRs for an international scientific conference, by five fictitious respondents (see Table 1). These orderings are similar to those exemplified in Figure 7; the only difference is that labels “$O_1$”, “$O_2$”, etc. are replaced with “$CR_1$”, “$CR_2$”, etc.. The application of the LCJ obviously originates the same scaling reported in Figure 8(d), which can be in turn rescaled in the range [0, 10] (through the transformation in Eq. 6), resulting in: 3.7 ($CR_1$), 2.3 ($CR_2$), 8.2 ($CR_3$) and 7.7 ($CR_4$); the scale values of the two dummy/anchor CRs are obviously 0 (*Z*) and 10 (*M*).

| Preference orderings by five respondents | | |
|---|---|---|
| $R_1$ | $M>CR_4\sim CR_3>CR_1>CR_2>Z$ | $\rightarrow I^{(1)}$: 3.7 ($CR_1$), 2.3 ($CR_2$), 8.2 ($CR_3$), 7.7 ($CR_4$), 0 ($Z$), 10 ($M$). |
| $R_2$ | $CR_3\sim CR_4\sim M>CR_1>CR_2>Z$ | |
| $R_3$ | $CR_4\sim M>CR_3>CR_2\sim CR_1>Z$ | |
| $R_4$ | $M>CR_3>CR_4\sim CR_1>Z\sim CR_2$ | |
| $R_5$ | $M>CR_3>CR_4>CR_2>CR_1>Z$ | |

*Z* is a dummy/anchor product/service denoting the zero importance;
*M* is a dummy/anchor product/service denoting the maximum-imaginable importance.

**Table 7. Preference orderings, formulated by five respondents ($R_1$ to $R_5$), on the relative importance of the four CRs in Table 1, and resulting Thurstone’s scaling; see also the intermediate steps in Figure 7 and Figure 8.**

Based on the considerations presented in Sect. “Anchoring the Thurstone’s scaling”, the new scale can be considered as a ratio one.

*New Phase 3*

**Customer satisfaction.** At first, the QFD team, with the predominant contribution of marketing experts identify three (or more) products/services to be benchmarked ($P_A$, $P_B$, …), as representative of the market segment of interest. Then, (potential) customers have to judge the level of satisfaction of each CR, formulating a preference ordering of the benchmarked products/services. The number of preference orderings by each respondent will therefore correspond to the number ($n$) of CRs. Preference orderings should also include two dummy/anchor products/services, $Z$ and $M$, which correspond respectively to the zero and the maximum-imaginable degree of satisfaction.

Returning to the previous example, let us assume that (potential) customers formulate their preference orderings of three conferences: ($P_A$) our existing conference, ($P_B$) the conference of the first competitor, and ($P_C$) the conference of the second competitor (see results in Table 8).

| | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $CR_1$ | $M > P_B > P_A > P_C > Z$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_B \sim M > P_C > P_A > Z$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_C > P_B > P_A > Z$ |
| $CR_2$ | $M > P_A > P_C \sim P_B > Z$ | $P_A \sim M > P_C \sim P_B > Z$ | $M > P_C \sim P_A > P_B > Z$ | $M > P_C > P_A \sim P_B > Z$ | $M > P_C > P_A > P_B > Z$ |
| $CR_3$ | $M > P_C \sim P_B > P_A > Z$ | $P_B \sim M > P_C \sim P_A > Z$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_C \sim P_A > P_B > Z$ |
| $CR_4$ | $P_C \sim M > P_B > Z \sim P_A$ | $P_C \sim M > P_A > P_B > Z$ | $P_C \sim M > P_B > Z \sim P_A$ | $M > P_C > P_B > Z \sim P_A$ | $M > P_C > P_B > P_A > Z$ |

$P_A$ is our existing international scientific conference;
$P_B$ is the existing conference of the first competitor;
$P_C$ is the existing conference of the second competitor;
$Z$ is a dummy/anchor product/service denoting the zero-satisfaction level;
$M$ is a dummy/anchor product/service denoting the maximum-imaginable satisfaction level.

**Table 8. Preference orderings formulated by five respondents ($R_1$ to $R_5$), on the level of satisfaction of three international scientific conferences (i.e., $P_A$, $P_B$ and $P_C$), with respect to the four CRs in Table 1. $Z$ and $M$ are two dummy/anchor conferences with zero and maximum-imaginable satisfaction level of the CRs.**

For each CR, the multi-respondent preference orderings are then aggregated through the following steps: (1) translation of preference orderings into paired-comparison relationships, (2) determination of the $f_{ij}$, $p_{ij}$ and $z_{ij}$ values (and relevant matrices, $\boldsymbol{F}$, $\boldsymbol{P}$ and $\boldsymbol{Z}$), (3) Thurstone's scaling, and (4) rescaling into the range [0, 10] (through the transformation in Eq. 6). Figure 9 reports the aggregation of the preference orderings and the resulting indicators of collective satisfaction ($S_A$, $S_B$, $S_C$), for the benchmarked conferences (see also Table 9). The comparability among these indicators is ensured by the presence of the dummy/anchor products/services ($Z$ and $M$), which are used to anchor the relevant scales with each other.

Consistently with the criticism in Sect. "Phase 3: Customer Competitive Benchmarking", the traditional procedure for determining the *improvement rate* ($I^{(2)}$) has been simplified significantly, with no need to introduce the indicator $T$.

(CR1)

(a) Respondent judgements

| Pair. comp. | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $(P_1, P_2)$ | < | < | < | < | < |
| $(P_1, P_3)$ | > | ~ | < | ~ | < |
| $(P_2, P_3)$ | > | > | > | > | < |
| $(P_1, Z)$ | > | > | > | > | > |
| $(P_2, Z)$ | > | > | > | > | > |
| $(P_3, Z)$ | > | > | > | > | > |
| $(P_1, M)$ | < | < | < | < | < |
| $(P_2, M)$ | < | ~ | ~ | < | < |
| $(P_3, M)$ | < | < | < | < | < |

(b) matrix ***F***

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 2.5 | 0.0 | 2.0 | 5.0 | 0.0 |
| $P_2$ | 5.0 | 2.5 | 4.0 | 5.0 | 1.0 |
| $P_3$ | 3.0 | 1.0 | 2.5 | 5.0 | 0.0 |
| $Z$ | 0.0 | 0.0 | 0.0 | 2.5 | 0.0 |
| $M$ | 5.0 | 4.0 | 5.0 | 5.0 | 2.5 |

(c) matrix ***P***

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.50 | 0.00* | 0.40 | 1.00* | 0.00* |
| $P_2$ | 1.00* | 0.50 | 0.80 | 1.00* | 0.20 |
| $P_3$ | 0.60 | 0.20 | 0.50 | 1.00* | 0.00* |
| $Z$ | 0.00* | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 0.80 | 1.00* | 1.00* | 0.50 |

(d) matrix ***Z*** and scaling

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.00 | 2.00 | 0.25 | -2.00 | 2.00 |
| $P_2$ | -2.00 | 0.00 | -0.84 | -2.00 | 0.84 |
| $P_3$ | -0.25 | 0.84 | 0.00 | -2.00 | 2.00 |
| $Z$ | 2.00 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -0.84 | -2.00 | -2.00 | 0.00 |
| $\Sigma_j$ | -2.25 | 3.99 | -0.59 | -7.98 | 6.83 |
| $\mu_j' = \Sigma_j / n$ | -0.45 | 0.80 | -0.12 | -1.60 | 1.37 |
| $\mu_j''$ [0, 10] | **3.9** | **8.1** | **5.0** | 0.0 | 10.0 |

(CR2)

| Pair. comp. | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $(P_1, P_2)$ | > | > | > | ~ | > |
| $(P_1, P_3)$ | > | > | ~ | < | < |
| $(P_2, P_3)$ | ~ | ~ | < | < | < |
| $(P_1, Z)$ | > | > | > | > | > |
| $(P_2, Z)$ | > | > | > | > | > |
| $(P_3, Z)$ | > | > | > | > | > |
| $(P_1, M)$ | < | ~ | < | < | < |
| $(P_2, M)$ | < | < | < | < | < |
| $(P_3, M)$ | < | < | < | < | < |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 2.5 | 4.5 | 2.5 | 5.0 | 0.5 |
| $P_2$ | 0.5 | 2.5 | 1.0 | 5.0 | 0.0 |
| $P_3$ | 2.5 | 4.0 | 2.5 | 5.0 | 0.0 |
| $Z$ | 0.0 | 0.0 | 0.0 | 2.5 | 0.0 |
| $M$ | 4.5 | 5.0 | 5.0 | 5.0 | 2.5 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.50 | 0.90 | 0.50 | 1.00* | 0.10 |
| $P_2$ | 0.10 | 0.50 | 0.20 | 1.00* | 0.00* |
| $P_3$ | 0.50 | 0.80 | 0.50 | 1.00* | 0.00* |
| $Z$ | 0.00* | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 0.90 | 1.00* | 1.00* | 1.00* | 0.50 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.00 | -1.28 | 0.00 | -2.00 | 1.28 |
| $P_2$ | 1.28 | 0.00 | 0.84 | -2.00 | 2.00 |
| $P_3$ | 0.00 | -0.84 | 0.00 | -2.00 | 2.00 |
| $Z$ | 2.00 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -1.28 | -2.00 | -2.00 | -2.00 | 0.00 |
| $\Sigma_j$ | 2.00 | -2.12 | 0.84 | -7.98 | 7.27 |
| $\mu_j' = \Sigma_j / n$ | 0.40 | -0.42 | 0.17 | -1.60 | 1.45 |
| $\mu_j''$ [0, 10] | **6.5** | **3.8** | **5.8** | 0.0 | 10.0 |

(CR3)

| Pair. comp. | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $(P_1, P_2)$ | < | ~ | < | < | > |
| $(P_1, P_3)$ | < | ~ | ~ | ~ | ~ |
| $(P_2, P_3)$ | ~ | > | > | > | < |
| $(P_1, Z)$ | > | > | > | > | > |
| $(P_2, Z)$ | > | > | > | > | > |
| $(P_3, Z)$ | > | > | > | > | > |
| $(P_1, M)$ | < | < | < | < | < |
| $(P_2, M)$ | < | ~ | < | < | < |
| $(P_3, M)$ | < | < | < | < | < |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 2.5 | 1.0 | 2.0 | 5.0 | 0.0 |
| $P_2$ | 4.0 | 2.5 | 3.5 | 5.0 | 0.5 |
| $P_3$ | 3.0 | 1.5 | 2.5 | 5.0 | 0.0 |
| $Z$ | 0.0 | 0.0 | 0.0 | 2.5 | 0.0 |
| $M$ | 5.0 | 4.5 | 5.0 | 5.0 | 2.5 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.50 | 0.20 | 0.40 | 1.00* | 0.00* |
| $P_2$ | 0.80 | 0.50 | 0.70 | 1.00* | 0.10 |
| $P_3$ | 0.60 | 0.30 | 0.50 | 1.00* | 0.00* |
| $Z$ | 0.00* | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 0.90 | 1.00* | 1.00* | 0.50 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.00 | 0.84 | 0.25 | -2.00 | 2.00 |
| $P_2$ | -0.84 | 0.00 | -0.52 | -2.00 | 1.28 |
| $P_3$ | -0.25 | 0.52 | 0.00 | -2.00 | 2.00 |
| $Z$ | 2.00 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -1.28 | -2.00 | -2.00 | 0.00 |
| $\Sigma_j$ | -1.09 | 2.08 | -0.27 | -7.98 | 7.27 |
| $\mu_j' = \Sigma_j / n$ | -0.22 | 0.42 | -0.05 | -1.60 | 1.45 |
| $\mu_j''$ [0, 10] | **4.5** | **6.6** | **5.1** | 0.0 | 10.0 |

(CR4)

| Pair. comp. | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ |
|---|---|---|---|---|---|
| $(P_1, P_2)$ | < | > | < | < | < |
| $(P_1, P_3)$ | < | < | < | < | < |
| $(P_2, P_3)$ | < | < | < | < | < |
| $(P_1, Z)$ | ~ | > | ~ | ~ | > |
| $(P_2, Z)$ | > | > | > | > | > |
| $(P_3, Z)$ | > | > | > | > | > |
| $(P_1, M)$ | < | < | < | < | < |
| $(P_2, M)$ | < | < | < | < | < |
| $(P_3, M)$ | ~ | ~ | ~ | < | < |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 2.5 | 1.0 | 0.0 | 3.5 | 0.0 |
| $P_2$ | 4.0 | 2.5 | 0.0 | 5.0 | 0.0 |
| $P_3$ | 5.0 | 5.0 | 2.5 | 5.0 | 1.5 |
| $Z$ | 1.5 | 0.0 | 0.0 | 2.5 | 0.0 |
| $M$ | 5.0 | 5.0 | 3.5 | 5.0 | 2.5 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.50 | 0.20 | 0.00* | 0.70 | 0.00* |
| $P_2$ | 0.80 | 0.50 | 0.00* | 1.00* | 0.00* |
| $P_3$ | 1.00* | 1.00* | 0.50 | 1.00* | 0.30 |
| $Z$ | 0.30 | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 1.00* | 0.70 | 1.00* | 0.50 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_1$ | 0.00 | 0.84 | 2.00 | -0.52 | 2.00 |
| $P_2$ | -0.84 | 0.00 | 2.00 | -2.00 | 2.00 |
| $P_3$ | -2.00 | -2.00 | 0.00 | -2.00 | 0.52 |
| $Z$ | 0.52 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -2.00 | -0.52 | -2.00 | 0.00 |
| $\Sigma_j$ | -4.31 | -1.15 | 5.46 | -6.51 | 6.51 |
| $\mu_j' = \Sigma_j / n$ | -0.86 | -0.23 | 1.09 | -1.30 | 1.30 |
| $\mu_j''$ [0, 10] | **1.7** | **4.1** | **9.2** | 0.0 | 10.0 |

↑ ↑ ↑

$\boldsymbol{S_A}$ $\boldsymbol{S_B}$ $\boldsymbol{S_C}$

Notes:
$Z$ is a dummy/anchor product denoting the zero-satisfaction level;
$M$ is a dummy/anchor product denoting the maximum-possible satisfaction level;
$n$=5 is the total number of products, including the two dummy ones;
(*)values of $p_{ij} \leq 0.023$ and $\geq 0.977$ have been conventionally associated with $z_{ij}$ = 1.995 and -1.995 respectively;
$f_{ij}$ denotes the number of times that $P_i$ is preferred to $P_j$;
$p_{ij}$ denotes the proportion of times that $P_i$ is preferred to $P_j$;
$z_{ij} = \Phi^{-1}(1 - p_{ij})$;
$S_A$, $S_B$ and $S_C$ (bold) are the resulting indicators of collective satisfaction.

**Figure 9. CR-by-CR results of the Thurstone's scaling for the customer-satisfaction preference orderings in Table 8.**

Given that (i) the relative positioning of the benchmarked products/services is considered when determining the $S_A$ values of our existing product/service and (ii) the dummy/anchor product/service $M$ = 10 depicts the maximum-imaginable satisfaction, to which our company should asymptotically aim, we suggest that:

$$I^{(2)} = M - S_A = 10 - S_A. \tag{7}$$

$I^{(2)}$ can be interpreted as an indicator of *potential improvement*, for a certain CR of our existing product/service. This does not necessarily mean that the new product/service should achieve the maximum-imaginable satisfaction level for each CR: this will depend on the mix of technical characteristics related to the CRs and on their specific values (i.e, Phases 8 and 9 of the HoQ). The resulting scale of $I^{(2)}$ will obviously be included between 0 (no potential improvement) and 10 (maximum-imaginable potential improvement); see the example in Table 9. It is worth remarking that this new logic simplifies the traditional procedure, eliminating $T$ and therefore any possible ambiguity in the interpretation of this indicator.

|  | $S_A$ | $S_B$ | $S_C$ | $I^{(2)}$ |
|---|---|---|---|---|
| $CR_1$ | 3.9 | 8.1 | 5.0 | 6.1 |
| $CR_2$ | 6.5 | 3.8 | 5.8 | 3.5 |
| $CR_3$ | 4.5 | 6.6 | 5.1 | 5.5 |
| $CR_4$ | 1.7 | 4.1 | 9.2 | 8.3 |

**Table 9. Summary of the results of the scaling process in Figure 9: $S_A$, $S_B$ and $S_C$ respectively depict the level of satisfaction of each CR for the benchmarked products/services ($P_A$, $P_B$ and $P_C$), while $I^{(2)}$ is calculated according to Eq. 7.**

**Sales points.** The QFD team should identify the CRs that mostly affect sales. Since this evaluation is subjective, we suggest the QFD experts express their individual judgments, which are then aggregated into a collective scaling. Again, the Thurstone’s LCJ seems well suited for this purpose. In practice, each expert formulates a preference ordering of the CRs, depending on their impact on sales; this ordering also includes the two dummy/anchor CRs, $Z$ and $M$, respectively corresponding to the absence and the maximum-imaginable degree of influence on sales. Subsequently, preference orderings are translated into paired-comparison relationships and the Thurstone’s scaling is applied. Figure 10 exemplifies this construction for the HoQ related to the international scientific conference. It can be noticed that the number of hierarchical levels in the preference orderings by three experts ($E_1$ to $E_3$) tends to be low, since just a few CRs are likely to be strategic.

As shown, the resulting $I^{(3)}$ values of $CR_1$ to $CR_4$ are respectively 1.8, 9.5, 5.3 and 1.5, denoting great impact on sales of $CR_2$, intermediate impact of $CR_3$, low impact of $CR_1$ and $CR_4$. The benefit of this procedure is to drive the QFD team to a collective choice, overcoming the conflicting opinions by individual experts. Likewise $I^{(1)}$ and $I^{(2)}$, $I^{(3)}$ can be considered as defined on a *ratio* scale.

(a) Expert ($E_1$ to $E_3$) judgements, paired-comparison relationships, and relevant values of $f_{ij}$, $p_{ij}$, and $z_{ij}$

| Expert | Preference ordering |
|---|---|
| $E_1$ | $M > CR_2 > CR_3 > CR_1 > Z \sim CR_4$ |
| $E_2$ | $CR_2 \sim M > CR_3 > CR_1 > Z \sim CR_4$ |
| $E_3$ | $CR_2 \sim M > CR_4 \sim CR_3 > Z \sim CR_1$ |

| Paired comparison | $E_1$ | $E_2$ | $E_3$ |
|---|---|---|---|
| ($CR_1$, $CR_2$) | < | < | < |
| ($CR_1$, $CR_3$) | < | < | < |
| ($CR_1$, $CR_4$) | > | > | < |
| ($CR_2$, $CR_3$) | > | > | > |
| ($CR_2$, $CR_4$) | > | > | > |
| ($CR_3$, $CR_4$) | > | > | ~ |
| ($CR_1$, $Z$) | > | > | ~ |
| ($CR_2$, $Z$) | > | > | > |
| ($CR_3$, $Z$) | > | > | > |
| ($CR_4$, $Z$) | ~ | ~ | > |
| ($CR_1$, $M$) | < | < | < |
| ($CR_2$, $M$) | < | ~ | ~ |
| ($CR_3$, $M$) | < | < | < |
| ($CR_4$, $M$) | < | < | < |

| $f_{ij}$ | $p_{ij}$ | $z_{ij}$ |
|---|---|---|
| 0 | 0.00* | 1.995 |
| 0 | 0.00* | 1.995 |
| 2 | 0.67 | -0.431 |
| 3 | 1.00* | -1.995 |
| 3 | 1.00* | -1.995 |
| 2.5 | 0.83 | -0.967 |
| 2.5 | 0.83 | -0.967 |
| 3 | 1.00* | -1.995 |
| 3 | 1.00* | -1.995 |
| 2 | 0.67 | -0.431 |
| 0 | 0.00* | 1.995 |
| 1 | 0.33 | 0.431 |
| 0 | 0.00* | 1.995 |
| 0 | 0.00* | 1.995 |

(b) matrix ***F***

| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $CR_1$ | 1.5 | 0.0 | 0.0 | 2.0 | 2.5 | 0.0 |
| $CR_2$ | 3.0 | 1.5 | 3.0 | 3.0 | 3.0 | 1.0 |
| $CR_3$ | 3.0 | 0.0 | 1.5 | 2.5 | 3.0 | 0.0 |
| $CR_4$ | 1.0 | 0.0 | 0.5 | 1.5 | 2.0 | 0.0 |
| $Z$ | 0.5 | 0.0 | 0.0 | 1.0 | 1.5 | 0.0 |
| $M$ | 3 | 2 | 3 | 3 | 3 | 1.5 |

(c) matrix ***P***

| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $CR_1$ | 0.50 | 0.00* | 0.00* | 0.67 | 0.83 | 0.00* |
| $CR_2$ | 1.00* | 0.50 | 1.00* | 1.00* | 1.00* | 0.33 |
| $CR_3$ | 1.00* | 0.00* | 0.50 | 0.83 | 1.00* | 0.00* |
| $CR_4$ | 0.33 | 0.00* | 0.17 | 0.50 | 0.67 | 0.00* |
| $Z$ | 0.17 | 0.00* | 0.00* | 0.33 | 0.50 | 0.00* |
| $M$ | 1.00* | 0.67 | 1.00* | 1.00* | 1.00* | 0.50 |

(d) matrix ***Z***

| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| $CR_1$ | 0.00 | 2.00 | 2.00 | -0.43 | -0.97 | 2.00 |
| $CR_2$ | -2.00 | 0.00 | -2.00 | -2.00 | -2.00 | 0.43 |
| $CR_3$ | -2.00 | 2.00 | 0.00 | -0.97 | -2.00 | 2.00 |
| $CR_4$ | 0.43 | 2.00 | 0.97 | 0.00 | -0.43 | 2.00 |
| $Z$ | 0.97 | 2.00 | 2.00 | 0.43 | 0.00 | 2.00 |
| $M$ | -2.00 | -0.43 | -2.00 | -2.00 | -2.00 | 0.00 |
| $\Sigma_j$ | -4.59 | 7.55 | 0.97 | -4.96 | -7.38 | 8.41 |
| $\mu_j' = \Sigma_j / n$ | -0.76 | 1.26 | 0.16 | -0.83 | -1.23 | 1.40 |
| $\mu_j''$ [0-10] $= \boldsymbol{I}^{(3)}$ | **1.8** | **9.5** | **5.3** | **1.5** | **0.0** | **10.0** |

Notes:
$Z$ is a dummy CR with zero importance for sales points;
$M$ is a dummy CR with maximum-possible importance for sales points;
$n$=6 is the total number of CRs, including the dummy ones;
$f_{ij}$ denotes the number of times that $CR_i$ is preferred to $CR_j$;
$p_{ij}$ denotes the proportion of times that $CR_i$ is preferred to $CR_j$;
$z_{ij} = \Phi^{-1}(1 - p_{ij})$;
$^{(*)}$values of $p_{ij} \leq 0.023$ and $\geq 0.977$ have been conventionally associated with $z_{ij}$ = 1.995 and -1.995 respectively.

**Figure 10. Preference orderings of three QFD experts ($E_1$ to $E_3$), on the importance of each CR for sales, and application of the LCJ.**

### *New phase 4*

The fact that the three (new) importance sub-indicators, $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$, are defined on ratio scales entails that their aggregation through a multiplicative model (like the one in Eq. 2) is no longer incorrect. Although we are aware of the possible advantages of multiplicative models with respect to the additive ones (Ebert and Welsch, 2004), we believe that a (weighted) additive model would be preferable:

$$I = w^{(1)} \cdot I^{(1)} + w^{(2)} \cdot I^{(2)} + w^{(3)} \cdot I^{(3)}. \tag{8}$$

This choice can be justified by the following reasons:

1. This model allows the QFD team to determine (strategy) weights ($w^{(1)}$, $w^{(2)}$ and $w^{(3)}$) of the three sub-indicators in a simple way. For example, considering the new product/service of an emerging company that struggles to gain market shares (e.g., improving its products/services with respect to those by competitors), it could be appropriate to rise $w^{(2)}$; on the other hand, considering the new product/service of a company with a consolidated brand image to preserve, it could be appropriate to rise $w^{(3)}$. Of course, the choice of weights is somehow "political" and should be made by the QFD team, depending on the design/strategic objectives. The scientific literature contains a variety of techniques to drive this operation (Vora et al., 2014; Wang et al.,

2014).

2. The *comparability* between $I^{(1)}$, $I^{(2)}$, and $I^{(3)}$ is ensured by the fact that these sub-indicators are defined on ratio scales with comparable zero and a conventional unit, corresponding to $(M - Z)/10$.
3. Although weights could theoretically be introduced into multiplicative models, such as,

$$I = \left[I^{(1)}\right]^{w^{(1)}} \cdot \left[I^{(2)}\right]^{w^{(2)}} \cdot \left[I^{(3)}\right]^{w^{(3)}}, \tag{9}$$

we think that it would be relatively difficult for the QFD team to control their influence on the final result (Chan and Wu, 2005; Van De Poel, 2007). For example, as discussed in Sect. "Critical Description of Customer Competitive Benchmarking", the use of multiplicative models (weighted or not) would make the substitution rate of sub-indicators change unpredictably.

Additionally, the model in Eq. 8 allows to analyse the contributions of the importance sub-indicators; Figure 11(a) shows the resulting $I$ values and the contributions of $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$ for the HoQ related to the international scientific conference. In this case, weights are conventionally set to $w^{(1)} = 50\%$, $w^{(2)} = 30\%$, and $w^{(3)} = 20\%$. In particular, the graph in Figure 11(b) may be used to visualize these contributions.

(a) Calculation of the importance contributions

| | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ | $w^{(1)} \cdot I^{(1)}$ | $w^{(2)} \cdot I^{(2)}$ | $w^{(3)} \cdot I^{(3)}$ | $I$ | |
|---|---|---|---|---|---|---|---|---|
| $CR_1$ | 3.7 | 6.1 | 1.8 | 1.9 | 1.8 | 0.4 | 4.1 | (18.8%) |
| $CR_2$ | 2.3 | 3.5 | 9.5 | 1.2 | 1.0 | 1.9 | 4.1 | (18.9%) |
| $CR_3$ | 8.2 | 5.5 | 5.3 | 4.1 | 1.6 | 1.1 | 6.2 | (31.4%) |
| $CR_4$ | 7.7 | 8.3 | 1.5 | 3.9 | 2.5 | 0.3 | 4.4 | (30.8%) |

Notes:
$I^{(1)}$ is the relative importance of a CR;
$I^{(2)}$ is the potential improvement for that CR, in terms of customer satisfaction;
$I^{(3)}$ is the strategic importance of that CR, for sales points;
Weights are conventionally set to $w^{(1)} = 50\%$, $w^{(2)} = 30\%$ and $w^{(3)} = 20\%$, respectively.

(b) Diagram of the importance contributions

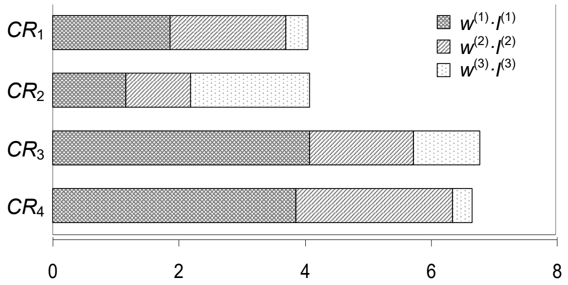

**Figure 11. Decomposition and visualization of the importance contributions to *I*, for the HoQ related to the international scientific conference.**

### *Summary*

The map in Table 10 summarized the new procedure, from the perspective of the relevant activities and responsibilities of the parties involved.

| HoQ Phase | QFD-team's experts | (Potential) customers | (Automatable) data processing |
|---|---|---|---|
| 2 - Relative Importance Ratings | | (i) Each customer formulates a preference ordering of the relative importance of CRs. | (ii) Application of the LCJ and determination of the $I^{(1)}$ values related to the CRs. |
| 3 - Customer Competitive Benchmarking | (i) Marketing experts of the QFD team identify the products/services to be benchmarked. | (ii) Each customer formulates a preference ordering of the level of satisfaction for the benchmarked products/services. The procedure is repeated for each CR. | (iii) Multiple application of the LCJ and determination of the indicators of collective satisfaction ($S_A$, $S_B$, ...) for the benchmarked products/services. |
| | | | (iv) New procedure to determine the $I^{(2)}$ values. |
| | (v) Each marketing expert of the QFD team formulates a preference ordering related to sales points, for the new product/service. | | (vi) Application of the LCJ to the preference orderings and determination of the $I^{(3)}$ values. |
| 4 - Final Importance Ratings | (i) Identification of the weights related to the importance dimensions: $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$. | | (ii) Aggregation of $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$, into $I$, through a weighted additive model. |

**Table 10. Map of the new procedure for Customer Competitive Benchmarking, with a synthetic description of the activities (in chronological order) and relevant responsibilities of the parties involved.**

## Real-life Application

This section shows a real-life application of the proposed procedure to design a civilian aircraft seat, from the perspective of passengers; see also (Franceschini and Maisano, 2015).

First, a QFD team of experts with cross-functional competences (e.g., marketing, design, quality, production, ...) is set up. Through market survey, a sample of $m = 30$ respondents – i.e., regular air passengers – are selected to identify the CRs by individual interview, focus groups and existing information. Finally, $n = 12$ relevant CRs (reported in Table 11) are identified to represent the major concerns of customers.

Subsequently, the QFD team, with the predominant contribution of marketing experts, identifies three existing products for the customer competitive benchmarking: ($P_A$) the aircraft seat of our company, ($P_B$) that of the first competitor, and ($P_C$) that of the second competitor.

| Abbr. | Description |
|---|---|
| $CR_1$ | Comfortable (does not give you back ache) |
| $CR_2$ | Enough leg room |
| $CR_3$ | Comfortable when you recline |
| $CR_4$ | Does not hit person behind when you recline |
| $CR_5$ | Comfortable seat belt |
| $CR_6$ | Seat belt feels safe |
| $CR_7$ | Arm rests not too narrow |
| $CR_8$ | Arm rest folds right away |
| $CR_9$ | Does not make you sweat |
| $CR_{10}$ | Does not soak up a spilt drink |
| $CR_{11}$ | Hole in tray for coffee cup |
| $CR_{12}$ | Magazines can be easily removed from rack |

**Table 11. List of the relevant CRs related to an aircraft seat, from the perspective of passengers.**

Through a questionnaire (see the model in Figure A.4, in the appendix), each respondent should then formulate a preference ordering about the importance of the CRs and $n = 12$ preference orderings on the satisfaction level of each of these CRs, for the benchmarked products: i.e., a total of 1+12 = 13 preference orderings.

The preference orderings related to the importance of CRs are reported in Table A.4 (in the appendix); it can be seen that they include the two dummy/anchor CRs, *Z* and *M*. Despite the possible difficulty in imaging the latter two CRs, respondents – thanks to the indications in Figure A.4 (in the appendix) – formulated their orderings correctly. These orderings are transformed into paired-comparison data and the Thurstone's LCJ is applied for obtaining a scaling of the CRs. The corresponding matrices $\boldsymbol{F}$, $\boldsymbol{P}$ and $\boldsymbol{Z}$ are shown respectively in Figures A.5, A.6 and A.7 (in the appendix), while the column "$I^{(1)}$", in Table 12, summarizes the resulting scaling. The unit and the origin of the resulting scale are transformed through Eq. 6, so as to be included in the interval [0, 10].

Next, the focus is on the level of satisfaction of CRs for the benchmarked products. Table A.5 (in the appendix) shows the $n·m = 12·30 = 360$ preference orderings formulated by the *m* respondents, while Figure A.8 (in the appendix) shows the relevant matrices $\boldsymbol{F}$, $\boldsymbol{P}$ and $\boldsymbol{Z}$, and the resulting CR-by-CR scaling. These results are also summarized in Table 12, in the form of collective satisfaction indicators ($S_A$, $S_B$, and $S_C$) for each of the benchmarked products.

Consistently with what explained in Sect. "New Procedure for Customer Competitive Benchmarking", the $I^{(2)}$ indicator is defined as the complement-to-ten of $S_A$ (see Eq. 7) Obviously, the CRs with higher values of $I^{(2)}$ are those with lower $S_A$ values, such as $CR_{10}$, $CR_{12}$ and $CR_{11}$ (see Table 12).

Ten marketing experts ($E_1$ to $E_{10}$) within the QFD team are then identified; these experts formulate their preference orderings on the strategic importance of CRs, from the perspective of sales. The resulting preference orderings are reported in Table A.6 (in the appendix). The LCJ is then applied: the relevant matrices $\boldsymbol{F}$, $\boldsymbol{P}$ and $\boldsymbol{Z}$ are reported respectively in Figures A.9, A.10 and A.11 (in the appendix). The resulting scaling is summarized in Table 12 (in the form of the $I^{(3)}$ values of the CRs). It can be seen that only four CRs have a relatively high score (i.e., $CR_1$, $CR_3$, $CR_7$, and $CR_9$), due to the low impact of the remaining ones for sales.

| | $S_A$ | $S_B$ | $S_C$ | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ |
|---|---|---|---|---|---|---|
| $CR_1$ | 8.6 | 5.6 | 2.5 | 9.1 | 1.4 | 9.7 |
| $CR_2$ | 3.8 | 5.5 | 6.7 | 8.4 | 6.2 | 1.5 |
| $CR_3$ | 7.7 | 8.2 | 1.9 | 4.6 | 2.3 | 9.6 |
| $CR_4$ | 5.0 | 3.5 | 9.1 | 3.4 | 5.0 | 2.4 |
| $CR_5$ | 2.2 | 8.3 | 6.4 | 8.2 | 7.8 | 2.0 |
| $CR_6$ | 2.7 | 3.8 | 8.0 | 7.1 | 7.3 | 0.8 |
| $CR_7$ | 5.0 | 7.0 | 3.4 | 7.5 | 5.0 | 7.7 |
| $CR_8$ | 6.2 | 6.3 | 4.3 | 3.8 | 3.8 | 1.4 |
| $CR_9$ | 7.8 | 1.9 | 7.8 | 5.8 | 2.2 | 8.8 |
| $CR_{10}$ | 3.5 | 5.6 | 7.3 | 1.2 | 6.5 | 1.5 |
| $CR_{11}$ | 2.3 | 3.7 | 2.4 | 1.8 | 7.7 | 0.8 |
| $CR_{12}$ | 4.0 | 6.1 | 7.3 | 1.3 | 6.0 | 1.4 |

**Table 12. $S_A$, $S_B$, $S_C$ values resulting from the competitive benchmarking of three aircraft seats (their calculation is reported in Figure A.6, in the appendix), and indicators $I^{(1)}$, $I^{(2)}$ and $I^{(3)}$.**

Once the values of the three importance sub-indicators are determined for each CR, the QFD team defines the weights to be used in their aggregation; for this application: $w^{(1)} = 50\%$, $w^{(2)} = 30\%$, and $w^{(3)} = 20\%$. A relatively high value of $w^{(2)}$ was determined, in order to encourage the improvement of the new product with respect to the competing ones.

Finally, the *I* values of the CRs are calculated through the formula in Eq. 8 (see results in Figure 12).

(a) Calculation of the importance contributions

| | $I^{(1)}$ | $I^{(2)}$ | $I^{(3)}$ | $w^{(1)}\cdot I^{(1)}$ | $w^{(2)}\cdot I^{(2)}$ | $w^{(3)}\cdot I^{(3)}$ | $I$ |
|---|---|---|---|---|---|---|---|
| $CR_1$ | 9.1 | 1.4 | 9.7 | 4.5 | 0.4 | 1.9 | 6.9 (11.7%) |
| $CR_2$ | 8.4 | 6.2 | 1.5 | 4.2 | 1.9 | 0.3 | 6.4 (10.8%) |
| $CR_3$ | 4.6 | 2.3 | 9.6 | 2.3 | 0.7 | 1.9 | 4.9 (8.3%) |
| $CR_4$ | 3.4 | 5.0 | 2.4 | 1.7 | 1.5 | 0.5 | 3.7 (6.3%) |
| $CR_5$ | 8.2 | 7.8 | 2.0 | 4.1 | 2.3 | 0.4 | 6.8 (11.6%) |
| $CR_6$ | 7.1 | 7.3 | 0.8 | 3.6 | 2.2 | 0.2 | 5.9 (10.0%) |
| $CR_7$ | 7.5 | 5.0 | 7.7 | 3.8 | 1.5 | 1.5 | 6.8 (11.5%) |
| $CR_8$ | 3.8 | 3.8 | 1.4 | 1.9 | 1.1 | 0.3 | 3.3 (5.6%) |
| $CR_9$ | 5.8 | 2.2 | 8.8 | 2.9 | 0.7 | 1.8 | 5.3 (9.0%) |
| $CR_{10}$ | 1.2 | 6.5 | 1.5 | 0.6 | 2.0 | 0.3 | 2.9 (4.8%) |
| $CR_{11}$ | 1.8 | 7.7 | 0.8 | 0.9 | 2.3 | 0.2 | 3.4 (5.7%) |
| $CR_{12}$ | 1.3 | 6.0 | 1.4 | 0.7 | 1.8 | 0.3 | 2.7 (4.6%) |

Notes:
$I^{(1)}$ is the relative importance of a CR;
$I^{(2)}$ is the potential improvement, in terms of customer satisfaction, for that CR;
$I^{(3)}$ is the strategic importance of that CR, for sales points;
Weights are set to $w^{(1)} = 50\%$, $w^{(2)} = 30\%$ and $w^{(3)} = 20\%$, respectively.

(b) Diagram of the importance contributions

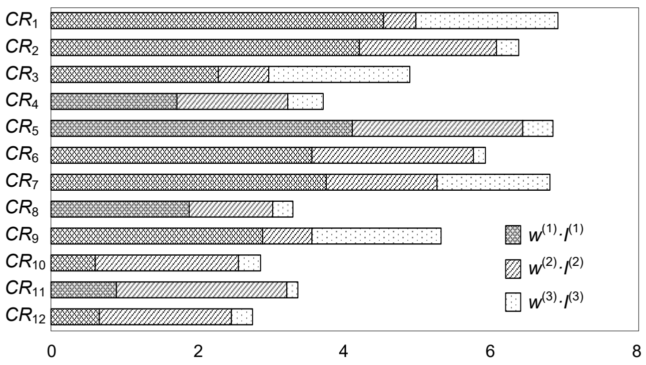

**Figure 12. Decomposition and visualization of the three importance contributions to *I*, for the CRs related to an aircraft seat.**

## Conclusions

This paper has critically analysed the traditional Customer Competitive Benchmarking of the QFD's HoQ. Several weaknesses are "hidden" in the activities of collection and aggregation of multi-respondent judgments, i.e., those concerning the relative importance of CRs, the level of satisfaction of the benchmarked products/services, and the identification of sales points. Some of the most significant weaknesses concern the subjective use of various (ordinal) response scales and their undue "promotion" to ratio scales. Additionally, the aggregation of the three importance sub-indicators ($I^{(1)}$, $I^{(2)}$, and $I^{(3)}$) through a multiplicative model is conceptually debatable.

This paper has then proposed an alternative procedure to overcome these weaknesses, based on multiple applications of the Thurstone's LCJ, in order to transform preference orderings by multiple individuals into a collective scaling. The comparability between several scales is ensured by the introduction of two dummy/anchor objects (*Z* and *M*) in preference orderings; this is certainly an important improvement with respect to a previous research in which the LCJ is used only for the prioritization of the HoQ's CRs. It was also suggested to aggregate $I^{(1)}$, $I^{(2)}$, and $I^{(3)}$ through a weighted additive model, which is intuitive and easy to manage by the QFD team. This alternative procedure does not compromise simplicity in data collection and processing, it is relatively effective and can be largely automated. In fact, it can be implemented using a common spreadsheet on a PC, allowing to obtain a solution with irrelevant processing time.

Apart from the limitations of the LCJ, which relies on some relatively strong assumptions/postulates, a limitation of the proposed procedure is that preference orderings may not be appropriate in some contexts (e.g., telephone or street interviews). In addition, the introduction of the two anchor objects ($Z$ and $M$) may complicate the formulation of preference orderings for respondents.

Regarding the future, we plan to generalize the proposed procedure and/or adapt it to more complex response modes, such as those consisting of (1) *partial* preference orderings (i.e., orderings that may also include relationships of *incomparability* (Nederpelt and Kamareddine, 2004)), or (2) judgements by respondents that are not necessarily equi-important (Franceschini and Maisano, 2017).

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## Appendix

### *Example of Improper Scale Promotion*

Let us focus the attention on the example in Figure 4. The application of the LCJ leads to a certain scaling (i.e., with arbitrary zero position) of the objects $O_1$, $O_2$, $O_3$ and $O_4$. The following transformation, i.e., the so-called *min-max normalization*, is sometimes applied pretending to "promote" this *interval* scale to a *ratio* one (Larose, 2014):

$$\frac{\mu_j''-0}{10-0}=\frac{\mu_j'-min(\mu_j')}{max(\mu_j')-min(\mu_j')}, \tag{A.1}$$

*min*() and *max*() being the minimum and maximum operator, respectively. The min-max normalization results into the new scale values reported in Table A.1.

| | Before | After |
|---|---|---|
| $O_1$ | -0.688 | 2.0 |
| $O_2$ | -1.129 | 0.0 |
| $O_3$ | 1.061 | 10.0 |
| $O_4$ | 0.756 | 8.6 |

**Table A.1. Scale values of four objects ($O_1$ to $O_4$), before and after applying the *min-max normalization* (in Eq. A.1).**

Although this transformation may seem apparently reasonable, it leads to an arbitrary zero assignment for the object with the minimum scale value ($O_2$), which does not necessarily imply the absence of the attribute considered.

To clarify the concept, let us assume to know in advance the "true" scale values of the four objects on the psychological continuum, which has a non-arbitrary zero point and a point corresponding to the maximum-imaginable degree of the attribute (see Figure A.1(a)); this scale can therefore be considered as a *ratio* one. Based on the paired-comparison relationships of these scale values (e.g., $O_1 > O_2$, $O_1 < O_3$, etc.), we then apply the LCJ and obtain an interval scale with arbitrary zero and unit (see Figure A.1(b)). Not surprisingly, the mutual distances in the latter scale reflect those in the former one, up to a certain scale factor, while the zero position has changed arbitrarily. We also notice that the LCJ takes into account the relative preferences between pairs of objects, ignoring "anchors", such as the absolute-zero point or the point corresponding to the maximum-imaginable degree.

Applying the *min-max normalization* in Eq. A.1, we obtain a third scale, whose zero corresponds to the object with lowest preference ($O_2$) and maximum value (10) to the object with highest preference ($O_3$) (see Figure A.1(c)). It may be noticed that the latter scale presents an arbitrary repositioning of the zero and a "contraction" of the unit, with respect to the initial one. As a conclusion, the proposed scale promotion is conceptually wrong and misleading.

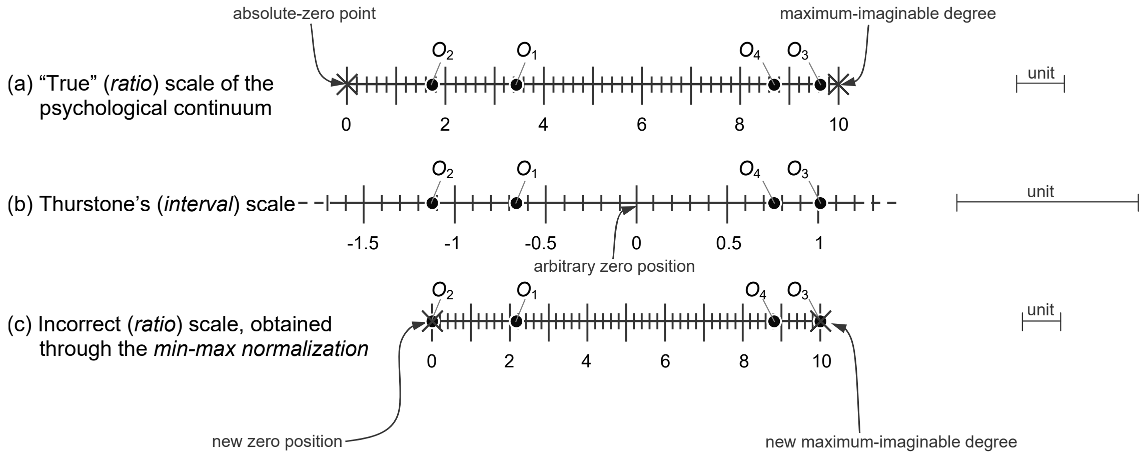

absolute-zero point
maximum-imaginable degree
(a) "True" (ratio) scale of the psychological continuum
O2
O1
O4
O3
0
2
4
6
8
10
unit
(b) Thurstone's (interval) scale
-1.5
-1
-0.5
0
0.5
1
arbitrary zero position
(c) Incorrect (ratio) scale, obtained through the min-max normalization
new zero position
new maximum-imaginable degree

**Figure A.1. Example of possible distortions due to the improper promotion of the Thurstone' (interval) scale to a ratio one, through the *min-max normalization*. E.g., the ratio between $O_4$ and $O_1$ is about 8.7/3.45 = 2.52 in the "true" scale (a), while being about 8.85/2.15 = 4.12 in the incorrect scale (c).**

### *Torgerson's anchoring*

This section exemplifies the anchoring technique by Torgerson (1958, page 196), applying it to the LCJ scaling in Figure 3 and Figure 4. The rationale of the Torgerson's anchoring is that results of the LCJ are (at least roughly) correlated with those resulting from the so-called *Method of Single Stimuli*, in which each judge directly assigns the objects' scale values, with respect to two anchors: (1) a (presumed) absolute zero, corresponding to the absence of the attribute, and (2) the *maximum-imaginable* degree of the attribute, conventionally set to 5. While aware of the difficulty and potential roughness of these direct assignments, Torgerson (1958, page 196) suggests their use just for the purpose of anchoring the LCJ scale.

Subsequently, judge assignments are aggregated – object by object – through a central tendency indicator, such as the mean or median value ($s$), and plot against the scale values ($x$) resulting from the LCJ. Then, a straight line to the points is fitted and the intercept on the horizontal axis ($s$=0) is taken as estimate of the position of the absolute-zero point ($Z$) and that on the horizontal line ($s$=5) as estimate of the position of the point with maximum-imaginable degree ($M$) of the attribute.

Considering the example in Figure 3, we hypothesize that the five judges directly assign the objects' scale values on a rating scale from 0 to 5, with unitary resolution; the zero point corresponds to the absence of the attribute while the maximum value (i.e., 5) corresponds to the maximum-imaginable degree of the attribute. Table A.2 collects these assignments.

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ |
|---|---|---|---|---|
| $J_1$ | 2 | 1 | 4 | 4 |
| $J_2$ | 4 | 1 | 5 | 5 |
| $J_3$ | 2 | 2 | 3 | 5 |
| $J_4$ | 2 | 0 | 4 | 2 |
| $J_5$ | 1 | 2 | 4 | 3 |
| Mean | 2.2 | 1.2 | 4 | 3.8 |

**Table A.2. Direct assignments of the scale values for four objects ($O_1$ to $O_4$), by five judges ($J_1$ to $J_5$). The rating scale in use is included between 0 (absence of the attribute) and 5 (maximum-imaginable degree) and has a unitary resolution.**

Assignments are then aggregated using the *arithmetic mean*. The graph in Figure A.2 plots the resulting mean values ($s$) against the scale values ($x$) resulting from the LCJ (see Figure 4). Then, a straight tendency line is fitted (through a linear least-squares regression) and the intersection of this line with the horizontal axis ($s$=0) determines an estimates of the absolute-zero point ($Z$, first anchor), while that with the horizontal line $s$=5 determines an estimate of the point ($M$, second anchor) of the maximum-imaginable degree of the attribute on the Thurstone’s scale. Next, the LCJ scale values are normalized in the conventional range [0, 10], through the linear transformation in Eq. 6. This scale can reasonably be considered as a ratio one (see Table A.3).

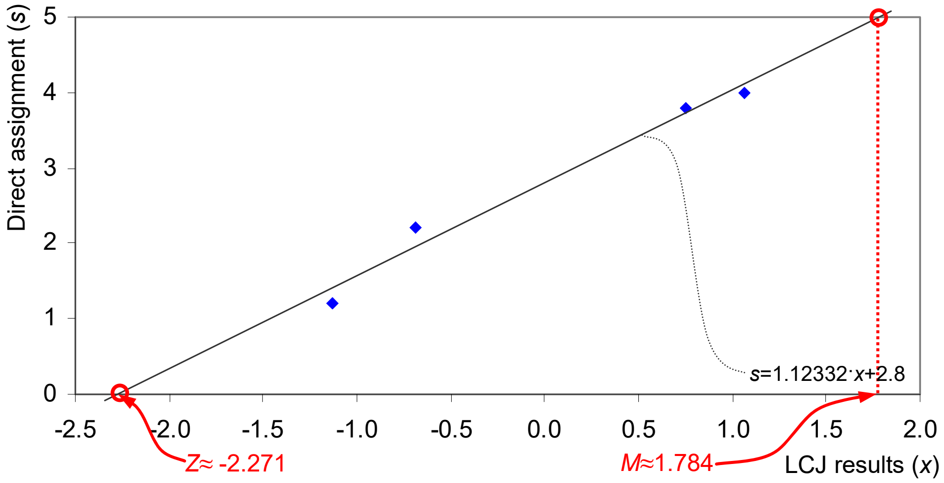
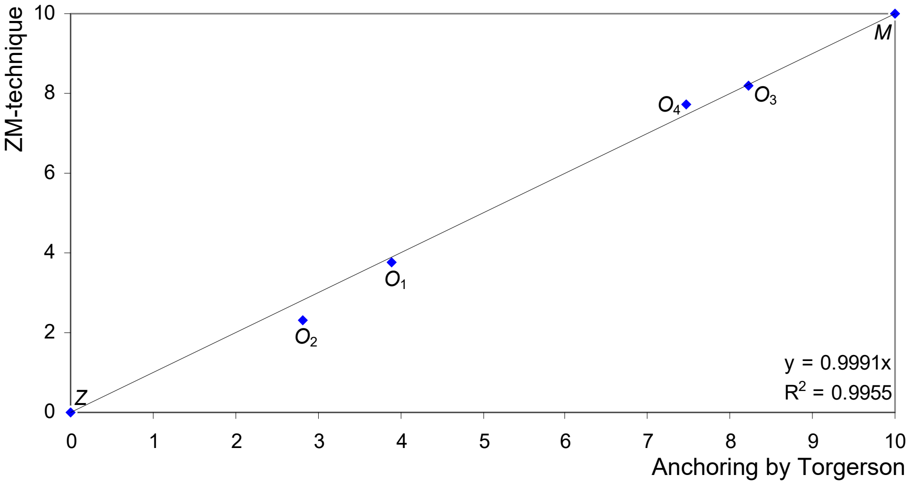


**Figure A.2. Comparison of the scale values resulting from the Thurstone’s LCJ and those resulting from a direct scale-value assignment (*Method of Single Stimuli*) for four objects ($O_1$ to $O_4$).**

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|
| Results of the LCJ | -0.688 | -1.129 | 1.061 | 0.756 | | |
| Anchor values | | | | | -2.271 | 1.784 |
| Scale values transformed into [0, 10] | **3.90** | **2.82** | **8.22** | **7.46** | **0** | **10** |

**Table A.3. Anchoring of the LCJ scale (in Figure A.1(d)), applying the technique by Torgerson.**

We have verified that the new anchoring technique (presented in the section “Anchoring the Thurstone’s Scaling: the ZM-technique”) provides results in line with those obtained from the Torgerson’s technique. E.g., Figure A.3 shows that, when applied to the same scaling problem, these two anchoring techniques are strongly correlated. Also, we have empirically observed that the correlation tends to increase for problems with a larger number of objects and/or respondents.



| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | *Z* | *M* |
|---|---|---|---|---|---|---|
| Anchoring by Torgerson | 3.9 | 2.8 | 8.2 | 7.5 | 0 | 10 |
| ZM-technique | 3.7 | 2.3 | 8.2 | 7.7 | 0 | 10 |

**Figure A.3. Comparison between two anchoring techniques (i.e., that by Torgerson, exemplified in Table A.3, and the ZM-technique, exemplified in Figure 8), with reference to the same LCJ-scaling problem.**

## *Further Material on the Real-life Application Example*

See the following Figures and Tables.

### Questionnaire

**Instructions for Respondent**

- A ***preference ordering*** is an ordered sequence of ***objects*** ($O_1$, $O_2$, …), depending on the degree of preference of a certain ***attribute***.
- The respondent has to position the objects, depending on the degree of preference of their attributes: most preferred objects at the top and least preferred at the bottom of the sequence.
- Two are the possible *relationships* between each pair of objects:
  1. *strict preference,* e.g., "$O_1$ is preferred to $O_2$", then $O_1$ is positioned at a higher hierarchical level than $O_2$;
  2. *indifference*, e.g., "$O_1$ has the same preference level of $O_2$", then the two objects are positioned at the same hierarchical level.
- The number of hierarchical levels is not fixed in advance, since it may depend on the number of objects and their mutual relationships.
- Apart from the *regular* objects ($O_1$, $O_2$, …), the respondent has to include two *dummy* objects in his/her preference ordering:

  ***Z*** object with a *zero* degree of preference of the attribute;
  ***M***, object with a *maximum-possible* degree of preference of the attribute.

  Regular objects with zero-preference degree should be positioned at the same hierarchical level of *Z* (indifference relationship) but **never below**, while objects with maximum-possible preference degree should be positioned at the same hierarchical level of *M*, but **never above**.

**Example**

Two respondents ($R_1$, $R_2$) construct their preference orderings on the **aesthetics** (i.e, the *attribute* of interest) of four **car models** (i.e, the *objects* of interest, $O_1$, $O_2$, $O_3$ and $O_4$).

As regards $R_1$, $O_4$ is preferred to $O_3$ and, in turn, to $O_1$ and to $O_2$; since $O_4$ reaches the maximum-possible degree of preference, it is considered indifferent to *M*.

As regards $R_2$, $O_3$ is preferred to $O_1$ and $O_4$ (tied), which are, in turn, preferred to $O_2$; since $O_2$ has a zero preference degree, it is considered indifferent to *Z*.

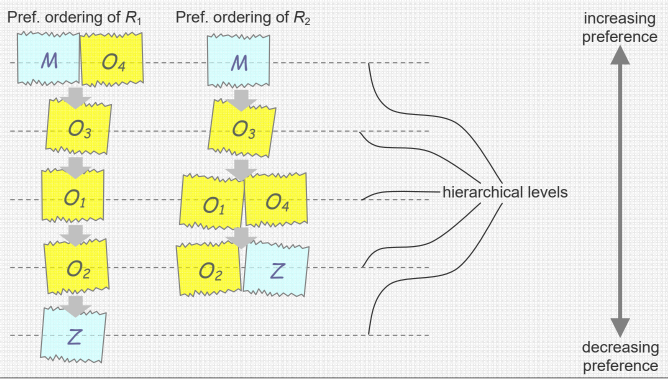


**Figure A.4. Example of questionnaire for the formulation of preference orderings.**

| Respondent | Preference ordering |
|---|---|
| $R_1$ | $CR_1 \sim CR_2 \sim CR_5 \sim CR_6 \sim CR_7 \sim M > CR_3 > CR_4 > CR_{10} \sim CR_8 > CR_{11} \sim CR_9 > Z \sim CR_{12}$ |
| $R_2$ | $CR_1 \sim CR_5 \sim CR_7 \sim M > CR_9 \sim CR_6 > CR_2 \sim CR_{11} \sim CR_8 > CR_4 \sim CR_{12} \sim CR_{10} > Z \sim CR_3$ |
| $R_3$ | $CR_1 \sim M > CR_3 \sim CR_2 > CR_6 \sim CR_5 > CR_7 > CR_8 \sim CR_9 \sim CR_4 > CR_{11} \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_4$ | $CR_1 \sim CR_2 \sim CR_7 \sim M > CR_3 \sim CR_5 > CR_6 > CR_8 \sim CR_9 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_4$ |
| $R_5$ | $CR_1 \sim CR_2 \sim CR_5 \sim M > CR_3 \sim CR_6 \sim CR_9 \sim CR4 > CR_7 \sim CR_8 > CR_{11} \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_6$ | $CR_1 \sim CR_5 \sim CR_6 \sim CR_7 \sim M > CR_2 \sim CR_9 \sim CR_3 > CR_{11} \sim CR_8 > CR_4 \sim CR_{10} > Z \sim CR_{12}$ |
| $R_7$ | $CR_2 \sim CR_7 \sim M > CR_1 > CR_5 \sim CR_8 \sim CR_6 > CR_9 \sim CR_{10} \sim CR_3 > CR_4 > CR_{12} \sim Z \sim CR_{11}$ |
| $R_8$ | $CR_1 \sim CR_5 \sim M > CR_6 \sim CR_7 \sim CR_9 \sim CR_{12} \sim CR_2 > CR_8 \sim CR_{11} > CR_3 \sim CR_4 > Z \sim CR_{10}$ |
| $R_9$ | $CR_1 \sim CR_4 \sim CR_5 \sim CR_7 \sim CR_9 \sim M > CR_2 > CR_6 > CR_{10} \sim CR_8 > CR_{11} \sim CR_{12} \sim CR_3 > Z$ |
| $R_{10}$ | $CR_2 \sim CR_5 \sim CR_6 \sim M > CR_7 > CR_1 > CR_3 > CR_{11} > CR_9 > CR_8 \sim CR_{10} > CR_{12} \sim Z \sim CR_4$ |
| $R_{11}$ | $CR_1 \sim M > CR_6 \sim CR_2 > CR_7 \sim CR_5 > CR_3 > CR_4 > CR_9 > CR_{11} > CR_{12} > CR_{10} \sim Z \sim CR_8$ |
| $R_{12}$ | $CR_2 \sim CR_7 \sim M > CR_1 > CR_9 \sim CR_3 > CR_8 \sim CR_6 \sim CR_4 > CR_5 > CR_{11} \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_{13}$ | $CR_1 \sim CR_2 \sim CR_5 \sim CR_8 \sim M > CR_7 \sim CR_3 \sim CR_6 > CR_9 \sim CR_{11} \sim CR_4 > CR_{12} \sim Z \sim CR_{10}$ |
| $R_{14}$ | $CR_2 \sim CR_5 \sim CR_6 \sim M > CR_1 > CR_3 > CR_9 > CR_7 > CR_4 > CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_8$ |

| | |
|---|---|
| $R_{15}$ | $CR_2 \sim CR_6 \sim CR_7 \sim M > CR_1 > CR_9 > CR_8 > CR_{11} \sim CR_{12} \sim CR_5 > CR_3 > CR_4 > Z \sim CR_{10}$ |
| $R_{16}$ | $CR_1 \sim CR_2 \sim CR_6 \sim CR_7 \sim M > CR_9 > CR_5 > CR_4 \sim CR_8 > CR_{11} > CR_3 > CR_{12} \sim Z \sim CR_{10}$ |
| $R_{17}$ | $CR_1 \sim CR_2 \sim M > CR_7 \sim CR_5 > CR_8 \sim CR_9 \sim CR_{10} \sim CR_4 > CR_6 \sim CR_{11} \sim CR_{12} \sim Z \sim CR_3$ |
| $R_{18}$ | $CR_1 \sim CR_2 \sim CR_3 \sim M > CR_9 > CR_7 \sim CR_8 \sim CR_4 \sim CR_5 > CR_{10} > CR_{11} \sim CR_6 > Z \sim CR_{12}$ |
| $R_{19}$ | $CR_1 \sim CR_2 \sim CR_6 \sim CR_9 \sim M > CR_3 \sim CR_7 \sim CR_4 \sim CR_5 > CR_8 > CR_{11} > CR_{12} \sim CR_{10} > Z$ |
| $R_{20}$ | $CR_1 \sim CR_2 \sim M > CR_5 > CR_6 \sim CR_3 > CR_7 \sim CR_4 > CR_9 > CR_{10} \sim CR_{11} \sim CR12 \sim Z \sim CR_8$ |
| $R_{21}$ | $CR_2 \sim CR_5 \sim M > CR_9 > CR_1 \sim CR_6 > CR_{12} \sim CR_8 > CR_4 \sim CR_{10} \sim CR_{11} \sim CR_7 \sim Z \sim CR_3$ |
| $R_{22}$ | $CR_6 \sim M > CR_5 > CR_2 > CR_9 > CR_1 > CR_3 > CR_4 > CR_7 \sim CR_8 > CR_{11} \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_{23}$ | $CR_1 \sim CR_2 \sim CR_5 \sim M > CR_6 \sim CR_7 \sim CR_9 \sim CR_4 > CR_3 \sim CR_{12} \sim CR_8 > CR_{10} \sim Z \sim CR_{11}$ |
| $R_{24}$ | $CR_1 \sim CR_6 \sim M > CR_5 > CR_3 \sim CR_7 \sim CR_8 \sim CR_2 > CR_9 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_4$ |
| $R_{25}$ | $CR_3 \sim CR_5 \sim CR_6 \sim M > CR_9 > CR_7 > CR_1 > CR_8 > CR_2 > CR_{11} > CR_{12} > CR_{10} \sim Z \sim CR_4$ |
| $R_{26}$ | $CR_1 \sim CR_2 \sim CR_5 \sim CR_7 \sim M > CR_8 \sim CR_9 \sim CR_{11} \sim CR_4 > CR_{12} \sim CR_6 > CR_3 > CR_{10} > Z$ |
| $R_{27}$ | $CR_5 \sim CR_7 \sim M > CR_6 > CR_1 \sim CR_9 \sim CR_2 > CR_3 > CR_4 \sim CR_8 > CR_{11} \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_{28}$ | $CR_1 \sim CR_6 \sim CR_7 \sim M > CR_5 \sim CR_9 \sim CR_2 > CR_3 \sim CR_{11} \sim CR_8 > CR_4 \sim CR_{12} \sim Z \sim CR_{10}$ |
| $R_{29}$ | $CR_7 \sim CR_9 \sim M > CR_1 > CR_5 > CR_2 > CR_{12} \sim CR_4 > CR_{11} \sim CR_3 \sim CR_6 > CR_8 > CR_{10} > Z$ |
| $R_{30}$ | $CR_5 \sim CR_6 \sim M > CR_7 \sim CR_1 > CR_3 \sim CR_4 \sim CR_8 \sim CR_9 \sim CR_{10} \sim CR_2 > CR_{12} \sim Z \sim CR_{11}$ |

**Table A.4. Preference orderings formulated by 30 (potential) customers (*R*₁ to *R*₃₀), relating to the importance of twelve CRs (*CR*₁ to *CR*₁₂), in the design of a civilian aircraft seat. The description of the CRs is reported in Table 11; *Z* is a dummy/anchor CR with zero degree of importance, while *M* is a dummy/anchor CR with maximum-imaginable degree of importance.**

$$F = \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
 & CR_1 & CR_2 & CR_3 & CR_4 & CR_5 & CR_6 & CR_7 & CR_8 & CR_9 & CR_{10} & CR_{11} & CR_{12} & Z & M \\
\hline
CR_1 & 15.0 & 17.0 & 28.5 & 29.5 & 18.5 & 19.5 & 18.5 & 29.5 & 24.5 & 30.0 & 30.0 & 30.0 & 30.0 & 9.5 \\
\hline
CR_2 & 13.0 & 15.0 & 26.5 & 28.5 & 15.5 & 18.0 & 17.5 & 27.0 & 23.0 & 29.5 & 29.5 & 29.5 & 30.0 & 8.5 \\
\hline
CR_3 & 1.5 & 3.5 & 15.0 & 20.5 & 4.5 & 7.0 & 9.0 & 18.0 & 13.5 & 25.5 & 22.5 & 22.5 & 28.5 & 1.0 \\
\hline
CR_4 & 0.5 & 1.5 & 9.5 & 15.0 & 2.5 & 6.5 & 5.0 & 14.5 & 8.0 & 23.5 & 20.5 & 23.0 & 27.0 & 0.5 \\
\hline
CR_5 & 11.5 & 14.5 & 25.5 & 27.5 & 15.0 & 18.0 & 18.0 & 26.5 & 23.0 & 30.0 & 29.5 & 29.5 & 30.0 & 7.5 \\
\hline
CR_6 & 10.5 & 12.0 & 23.0 & 23.5 & 12.0 & 15.0 & 16.0 & 25.0 & 20.5 & 28.0 & 27.5 & 27.5 & 29.5 & 6.0 \\
\hline
CR_7 & 11.5 & 12.5 & 21.0 & 25.0 & 12.0 & 14.0 & 15.0 & 26.0 & 21.0 & 29.5 & 29.5 & 28.5 & 29.5 & 6.5 \\
\hline
CR_8 & 0.5 & 3.0 & 12.0 & 15.5 & 3.5 & 5.0 & 4.0 & 15.0 & 6.5 & 25.5 & 23.0 & 24.5 & 28.0 & 0.5 \\
\hline
CR_9 & 5.5 & 7.0 & 16.5 & 22.0 & 7.0 & 9.5 & 9.0 & 23.5 & 15.0 & 26.5 & 26.5 & 28.5 & 29.0 & 1.5 \\
\hline
CR_{10} & 0.0 & 0.5 & 4.5 & 6.5 & 0.0 & 2.0 & 0.5 & 4.5 & 3.5 & 15.0 & 11.5 & 15.0 & 21.0 & 0.0 \\
\hline
CR_{11} & 0.0 & 0.5 & 7.5 & 9.5 & 0.5 & 2.5 & 0.5 & 7.0 & 3.5 & 18.5 & 15.0 & 19.0 & 23.0 & 0.0 \\
\hline
CR_{12} & 0.0 & 0.5 & 7.5 & 7.0 & 0.5 & 2.5 & 1.5 & 5.5 & 1.5 & 15.0 & 11.0 & 15.0 & 20.5 & 0.0 \\
\hline
Z & 0.0 & 0.0 & 1.5 & 3.0 & 0.0 & 0.5 & 0.5 & 2.0 & 1.0 & 9.0 & 7.0 & 9.5 & 15.0 & 0.0 \\
\hline
M & 20.5 & 21.5 & 29.0 & 29.5 & 22.5 & 24.0 & 23.5 & 29.5 & 28.5 & 30.0 & 30.0 & 30.0 & 30.0 & 15.0 \\
\hline
\end{array}$$

**Figure A.5. Matrix *F*, obtained from the preference orderings in Table A.4.**

$P =$

| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $CR_5$ | $CR_6$ | $CR_7$ | $CR_8$ | $CR_9$ | $CR_{10}$ | $CR_{11}$ | $CR_{12}$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CR_1$ | 0.500 | 0.567 | 0.950 | 1.000* | 0.617 | 0.650 | 0.617 | 1.000* | 0.817 | 1.000* | 1.000* | 1.000* | 1.000* | 0.317 |
| $CR_2$ | 0.433 | 0.500 | 0.883 | 0.950 | 0.517 | 0.600 | 0.583 | 0.900 | 0.767 | 1.000* | 1.000* | 1.000* | 1.000* | 0.283 |
| $CR_3$ | 0.050 | 0.117 | 0.500 | 0.683 | 0.150 | 0.233 | 0.300 | 0.600 | 0.450 | 0.850 | 0.750 | 0.750 | 0.950 | 0.033 |
| $CR_4$ | 0.000* | 0.050 | 0.317 | 0.500 | 0.083 | 0.217 | 0.167 | 0.483 | 0.267 | 0.783 | 0.683 | 0.767 | 0.900 | 0.000* |
| $CR_5$ | 0.383 | 0.483 | 0.850 | 0.917 | 0.500 | 0.600 | 0.600 | 0.883 | 0.767 | 1.000* | 1.000* | 1.000* | 1.000* | 0.250 |
| $CR_6$ | 0.350 | 0.400 | 0.767 | 0.783 | 0.400 | 0.500 | 0.533 | 0.833 | 0.683 | 0.933 | 0.917 | 0.917 | 1.000* | 0.200 |
| $CR_7$ | 0.383 | 0.417 | 0.700 | 0.833 | 0.400 | 0.467 | 0.500 | 0.867 | 0.700 | 1.000* | 1.000* | 0.950 | 1.000* | 0.217 |
| $CR_8$ | 0.000* | 0.100 | 0.400 | 0.517 | 0.117 | 0.167 | 0.133 | 0.500 | 0.217 | 0.850 | 0.767 | 0.817 | 0.933 | 0.000* |
| $CR_9$ | 0.183 | 0.233 | 0.550 | 0.733 | 0.233 | 0.317 | 0.300 | 0.783 | 0.500 | 0.883 | 0.883 | 0.950 | 0.967 | 0.050 |
| $CR_{10}$ | 0.000* | 0.000* | 0.150 | 0.217 | 0.000* | 0.067 | 0.000* | 0.150 | 0.117 | 0.500 | 0.383 | 0.500 | 0.700 | 0.000* |
| $CR_{11}$ | 0.000* | 0.000* | 0.250 | 0.317 | 0.000* | 0.083 | 0.000* | 0.233 | 0.117 | 0.617 | 0.500 | 0.633 | 0.767 | 0.000* |
| $CR_{12}$ | 0.000* | 0.000* | 0.250 | 0.233 | 0.000* | 0.083 | 0.050 | 0.183 | 0.050 | 0.500 | 0.367 | 0.500 | 0.683 | 0.000* |
| $Z$ | 0.000* | 0.000* | 0.050 | 0.100 | 0.000* | 0.000* | 0.000* | 0.067 | 0.033 | 0.300 | 0.233 | 0.317 | 0.500 | 0.000* |
| $M$ | 0.683 | 0.717 | 0.967 | 1.000* | 0.750 | 0.800 | 0.783 | 1.000* | 0.950 | 1.000* | 1.000* | 1.000* | 1.000* | 0.500 |

**Figure A.6. Matrix *P*, obtained from the matrix *F* in Figure A.5.**

$Z =$

| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $CR_5$ | $CR_6$ | $CR_7$ | $CR_8$ | $CR_9$ | $CR_{10}$ | $CR_{11}$ | $CR_{12}$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CR_1$ | 0.000 | -0.168 | -1.645 | -1.995 | -0.297 | -0.385 | -0.297 | -1.995 | -0.903 | -1.995 | -1.995 | -1.995 | -1.995 | 0.477 |
| $CR_2$ | 0.168 | 0.000 | -1.192 | -1.645 | -0.042 | -0.253 | -0.210 | -1.282 | -0.728 | -1.995 | -1.995 | -1.995 | -1.995 | 0.573 |
| $CR_3$ | 1.645 | 1.192 | 0.000 | -0.477 | 1.036 | 0.728 | 0.524 | -0.253 | 0.126 | -1.036 | -0.674 | -0.674 | -1.645 | 1.834 |
| $CR_4$ | 1.995 | 1.645 | 0.477 | 0.000 | 1.383 | 0.784 | 0.967 | 0.042 | 0.623 | -0.784 | -0.477 | -0.728 | -1.282 | 1.995 |
| $CR_5$ | 0.297 | 0.042 | -1.036 | -1.383 | 0.000 | -0.253 | -0.253 | -1.192 | -0.728 | -1.995 | -1.995 | -1.995 | -1.995 | 0.674 |
| $CR_6$ | 0.385 | 0.253 | -0.728 | -0.784 | 0.253 | 0.000 | -0.084 | -0.967 | -0.477 | -1.501 | -1.383 | -1.383 | -1.995 | 0.842 |
| $CR_7$ | 0.297 | 0.210 | -0.524 | -0.967 | 0.253 | 0.084 | 0.000 | -1.111 | -0.524 | -1.995 | -1.995 | -1.645 | -1.995 | 0.784 |
| $CR_8$ | 1.995 | 1.282 | 0.253 | -0.042 | 1.192 | 0.967 | 1.111 | 0.000 | 0.784 | -1.036 | -0.728 | -0.903 | -1.501 | 1.995 |
| $CR_9$ | 0.903 | 0.728 | -0.126 | -0.623 | 0.728 | 0.477 | 0.524 | -0.784 | 0.000 | -1.192 | -1.192 | -1.645 | -1.834 | 1.645 |
| $CR_{10}$ | 1.995 | 1.995 | 1.036 | 0.784 | 1.995 | 1.501 | 1.995 | 1.036 | 1.192 | 0.000 | 0.297 | 0.000 | -0.524 | 1.995 |
| $CR_{11}$ | 1.995 | 1.995 | 0.674 | 0.477 | 1.995 | 1.383 | 1.995 | 0.728 | 1.192 | -0.297 | 0.000 | -0.341 | -0.728 | 1.995 |
| $CR_{12}$ | 1.995 | 1.995 | 0.674 | 0.728 | 1.995 | 1.383 | 1.645 | 0.903 | 1.645 | 0.000 | 0.341 | 0.000 | -0.477 | 1.995 |
| $Z$ | 1.995 | 1.995 | 1.645 | 1.282 | 1.995 | 1.995 | 1.995 | 1.501 | 1.834 | 0.524 | 0.728 | 0.477 | 0.000 | 1.995 |
| $M$ | -0.477 | -0.573 | -1.834 | -1.995 | -0.674 | -0.842 | -0.784 | -1.995 | -1.645 | -1.995 | -1.995 | -1.995 | -1.995 | 0.000 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Sigma_j$ | 15.190 | 12.592 | -2.324 | -6.641 | 11.814 | 7.568 | 9.130 | -5.369 | 2.390 | -15.299 | -13.066 | -14.823 | -19.963 | 18.801 |
| $\mu_j'=\Sigma_j/\mathrm{n}$ | 1.085 | 0.899 | -0.166 | -0.474 | 0.844 | 0.541 | 0.652 | -0.384 | 0.171 | -1.093 | -0.933 | -1.059 | -1.426 | 1.343 |
| $f^{(1)}=\mu''$ | **9.1** | **8.4** | **4.6** | **3.4** | **8.2** | **7.1** | **7.5** | **3.8** | **5.8** | **1.2** | **1.8** | **1.3** | **0.0** | **10.0** |

**Figure A.7. Matrix *Z*, obtained from the matrix *P* in Figure A.6, and results of the Thurtsone's scaling. Values of $p_{ij} \leq 0.023$ and $\geq 0.977$ (marked with "*" in Figure A.4) have been conventionally associated with $z_{ij} = 1.995$ and -1.995.**

| Resp. | Preference orderings | | | | | |
|---|---|---|---|---|---|---|
| | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $CR_5$ | $CR_6$ |
| $R_1$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_C \sim P_B > Z \sim P_A$ | $P_B \sim M > P_A > Z \sim P_C$ | $P_C \sim M > P_B > P_A > Z$ | $P_B \sim P_C \sim M > Z \sim P_A$ | $M > P_C > P_B > Z \sim P_A$ |
| $R_2$ | $M > P_B \sim P_A > Z \sim P_C$ | $M > P_C \sim P_B > P_A > Z$ | $P_A \sim M > P_B > Z \sim P_C$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_C > P_B > P_A > Z$ | $P_C \sim M > P_B > Z \sim P_A$ |
| $R_3$ | $M > P_B \sim P_A > Z \sim P_C$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_B \sim M > P_A > P_C > Z$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_B \sim P_C \sim P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_4$ | $M > P_A > P_C > Z \sim P_B$ | $M > P_C > P_A > Z \sim P_B$ | $M > P_A > P_B > P_C > Z$ | $M > P_C \sim P_B > P_A > Z$ | $M > P_B \sim P_C \sim P_A > Z$ | $M > P_C \sim P_A > Z \sim P_B$ |
| $R_5$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_C > P_A > P_B > Z$ | $M > P_B \sim P_A > P_C > Z$ | $M > P_C > P_A > Z \sim P_B$ | $P_B \sim P_C \sim M > Z \sim P_A$ | $M > P_C \sim P_A > P_B > Z$ |
| $R_6$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_B > P_A > Z$ | $P_A \sim M > P_B > Z \sim P_C$ | $P_C \sim M > P_B > P_A > Z$ | $P_B \sim M > P_C \sim P_A > Z$ | $M > P_C > P_A \sim P_B > Z$ |
| $R_7$ | $P_A \sim M > P_B > Z \sim P_C$ | $M > P_A > P_C > P_B > Z$ | $M > P_B > P_A > Z \sim P_C$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim M > P_A > P_C > Z$ | $M > P_A > P_C \sim P_B > Z$ |
| $R_8$ | $P_A \sim M > P_B > P_C > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_A \sim M > P_B > P_C > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim M > P_A > P_C > Z$ | $P_B \sim P_C \sim M > P_A > Z$ |
| $R_9$ | $P_A \sim P_B \sim M > P_C > Z$ | $M > P_C \sim P_A > P_B > Z$ | $P_A \sim M > P_B > P_C > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{10}$ | $P_A \sim M > P_C \sim P_B > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_B \sim M > P_A > P_C > Z$ | $P_C \sim M > P_A > Z \sim P_B$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_C > P_A > P_B > Z$ |
| $R_{11}$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_C > P_B > P_A > Z$ | $M > P_A > P_B > P_C > Z$ | $M > P_C > P_A > Z \sim P_B$ | $M > P_B > P_C > Z \sim P_A$ | $M > P_B > P_C > P_A > Z$ |
| $R_{12}$ | $P_A \sim M > P_C > Z \sim P_B$ | $M > P_B > P_C > P_A > Z$ | $P_A \sim M > P_B > Z \sim P_C$ | $P_C \sim M > P_A \sim P_B > Z$ | $M > P_B > P_C > Z \sim P_A$ | $P_C \sim M > P_B > Z \sim P_A$ |
| $R_{13}$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_B > P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_A \sim P_B \sim P_C \sim M > Z$ | $P_B \sim M > P_C > P_A > Z$ | $M > P_C > P_B > Z \sim P_A$ |
| $R_{14}$ | $M > P_A > P_B > Z \sim P_C$ | $M > P_C \sim P_B > Z \sim P_A$ | $M > P_A > P_B > P_C > Z$ | $M > P_C > P_A > P_B > Z$ | $M > P_B > P_C > P_A > Z$ | $M > P_C > P_A \sim Z \sim P_B$ |
| $R_{15}$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_C > P_B > P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_B > P_A > Z$ | $P_C \sim M > P_B > Z \sim P_A$ | $P_C \sim M > P_A > Z \sim P_B$ |
| $R_{16}$ | $M > P_A > P_B > P_C > Z$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_B > P_A > P_C > Z$ | $P_C \sim M > P_A \sim P_B > Z$ | $P_B \sim M > P_C > P_A > Z$ | $P_C \sim M > P_A \sim Z \sim P_B$ |
| $R_{17}$ | $M > P_B \sim P_A > Z \sim P_C$ | $M > P_C \sim P_B > P_A > Z$ | $M > P_B \sim P_A > Z \sim P_C$ | $P_C \sim M > P_A > Z \sim P_B$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_B \sim P_C \sim P_A > Z$ |
| $R_{18}$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_C > P_A > P_B > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_A \sim P_C \sim M > P_B > Z$ | $M > P_C \sim P_B > Z \sim P_A$ | $P_C \sim M > P_A \sim P_B > Z$ |
| $R_{19}$ | $P_A \sim M > P_B > P_C > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_B \sim M > P_A > Z \sim P_C$ | $P_A \sim P_C \sim M > P_B > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{20}$ | $P_A \sim M > P_B > P_C > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_C \sim P_B > P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{21}$ | $M > P_B \sim P_A > P_C > Z$ | $M > P_C > P_A \sim Z \sim P_B$ | $M > P_B \sim P_A > Z \sim P_C$ | $P_C \sim M > P_A > Z \sim P_B$ | $M > P_B > P_C > Z \sim P_A$ | $M > P_C > P_A \sim P_B > Z$ |
| $R_{22}$ | $P_A \sim M > P_B > Z \sim P_C$ | $M > P_B > P_C > P_A > Z$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_C > P_A > Z \sim P_B$ | $M > P_B > P_C \sim Z \sim P_A$ | $M > P_C > P_B > Z \sim P_A$ |
| $R_{23}$ | $M > P_B \sim P_A > Z \sim P_C$ | $M > P_C \sim P_B > P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_A > Z \sim P_B$ | $M > P_C \sim P_B > Z \sim P_A$ | $M > P_C > P_B > Z \sim P_A$ |
| $R_{24}$ | $M > P_A > P_C \sim Z \sim P_B$ | $M > P_B > P_C > Z \sim P_A$ | $P_A \sim M > P_B > Z \sim P_C$ | $M > P_C \sim P_A > P_B > Z$ | $M > P_C \sim P_B > Z \sim P_A$ | $M > P_C > P_A \sim Z \sim P_B$ |
| $R_{25}$ | $M > P_A > P_C \sim P_B > Z$ | $M > P_B > P_C > Z \sim P_A$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_C > P_A > P_B > Z$ | $P_B \sim M > P_C > P_A > Z$ | $P_C \sim M > P_B > Z \sim P_A$ |
| $R_{26}$ | $P_A \sim P_B \sim P_C \sim M > Z$ | $M > P_B > P_C > P_A > Z$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_B > P_C > Z \sim P_A$ | $P_B \sim P_C \sim M > P_A > Z$ |
| $R_{27}$ | $P_A \sim M > P_C \sim P_B > Z$ | $M > P_B \sim P_C \sim P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $M > P_C \sim P_B > P_A > Z$ | $P_B \sim P_C \sim M > Z \sim P_A$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{28}$ | $P_A \sim M > P_C > P_B > Z$ | $M > P_C > P_A \sim P_B > Z$ | $M > P_C \sim P_B > P_A > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim M > P_A > P_C > Z$ | $P_C \sim M > P_A > Z \sim P_B$ |
| $R_{29}$ | $P_A \sim M > P_B > P_C > Z$ | $P_B \sim M > P_C > P_A > Z$ | $P_B \sim M > P_A > P_C > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{30}$ | $P_B \sim M > P_A > P_C > Z$ | $P_A \sim M > P_C > Z \sim P_B$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_C \sim M > P_A \sim P_B > Z$ | $P_B \sim M > P_C > Z \sim P_A$ | $M > P_C > P_A \sim P_B > Z$ |

| Resp. | Preference orderings | | | | | |
|---|---|---|---|---|---|---|
| | $CR_7$ | $CR_8$ | $CR_9$ | $CR_{10}$ | $CR_{11}$ | $CR_{12}$ |
| $R_1$ | $P_B \sim M > P_A > P_C > Z$ | $M > P_C \sim P_A > P_B > Z$ | $P_A \sim P_C \sim M > P_B > Z$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_A > P_C > Z \sim P_B$ | $M > P_A > P_C \sim P_B > Z$ |
| $R_2$ | $M > P_B > P_A > Z \sim P_C$ | $P_B \sim M > P_A > Z \sim P_C$ | $P_A \sim M > P_C > Z \sim P_B$ | $P_C \sim M > P_B > Z \sim P_A$ | $M > P_B > P_C \sim Z \sim P_A$ | $P_B \sim P_C \sim M > P_A > Z$ |
| $R_3$ | $M > P_C \sim P_B > P_A > Z$ | $M > P_B > P_A > P_C > Z$ | $P_A \sim M > P_C \sim P_B > Z$ | $M > P_B > P_C > P_A > Z$ | $M > P_C > P_A \sim Z \sim P_B$ | $M > P_C > P_B > P_A > Z$ |
| $R_4$ | $M > P_B \sim P_A > Z \sim P_C$ | $M > P_C \sim P_A > P_B > Z$ | $P_A \sim M > P_C > Z \sim P_B$ | $M > P_C > P_A > P_B > Z$ | $M > P_C > P_A \sim P_B > Z$ | $M > P_C > P_B > Z \sim P_A$ |
| $R_5$ | $M > P_C > P_A \sim P_B > Z$ | $M > P_C \sim P_B > P_A > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_C \sim M > P_B > Z \sim P_A$ | $M > P_B > P_C \sim P_A > Z$ | $P_C \sim M > P_A \sim P_B > Z$ |
| $R_6$ | $P_B \sim M > P_A > P_C > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ | $P_A \sim P_C \sim M > P_B > Z$ | $M > P_A > P_C > P_B > Z$ | $P_B \sim P_C \sim M > P_A > Z$ |
| $R_7$ | $P_B \sim M > P_A > P_C > Z$ | $P_B \sim M > P_A > P_C > Z$ | $M > P_A > P_C > Z \sim P_B$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_C \sim P_B > Z \sim P_A$ | $P_B \sim M > P_C > P_A > Z$ |
| $R_8$ | $M > P_B > P_C \sim P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim M > P_C > P_A > Z$ | $M > P_C > P_B > P_A > Z$ | $P_A \sim M > P_C > P_B > Z$ |
| $R_9$ | $P_A \sim P_B \sim M > P_C > Z$ | $M > P_C \sim P_B > P_A > Z$ | $P_C \sim M > P_A > P_B > Z$ | $P_B \sim M > P_C > P_A > Z$ | $P_B \sim M > P_A > Z \sim P_C$ | $M > P_B \sim P_C \sim P_A > Z$ |
| $R_{10}$ | $P_B \sim M > P_A > P_C > Z$ | $M > P_A > P_B > P_C > Z$ | $M > P_C > P_A > P_B > Z$ | $M > P_C > P_B > P_A > Z$ | $M > P_B > P_A > Z \sim P_C$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{11}$ | $M > P_A > P_B > P_C > Z$ | $M > P_C > P_B > P_A > Z$ | $M > P_C > P_A > Z \sim P_B$ | $M > P_A > P_C \sim P_B > Z$ | $M > P_B > P_C > P_A > Z$ | $P_B \sim M > P_C > P_A > Z$ |
| $R_{12}$ | $M > P_B > P_A > P_C > Z$ | $M > P_A > P_B > P_C > Z$ | $P_A \sim M > P_C > Z \sim P_B$ | $P_C \sim M > P_B > Z \sim P_A$ | $M > P_C > P_B > Z \sim P_A$ | $M > P_B > P_C \sim P_A > Z$ |
| $R_{13}$ | $P_A \sim M > P_C \sim P_B > Z$ | $P_A \sim M > P_B > Z \sim P_C$ | $P_A \sim P_C \sim M > Z \sim P_B$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_B \sim P_C \sim P_A > Z$ |
| $R_{14}$ | $M > P_B > P_C > Z \sim P_A$ | $M > P_B > P_A > P_C > Z$ | $M > P_A > P_C > Z \sim P_B$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_C > P_A \sim Z \sim P_B$ | $M > P_A > P_C \sim P_B > Z$ |
| $R_{15}$ | $M > P_C > P_A > P_B > Z$ | $M > P_A > P_C > P_B > Z$ | $M > P_C > P_A > P_B > Z$ | $M > P_C > P_B > Z \sim P_A$ | $M > P_B > P_C \sim Z \sim P_A$ | $P_C \sim M > P_A \sim P_B > Z$ |
| $R_{16}$ | $M > P_B \sim P_A > P_C > Z$ | $P_A \sim M > P_C > P_B > Z$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_C > P_A > P_B > Z$ | $M > P_B > P_C \sim Z \sim P_A$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{17}$ | $M > P_B > P_C \sim Z \sim P_A$ | $M > P_B \sim P_A > P_C > Z$ | $P_C \sim M > P_A > Z \sim P_B$ | $M > P_C > P_B > Z \sim P_A$ | $M > P_B > P_C \sim P_A > Z$ | $P_C \sim M > P_A \sim P_B > Z$ |
| $R_{18}$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_A > P_B > Z \sim P_C$ | $P_C \sim M > P_A > P_B > Z$ | $P_C \sim M > P_B > P_A > Z$ | $M > P_A > P_C > Z \sim P_B$ | $M > P_B \sim P_C \sim P_A > Z$ |
| $R_{19}$ | $P_B \sim M > P_C > P_A > Z$ | $P_A \sim M > P_C \sim P_B > Z$ | $M > P_C \sim P_A > P_B > Z$ | $M > P_C > P_A \sim P_B > Z$ | $M > P_B \sim P_C \sim P_A > Z$ | $P_C \sim M > P_B > P_A > Z$ |
| $R_{20}$ | $M > P_A > P_C \sim P_B > Z$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_A > P_C \sim P_B > Z$ | $P_C \sim M > P_B > Z \sim P_A$ | $M > P_B \sim P_C \sim Z \sim P_A$ | $M > P_C \sim P_B > P_A > Z$ |
| $R_{21}$ | $M > P_B > P_C > P_A > Z$ | $M > P_A > P_C \sim P_B > Z$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_B > P_C \sim P_A > Z$ | $M > P_B > P_C \sim Z \sim P_A$ | $M > P_C \sim P_B > P_A > Z$ |
| $R_{22}$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_A > P_C > Z \sim P_B$ | $M > P_B > P_C > Z \sim P_A$ | $M > P_B > P_C \sim Z \sim P_A$ | $M > P_C > P_B > P_A > Z$ |
| $R_{23}$ | $P_B \sim M > P_A > Z \sim P_C$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_A \sim M > P_C > Z \sim P_B$ | $M > P_C \sim P_B > Z \sim P_A$ | $M > P_B > P_C \sim Z \sim P_A$ | $P_C \sim M > P_B > Z \sim P_A$ |
| $R_{24}$ | $M > P_B \sim P_C \sim Z \sim P_A$ | $M > P_C > P_A \sim P_B > Z$ | $P_A \sim P_C \sim M > P_B > Z$ | $M > P_B \sim P_C \sim P_A > Z$ | $M > P_A > P_C \sim Z \sim P_B$ | $M > P_B > P_C > Z \sim P_A$ |
| $R_{25}$ | $M > P_A > P_C \sim P_B > Z$ | $M > P_C \sim P_B > P_A > Z$ | $M > P_C > P_A > P_B > Z$ | $P_C \sim M > P_A > P_B > Z$ | $M > P_B > P_A > Z \sim P_C$ | $M > P_C > P_A > P_B > Z$ |
| $R_{26}$ | $P_B \sim M > P_C \sim P_A > Z$ | $M > P_B > P_C > P_A > Z$ | $P_A \sim P_C \sim M > P_B > Z$ | $M > P_A > P_C > P_B > Z$ | $P_B \sim M > P_C > P_A > Z$ | $P_A \sim P_C \sim M > P_B > Z$ |
| $R_{27}$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $P_C \sim M > P_A > Z \sim P_B$ | $P_C \sim M > P_A \sim P_B > Z$ | $M > P_C \sim P_A > Z \sim P_B$ | $P_B \sim M > P_C > Z \sim P_A$ |
| $R_{28}$ | $M > P_C \sim P_B > P_A > Z$ | $P_A \sim P_B \sim M > P_C > Z$ | $M > P_A > P_C \sim P_B > Z$ | $M > P_C \sim P_A > P_B > Z$ | $M > P_B \sim P_C \sim Z \sim P_A$ | $P_C \sim M > P_B > Z \sim P_A$ |
| $R_{29}$ | $P_B \sim M > P_C \sim P_A > Z$ | $P_A \sim M > P_C \sim P_B > Z$ | $P_A \sim M > P_C \sim P_B > Z$ | $P_B \sim P_C \sim M > P_A > Z$ | $M > P_B > P_C > Z \sim P_A$ | $P_C \sim M > P_A > P_B > Z$ |
| $R_{30}$ | $P_A \sim M > P_B > P_C > Z$ | $M > P_B \sim P_C \sim Z \sim P_A$ | $P_A \sim P_C \sim M > Z \sim P_B$ | $P_B \sim M > P_C > P_A > Z$ | $M > P_A > P_C \sim Z \sim P_B$ | $P_B \sim M > P_C > Z \sim P_A$ |

**Table A.5. Preference orderings formulated by 30 (potential) customers ($R_1$ to $R_{30}$), relating to the degree of satisfaction of each CR ($CR_1$ to $CR_{12}$) for the products/services benchmarked (i.e., $P_A$, $P_B$ and $P_C$). $Z$ is a dummy/anchor product with zero satisfaction level, while $M$ is a dummy/anchor product with maximum-imaginable degree of satisfaction.**

(a) $CR_i$ (b) matrix ***F*** (c) matrix ***P*** (d) matrix ***Z*** (e) Scaling

$CR_1$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 23.5 | 29.0 | 30.0 | 9.0 |
| $P_B$ | 6.5 | 15.0 | 24.5 | 28.5 | 2.5 |
| $P_C$ | 1.0 | 5.5 | 15.0 | 26.0 | 0.5 |
| $Z$ | 0.0 | 1.5 | 4.0 | 15.0 | 0.0 |
| $M$ | 21.0 | 27.5 | 29.5 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.78 | 0.97 | 1.00* | 0.30 |
| $P_B$ | 0.22 | 0.50 | 0.82 | 0.95 | 0.08 |
| $P_C$ | 0.03 | 0.18 | 0.50 | 0.87 | 0.00* |
| $Z$ | 0.00* | 0.05 | 0.13 | 0.50 | 0.00* |
| $M$ | 0.70 | 0.92 | 1.00* | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | -0.78 | -1.83 | -2.00 | 0.52 |
| $P_B$ | 0.78 | 0.00 | -0.90 | -1.64 | 1.38 |
| $P_C$ | 1.83 | 0.90 | 0.00 | -1.11 | 2.00 |
| $Z$ | 2.00 | 1.64 | 1.11 | 0.00 | 2.00 |
| $M$ | -0.52 | -1.38 | -2.00 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | 4.09 | 0.38 | -3.62 | -6.75 | 5.90 |
| $\mu_j' = \Sigma_j / n$ | 0.82 | 0.08 | -0.72 | -1.35 | 1.18 |
| $\mu_j''$ [0-10] | **8.6** | **5.6** | **2.5** | 0.0 | 10.0 |

$CR_2$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 10.0 | 4.5 | 27.5 | 1.0 |
| $P_B$ | 20.0 | 15.0 | 12.0 | 28.5 | 2.5 |
| $P_C$ | 25.5 | 18.0 | 15.0 | 30.0 | 3.0 |
| $Z$ | 2.5 | 1.5 | 0.0 | 15.0 | 0.0 |
| $M$ | 29.0 | 27.5 | 27.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.33 | 0.15 | 0.92 | 0.03 |
| $P_B$ | 0.67 | 0.50 | 0.40 | 0.95 | 0.08 |
| $P_C$ | 0.85 | 0.60 | 0.50 | 1.00* | 0.10 |
| $Z$ | 0.08 | 0.05 | 0.00* | 0.50 | 0.00* |
| $M$ | 0.97 | 0.92 | 0.90 | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.43 | 1.04 | -1.38 | 1.83 |
| $P_B$ | -0.43 | 0.00 | 0.25 | -1.64 | 1.38 |
| $P_C$ | -1.04 | -0.25 | 0.00 | -2.00 | 1.28 |
| $Z$ | 1.38 | 1.64 | 2.00 | 0.00 | 2.00 |
| $M$ | -1.83 | -1.38 | -1.28 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -1.92 | 0.44 | 2.00 | -7.02 | 6.49 |
| $\mu_j' = \Sigma_j / n$ | -0.38 | 0.09 | 0.40 | -1.40 | 1.30 |
| $\mu_j''$ [0-10] | **3.8** | **5.5** | **6.7** | 0.0 | 10.0 |

$CR_3$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 14.5 | 28.0 | 30.0 | 6.5 |
| $P_B$ | 15.5 | 15.0 | 29.5 | 30.0 | 6.5 |
| $P_C$ | 2.0 | 0.5 | 15.0 | 25.0 | 0.0 |
| $Z$ | 0.0 | 0.0 | 5.0 | 15.0 | 0.0 |
| $M$ | 23.5 | 23.5 | 30.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.48 | 0.93 | 1.00* | 0.22 |
| $P_B$ | 0.52 | 0.50 | 1.00* | 1.00* | 0.22 |
| $P_C$ | 0.07 | 0.00* | 0.50 | 0.83 | 0.00* |
| $Z$ | 0.00* | 0.00* | 0.17 | 0.50 | 0.00* |
| $M$ | 0.78 | 0.78 | 1.00* | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.04 | -1.50 | -2.00 | 0.78 |
| $P_B$ | -0.04 | 0.00 | -2.00 | -2.00 | 0.78 |
| $P_C$ | 1.50 | 2.00 | 0.00 | -0.97 | 2.00 |
| $Z$ | 2.00 | 2.00 | 0.97 | 0.00 | 2.00 |
| $M$ | -0.78 | -0.78 | -2.00 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | 2.67 | 3.25 | -4.52 | -6.95 | 5.56 |
| $\mu_j' = \Sigma_j / n$ | 0.53 | 0.65 | -0.90 | -1.39 | 1.11 |
| $\mu_j''$ [0-10] | **7.7** | **8.2** | **1.9** | 0.0 | 10.0 |

$CR_4$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 20.0 | 2.0 | 30.0 | 1.5 |
| $P_B$ | 10.0 | 15.0 | 2.0 | 26.5 | 1.0 |
| $P_C$ | 28.0 | 28.0 | 15.0 | 30.0 | 11.0 |
| $Z$ | 0.0 | 3.5 | 0.0 | 15.0 | 0.0 |
| $M$ | 28.5 | 29.0 | 19.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.67 | 0.07 | 1.00* | 0.05 |
| $P_B$ | 0.33 | 0.50 | 0.07 | 0.88 | 0.03 |
| $P_C$ | 0.93 | 0.93 | 0.50 | 1.00* | 0.37 |
| $Z$ | 0.00* | 0.12 | 0.00* | 0.50 | 0.00* |
| $M$ | 0.95 | 0.97 | 0.63 | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | -0.43 | 1.50 | -2.00 | 1.64 |
| $P_B$ | 0.43 | 0.00 | 1.50 | -1.19 | 1.83 |
| $P_C$ | -1.50 | -1.50 | 0.00 | -2.00 | 0.34 |
| $Z$ | 2.00 | 1.19 | 2.00 | 0.00 | 2.00 |
| $M$ | -1.64 | -1.83 | -0.34 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -0.72 | -2.57 | 4.66 | -7.18 | 5.81 |
| $\mu_j' = \Sigma_j / n$ | -0.14 | -0.51 | 0.93 | -1.44 | 1.16 |
| $\mu_j''$ [0-10] | **5.0** | **3.5** | **9.1** | 0.0 | 10.0 |

$CR_5$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 1.0 | 5.5 | 23.5 | 0.0 |
| $P_B$ | 29.0 | 15.0 | 21.0 | 30.0 | 7.0 |
| $P_C$ | 24.5 | 9.0 | 15.0 | 29.5 | 4.0 |
| $Z$ | 6.5 | 0.0 | 0.5 | 15.0 | 0.0 |
| $M$ | 30.0 | 23.0 | 26.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.03 | 0.18 | 0.78 | 0.00* |
| $P_B$ | 0.97 | 0.50 | 0.70 | 1.00* | 0.23 |
| $P_C$ | 0.82 | 0.30 | 0.50 | 1.00* | 0.13 |
| $Z$ | 0.22 | 0.00* | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 0.77 | 0.87 | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 1.83 | 0.90 | -0.78 | 2.00 |
| $P_B$ | -1.83 | 0.00 | -0.52 | -2.00 | 0.73 |
| $P_C$ | -0.90 | 0.52 | 0.00 | -2.00 | 1.11 |
| $Z$ | 0.78 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -0.73 | -1.11 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -3.95 | 3.63 | 1.26 | -6.77 | 5.83 |
| $\mu_j' = \Sigma_j / n$ | -0.79 | 0.73 | 0.25 | -1.35 | 1.17 |
| $\mu_j''$ [0-10] | **2.2** | **8.3** | **6.4** | 0.0 | 10.0 |

$CR_6$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 10.0 | 2.5 | 25.0 | 0.0 |
| $P_B$ | 20.0 | 15.0 | 3.0 | 27.0 | 1.0 |
| $P_C$ | 27.5 | 27.0 | 15.0 | 30.0 | 7.5 |
| $Z$ | 5.0 | 3.0 | 0.0 | 15.0 | 0.0 |
| $M$ | 30.0 | 29.0 | 22.5 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.33 | 0.08 | 0.83 | 0.00* |
| $P_B$ | 0.67 | 0.50 | 0.10 | 0.90 | 0.03 |
| $P_C$ | 0.92 | 0.90 | 0.50 | 1.00* | 0.25 |
| $Z$ | 0.17 | 0.10 | 0.00* | 0.50 | 0.00* |
| $M$ | 1.00* | 0.97 | 0.75 | 1.00* | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.43 | 1.38 | -0.97 | 2.00 |
| $P_B$ | -0.43 | 0.00 | 1.28 | -1.28 | 1.83 |
| $P_C$ | -1.38 | -1.28 | 0.00 | -2.00 | 0.67 |
| $Z$ | 0.97 | 1.28 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -1.83 | -0.67 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -2.84 | -1.40 | 3.99 | -6.24 | 6.50 |
| $\mu_j' = \Sigma_j / n$ | -0.57 | -0.28 | 0.80 | -1.25 | 1.30 |
| $\mu_j''$ [0-10] | **2.7** | **3.8** | **8.0** | 0.0 | 10.0 |

$CR_7$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 8.5 | 20.0 | 28.5 | 1.5 |
| $P_B$ | 21.5 | 15.0 | 25.0 | 29.5 | 5.0 |
| $P_C$ | 10.0 | 5.0 | 15.0 | 26.5 | 0.0 |
| $Z$ | 1.5 | 0.5 | 3.5 | 15.0 | 0.0 |
| $M$ | 28.5 | 25.0 | 30.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.28 | 0.67 | 0.95 | 0.05 |
| $P_B$ | 0.72 | 0.50 | 0.83 | 0.98 | 0.17 |
| $P_C$ | 0.33 | 0.17 | 0.50 | 0.88 | 0.02 |
| $Z$ | 0.05 | 0.02 | 0.12 | 0.50 | 0.02 |
| $M$ | 0.95 | 0.83 | 0.98 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.57 | -0.43 | -1.64 | 1.64 |
| $P_B$ | -0.57 | 0.00 | -0.97 | -2.00 | 0.97 |
| $P_C$ | 0.43 | 0.97 | 0.00 | -1.19 | 2.00 |
| $Z$ | 1.64 | 2.00 | 1.19 | 0.00 | 2.00 |
| $M$ | -1.64 | -0.97 | -2.00 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -0.14 | 2.57 | -2.20 | -6.83 | 6.60 |
| $\mu_j' = \Sigma_j / n$ | -0.03 | 0.51 | -0.44 | -1.37 | 1.32 |
| $\mu_j''$ [0-10] | **5.0** | **7.0** | **3.4** | 0.0 | 10.0 |

$CR_8$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 14.0 | 20.5 | 29.5 | 3.5 |
| $P_B$ | 16.0 | 15.0 | 20.0 | 29.5 | 3.5 |
| $P_C$ | 9.5 | 10.0 | 15.0 | 27.5 | 0.5 |
| $Z$ | 0.5 | 0.5 | 2.5 | 15.0 | 0.0 |
| $M$ | 26.5 | 26.5 | 29.5 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.47 | 0.68 | 0.98 | 0.12 |
| $P_B$ | 0.53 | 0.50 | 0.67 | 0.98 | 0.12 |
| $P_C$ | 0.32 | 0.33 | 0.50 | 0.92 | 0.02 |
| $Z$ | 0.02 | 0.02 | 0.08 | 0.50 | 0.02 |
| $M$ | 0.88 | 0.88 | 0.98 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.08 | -0.48 | -2.00 | 1.19 |
| $P_B$ | -0.08 | 0.00 | -0.43 | -2.00 | 1.19 |
| $P_C$ | 0.48 | 0.43 | 0.00 | -1.38 | 2.00 |
| $Z$ | 2.00 | 2.00 | 1.38 | 0.00 | 2.00 |
| $M$ | -1.19 | -1.19 | -2.00 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | 1.20 | 1.32 | -1.52 | -7.37 | 6.37 |
| $\mu_j' = \Sigma_j / n$ | 0.24 | 0.26 | -0.30 | -1.47 | 1.27 |
| $\mu_j''$ [0-10] | **6.2** | **6.3** | **4.3** | 0.0 | 10.0 |

$CR_9$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 29.0 | 14.0 | 30.0 | 5.5 |
| $P_B$ | 1.0 | 15.0 | 2.0 | 24.0 | 0.0 |
| $P_C$ | 16.0 | 28.0 | 15.0 | 30.0 | 7.0 |
| $Z$ | 0.0 | 6.0 | 0.0 | 15.0 | 0.0 |
| $M$ | 24.5 | 30.0 | 23.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.97 | 0.47 | 0.98 | 0.18 |
| $P_B$ | 0.03 | 0.50 | 0.07 | 0.80 | 0.02 |
| $P_C$ | 0.53 | 0.93 | 0.50 | 0.98 | 0.23 |
| $Z$ | 0.02 | 0.20 | 0.02 | 0.50 | 0.02 |
| $M$ | 0.82 | 0.98 | 0.77 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | -1.83 | 0.08 | -2.00 | 0.90 |
| $P_B$ | 1.83 | 0.00 | 1.50 | -0.84 | 2.00 |
| $P_C$ | -0.08 | -1.50 | 0.00 | -2.00 | 0.73 |
| $Z$ | 2.00 | 0.84 | 2.00 | 0.00 | 2.00 |
| $M$ | -0.90 | -2.00 | -0.73 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | 2.84 | -4.49 | 2.85 | -6.83 | 5.62 |
| $\mu_j' = \Sigma_j / n$ | 0.57 | -0.90 | 0.57 | -1.37 | 1.12 |
| $\mu_j''$ [0-10] | **7.8** | **1.9** | **7.8** | 0.0 | 10.0 |

$CR_{10}$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 9.5 | 4.5 | 26.0 | 0.5 |
| $P_B$ | 20.5 | 15.0 | 9.0 | 30.0 | 2.0 |
| $P_C$ | 25.5 | 21.0 | 15.0 | 30.0 | 6.0 |
| $Z$ | 4.0 | 0.0 | 0.0 | 15.0 | 0.0 |
| $M$ | 29.5 | 28.0 | 24.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.32 | 0.15 | 0.87 | 0.02 |
| $P_B$ | 0.68 | 0.50 | 0.30 | 0.98 | 0.07 |
| $P_C$ | 0.85 | 0.70 | 0.50 | 0.98 | 0.20 |
| $Z$ | 0.13 | 0.02 | 0.02 | 0.50 | 0.02 |
| $M$ | 0.98 | 0.93 | 0.80 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.48 | 1.04 | -1.11 | 2.00 |
| $P_B$ | -0.48 | 0.00 | 0.52 | -2.00 | 1.50 |
| $P_C$ | -1.04 | -0.52 | 0.00 | -2.00 | 0.84 |
| $Z$ | 1.11 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -2.00 | -1.50 | -0.84 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -2.40 | 0.45 | 2.71 | -7.10 | 6.33 |
| $\mu_j' = \Sigma_j / n$ | -0.48 | 0.09 | 0.54 | -1.42 | 1.27 |
| $\mu_j''$ [0-10] | **3.5** | **5.6** | **7.3** | 0.0 | 10.0 |

$CR_{11}$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 9.0 | 15.0 | 23.5 | 0.0 |
| $P_B$ | 21.0 | 15.0 | 18.0 | 25.5 | 1.0 |
| $P_C$ | 15.0 | 12.0 | 15.0 | 23.0 | 0.0 |
| $Z$ | 6.5 | 4.5 | 7.0 | 15.0 | 0.0 |
| $M$ | 30.0 | 29.0 | 30.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.30 | 0.50 | 0.78 | 0.02 |
| $P_B$ | 0.70 | 0.50 | 0.60 | 0.85 | 0.03 |
| $P_C$ | 0.50 | 0.40 | 0.50 | 0.77 | 0.02 |
| $Z$ | 0.22 | 0.15 | 0.23 | 0.50 | 0.02 |
| $M$ | 0.98 | 0.97 | 0.98 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.52 | 0.00 | -0.78 | 2.00 |
| $P_B$ | -0.52 | 0.00 | -0.25 | -1.04 | 1.83 |
| $P_C$ | 0.00 | 0.25 | 0.00 | -0.73 | 2.00 |
| $Z$ | 0.78 | 1.04 | 0.73 | 0.00 | 2.00 |
| $M$ | -2.00 | -1.83 | -2.00 | -2.00 | 0.00 |

|  | $P_1$ | $P_2$ | $P_3$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -1.74 | -0.02 | -1.52 | -4.54 | 7.82 |
| $\mu_j' = \Sigma_j / n$ | -0.35 | 0.00 | -0.30 | -0.91 | 1.56 |
| $\mu_j''$ [0-10] | **2.3** | **3.7** | **2.4** | 0.0 | 10.0 |

$CR_{12}$

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 15.0 | 9.0 | 5.5 | 27.0 | 1.0 |
| $P_B$ | 21.0 | 15.0 | 10.5 | 30.0 | 3.0 |
| $P_C$ | 24.5 | 19.5 | 15.0 | 30.0 | 6.0 |
| $Z$ | 3.0 | 0.0 | 0.0 | 15.0 | 0.0 |
| $M$ | 29.0 | 27.0 | 24.0 | 30.0 | 15.0 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.50 | 0.30 | 0.18 | 0.90 | 0.03 |
| $P_B$ | 0.70 | 0.50 | 0.35 | 0.98 | 0.10 |
| $P_C$ | 0.82 | 0.65 | 0.50 | 0.98 | 0.20 |
| $Z$ | 0.10 | 0.02 | 0.02 | 0.50 | 0.02 |
| $M$ | 0.97 | 0.90 | 0.80 | 0.98 | 0.50 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $P_A$ | 0.00 | 0.52 | 0.90 | -1.28 | 1.83 |
| $P_B$ | -0.52 | 0.00 | 0.39 | -2.00 | 1.28 |
| $P_C$ | -0.90 | -0.39 | 0.00 | -2.00 | 0.84 |
| $Z$ | 1.28 | 2.00 | 2.00 | 0.00 | 2.00 |
| $M$ | -1.83 | -1.28 | -0.84 | -2.00 | 0.00 |

|  | $P_A$ | $P_B$ | $P_C$ | $Z$ | $M$ |
|---|---|---|---|---|---|
| $\Sigma_j$ | -1.98 | 0.85 | 2.44 | -7.27 | 5.95 |
| $\mu_j' = \Sigma_j / n$ | -0.40 | 0.17 | 0.49 | -1.45 | 1.19 |
| $\mu_j''$ [0-10] | **4.0** | **6.1** | **7.3** | 0.0 | 10.0 |

$S_A$ $S_B$ $S_C$

**Figure A.8. Matrices *F*, *P*, *Z* and Thurstone's scaling related to the preference orderings in Table A.5. *Z* is a dummy/anchor product with zero satisfaction level, while *M* is a dummy/anchor product with maximum-imaginable degree of satisfaction.**

| Respondent | Preference ordering |
|---|---|
| $E_1$ | $CR_9 \sim M > CR_1 \sim CR_3 > CR_4 \sim CR_{10} \sim CR_{12} \sim CR_7 > CR_5 \sim CR_6 \sim CR_{11} \sim CR_2 \sim Z \sim CR_8$ |
| $E_2$ | $CR_1 \sim CR_3 \sim CR_9 \sim M > CR_7 > CR_{10} > CR_{11} \sim CR_4 > CR_5 \sim CR_6 \sim CR_2 \sim CR_{12} \sim Z \sim CR_8$ |
| $E_3$ | $CR_1 \sim M > CR_7 \sim CR_3 > CR_5 > CR_9 > CR_4 \sim CR_8 \sim CR_2 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_6$ |
| $E_4$ | $CR_1 \sim M > CR_3 > CR_9 > CR_7 > CR_{12} \sim CR_4 > CR_8 \sim CR_5 \sim CR_{10} \sim CR_{11} \sim CR_6 \sim Z \sim CR_2$ |
| $E_5$ | $CR_1 \sim CR_3 \sim CR_7 \sim M > CR_9 > CR_8 \sim CR_5 \sim CR_2 > CR_4 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_6$ |
| $E_6$ | $CR_1 \sim CR_3 \sim CR_7 \sim CR_9 \sim M > CR_5 \sim CR_{10} \sim CR_{11} \sim CR_8 > CR_4 > CR_{12} > CR_6 \sim Z \sim CR_2$ |
| $E_7$ | $CR_3 \sim M > CR_9 > CR_1 > CR_2 \sim CR_7 > CR_4 > CR_8 \sim CR_6 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_5$ |
| $E_8$ | $CR_1 \sim CR_3 \sim M > CR_9 \sim CR_7 > CR_5 \sim CR_{12} \sim CR_2 > CR_8 > CR_{10} > CR_{11} \sim CR_4 \sim Z \sim CR_6$ |
| $E_9$ | $CR_1 \sim CR_3 \sim CR_9 \sim M > CR_7 > CR_6 > CR_4 > CR_5 > CR_2 > CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_8$ |
| $E_{10}$ | $CR_1 \sim CR_3 \sim CR_9 \sim M > CR_7 > CR_8 \sim CR_6 \sim CR_4 > CR_5 \sim CR_{10} \sim CR_{11} \sim CR_{12} \sim Z \sim CR_2$ |

**Table A.6. Preference orderings, formulated by 10 experts (*$E_1$* to *$E_{10}$*) of the QFD team, about the impact of CRs (*$CR_1$* to *$CR_{12}$*) on sales, for a new civilian aircraft seat. *Z* is a dummy/anchor CR with zero importance for sales-points, while *M* is a dummy/anchor CR with maximum-imaginable importance for sales.**

$$F = \begin{array}{c|cccccccccccccc|}
 & CR_1 & CR_2 & CR_3 & CR_4 & CR_5 & CR_6 & CR_7 & CR_8 & CR_9 & CR_{10} & CR_{11} & CR_{12} & Z & M \\
\hline
CR_1 & 5.0 & 10.0 & 5.5 & 10.0 & 10.0 & 10.0 & 9.0 & 10.0 & 6.0 & 10.0 & 10.0 & 10.0 & 10.0 & 4.0 \\
CR_2 & 0.0 & 5.0 & 0.0 & 3.5 & 4.0 & 5.5 & 0.5 & 5.5 & 0.0 & 5.5 & 6.0 & 5.0 & 7.0 & 0.0 \\
CR_3 & 4.5 & 10.0 & 5.0 & 10.0 & 10.0 & 10.0 & 8.5 & 10.0 & 7.0 & 10.0 & 10.0 & 10.0 & 10.0 & 3.5 \\
CR_4 & 0.0 & 6.5 & 0.0 & 5.0 & 6.0 & 7.0 & 0.5 & 6.0 & 0.0 & 5.5 & 7.0 & 7.0 & 8.5 & 0.0 \\
CR_5 & 0.0 & 6.0 & 0.0 & 4.0 & 5.0 & 6.0 & 0.0 & 6.0 & 1.0 & 6.0 & 6.5 & 6.0 & 7.5 & 0.0 \\
CR_6 & 0.0 & 4.5 & 0.0 & 3.0 & 4.0 & 5.0 & 0.0 & 4.0 & 0.0 & 4.0 & 5.0 & 4.0 & 6.0 & 0.0 \\
CR_7 & 1.0 & 9.5 & 1.5 & 9.5 & 10.0 & 10.0 & 5.0 & 10.0 & 3.0 & 9.5 & 10.0 & 9.5 & 10.0 & 1.0 \\
CR_8 & 0.0 & 4.5 & 0.0 & 4.0 & 4.0 & 6.0 & 0.0 & 5.0 & 0.0 & 5.5 & 6.0 & 5.0 & 7.0 & 0.0 \\
CR_9 & 4.0 & 10.0 & 3.0 & 10.0 & 9.0 & 10.0 & 7.0 & 10.0 & 5.0 & 10.0 & 10.0 & 10.0 & 10.0 & 2.5 \\
CR_{10} & 0.0 & 4.5 & 0.0 & 4.5 & 4.0 & 6.0 & 0.5 & 4.5 & 0.0 & 5.0 & 6.5 & 5.0 & 7.0 & 0.0 \\
CR_{11} & 0.0 & 4.0 & 0.0 & 3.0 & 3.5 & 5.0 & 0.0 & 4.0 & 0.0 & 3.5 & 5.0 & 4.5 & 6.0 & 0.0 \\
CR_{12} & 0.0 & 5.0 & 0.0 & 3.0 & 4.0 & 6.0 & 0.5 & 5.0 & 0.0 & 5.0 & 5.5 & 5.0 & 7.0 & 0.0 \\
Z & 0.0 & 3.0 & 0.0 & 1.5 & 2.5 & 4.0 & 0.0 & 3.0 & 0.0 & 3.0 & 4.0 & 3.0 & 5.0 & 0.0 \\
M & 6.0 & 10.0 & 6.5 & 10.0 & 10.0 & 10.0 & 9.0 & 10.0 & 7.5 & 10.0 & 10.0 & 10.0 & 10.0 & 5.0 \\
\hline
\end{array}$$

**Figure A.9. Matrix *F*, obtained from the paired-comparison data in Table A.6.**

| $P =$ | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $CR_5$ | $CR_6$ | $CR_7$ | $CR_8$ | $CR_9$ | $CR_{10}$ | $CR_{11}$ | $CR_{12}$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CR_1$ | 0.500 | 1.000* | 0.550 | 1.000* | 1.000* | 1.000* | 0.900 | 1.000* | 0.600 | 1.000* | 1.000* | 1.000* | 1.000* | 0.400 |
| $CR_2$ | 0.000* | 0.500 | 0.000* | 0.350 | 0.400 | 0.550 | 0.050 | 0.550 | 0.000* | 0.550 | 0.600 | 0.500 | 0.700 | 0.000* |
| $CR_3$ | 0.450 | 1.000* | 0.500 | 1.000* | 1.000* | 1.000* | 0.850 | 1.000* | 0.700 | 1.000* | 1.000* | 1.000* | 1.000* | 0.350 |
| $CR_4$ | 0.000* | 0.650 | 0.000* | 0.500 | 0.600 | 0.700 | 0.050 | 0.600 | 0.000* | 0.550 | 0.700 | 0.700 | 0.850 | 0.000* |
| $CR_5$ | 0.000* | 0.600 | 0.000* | 0.400 | 0.500 | 0.600 | 0.000* | 0.600 | 0.100 | 0.600 | 0.650 | 0.600 | 0.750 | 0.000* |
| $CR_6$ | 0.000* | 0.450 | 0.000* | 0.300 | 0.400 | 0.500 | 0.000* | 0.400 | 0.000* | 0.400 | 0.500 | 0.400 | 0.600 | 0.000* |
| $CR_7$ | 0.100 | 0.950 | 0.150 | 0.950 | 1.000* | 1.000* | 0.500 | 1.000* | 0.300 | 0.950 | 1.000* | 0.950 | 1.000* | 0.100 |
| $CR_8$ | 0.000* | 0.450 | 0.000* | 0.400 | 0.400 | 0.600 | 0.000* | 0.500 | 0.000* | 0.550 | 0.600 | 0.500 | 0.700 | 0.000* |
| $CR_9$ | 0.400 | 1.000* | 0.300 | 1.000* | 0.900 | 1.000* | 0.700 | 1.000* | 0.500 | 1.000* | 1.000* | 1.000* | 1.000* | 0.250 |
| $CR_{10}$ | 0.000* | 0.450 | 0.000* | 0.450 | 0.400 | 0.600 | 0.050 | 0.450 | 0.000* | 0.500 | 0.650 | 0.500 | 0.700 | 0.000* |
| $CR_{11}$ | 0.000* | 0.400 | 0.000* | 0.300 | 0.350 | 0.500 | 0.000* | 0.400 | 0.000* | 0.350 | 0.500 | 0.450 | 0.600 | 0.000* |
| $CR_{12}$ | 0.000* | 0.500 | 0.000* | 0.300 | 0.400 | 0.600 | 0.050 | 0.500 | 0.000* | 0.500 | 0.550 | 0.500 | 0.700 | 0.000* |
| $Z$ | 0.000* | 0.300 | 0.000* | 0.150 | 0.250 | 0.400 | 0.000* | 0.300 | 0.000* | 0.300 | 0.400 | 0.300 | 0.500 | 0.000* |
| $M$ | 0.600 | 1.000* | 0.650 | 1.000* | 1.000* | 1.000* | 0.900 | 1.000* | 0.750 | 1.000* | 1.000* | 1.000* | 1.000* | 0.500 |

**Figure A.10. Matrix *P* obtained from the matrix *F* in Figure A.9.**

| $Z =$ | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ | $CR_5$ | $CR_6$ | $CR_7$ | $CR_8$ | $CR_9$ | $CR_{10}$ | $CR_{11}$ | $CR_{12}$ | $Z$ | $M$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CR_1$ | 0.000 | -1.995 | -0.126 | -1.995 | -1.995 | -1.995 | -1.282 | -1.995 | -0.253 | -1.995 | -1.995 | -1.995 | -1.995 | 0.253 |
| $CR_2$ | 1.995 | 0.000 | 1.995 | 0.385 | 0.253 | -0.126 | 1.645 | -0.126 | 1.995 | -0.126 | -0.253 | 0.000 | -0.524 | 1.995 |
| $CR_3$ | 0.126 | -1.995 | 0.000 | -1.995 | -1.995 | -1.995 | -1.036 | -1.995 | -0.524 | -1.995 | -1.995 | -1.995 | -1.995 | 0.385 |
| $CR_4$ | 1.995 | -0.385 | 1.995 | 0.000 | -0.253 | -0.524 | 1.645 | -0.253 | 1.995 | -0.126 | -0.524 | -0.524 | -1.036 | 1.995 |
| $CR_5$ | 1.995 | -0.253 | 1.995 | 0.253 | 0.000 | -0.253 | 1.995 | -0.253 | 1.282 | -0.253 | -0.385 | -0.253 | -0.674 | 1.995 |
| $CR_6$ | 1.995 | 0.126 | 1.995 | 0.524 | 0.253 | 0.000 | 1.995 | 0.253 | 1.995 | 0.253 | 0.000 | 0.253 | -0.253 | 1.995 |
| $CR_7$ | 1.282 | -1.645 | 1.036 | -1.645 | -1.995 | -1.995 | 0.000 | -1.995 | 0.524 | -1.645 | -1.995 | -1.645 | -1.995 | 1.282 |
| $CR_8$ | 1.995 | 0.126 | 1.995 | 0.253 | 0.253 | -0.253 | 1.995 | 0.000 | 1.995 | -0.126 | -0.253 | 0.000 | -0.524 | 1.995 |
| $CR_9$ | 0.253 | -1.995 | 0.524 | -1.995 | -1.282 | -1.995 | -0.524 | -1.995 | 0.000 | -1.995 | -1.995 | -1.995 | -1.995 | 0.674 |
| $CR_{10}$ | 1.995 | 0.126 | 1.995 | 0.126 | 0.253 | -0.253 | 1.645 | 0.126 | 1.995 | 0.000 | -0.385 | 0.000 | -0.524 | 1.995 |
| $CR_{11}$ | 1.995 | 0.253 | 1.995 | 0.524 | 0.385 | 0.000 | 1.995 | 0.253 | 1.995 | 0.385 | 0.000 | 0.126 | -0.253 | 1.995 |
| $CR_{12}$ | 1.995 | 0.000 | 1.995 | 0.524 | 0.253 | -0.253 | 1.645 | 0.000 | 1.995 | 0.000 | -0.126 | 0.000 | -0.524 | 1.995 |
| $Z$ | 1.995 | 0.524 | 1.995 | 1.036 | 0.674 | 0.253 | 1.995 | 0.524 | 1.995 | 0.524 | 0.253 | 0.524 | 0.000 | 1.995 |
| $M$ | -0.253 | -1.995 | -0.385 | -1.995 | -1.995 | -1.995 | -1.282 | -1.995 | -0.674 | -1.995 | -1.995 | -1.995 | -1.995 | 0.000 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Sigma_j$ | 19.366 | -9.110 | 19.008 | -5.999 | -7.190 | -11.387 | 12.432 | -9.453 | 16.317 | -9.094 | -11.651 | -9.501 | -14.292 | 20.553 |
| $\mu_j'=\Sigma_j / n$ | 1.383 | -0.651 | 1.358 | -0.429 | -0.514 | -0.813 | 0.888 | -0.675 | 1.165 | -0.650 | -0.832 | -0.679 | -1.021 | 1.468 |
| $f^{(3)}=\mu''$ | **9.7** | **1.5** | **9.6** | **2.4** | **2.0** | **0.8** | **7.7** | **1.4** | **8.8** | **1.5** | **0.8** | **1.4** | **0.0** | **10.0** |

**Figure A.11. Matrix *Z*, obtained from the matrix *P* in Figure A.10, and results of the Thurtsone's scaling. Values of $p_{ij} \leq 0.023$ and $\geq 0.977$ (marked with "*" in Figure A.8) have been conventionally associated with $z_{ij} = 1.995$ and -1.995.**