Evaluation of Decentralized Feedback Traffic Light Control with Dynamic Cycle Length

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Abstract: An established rule of thumb in the field of traffic light control prescribes that, during periods of higher demand, it is convenient to have longer cycles. This is in order to reduce the fraction of the cycle length when no incoming lanes receive green light. In this paper, we simulate a novel, provably stable, decentralized feedback traffic light control policy with variable cycle length. The proposed control strategy is fully decentralized and does not require any information about the network structure or the turning rates. Through simulations on a micro simulator, we compare the performance of our variable cycle length policy to a similar feedback policy with fixed cycle length and with a fixed-time control policy. The simulations show that having dynamic cycle lengths allows one to significantly reduce the overall queue lengths in the network, in both medium and low demands.

Keywords: Traffic light control, Feedback traffic control, Urban traffic networks

1. INTRODUCTION

Classical traffic light control relies on fixed-time control schemes where the traffic lights are preprogrammed with information on how long they should give green light to each incoming lane in a junction. One drawback of this open loop setting is that even if one has good knowledge about the right tuning for the typical scenarios, a rapid change of the traffic load or of the drivers' behavior can make the traffic light less effective. Such rapid changes can be, e.g., an accident or one commonly used navigation system suddenly suggesting a new route choice based on the state of the traffic network.

In order to improve the traffic lights' performance under changing conditions, adaptive signal control systems, such as SCOOT, have been developed. However, those systems are relatively complex, they make centralized decisions and their complexity makes them difficult to analyze from a control theoretical perspective.

Due to the recent rapid development of traffic sensors, it is now often easier to acquire real-time data about the traffic state. This idea of using current state feedback information in order to determine the control strategy in real time has previously been analyzed –with proven stability properties– in Varaiya (2013b), Varaiya (2013a), Wongpiromsarn et al. (2012) and Le et al. (2015). The strategies considered in these works are commonly referred to as MaxPressure/BackPressure and are based on an idea, originally proposed in Tassiulas and Ephremides (1992) for radio networks. When this strategy is adapted to traffic lights control as in the aforementioned papers, the resulting traffic light controller is dependable on either exogenous information or estimates on how the traffic flow propagates through the network at each junction, i.e., the turning rates. Under the assumption that such turning rates are known, other feedback based control strategies have been developed as well, e.g., in Grandinetti et al. (2015) the authors develop an optimal control policy based on one-step predictions.

In all the mentioned control strategies, the cycle length is fixed. However, when designing a signal timing in an open loop control setting, standard formulas suggest that during high demands, the cycle lengths should be longer, see e.g. Roess et al. (2011). In Kovács et al. (2016) the authors derive an expression for computing the optimal cycle length, and propose that the cycle length should be adjusted during time, but such adjustment is done on a much longer time scale compared to the cycle length, so that the cycle lengths dynamics and phase activation dynamics have not been analyzed simultaneously.

In this paper, we study a decentralized feedback traffic light control policy with variable cycle length. The proposed policy is based on a generalized proportional allocation principle, that is an adaptation of the proportional fairness notion, see. e.g., Massoulié (2007) and Walton (2014). At every junction in the urban traffic network, at the beginning of each cycle, the proposed traffic light control sets the length of the new cycle as a proper increasing function of the current aggregate queue lengths of all lanes, and allocates time to the different phases proportionally to the aggregate queue lengths of the lanes activated in

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the phases. Such feedback traffic light control policy is (i) completely decentralized, in the sense that cycle length and phase allocations at each junction are determined dynamically as a function of the current queue lengths at the incoming lanes at that junction only; and (ii) universal, as it does not require any information the drivers' turning rates nor any knowledge of the network structure except for the local lanes and phases. These properties make it resilient to exogenous perturbations and shocks that may affect the transportation network, c.f., Como et al. (2013a, 2015); Como (2017).

The contribution of the paper is that we evaluate a control policy that simultaneously decides the cycle length and the phase activation. The stability of this controller has been analyzed in Nilsson et al. (2017) for a single junction, where it is shown that the controller is stabilizing in the sense that it will keep all the queue lengths bounded. A continuous approximation for a network setting of the same idea has previously been analyzed, with stability proof, in Nilsson et al. (2015) and Nilsson and Como (2017). Apart from desirable stability properties, the control strategy we are evaluating is fully decentralized and does not require any exogenous information about the network topology. Those properties make the controller easy to implement. To the best of the author's knowledge, this is the first time that a feedback-based controller with proven stability properties, that both decide the cycle length and the phase activation instantaneously is evaluated in a traffic simulator.

The rest of the paper is organized as follows: In Section 2 we introduce the proposed control strategy with dynamic cycle lengths. In Section 3 the simulation scenario is presented, together with the other control strategies. Discussion of the simulation results is done in Section 4 and some points of future research are given in Section 5.

2. DECENTRALIZED FEEDBACK TRAFFIC LIGHT CONTROL WITH VARIABLE CYCLE LENGTH

In this section, we introduce a decentralized feedback traffic light control policy with variable cycle length to be used in the rest of the paper. The proposed policy is based on a generalized proportional allocation principle, whereby, at every junction in the urban traffic network, the time each phase is activated during one cycle is allocated proportionally to the aggregate queue lengths of the lanes activated in the phases, as measured at the beginning of the cycle. Then, the cycle length is also determined by the traffic light controller as a proper increasing function of the current aggregate queue lengths of all lanes. Such feedback traffic light control policy is completely decentralized, in the sense that cycle length and phase allocations at each junction are determined dynamically as a function of the current queue lengths at the incoming lanes at that junction only. Moreover, this policy is universal in that it does not require any information the drivers' turning rates nor any knowledge of the network structure except for the local lanes and phases.

Formally, for a junction with n incoming lanes and p phases, let $P \in \{0,1\}^{n\times p}$ be the phase matrix, where

$$P_{ij} = \begin{cases} 1 \text{ if lane } i \text{ belongs to } j\text{-th phase} \\ 0 \text{ if otherwise}. \end{cases}$$

We assume that the order of activation of the different phases during a cycle is fixed and let $T_w > 0$ be the total clearance time, i.e., the aggregate time needed for shifts between consecutive phases (i.e., the time between the deactivation of one phase and the activation of the next phase) during a cycle. Let

$$0 = t_0 < t_1 < t_2 < \dots$$

be the starting times of the consecutive cycles at the junction and, for $k \geq 0$, let $x(t_k) \in \mathbb{Z}_+^n$ be the vector whose entries $x_i(t_k)$ correspond to the number of vehicles queueing up at each lane i at the beginning of the k-th cycle. At the beginning t_k of the k-th cycle, a decentralized feedback traffic light control policy with variable cycle length determines, as a function of the current local queue length vector $x(t_k)$, both (i) the k-th cycle length

$$T(x(t_k)) \geq T_w \,,$$

so that

$$t_{k+1} = t_k + T(x(t_k)) \,,$$

and (ii) the vector $u(x(t_k)) \in \mathbb{R}_+^p$ whose entries $u_j(x(t_k))$ correspond to the fractions of time that will be allocated to each phase j during the k-th cycle. The total fraction of time allocated to the different phases is determined by the cycle length and the total clearance time as

$$\sum_{j=1}^{p} u_j(x(t_k)) = 1 - \frac{T_w}{T(x(t_k))} \,.$$

We are now ready to define the decentralized feedback traffic light control policy with variable cycle length studied in this paper, that is based on a generalized proportional allocation rule. For simplicity of exposition we will first consider the case of orthogonal phases, i.e., when every lane belongs to only one phase so that

$$\sum_{j=1}^{p} P_{ij} = 1 \,, \qquad i = 1, \dots, n \,,$$

whereas we refer to Remark 2 for the general case of non orthogonal phases. In the orthogonal phases case, for all $k \geq 0$, the proposed policy sets the length of the upcoming k-th cycle as

$$T(x(t_k)) = T_w + \frac{T_w}{\kappa} \sum_{1 \leq i \leq n} x_i(t_k) \tag{1}$$

and the fractions of time to be allocated to the the different phases to

$$u_j(x(t_k)) = \frac{\sum_{1\leq i\leq n} P_{ij} x_i(t_k)}{\kappa + \sum_{1 \leq i \leq n} x_i(t_k)} \,, \qquad j = 1, 2, \dots, p \,, \tag{2}$$

where $\kappa > 0$ is a design parameter. A larger value of κ yields shorter cycles, while a smaller value of κ yields longer cycles. Later in the paper it will be discussed what reasonable choices of such design parameter κ are. Notice that, regardless of the choice of $\kappa > 0$, it holds that if $\tilde{x}(k) > x(k)$ then $T(\tilde{x}(k)) > T(x(k))$ and hence the cycle length will be longer when the demand is higher.

Example 1. Consider the blue phase in Fig. 2a. If the incoming are numbered clockwise, where the north-leftmost lane is numbered as 1, then the fraction of time allocated to the blue phase is

$$u_1(x(t_k)) = \frac{x_1(t_k) + x_5(t_k)}{\kappa + \sum_{1 \le i \le 8} x_i(t_k)} \,.$$

Remark 1. For the stability analysis in Nilsson et al. (2017), an upper bound of the cycle length is required to guarantee stability. This to guarantee that the cycle length will be bounded. However, in practice, only a limited area of the lane is covered with sensors, so the measured queue length will be bounded and hence the maximum cycle time will be bounded as well. That the measurements can saturate will also limit the set of inflows that the controller can stabilize.

Remark 2. Although the setting in this paper is restricted to orthogonal phases, i.e., each lane belongs to just one phase, it is possible to extend the control-strategy to handle an arbitrary set of phases. The fraction of the time that each phase should be activated, $\nu \in [0,1]^p$, and the fraction of the cycle that should be allocated to phase shifts $w \ge 0$, is then computed by solving the following convex optimization problem:

$$\begin{aligned} \underset{\nu \in \mathbb{R}^p_+ ,\ w \in \mathbb{R}_+}{\text{maximize}} \quad & \sum_{1 \le i \le n} x_i(t_k) \log\left((P\nu)_i\right) + \kappa \log(w) \,, \\ \text{subject to} \quad & \sum_{1 \le i \le n} \nu_i + w = 1 \,. \end{aligned}$$

The cycle length and the fractions of time allocated to the different phases are then computed as

$$T(x(t_k)) = \frac{T_w}{w^*}$$

and

$$u_j(x(t_k)) = \nu_j^* \,, \qquad 1 \le j \le p \,,$$

where (w^*, ν^*) is a maximizer in the optimization above. In fact, (1) and (2) is the explicit solution to the optimization problem above, in the special case of orthogonal phases. See Nilsson et al. (2017) for further details.

Remark 3. For the proposed controller, the activation order of the phases is not specified. The stability analyses Nilsson et al. (2017) holds for any activation order, and one can use other tuning methods to decide the activation order within each cycle, e.g., to have green waves.

3. SIMULATION

The simulations are done with the open source micro simulator SUMO, Krajzewicz et al. (2012) on a scenario similar, but not identical, to one presented in Gregoire et al. (2015).

3.1 Network topology

The network is constructed as a Manhattan-like grid, see Fig. 1, with eleven bidirectional north to south streets (indexed A to K) and eleven bidirectional east to west streets (indexed 1 to 11). All streets with an odd number or indexed by letter A, C, E, G, I or K consist of one lane in each direction, while the others consist of two lanes in each direction. The speed limit on each lane is 50 km/h. The distance between each junction is three hundred meters. Fifty meters before each junction, every street has an additional lane, reserved for vehicles that want to turn left. Due to the varying number of lanes, four different junction topologies exist, all shown in Fig. 2, together with the set of possible phases. Each junction is equipped with sensors on the incoming lanes that are able to measure the number of vehicles queuing up to fifty meters from the junction. The sensors measure the queue lengths by the number of stopped vehicles.

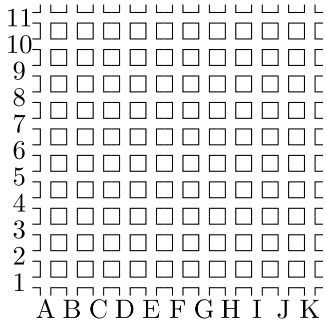
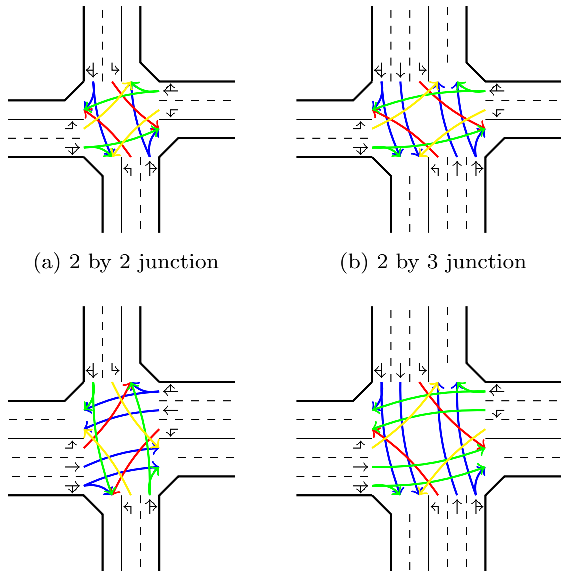


Fig. 1. The Manhattan-like network used in the simulation.



(a) 2 by 2 junction

(b) 2 by 3 junction

(c) 3 by 2 junction

(d) 3 by 3 junction

Fig. 2. The four different types of junctions.

Between the phases, there will be a fixed five seconds clearance time, before the next phase is activated.

3.2 Traffic demand

To generate the traffic demand, the tool *ActivityGen*[1] is used. The tool is a supporting tool for SUMO and generates a fictive traffic demand based on a given population and work/residential ratio among the roads. For the specific network in Section 3.1, we assume that the northern part of the city, streets 7-11, is a working zone, i.e., the ratio between the population among the street and workplaces among the street is 1 to 10. For the southern part of the city, streets 1-6, the ratio is the opposite. We evaluate the proposed control strategy for four different population scenarios: 1 000, 5 000, 10 000 and 20 000 citizens. We will study the traffic during the morning, from 6 am until 11 am. The number of vehicles that departures

[1] http://sumo.dlr.de/wiki/ACTIVITYGEN

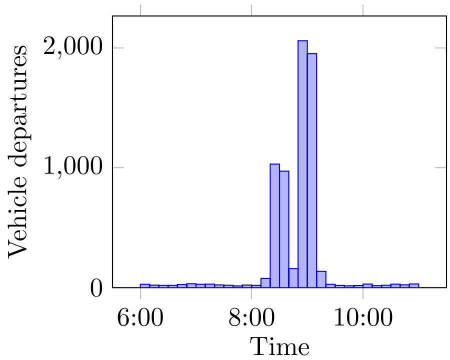


Fig. 3. The number of vehicle departures during the morning for the scenario with a population of 10 000.

during that time is shown in Fig. 3, for the case when the population is 10 000. For the other population scenarios, the number departures look similar but scale accordingly.

3.3 Control strategies

For each of the population scenarios, we compare three different control strategies:

(1) *Proportional control with different κ* The controller described in Section 2 where the values of κ will be 0.1, 1, 5, 10, 20 and 40. The value κ is chosen to be the same for all the junctions in the network. The clearance time between each phase shift is 5 seconds, so $T_w = 20$.
(2) *Proportional control with fixed cycle length* The cycle length will be the same for every cycle, 110 seconds, but the fraction of the cycle where each phase is activated, will be split according to (2) with $\kappa = 0$. The clearance time between each phase is 5 seconds. In the case that no vehicles are queueing when the cycle starts, all phases will be activated for an equal amount of time during the upcoming cycle.
(3) *Fixed timing* The first and third phases (blue respectively green in Fig. 2) are activated for 30 seconds each, while the second and fourth phases (red respectively yellow in Fig. 2) are activated for 15 seconds each. The clearance time between each phase is 5 seconds.

3.4 Results

The total queue lengths for different choices of κ is shown in Fig. 5 (last page) and the comparison of the different control strategies are shown in Fig. 6 (last page). Also, in Fig. 4 it shown how the cycle length varies in time for one specific junction, in this case junction E6. How the proportional controller reduces the queue lengths in comparison to a fixed time control strategy is also shown in Table 1. In Table 2 and Table 3 it is shown how much less or more time the vehicles spend queueing when using the proportional control strategy with dynamic respectively fixed cycle length compared to a fixed time control strategy.

Table 1. The relative length of the overall queue length with the proportional controller ($\kappa = 5$) compared to fixed signal timing.

	1 000	5 000	10 000	20 000
6:00 - 8:00	55 %	52 %	53 %	53 %
8:00 - 10:00	52 %	64 %	81 %	112 %
10:00 - 11:00	58 %	52 %	51 %	145 %

Table 2. The overall relative queuing time with the proportional controller ($\kappa = 5$) compared to fixed signal timing.

	1 000	5 000	10 000	20 000
6:00 - 8:00	23 %	22 %	23 %	24 %
8:00 - 10:00	24 %	48 %	71 %	122 %
10:00 - 11:00	26 %	21 %	22 %	256 %

Table 3. The overall relative queuing time with the proportional controller with fixed cycle length compared to fixed signal timing.

	1 000	5 000	10 000	20 000
6:00 - 8:00	116 %	119 %	125 %	164 %
8:00 - 10:00	259 %	312 %	283 %	260 %
10:00 - 11:00	117 %	110 %	133 %	6 995 %

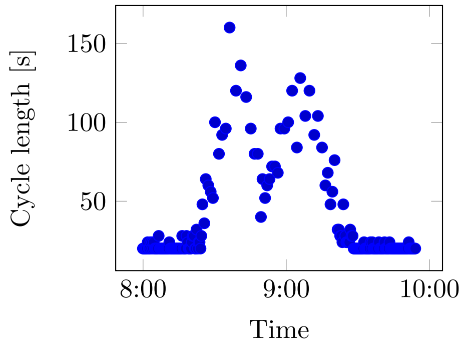


Fig. 4. How the cycle length for junction E6 varies with time when $\kappa = 5$ and the population is 10 000.

4. DISCUSSION

4.1 Choosing the parameter κ

Too small κ will yield too long cycles, which have some disadvantages. It may result in that more green light is given than necessary, and even if not, the drivers expect to see some movements of the queue within a reasonable amount of time. The problem with over-allocating green light can probably be solved by activating the next phase when the lane is empty, something that is a direction of future research. Another disadvantage of having long cycles is that it may cause unwanted spill backs, both by making the queues too long, but also by releasing too many vehicles at once.

On the other hand, too large a κ will give very short cycles may waste a too big fraction of the cycle to phase shifts. Since there is a maximum number of vehicles each sensor can detect, it may happen that a large value of κ makes the cycle and hence the phase activation time so small, so even when all the sensors are fully covered there is not enough time for the drivers to react.

For the simulations performed, it looks like that letting κ be between 5 and 10 gives the best performance. In Fig. 4 it can be seen that $\kappa = 5$ yields reasonable cycle times that adapts well to the fact of having two morning demand peaks in the departures, as seen in Fig. 3. From the expression in (1) it can also be seen that if κ is in the order of magnitude or less of the expected number of vehicles queueing, the number of vehicles have a larger influence on the cycle length.

4.2 Comparison between the strategies

In most of the scenarios, the proportional control with dynamic cycle lengths performs better than the other control strategies. One should keep in mind that the feedback controllers only have limited information to act upon, for instance, it is not able to see if vehicles are queueing up beyond the sensor covered area. This means that during high demands, when all the sensors are fully covered, the control action will be de same, independent of the state in the network. This is at least partly the explanation why the controller does not shorter the average queue lengths during high demands and hence also why the vehicles have to spend more time queueing compared to a fixed signal timing in this scenario. Worth mentioning is also that proportional fairness strategies without dynamic cycle lengths seem to give worse performance than the fixed control strategy, see Fig. 6, Table 2, and Table 3.

5. CONCLUSIONS

In this paper, we have verified that dynamic cycle lengths can improve the performance of traffic light controls. In the future, we plan to develop tuning rules for the parameter κ both based on the junction topology, as well as auto-tuning strategies using traffic state information.

Another point of interest is to introduce some weighting of phases. As it is now, when all the sensors are fully covered with vehicles, it follows from (2), that the phases will all be activated in proportion to the number of lanes in the phase. By introducing weighting, it should be possible to achieve different desired behaviors, in the case when the incoming lanes are fully occupied. Also introducing weighting of different lines would be interesting if it is known beforehand that a subset of lanes is critical.

In order to make the simulations more realistic, we plan to apply user adoption for the drivers' route choices. At the moment it is assumed that the drivers stick to their planned path, independent of the state of the network. In reality, the drivers will find better ways both by their a priori knowledge about the usual traffic situation and real-time congestion information. c.f. Como et al. (2013b).

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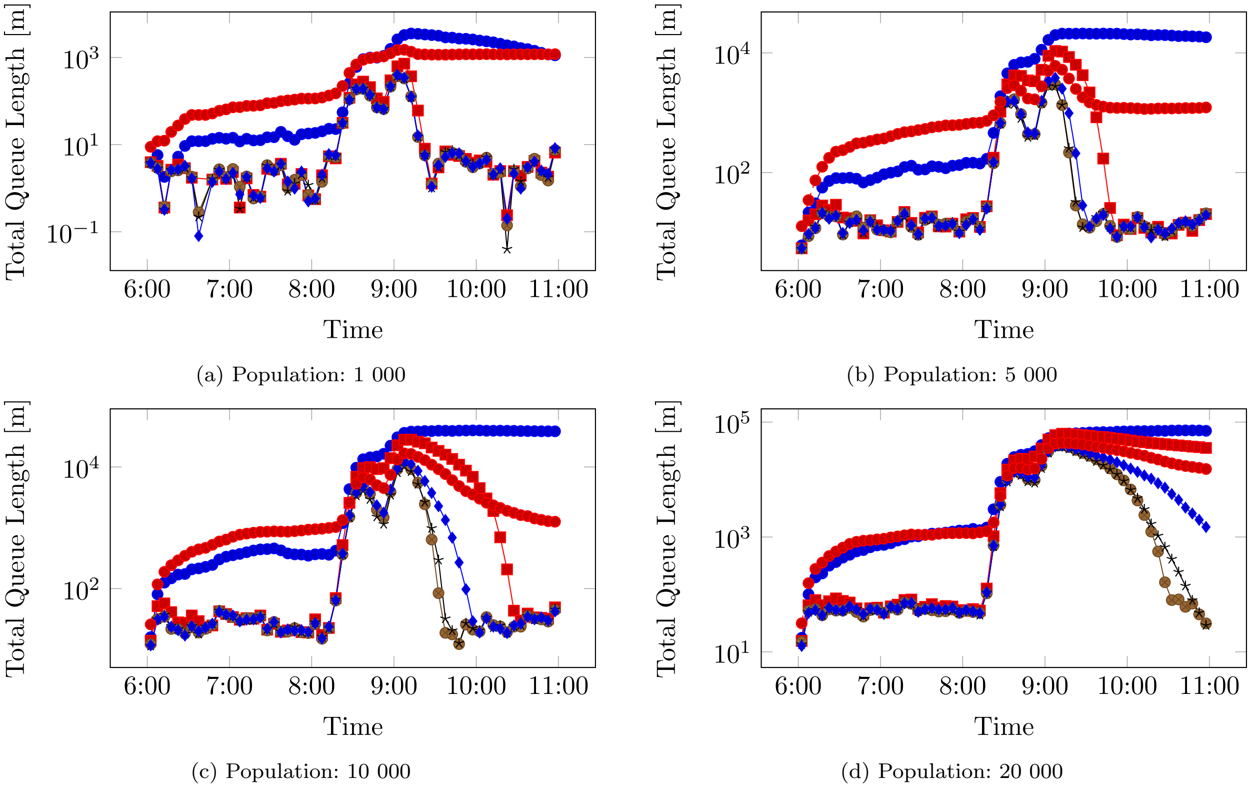


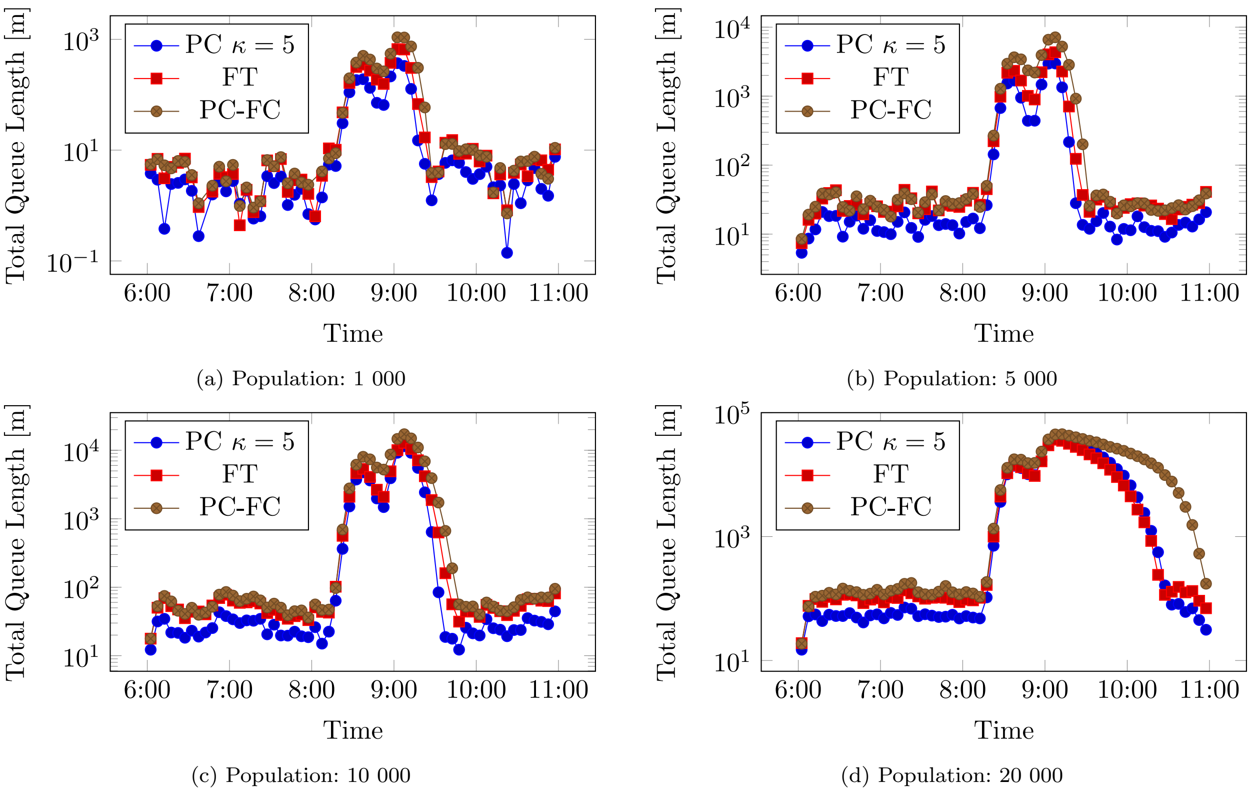
(a) Population: 1 000

(b) Population: 5 000

(c) Population: 10 000

(d) Population: 20 000

Fig. 5. The total queue lengths for different choices of κ for the four different scenarios. To improve readability, the queue lengths are averaged over a five-minute interval and the y-axis is in log-scale. The values of κ are 0.1 (—●—), 1 (—■—), 5 (—●—), 10 (—∗—), 20 (—♦—) and 40 (- -●- -).



(a) Population: 1 000

(b) Population: 5 000

(c) Population: 10 000

(d) Population: 20 000

Fig. 6. Comparison of the proportional control with dynamic cycle length (PC) with $\kappa = 5$, proportional control with fixed cycle length (PC-FC) and fixed signal timing (FT) for the four different scenarios. To improve readability the queue lengths are averaged over a five-minute interval and the y-axis is in log-scale.