Modelling cascading failures in lifelines using temporal networks

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Abstract: Lifelines are critical infrastructure systems with high interdependency. During a disaster, the interdependency between the lifelines can lead to cascading failures. In the literature, the approaches used to analyze infrastructure interdependencies within the social, political, and economic domains do not properly describe the infrastructures’ emergency management. During an emergency, the response phase is very condensed in time, and the failures that occur are usually amplified through cascading effects in the long-term period. Because of these peculiarities, interdependencies need to be modeled considering the time dimension. The methodology proposed in this paper is based on a modified version of the Input-output Inoperability Model. The lifelines are modeled using graph theory, and perturbations are applied to the elements of the graph, simulating natural or man-made disasters. The cascading effect among the interdependent networks has been simulated using a spatial multilayer approach. The adjacency tensor has been used to for the temporal dimension and its effects. Finally, the numerical results of the simulations with the proposed model are represented by probabilities of failure for each node of the system. As a case study, the methodology has been applied to a nuclear power plant. The model can be adopted to run analysis at different scales, from the regional to the local scales.

Keywords: Resilience; Interdependency; Lifelines; Temporal Networks.

1 Introduction

Lifelines can be defined as critical infrastructure systems, which provide a reliable flow of services and goods that are essential to the economic, social, and political security communities. From the civil engineering point of view, lifelines can be grouped into five principal systems: electric power, gas and liquid fuels, telecommunications, transportation, and water supply. The links among different networks increase the potential of cascading failures, which can bring up catastrophic amplification of the impact.

Critical infrastructure systems are dependent and interdependent in multiple ways, where dependency refers to the unidirectional relationship and interdependency indicates the bidirectional interaction. They usually present upstream-downstream relationships and loop relationships, which turn the infrastructures’ behavior into non-linear and non-stationary behavior. Several authors have provided different classifications (Table 1) of lifelines interdependencies. The first classification, which is still widely accepted, is the one given by Rinaldi [1]. In accordance with the period in which it was published, this classification refers only to interdependencies among physical lifelines. More recent classifications, like the one suggested by Cimellaro [2], take into account the interdependencies between both physical and
non-physical lifelines, and they are more appropriate for the evaluation of the overall level of resilience of a community.

Table 1: Types of interdependencies according to different authors

<table>
<thead>
<tr>
<th>Authors</th>
<th>Types of interdependence</th>
</tr>
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<tbody>
<tr>
<td>Rinaldi et al. [1]</td>
<td>Physical, Cyber, Geographic, Logical</td>
</tr>
<tr>
<td>Dudenhoeffer et al. [4]</td>
<td>Physical, Geospatial, Policy, Informational</td>
</tr>
<tr>
<td>Wallace et al. [5]</td>
<td>Input, Mutual, Shared, Exclusive, Co-located</td>
</tr>
<tr>
<td>Cimellaro et al. [2]</td>
<td>Physical, Cyber, Geographicalal, Policy/Procedural, Societal, Budgetary, Market &amp; Economy</td>
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Different modeling and simulation approaches have been developed to analyze the interdependency. The approaches are mainly classified into five groups: (i) system dynamics based models; (ii) network based models; (iii) empirical approaches; (iv) agent based models; (v) economic theory based models. A description of these types of models with their advantages and disadvantages can be found in Cimellaro (2016) [7] and in Ouyang (2014) [8].

However, none of the previous classifications and models analyzes the effect of the time dimension. Time dependent analyses are required when the temporal inhomogeneity matters and the sequence of events is important. This is usually the case of emergencies, where the importance of dependencies changes according to the needs of the community [9].

This paper proposes a new method based on the Input-output Inoperability Method. The method falls under the category of the economic theory based models [10]. The classic IIM is a static model, thus it is not able to manage dynamic dependencies. Many authors have overcome this limitation with extensions of the original IIM [11] [12] [13], while in the proposed approach the IIM has been modified using the approach of temporal networks. In the literature, there are several studies on temporal networks, which are summarized in Holme and Saramäky’s work [14]. Here, the use of temporal networks is adopted to model the cascading effects between critical infrastructures using a spatial multilayer environment.

2 Modelling temporal networks

To consider the effect of time on networks, it is required to have a model capable of representing the condition of the system at every time step. The methodology proposed in this paper is presented below.

2.1 Input-output Inoperability Method and its limitations

Developed by Haines and Jiang [10], the IIM model is an adaptation of the Leontief’s input-output (I-O) analysis of economic interdependencies [15]. The IIM proposed in this paper is intended to simulate the propagation of the inoperability risk in the infrastructure sector. The inoperability is defined as the inability for a system to perform its intended function. Mathematically, it ranges between 0 and 1: when 0 means that the element is functioning at full capacity, while 1 signifies a completely inoperativity of the element. The Equation describing the IIM is presented below:

\[ q = (I - A)^{-1} \cdot c \]  

(1)

where \( q \) is the damage vector which contains the inoperability values for the \( n \) infrastructures considered; \( A \) is a matrix which depicts the extent of interdependence among infrastructures and it is the transpose of the adjacency matrix describing the topology of the system; \( I \) is the
identity matrix; $c$ is the scenario vector that includes the effects of the disruption event (e.g. natural disaster, man-made attack, intrinsic failure, etc.) on each infrastructure. The damage vector $q$ is the output of the model and it represents the level of inoperability of the infrastructures composing the system according to the topology described by the $A$-matrix. Each element of this matrix indicates the influence of the $j$-th infrastructure on the $i$-th infrastructure, and can range between 1 (i.e. complete propagation of the scenario from $j$ to $i$) and 0 (i.e. no propagation).

To give an example of which are the outputs of the IIM, the case of a six-node network developed by Valencia [16] is reported (from now onwards it will be referred to as Example 1). There are two networks, an electric and a water network, serving three buildings (Figure 1a). The hazard considered is infrastructure aging. To measure the impact of any node decay across the network, the column summation of the damage vector $q$ of each node $i$ at each time $t$ is computed. This is the decay score (Equation (2)):

$$dc_s(t) = \sum_{i=1}^{n} q_i(t)$$

This approach, which is applied to a complex infrastructure network, presents three important limitations: (i) it does not take into account the redundancies of the system; (ii) it does not consider the temporal evolution of the system; (iii) its inputs and outputs are not significant probabilistic quantities.

If in the system of Figure 1a, a new pump house is added in parallel to the first one, the network presents a redundancy. Figure 1b shows the new topology of Example 2. It is clear that the performance of the system is improved with respect to the previous case. We expect that the impact of the water tower and of the electrical source remains the same, while the impact of each of the pump houses decreases. But if we compare the results obtained with the one-pump case, the expected trends are not met.

To solve the problems related to redundancies, probabilities of nodes in parallel can be combined properly. In the previous case, the $dc_s$ of the electrical source and the water tower are increasing because the algorithm sees another node (the new pump) that needs to be powered by them. To avoid this, the Series-Parallel Vector is introduced (Equation (3)):

$$SP = \begin{pmatrix} 
1/n_1 \\
1/n_2 \\
\vdots \\
1 \\
\end{pmatrix} \quad (forBLD)$$

where $n_i$ is the number of redundant nodes of node $i$. After having expanded it to the n-dimension, it is possible to add it to the damage vector equation (Equation (4)):
\[ q_i(t) = \left[ I - AS^p \right]^{-1}c_i \]  

(4)

After this operation, the values of the \( dc_s \) of the electrical source and water tower return to the expected values.

Now it is necessary to introduce an index able to represent the performance of the entire system: the system score. It is a dimensionless risk index that varies in the range \( 0 \div \infty \). It expresses the rating of a system of infrastructures, at the time \( t \). Equation (5) defines it:

\[ sys_s(t) = \sum_k \frac{\sum_{i} dc_s_k(t)}{n_k} \]  

(5)

where \( k \) is the type of node (i.e. electrical sources, water towers and pump houses). The final targets (i.e. buildings) are not considered when calculating the \( sys_s \). A low value of the \( sys_s \) indicates that the system of infrastructures has low risk of target nodes’ failure, while a high value indicates high risk.

2.2 Modified IIM for Temporal Networks

Graph theory has been used to model the infrastructure networks. The geographical, topological, and flow information of a network can be represented with a graph \( G(V, E) \) which is formed by a set \( V \) of vertices (or nodes) and a set \( E \) of edges (or links). The characterization of the nodes depends on the spatial scale of the considered problem, which might be an entire infrastructure [17], a sub-system or even a unit. To each node specific features such as hierarchy, resistance and autonomy can be attributed, while edges do not have any features assigned, but they are oriented. Moreover, the edges can link nodes intra-network (i.e. within a specific infrastructure) or inter-networks (i.e. across different infrastructures). The last one represents the interdependencies described in the \( A \)-matrix. Any inter-network link will be specified as Boolean, either 0 or 1. Thus \( a_{xy} \) values will be 0 if the \( x \)-th node belonging to the \( i \)-th infrastructure is not dependent on the \( y \)-th node belonging to the \( j \)-th infrastructure.

In addition to the existing formulations, the concept of chains has been introduced in the model. A chain is a sequence of nodes from one vertex to another through some edges. The chains of interest are those that connect a source (i.e. a node without inflows) to a sink (i.e. a node without outflows). It is assumed that every node of a chain must have at most one inflow edge, but can have multiple outflow edges. The hierarchy of the chains is defined by the design of the infrastructure.

The proposed methodology modifies the IIM deterministic formulation in probabilistic terms. The probability of failure of a single node is obtained by combining the natural hazard with the infrastructure vulnerability and it refers to the status of the node itself after the perturbation. Hereinafter it will be called self-failure probability \( (P_s) \) and will substitute the scenario vector \( c \). The hazard component is represented by an event vector \( E(n \times 1) \), where \( n \) is the number of nodes in the system. The elements of the \( E \)-vector can be physical quantities such as the \( pga, pgv, pgd \), the wave height of a tsunami, etc. Moreover, they can change from node to node, because infrastructures usually have a large spatial extension (Figure 2a). By performing different simulations, using different \( E \)-vectors it is possible to approach the problem in probabilistic terms. Each simulation has a weight, which is directly taken from the hazard curves. The vulnerability of each node is represented by the fragility curves (Figure 2b), and for each node there are as many fragility curves as the types of hazard acting. The probability of failure \( P_f \) of a node is obtained inserting the value of the \( E \)-vector in the fragility curve of the node. The approach proposed by Valencia [16] of summing up the elements of the \( q \)-vector to obtain a final score to evaluate the interdependency performances has obvious limitations, because they are not normalized to the dimension of the system. In
addition, it does not take into account the benefits given by the redundancies. In the modified IIM here proposed, the probability of failure $P_f$ of every node is obtained combining the $P_{sf}$ with the cascading failure probability $P_{cf}$ which is calculated using a step by step approach considering the topology of the system (Figure 2c).

![Flowchart of the probabilistic approach](image)

**Figure 2**: Flowchart of the probabilistic approach. Starting from a network perturbation (a) probabilities of failure of nodes are computed on fragility curves (b) and then propagated according to the topology of the network (c).

The more intuitive approach for analyzing a system of infrastructures is solving each network separately and then considering their interaction. Then, infrastructure networks are shown as layers, which overlap each other and share some nodes. Considering Example 1, the element pump needs both electricity and water. Using layer’s visualization, a single node will be projected in the two layers and a virtual edge will link the two projections (Figure 3).

Another issue to deal with is that the IIM can only use square matrices, while the inter-networks matrices are usually rectangular. To overcome this limitation, Valencia [16] suggests the introduction of a rectangular $I$-matrix. The idea is to increase the values of $c$-vector of the $i$-th infrastructure, by adding the $q$-vector computed for the $j$-th network (Equation (6)).

$$c_{j \rightarrow i} = I_{j \rightarrow i} \cdot q_j + c_i$$  \hspace{1cm} (6)  

![Example of layer subdivision for interdependent networks](image)

**Figure 3**: Example of layer subdivision for interdependent networks

Inserting the output of the first network into the input of the second one is the correct approach for the evaluation of the cascading effects. However, this formulation starts from the same deterministic values of before. The current method involves the combination of the $P_{cf}$ of upstream and downstream networks (Equation (7)).
\[ P^*_d = \left( I_{j \rightarrow i} \cdot P_{d_j} \right) \cup P_{d_i} \]

where the \( P^*_d \) can be considered cascading-failure probability which incorporates in the node all the information coming from upstream networks and nodes.

What has not been addressed yet is the temporal dimension. Therefore a timeline \( \tau = \{ t_0, t_1, t_2, \cdots, T \} \) is introduced. The time step \( \Delta t \) of the elements of the \( \tau \)-vector represents the time necessary for the propagation of the events across the entire system. Given this timeline, it is clear that to each event must be associated a time and that the model must run at every time step. Now the model is not stationary but is made up of temporal networks, denoted as a time dependent graph \( G(t) = G(V, E(t)) \). The \( P_d \) of nodes changes over time, in accordance with the sequence of events.

![Temporal variance of Operability Labels. Disruptive events tend to change their value and decrease the overall probability of operability (ΣPocc)](image)

The adopted strategy is to pass from bi-dimensional matrixes to a tri-dimensional tensor notation. The topology of every network is now described by an adjacency tensor, whose elements are \( a_{x,t} \). Each different temporal layer of the \( A \)-tensor represents a possible chain. To better understand which of the chains is active at the time \( t \), the probability of occurrence of a specific configuration \( P_{occ} \) is assigned to every layer (Figure 4). This value expresses if the layer is on \( (P_{occ} = 1) \), or off \( (P_{occ} = 0) \) at the considered time step. The condition to be on is that, in the current configuration, target nodes do not fail and that configurations with higher degree of hierarchy are off. Transferring this concept to the probabilistic model means that the values of \( P_{occ} \) become probabilities of being active. The sum of the probability of occurrence of a network is \( 0 \leq \Sigma P_{occ} \leq 1 \) and the term \( 1 - \Sigma P_{occ} \) represents the loss of capacity of the network (LoC).

### 3 Case study

The 2011 Fukushima nuclear power plant disaster has been selected to conduct the case study. The earthquake and tsunami that struck Japan’s Fukushima Daiichi nuclear power station on March 11, 2011, knocked out backup power systems that were needed to cool the...
reactors, causing three of them to be subjected to fuel melting, hydrogen explosions, and radiations.

3.1 Modelling the Nuclear Power Plant

The aim is to obtain a model of a nuclear power plant equipped with a Boiling Water Reactor (BWR), similar to the one in Fukushima. The topology and data regarding the disrupting events affecting the system are inspired by the Fukushima case study. However, the parameters of the system’s components are generic and taken from the literature.

The plant scheme of the Unit 1 (furnished by TEPCO) was used as a reference for building the nuclear power plant model. In this paragraph, the water and the electric networks are taken into account. The first networks is at the local scale, while the second one expands from the regional to the local scale.

![Diagram](image)

*Figure 5: Simplified model for lifelines serving a nuclear power plant*

Two different models are here presented: the first is simplified, the second is more detailed. The simplified model is shown in Figure 5 and is composed of an electric and a water network. The first emergency cooling systems consists of the Isolation Condenser (IC), which cools the steam coming from the reactor in a pool and does not need electricity. After this step, the High Pressure Coolant Injection system (HPCI) can cool the core in emergency conditions. It can draw water from the Condensate Storage Tank (CST) or from the Suppression Pool (SP) and then injects it into the core. The pump used by this system is steam-driven. The IC and HPCI systems were considered not dependent on electricity in the simplified model. However, in reality they are indirectly dependent on it because of the presence of electric valves.

To better model the eventual human interventions on the system, other three networks have been added: the telecommunication, the transportation, and the emergency services network. All the layers of this new detailed model are interdependent, as shown in Figure 6. Nodes have been classified according to their location and altitude, so that for each one it is possible to estimate the intensity of the event. Regarding earthquake and tsunami fragility functions, most of them has been taken from ATC-13 Earthquake damage evaluation data for California [18]. Other earthquake fragility curves have been taken from the ALA report [19] and from the HAZUS database [20]. Tsunami fragility curves have been considered as linear functions between two values from the ATC-13 recommendations. The autonomy curves have been modelled as step functions where the step is located in correspondence to the nominal value
indicated by Hitachi-GE [21].

Figure 6: Interdependent layers of the detailed model for lifelines serving a nuclear power plant

3.2 Analysis of the system

This section shows the results relative to the electric network of both the simplified and the detailed models. In the simplified model, the earthquake is responsible for the collapse of the AC transmission line. This leads to the loss of off-site AC power, which represents the first configuration of the electric network. The electric network assumes another configuration so that the power supply is still guaranteed to the water network. The arrival of the tsunami drastically changes the situation. The Diesel generators and the ordinary cooling line are out of order. Batteries are damaged too, but there is no need of them anymore since the pumps that they were feeding are failed. IC backup cooling system is kicked off and the probability of failure of the reactor core cooling is still close to 0. Ten hours after the earthquake, the autonomy of the IC starts to decrease and it is substituted by the HPCI system. As the probability of failure of the IC increases, the probability of failure of the reactor core increases as well, because it now relies on the HPCI system, which was seriously damaged by the earthquake and the tsunami. Figure 7 contains all these information.

Similar to the previous model, in the detailed model, the earthquake is responsible for the shutdown of the NPP turbine and to the off-site AC power. Electricity is still provided by diesel generators which feed the RHR system and the control room. Damages caused by the earthquake to emergency cooling systems imply that the first three backup lines have a probability of occurrence $P_{occ} \neq 0$, but on the other hand the most likely to be active is the RHR. After the tsunami, the diesel tanks, CSTs, and seawater pumps are completely damaged. Afterwards, the water network switches to the IC and HPCI configurations, but the control of their valves DC power is needed. The batteries have a relevant probability of failure, so the loss of capacity sharply increases. After three hours, valves are manually opened and the cooling is provided by the IC, until its autonomy runs out and brings to a complete loss of capacity (Figure 8).

Results show that both the models can effectively reproduce the ongoing situation for the electric network. There are no large differences because the differences between the two models are just the number of power panels and the split of target nodes’ supply lines. Target nodes cannot propagate upstream, and therefore this has no effect. Looking at the water network’s results, there are some differences because different sources feed different configurations of the water network. For this reason, the simplified model is not reliable.
4 Concluding remarks

Lifelines are a crucial part of society and their operability is fundamental for the wellbeing of the community. To reduce costs related to their failure and recovery, it is important to intervene on their vulnerability. Considering the system’s components as independent elements is not realistic because the interdependencies and the cascading effects play a major role. This work introduces a modified Input-output Inoperability Method containing three implementations of the traditional IIM. Firstly, a probabilistic approach is used. The model deals with hazard curves, fragility functions and probabilities of failure, which are quantities much easier to combine and understand than the ones of the original IIM. The second step is the adoption of a multilayer approach for modelling different interdependent networks. This allows a rapid and intuitive combination of analyses of separate networks, and at the same time gives the perception of the interdependencies’ role in the dynamics of failure. The third and major implementation is the adoption of a tensor notation with the aim of taking into account the temporal dimension of the problem.

The methodology has been tested on a real case study. Lifelines serving the Fukushima nuclear power plant has been modeled, from the regional to the building scale. To validate the model, results has been compared with the information related to the 2011 disaster. The results obtained with a simplified model are not entirely reliable, while a detailed mode could fit the situations occurred with a relatively low approximation.

Acknowledgements

The research leading to these results has received funding from the European Research Council under the Grant Agreement n° ERC_IDEAL RESCUE_637842 of the project IDEAL RESCUE—Integrated Design and Control of Sustainable Communities during Emergencies.
References