Identification of a lumped parameters numerical model of gas bearings: Analysis of 1d parallel plates

Identification of a lumped parameters numerical model of gas bearings: analysis of 1D parallel plates

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Abstract. Interest on numerical models able to estimate the damping capacity of gas bearings has been recently grown up. These models are based on the discretization of Reynolds equation with distributed parameters (DP) or can also be simplified models taking into account a few lumped parameters (LP), which are able to describe the dynamic characteristics of the bearing as well. In this paper a simple LP model is compared with the analytical solution for the case of the infinite parallel plates. It is shown that the model is able to predict correctly the damping capacity.

Keywords: gas lubrication, damping capacity, lumped parameters.

1 Introduction

Different are the areas of technology in which gas bearings are employed. In metrology applications aerostatic bearings are widely used to sustain the measuring machines. More in general, precision devices can successfully employ these components to ensure a frictionless and precise motion. In literature, several works analyze the dynamic characteristics of aerostatic bearings. For example in [1,2] the pneumatic hammer is studied evaluating the stiffness and damping coefficients. Pocketed and inherently compensated aerostatic thrust bearings were analyzed in [3-6] evaluating the effect of different design parameters on the dynamic characteristics.

Simple lumped parameters (LP) models can be successfully used to predict with good accuracy the static and dynamic characteristics of gas bearings. For their simplicity, they are suitable for sensitivity analyses and optimization of the geometry of the bearings. The authors in [7] developed a LP static model for rectangular aerostatic pads with multiple supply holes and in [8,9] they obtained analytically the expression of the dynamic stiffness. Regarding the squeeze effect, they also analyzed the dynamic stiffness of a rectangular pad with no supply holes, see [10].

In these previous works, the LP models were compared with DP models, but no exact solution was available due to the complex geometry involved. In order to prove the feasibility of a LP model in obtaining results with good accuracy respect to the exact solution, in this paper the LP dynamic model of the infinite length parallel plates is considered. In this case the analytical solution is known, as in [11]. The comparison proves the accuracy of the LP model respect to the exact solution.

2 The LP model

Two infinite length parallel plates of width B are considered, see Fig. 1. The pressure between the two plates is generated by the oscillatory squeeze motion of one plate respect to the other. The only coordinate on which the pressure depends is x, as along y direction there are no pressure gradients. A pressure distribution as depicted in the left of figure 1 is assumed in dynamic conditions, as a result of the squeeze motion. Coefficient α is variable in range $0<\alpha<0.5$ and will be later identified. In case $\alpha=0$ the pressure distribution is constant; in case $\alpha=0.5$ it is linear with its maximum in the center of the pad. In the right of figure 1 the equivalent pneumatic circuit is represented. A capacitance is considered in the middle of 2 equal resistances. The mass air flow rate G passes through the resistances as a result of the air gap variation in time.

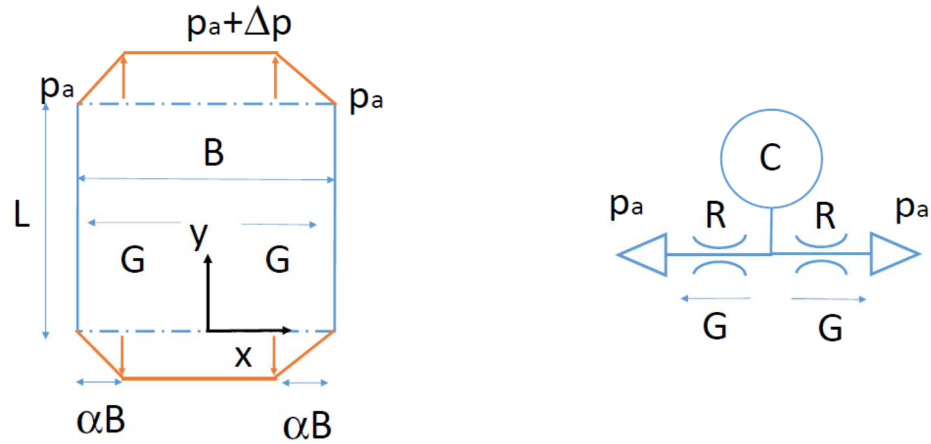

Fig. 1. In the left sketch of the geometry under study and of the pressure distribution assumed; in the right scheme of the equivalent pneumatic circuit

The following dynamic air gap and pressure are defined, which are the sum of the static values and the perturbations:

$$h' = h_0 + \Delta h \tag{1}$$

$$p' = p_a + \Delta p \tag{2}$$

The air flow *G* relative to the section of length *L* is calculated using the gas lubrication formula for the mono-dimensional flow:

$$G = \frac{h'^3}{24\mu RT}[p'^2 - p_a^2]\frac{L}{\alpha B} \tag{3}$$

The mass of air between the plates in the section of length *L* is

$$m = \frac{p'h'}{RT}BL(1-\alpha) + \frac{p_a}{RT}h'\alpha BL \tag{4}$$

The mass continuity equation can be written in dynamic conditions:

$$-2G = \dot{m} \tag{5}$$

In order to obtain the transfer function between the air gap h and the load capacity F the previous terms are linearized:

$$G \cong \frac{\partial G}{\partial h}\Delta h + \frac{\partial G}{\partial p}\Delta p = \frac{h_0^3}{24\mu RT}\frac{L}{\alpha B}2p_a\Delta p \tag{6}$$

$$\dot{m} = \frac{p_a}{RT}BL\Delta\dot{h} + \frac{h_0}{RT}BL(1-\alpha)\Delta\dot{p} \tag{7}$$

The linearized continuity equation turns to be

$$-\frac{h_0^3}{6\mu RT}\frac{L}{\alpha B}p_a\Delta p = \frac{p_a}{RT}BL\Delta\dot{h} + \frac{h_0}{RT}BL(1-\alpha)\Delta\dot{p} \tag{8}$$

The Laplace transform is now introduced

$$\Delta p(s)\left(\frac{h_0}{RT}BL(1-\alpha)s + \frac{h_0^3}{6\mu RT}\frac{L}{\alpha B}p_a\right) = -\Delta h(s)\frac{p_a}{RT}BLs \tag{9}$$

Considering that the perturbation of load capacity is related to the perturbation of pressure by

$$\Delta F = \frac{BL}{2}\Delta p \tag{10}$$

the transfer function between air gap and load capacity is then obtained:

$$\frac{\Delta F(s)}{\Delta h(s)} = -\frac{As}{\tau s+1} \tag{11}$$

where

$$A = \frac{3\mu\alpha B^3 L}{h_0^3} \tag{12}$$

$$\tau = \frac{6\mu\alpha(1-\alpha)B^2}{h_0^2 p_a} \tag{13}$$

The stiffness coefficient at zero frequency is null, while at infinite frequency it is A/τ. The damping coefficient at zero frequency is equal to A, while at infinite frequency it is null.

3 Analytical solution

The Reynolds equation for the infinite length parallel plates is

$$h^3\frac{d^2p^2}{dx^2} = 24\mu\frac{d(ph)}{dt} \tag{14}$$

In the middle of the plates (x=0) the boundary condition due to symmetry is imposed:

$\left.\frac{dp}{dx}\right|_{x=0} = 0$, while at the edges of the plates ambient pressure is imposed: $p_{x=B/2} = p_a$. The analytical solution can be obtained under the assumption of low amplitude of the squeeze motion [11]. The perturbed air gap is

$$h(t) = h_0 + \Delta h = h_0 + Re\left(\overline{\Delta h} e^{i\omega t}\right) \tag{15}$$

while the perturbed pressure is

$$p(x,t) = p_a + \Delta p = p_a + Re(\overline{\Delta p} e^{i\omega t}) \tag{16}$$

The linearized Reynolds equation yields

$$\frac{d^2(\Delta p)}{dx^2} = \frac{12\mu i\omega}{p_a h_0^2}\Delta p + \frac{12\mu i\omega}{h_0^3}\Delta h \tag{17}$$

which analytic solution is

$$\Delta p = \frac{p_a \Delta h}{h_0}\left[\frac{\cosh\left(\sqrt{\sigma} e^{\frac{i\pi}{4}}\frac{x}{B}\right)}{\cosh\left(\frac{1}{2}\sqrt{\sigma} e^{\frac{i\pi}{4}}\right)} - 1\right] \tag{18}$$

where σ is the squeeze number: $\sigma = \frac{12\mu\omega B^2}{p_a h_0^2}$.

Integrating the pressure field over the area the dynamic load is obtained:

$$\Delta F = 2L\int_0^{B/2}(p - p_a)\, dx \tag{19}$$

The coefficients of stiffness and damping are the real and imaginary parts of the solution:

$$k = \frac{LBp_a}{h_0} Re\left[1 - 2\frac{\tanh(\sqrt{\sigma} e^{i\pi/4}/2)}{\sqrt{\sigma} e^{i\pi/4}}\right] \tag{20}$$

$$c = \frac{LBp_a}{h_0\omega} Imag\left[1 - 2\frac{\tanh(\sqrt{\sigma} e^{i\pi/4}/2)}{\sqrt{\sigma} e^{i\pi/4}}\right] \tag{21}$$

4 Comparison between LP model and analytical solution

The dynamic coefficients of stiffness and damping calculated with the LP model and the analytical solution are here compared.

The following dimensionless coefficients are defined:

$$\bar{k} = \frac{kh_0}{LBp_a} \tag{22}$$

$$\bar{c} = \frac{ch_0\omega}{LBp_a\sigma} = \frac{ch_0^3}{12\mu B^3 L} \tag{23}$$

where h_0 is the static value of the air gap.

Fig. 2 represents the dimensionless coefficients vs the squeeze number. The LP results are relative to three different values of factor α. It can be seen that choosing value α=0.33 allows to obtain a very good estimation on damping. The explanation is that with this value of α, the LP pressure distribution better approximates the analytical one. About stiffness, the estimation is also good until σ=20, then the LP model diverges from the analytical solution. The difference at high frequency between the analytical solution and the LP model results is due to the low order of the transfer function of the LP model, which is of first order. A more accurate solution could be obtained at high frequencies increasing the order of the transfer function.

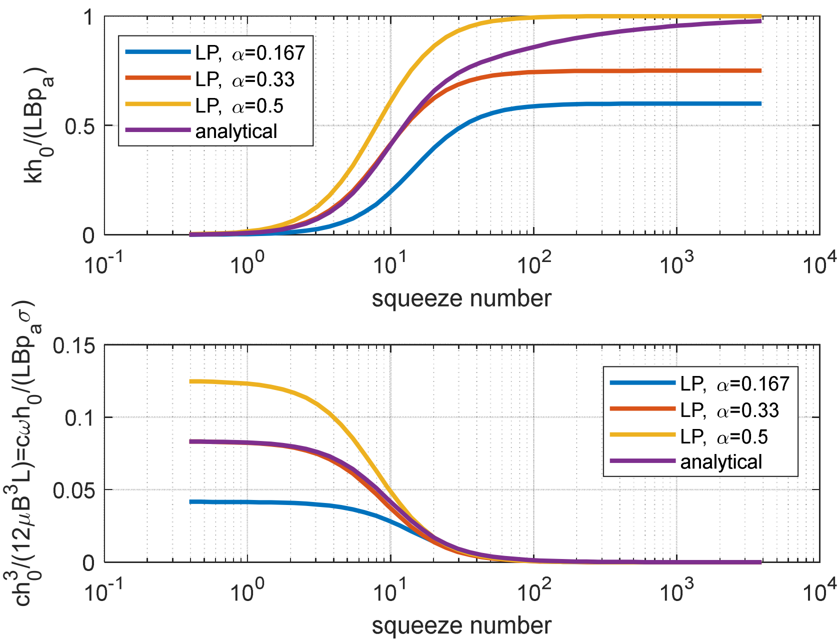

Fig. 2. Comparison between LP model and analytical model

5 Conclusions

The lumped parameters model of the mono-dimensional pad with no supply holes is in good accordance with the analytical solution, which is available for this simple geometry. The paper demonstrates that this methodology can be usefully employed in approximating the solution of a DP problem, of which in this case the analytical solution is available. More in general, the same methodology can be employed to study more realistic and complex geometries of aerostatic pads and to perform sensitivity analysis and optimization.

Nomenclature

α	coefficient of the approximated pressure distribution	k	stiffness coefficient
B	width of the plates	L	length of the plates
c	damping coefficient	μ	air viscosity
F	load capacity of the plates	p_a	ambient pressure
G	mass air flow rate	R	gas constant of gas
h_0	static air gap	σ	squeeze number
h'	perturbed air gap	T	absolute temperature
		ω	frequency of perturbation

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