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Gravity water wheels as micro hydropower energy source: a review based on historic data, design methods, efficiencies and modern optimizations

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Abstract

Nowadays, due to the need of clean energy and sustainable electricity production, hydropower is playing a central role in satisfying the energy demand. Particularly, low head micro hydropower plants (installed power less than 100 kW and few meters head) are spreading worldwide, due to their low payback periods and good environmental sustainability. Gravity water wheels are micro hydropower converters typically used in sites with heads less than 6 m and discharges of few cubic meters per second. Water wheels started to be scientifically investigated in the eighteenth century. After their scientific oblivion occurred in the twentieth century, in the last two decades scientific research on water wheels has undergone a revival.

In this paper a review on gravity water wheels was presented. Water wheels technology was discussed focusing on the geometric and hydraulic design; data and engineering equations found in historic books of the nineteenth century were also presented. Water wheels performance was described examining experimental results, and modern theoretical models for efficiency estimation were discussed. Finally, results achieved through experiments and numerical simulations were discussed with the aim of optimizing the performance of gravity water wheels.

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Results showed that the maximum efficiency of overshot and undershot water wheels can be identified in around 85%, while that of breastshot water wheels can be quantified between 75% to 80%, depending on the inflow configuration. Maximum efficiency of modern water wheels can be maintained at such high values over a wider range of flow rates and hydraulic conditions with respect to older installations. Hence well designed water wheels can be considered as efficient and cost-effective micro hydropower converters.

Keywords: gravity machine; hydrostatic pressure converter; water wheel; micro-hydro; water mill; renewable energy.

1. Introduction

Renewable energy sources for electricity generation in large scale have become an important purpose for meeting the renewable energy targets and for reducing greenhouse gas emissions [1] [2] [3]. Indeed, it is estimated that only 8% of the world consumed energy is generated from renewable sources, while 92% from non-renewable ones [4].

Table 1 shows the installed power of renewable energy sources in terms of GW at end of year 2013 [5]. It can be seen that among renewable energy sources (like biomass heating, solar heating system, wind power plants), hydropower plays a significant role in supplying the electricity demand, and large hydropower plants (installed power higher than 10 MW) are the most contributory renewable energy source. Furthermore, Asia, Africa and South America still have a large technical potential for hydropower which has not been exploited yet, and equal to about 12,000 TWh/year; in Europe the hydropower technical potential has been estimated in 1,000 TWh/year [5].

Hydropower exhibits some advantages over the other renewable solutions, like wind and solar power plants (from Tab.1 wind and solar plants are the second and third most diffused renewable sources) [6] [7]. For example, hydropower is more responsive to load management requirements, while pumping plants can consume electricity in low demand and low price periods, and provide it during periods with high energy demand. Hydropower output can be predicted more easily than solar and wind power plants, because hydro plants can be managed by human control, except in the case of long dry periods [6].

However, hydropower potential on large scale has been exploited in almost every part of industrialized countries, especially in Europe. Meanwhile, environmental impacts caused by dams of large hydropower plants in emerging countries are generally hardly accepted: flooding of large areas upstream, interruption of longitudinal connectivity of a river and problems with trapped sediments [6][8]. As a consequence, two main strategies can be identified for the future of hydropower development. The first is a better management of the output of existing big plants, with the installation of pumping stations for load management [9]. The second strategy consists in the installation of smaller hydropower plants, like micro hydro plants [6]. In this context, UNIDO, the USA Organization for the Industrial Development, classifies hydropower plants into the following categories: *large* for installed power more than 10 MW; *small* for installed power less than 10 MW; *mini* for installed power less than 1 MW; *micro* for installed power less than 100 kW; *pico* for installed power less than 5 kW.

Micro hydro plants exploit sites with heads of the order of meters or few tens of meters, and discharges of few cubic meters per second or less, as it can be observed in Fig.1 (the range $\leq$ 100 kW). Sites suitable for micro hydro are present in almost all countries [5]; for example, it is estimated that in Europe 350,000 sites suitable for micro hydro plants are available [10]. Micro hydro plants are very attractive because of their eco-sustainability and wide applicability on the territory, especially in rural and decentralized areas [6]. Indeed, the installation of micro hydro plants is considered as the most economical option for rural electrification [5]. When existing structures are used, only few new works are required, so that infrastructures costs can be reduced [11].

Micro hydro technology is a reliable and easy operation technology; the estimated life cycle is of more than fifty years and the global efficiency ranges from 60% to 90% [5]. In industrialized countries, micro hydropower can contribute to meet the non-fossil fuel targets imposed by public authorities. In emerging countries it can help to satisfy the increasing request of decentralized electricity. The additional advantages of micro hydropower plants are numerous and include grid stability, reduced land requirements, good opportunities for technology export and economic development at the local scale [12].

1.1. Micro hydropower turbines

In the hydropower field different machine types can be used to convert hydro energy into mechanical one [13]. Hydropower machines can be classified into 1) action turbines, like stream water wheels and vertical axis water wheels, Turgo, Pelton and Cross Flow turbines [11][14] [15] [16]; 2) reaction turbines, like Kaplan and Francis turbines [11] [17]; 3) hydrostatic pressure converters (HPC), like gravity water wheels (undershot, breastshot and overshot) and Archimedes screws [18]. Action turbines exploit the kinetic energy of the flow, hence the flow momentum. Reaction turbines exploit also the water pressure since they are installed inside closed and pressurized pipes. Hydrostatic pressure converters are driven by the hydrostatic force of water and operate in open air. The operational range of action turbines, reaction turbines and HPC is reported in Fig.1. It can be seen that, in the micro hydropower field, stream water wheels, gravity water wheels and Archimedes screws are the most suitable option.

Stream water wheels are installed in flowing water, and they exploit the flow kinetic energy with maximum power coefficient of 40% [19] [20]. This implies that very high flow rates and wheel dimensions are required to generated appreciable power output. Therefore, in the last years, an improved design of stream wheel has been introduced [21]. The new stream wheel is designed to self generate the required head, acting like a weir. Water level upstream is increased [22] [23], so that the hydrostatic force is mainly exploited instead of the flow kinetic energy. Hence the stream wheel becomes an HPC. The most representative machine in this context is the Hydrostatic Pressure Machine [21] [24], with maximum efficiency of 65%.

Instead, in Archimedes screws and gravity water wheels (the two most diffused kinds of HPC) the hydrostatic force is generated by the water weight contained inside the machine buckets. Thereby, they are called gravity machine. Archimedes screws rotate around an axle inclined on the horizontal of about 22° to 35°; they are called hydrodynamic screws when the external tube does not rotate with the screw, but it is fixed and acts only as a support [25] [26]. Gravity water wheels rotate around an horizontal axle. Maximum efficiencies of gravity machines are included between 70% to 90% [18] [27].

Three main types of gravity water wheels can be identified: *overshot*, where the water enters from the top, *breastshot* and *undershot*, where the water enters from the upstream side. Depending on the water entry point, breastshot wheels can be distinguished in high, middle and low. High breastshot wheels receive water over the rotation axis; middle breastshot wheels near the axis, and the low ones under the axis. Low breastshot water wheels can be also called undershot water wheels. Schematic historic representations are depicted in Fig.2. The most efficient kinds of undershot water wheels are *Sagebien* water wheels with forwards flat blades and *Zuppinger* water wheels with curved blades. *Sagebien* wheels are optimized for minimizing the inflow power losses, ensuring a gentle entry of blades into the upstream water. *Zuppinger* wheels are designed with blade shape to reduce the outflow power losses, reducing the portion of water that is uplift over the downstream water surface. Schematic historic pictures are depicted in Fig.3. Gravity water wheels will be the aim of the present paper.

1.2. Eco-compatibility and cost effectiveness of hydrostatic pressure converters

In relation to their efficiency, HPC can be considered the most eco-friendly and cost effective hydropower converters for the exploitation of very low heads and discharges [17] [28] [29].

With regards to the eco-compatibility, HPC operate with atmospheric pressure, with no pressurized pipes or draft tubes. Maximum rotational speeds are few tens of revolutions per minute (i.e. tangential speed of 1-2 m/s). These characteristics, combined with the large cells, are expected to make downstream passage of small fish possible, with a good behavior in relation to sediment passage. However, the effect of blade strike and cutting action when the blade enters the curved bed of the canal needs to be considered [30].

Research conducted at the end of the nineteenth century, the years when water wheels were still in large use and modern action and reaction turbines were just being introduced, indicates that gravity water wheels did not damage fish, as opposed to new turbines [30]. Indeed, recent tests showed that 75% of fish passed unharmed through water wheels, 92% through Archimedes screws, and only 45% through Francis turbines [29]. Tests conducted in Germany showed that fish were more able to pass through water wheels than Archimedes screws [31]. Instead, Ely et al. (2013)[32] claimed that *Sagebien* water wheels are unlikely to be used for transporting fish downstream due to their unattractiveness to fish.

Finally, it is worthwhile to write a general consideration regarding the impact of hydraulic structures of hydro plants on fish migration, with a focus on water wheels. When an hydro plant is installed in a flowing river, the dam generates impacts on ecosystems, in particular it affects fish migration and leads to flooding upstream [33][34]. Instead, hydro plants equipped with water wheels and Archimedes screws are generally installed in sites where small head differences already exist, and backwater propagation phenomena are limited. In these cases environmental impacts on the upstream migration of fish are minimized. On the other side, some harmful effects could be generated on fish if they get in contact with the machine blades, as discussed in [32]. Other environmental impacts related to water wheels (and Archimedes screws) could derive from the removal of water from rivers, or from the weirs construction for the generation of the head difference. For example, a reduction of 90% of salmon populations was found throughout North-Western Europe before 1600, due to improvements in watermill technology and their geographical expansion [35]. Nowadays, impacts can be minimized by installing water wheels in existing channels (e.g. irrigation canals) and in old water mills. As a reference, in Europe there is a huge potential concerning with this strategy [10].

Instead, the generation of noise during wheel operation could be a problem, especially when the blades are not well designed. For example, an overshot wheel in Pader (Gemany) suspended its operation due to the neighboring residents complaints about the wheel pulsating noise. Bristle elements were installed in the paddles to reduce the noise [36]. Furthermore, a Zuppinger water wheel in Germany had blades that slammed on the upstream free surface, generating a pulsating noise [31]. Such problem was also found during experimental tests on Zuppinger water wheels, but it did not occur for Sagebien water wheels [31]. Therefore, although a noise evaluation could be important in micro hydro schemes [37], the problem of noise generation of water wheels can be minimized by the optimal blade design.

For what concerns with the cost-effectiveness, for example in Germany overshot water wheels are currently built (including installation and grid connection) for 3900÷4400 €/kW of installed capacity, undershot wheels for 6900÷8700 €/kW, Archimedes screws approximately 7400÷7800 €/kW of installed capacity. For comparison, low head Kaplan turbines cost 13000÷13900 €/kW, hence water wheels cost is between 30% and 66% of Kaplan turbine ones [27] [38]. Maximum payback periods can be estimated as 14.4÷15.4 for Archimedes screws, 7.5÷8.5 years for overshot and 12÷17 years for undershot wheels (with expected life time of 30 years), which are very low if compared to Kaplan turbine installations for the same head, where payback periods of 25 ÷ 30 years can be expected [27] [38]. Furthermore, costs can be reduced when the existing hydraulic structures of abandoned water mills or irrigation canals are used. When HPC are installed at the outlet of wastewater treatment plants, the cost-effectiveness of the whole plant (treatment and hydro plant) can be optimized [39]. Table 2 summarizes the previous data.

1.3. Scope of the work

Scope of the present study is to present the state of the art on gravity water wheels. This is justified by the fact that gravity water wheels can represent an attractive solution in the micro hydropower field. This is due to the large worldwide diffusion of sites suitable for water wheels, mainly in rural areas, and their multi purposes [10]. Indeed, in addition to electricity generation and their lower costs, when installed in old water mills, water wheels can contribute to the preservation of the cultural heritage, the increase of tourism, the promotion of local manufacture (they can be used for grinding grain, forging iron, pumping water, sawing wood and stones, for metalworking and leather tanning) and the creation of employment.

In this review, historic books and manuals of the nineteenth century were firstly reviewed and discussed, and modern data on their performance (obtained mainly from works performed in the last two decades) were then presented and compared to older ones. Experimental, theoretical and numerical data were shown, and commented in light of water wheels practical application.

The paper is divided as it follows. In section 2 gravity water wheels history is briefly described. Then, gravity wheels are deeply examined, subdivided into overshot (section 3), breastshot (section 4) and undershot (section 5) water wheels. Each section is subdivided into four subsections. The *Design prescriptions* subsection illustrates some design rules for each wheel. The *Measured efficiency* one shows experimental data concerning with the hydraulic efficiency. The *Efficiency estimation* subsection discusses theoretical models to estimate the efficiency. Finally subsection *Performance improvement* shows some strategies to optimize the efficiency. A list of practical examples of water wheels in operation is also included in the paper.

2. Brief scientific history of water wheels

The first technical treatise dealing with water wheels was *Pneumatica* by Philo of Byzantium of the third century B.C., but a clear description of water wheels was made only by *Vitruvius* in *De Architectura* in the first century B.C. [40]. Water wheels spread considerably during Middle Ages, as a component of water mills [41].

Water wheels started to be scientifically investigated for engineering purposes in the eighteenth century. Stream water wheels were analyzed by many engineers and scientists like *Parent*, *de Borda* and *Smeaton* [21] [42]. From the eighteenth century onwards, stream wheels were frequently employed in order to generate mechanical energy. They were considered cost effective since little engineering work was required [27].

In 1704 *Antoine Parent* published his theory on jets and calculated the efficiency of stream wheels, estimating a maximum efficiency of $\eta = 8/27$ when the tangential speed of the blades was one third of the absolute flow velocity, but, as a consequence of a mistake, *Parent* limited the hydraulic efficiency of stream water wheels to just $\eta = 4/27$.

In 1767, *de Borda* published his theory and corrected *Parent* analysis, estimating the maximum efficiency in $\eta = 1/2$, when the blade speed was one half of the absolute flow velocity.

John Smeaton published then experimental data which demonstrated a maximum efficiency of stream water wheels of $\eta = 1/3$, greater than that provided by *Parent* ($\eta = 4/27$) but lower than that provided by *de Borda* ($\eta = 1/2$) [42]. In 1759 *John Smeaton* published also experimental data on gravity wheels [42], demonstrating the higher efficiency of gravity wheels over the efficiency of stream wheels.

Later, in the early nineteenth century, *J. V. Poncelet* performed a new blade design for the stream water wheel, increasing the maximum efficiency up to $\eta = 0.55 \div 0.6$ [27] [43]. The blades of the *Poncelet* wheel were shaped in order to minimize power losses during water entry; the blades were curved so that the water could flow from the tip of the blade toward the root, pushing on the blades also by its weight.

In the nineteenth century and at the beginning of the twentieth century, additional theories were developed, experimental tests on water wheels were conducted and water wheel use spread considerably [44] [45][46] [47][48] [49][50]. By 1820 France had 60,000 water wheels, by 1850 England had 25-30,000 water wheels, and by 1925 Germany had 33,500 water wheels [27] [43]. In Tab.3 some historic literature books are reported, highlighting what kind of information each book includes (theoretical models, experimental results, design prescriptions). Most of the information concerned with practical design suggestions, supported by theoretical considerations aimed at the estimation of power losses and efficiency. Some values on the efficiency were also reported, although it was not always clear if these values were obtained from experiments or empirically from real installations. However, theories developed during these years were generally not validated on experimental tests, although they had a good level of detail. Several prescriptions on water wheels design were empirical, and not based on scientific evidence. Furthermore, experimental tests conducted in those years had several uncertainty.

The last significant improvements in water wheels design were introduced in the middle of the nineteenth century, with the introduction of two particular kinds of undershot water wheels: the *Sagebien* water wheel, with flat and forward blades, and the *Zuppinger* water wheel, with curved blades (Fig.3). They took the name by their inventors, and replaced the classical less efficient undershot wheel with straight and radial blades. *Zuppinger* wheels were sometimes also used as breastshot water wheels.

At the end of the nineteenth century, with the advent of modern turbines (*Pelton*, *Francis* and *Kaplan* turbines) and big hydroelectric plants, the scientific interest on water wheels declined. Although water wheels continued to operate (but less than during the previous century), they were considered bygone and ancient hydraulic machines.

Nowadays, due to the new interest in micro hydropower, the scientific research on water wheels is experiencing a revival. There are now some companies that manufacture water wheels [27], and research centers that are carrying out research on them, as summarized in Tab.4. From Tab.4 it can be seen that a lot of experimental work has been performed on overshot water wheels. Theoretical considerations and experiments have been also developed for undershot and breastshot wheels, considering that experimental results and theories on breastshot wheels can also be extended to undershot ones. Numerical simulations (generally by using Computational Fluid Dynamic -CFD- tools) have been developing during the last years. For what concerns with real installations, Tab.5 reports some water wheel examples in operation, while real installations of water wheels are shown in Fig.4. It is worthwhile to note that a lot of water wheels are installed in old water mills.

3. Overshot water wheels

In overshot wheels water enters into the wheel from the top. They are generally used for head differences between 2.5 and 6 m, with maximum efficiency of 85% [51]. Typical flow rates per metre width are between 0.12 m^3/s and 0.3 m^3/s (Tab.5), so that the maximum flow rate recommended from literature is 0.2 m^3/s per metre width [27]. A typical overshot water wheel, with his theoretical sketch, is shown in Fig.5.

3.1. Design prescriptions

Overshot water wheels exploit the weight of water, by lowering the water within the cells from the upstream channel to the tailrace. The blade profile should be shaped as the curvature of the free jet during the inflow process. The cells should be designed also for retaining water inside them until the lowest position, where they empty rapidly. The cells should be filled with water at 30-50$\%$ of their volume, that means a filling ratio (water volume inside the bucket to the volume of the bucket) of 0.3-0.5. The opening of each cell is slightly wider than the jet, in order to let the air escape [27].

Maximum rotational speeds have been identified in $u/v < 0.6$ [27] and $u/v = 0.75$ [51], where u is the wheel tangential speed and v the inflow velocity. A critical rotational speed N_{cr} was also deduced, which is the maximum rotational speed after that the efficiency started to decrease [51]. The critical speed in revolutions per minute can be expressed as $N_{cr} = 31.3/\sqrt{D}$, where $D = 2R$ is the diameter and R the wheel radius. Instead, from Williams and Bromley (2000), $N_{cr} = 27.2/\sqrt{D}$ [52], so that a practical rule can be $N_{cr} = 30/\sqrt{D}$.

The number of buckets/blades n generally ranges between 20 to 50, for diameters between 3 and 6 meters [47][53]. A practical rule can be $n = 16R$ with R in meters [54], or $l = 0.75d + 0.1$ m [55], with l the distance between two blades and d the bucket depth in radial direction. In Fig. 6, previous equations and additional ones (see also Tab.6) are plotted, and blades data related to existing water wheels are also shown; real wheels data well fit the literature equations, with an interpolating equation equal to $n = 14.8R + 6.3$, valid for wheel radius ranging between 1 m and 3-4 m. Furthermore, the number of blades is similarly predicted by empirical design rules.

The depth of the buckets (i.e. the length of the cell along the radial direction) is generally $d = 0.2 - 0.35$ m [47][56][57]. Additional equations for the blades depth are reported in Tab.6 and plotted in Fig. 7 for a better comparison. As it can be seen from the figure, some discrepancies can be found for what concerns with the depth of the buckets in Fig.7.

3.2. Measured efficiency

Based on an historic literature review, maximum efficiency of overshot water wheels was identified in 80-85% [27]. This efficiency exhibited an almost constant trend over a wide range of operative conditions, in particular between $0.2Q_{max}$ and Q_{max}, where Q_{max} was the flow rate at maximum efficiency. This result has been also confirmed in Quaranta and Revelli (2015) [51]. Then, efficiency reduced at 80% at $1.5Q_{max}$. Over $1.5Q_{max}$ volumetric losses made the efficiency significantly decrease [51] (Fig.8). Maximum efficiencies occurred at rotational speed lower than the critical speed. Such high efficiencies have been also found in other experimental tests [58] [59].

3.3. Efficiency estimation

Denny (2004) [43] proposed a simplified method to estimate the efficiency. The proposed method can be considered accurate at low rotational speeds and flow rates, when volumetric losses at the top of the wheel are negligible. Instead, at rotational speeds higher than the critical one (see section 3.1), the previous estimate of such volumetric losses is advisable. With this combined strategy, the power output can be estimated with a discrepancy of 19% (based on experimental results [51]).

An alternative and more accurate method to estimate the efficiency is to calculate the power losses, that subtracted to the power input allow to obtain the power output and efficiency. This is a general method that is used in hydraulic machines, both for water wheels and Archimedes screws [60]. With reference to overshot water wheels, from Fig.5 power losses can be distinguished in: 1) inflow power losses (impact power losses L_{imp} due to the impact of water flow into the blades and volumetric water losses at the wheel top L_{Q_u}, i.e. water that does not fill the buckets); 2) outflow power losses (water that spills out from the bucket during rotation L_{Q_r}, and blade impact into the downstream water L_t); 3) volumetric losses (again L_{Q_u} and L_{Q_r}, already described) and 4) mechanical friction at the shaft (L_g) due the the weight of the rotating wheel. In all of these models the water inside the cells is supposed to be at rest, the hydraulic flow field is supposed one dimensional, and the effect of the centrifugal force, which makes the surface profile of the water not to be horizontal, is taken into account.

Such a power losses model has been developed and applied to experimental results with average discrepancy of 8.2% [51]. The most significant power losses are volumetric losses L_{Q_u} and L_{Q_r}, that can reach a maximum of 71% and 32% with respect to the power input, respectively. The model discussed in [51] can be considered a modern version of those developed in the past (see for example [47][48][61]), where volumetric water losses at the wheel top were not considered.

3.4. Performance improvement

Based on research conducted in recent times, it is possible to draw up some strategies to improve the performance of overshot water wheels [18][52] [62].

For example, a modified design of overshot water wheels has been proposed to increase the efficiency at flow rates and rotational speeds higher than the optimal ones, when a significant portion of flow rate flows away from the buckets. The design consists in a wall located around the periphery of the wheel, with the aim of reducing volumetric losses and improving wheel efficiency. The design of the wall is conceived in order to not affect the upstream conditions, guaranteeing atmospheric pressure at the top of the wheel. Thereby, the clearance between the wall and the wheel at the top should ensure that the wall does not enter in contact with the upstream water flow. The performance improvement was identified in more than 20% [18].

Wayudi et al. (2013) proposed a different method to increase the efficiency of overshot wheels [62]. The overflow of water from the buckets and the volumetric losses at the top of the wheel were converted into a water jet with high kinetic energy through a nozzle, and then squirt against the lowest blade. The efficiency increased from 61.6% to 73.5%. But considering that the wall of the first method can be done with a simple steel plate, the first improvement strategy is supposed to be more cost effective.

Furthermore, the effect of jet velocity has been investigated changing the slope of the conveying channel [52]. In the optimal range of rotational speeds (lower than the critical rotational speed), passing from a channel slope of 0° to 20° the power output increased of 12.5-30%. At higher rotational speeds, the power output increased more. However, the performance increase at higher rotational speeds should be mainly attributed to the fact that using a steeper channel a bigger amount of flow rate could enter into the buckets (volumetric losses reduction), and only partially to the better exploitation of the kinetic term.

As a final suggestion, it is recommended to not exceed the critical velocity, already discussed in previous sections.

4. Breastshot water wheels

In breastshot water wheels the flow fills the buckets entering from the upstream side of the wheel. Breastshot wheels rotate in the opposite direction with respect to overshot wheels [63], and they are usually employed for head differences lower than 4 m. The typical flow rate per metre width is between 0.5 m^3/s and 0.75 m^3/s (see Tab.5), so that a common flow rate is 0.6 m^3/s per meter width [27].

The inflow configuration of breastshot water wheels can be regulated using an overflow weir (*slow* breastshot wheels) or a sluice gate (*fast* breastshot wheels), with the aim of regulating the upstream water depth and the flow velocity to the wheel [49] [64]. In slow breastshot wheels, the flow kinetic energy is generally negligible and it does not contribute significantly to the torque. Inflow water depths are comparable with the blade height, so that the blades may experience a resistive drag force interacting with it. In fast breastshot wheels, the flow kinetic energy is significant, so that the flow momentum contributes to the torque. Inflow water depths are generally smaller than blade height. The drag experienced by the blades is generally negligible, but if the blades are not well designed [65], the flow momentum can be lost, and, sometimes, it can contribute negatively to power production, decreasing the power output. Also undershot water wheels can be classified into slow and fast. A typical breastshot water wheel is shown in Fig.9.

4.1. Design prescriptions

High breastshot wheels have generally diameters of $D = H+1$ [44] [66], with H the upstream-downstream water level difference. Middle breastshot wheels are generally built with diameters D slightly higher than $2H$, thus a radius slightly longer than the head H [49]. Low breastshot water wheels have generally diameters higher than 6 meters, and ratio D/H typically higher than 4. As already said, low breastshot wheels can be also considered undershot water wheels. The previous design rules are depicted in Fig. 10, where further design rules are illustrated.

The filling ratio of the buckets is well agreed in $0.3 - 0.5$, as for overshot water wheels, that can be extended up to 0.75 for low breastshot wheels [31].

Bresse [56] suggested a peripheral distance between two blades (l) of about $1.3 - 1.5$ times the upstream water depth (as confirmed in recent studies [67]), while in Chaudy (1896) [44] and in Garuffa (1897) [49] it was suggested $l = 0.4$ m. The depth of the cells was recommended to be $d = (0.4 - 0.5)(D/H)^{1/3}$ [48] or $d = (0.4 - 0.5)(D/4)^{1/3}$ [49].

Historically, the rotational speed was identified in $u/v = 0.4 - 0.6$ [44][66]. *Cullen* [54] suggested optimal rotational speeds of 8−10.6 rpm for diameters of 4.2−5.7 m, without considering the hydraulic conditions. But the optimal rotational speed depends on the hydraulic conditions, that is flow rate and flow velocity. Indeed, from recent results, the optimal rotational speed of a fast middle-low breastshot wheel was identified in:

$$u_v/v = (-1.24a^* - 0.22)Q^* + (1.73a^* + 0.19) \tag{1}$$

where $Q^* = Q/(u \cdot H_g^2)$, with H_g the canal drop [68], and $a^* = a/H_g$. a is the opening of the sluice gate and u_v is the blade tangential speed in the direction of the inflow velocity v. Equation 1 can be solved iteratively [64].

Instead, for slow breastshot/undershot wheels, where the flow kinetic energy is generally negligible with respect to the potential one, the rotational speed can be chosen as:

$$u = (0.2 - 0.4)\sqrt{2gH} \tag{2}$$

where H is the neat hydraulic head [31][64].

4.2. *Measured efficiency*

The performance of a slow breastshot wheel has been investigated by experimental tests by Müller and Wolter (2004), showing efficiency of 85% and constant from 0.2 Q_{max} up to Q_{max} (Q_{max} is the highest flow rate at maximum efficiency) [69]. Considering the cases with the sluice gate (fast breastshot wheel), the maximum efficiency was 75% between $(0.56 \div 0.6)Q_{max}$ and Q_{max}, where Q_{max} is the maximum flow rate in the range of constant efficiency for each geometric inflow configuration [64]. Previous data are depicted in Fig.11, where efficiencies at different sluice gate opening (*a*) for the fast wheel [64] and the efficiency of the slow wheel [69] are shown. The efficiency of the slow breatshot wheel is higher because of the lower power losses related to kinetic terms and water velocity.

4.3. *Efficiency estimation*

With regards to the efficiency estimation, one representative and simplified historic model to quantify the power output of a breastshot water wheel is that developed in 1843 by *Morin* [46][63], who made experiments on different breastshot wheels. He resumed his results by the following equation:

$$P_M = \chi \cdot \rho g \cdot Q \cdot \left[\frac{(v_e \cdot cos\alpha - u)u}{g} + H \right] \tag{3}$$

where $\chi = 0.77$ for fast wheels and $\chi = 0.8$ for slow breastshot wheels. H was the head difference (excluding kinetic terms) and α was the angle between the tangential wheel velocity u and the entry water velocity v_e. However, using the experimental data of [63] for a fast breastshot water wheel, it was found that χ could be expressed as a function of a dimensionless rotational speed of the wheel u^*. Therefore, it is possible to modify the coefficient $\chi = 0.77$ into $\chi = 1.23tanh(2.37u^*)$, where $u^* = \frac{u}{\sqrt{2gH}}$. Hence the tangential velocity u of blades tip was normalized to the diameter. Such normalized velocity was considered in order to generalize the results, taking into account that larger wheels rotate slower. In this way, using the modified *Morin* equation, the discrepancy with experiments reduces from 18% to 11% [70].

Finally, a dimensional analysis has been conducted from Vidali et al. (2016) to achieve a relationship to estimate the maximum power output, that is the power output occurring at the optimal rotational speed [68], valid for wheels geometrically similar to the investigated one.

The performance of a breastshot (and undershot) water wheel can be also quantified by power losses models. With reference to Fig.9, power losses can be classified into: 1) inflow power losses (head losses in the canal L_c due to turbulence and friction, impact losses L_{imp} of water flow into the blades, water losses L_{Q_u} due the the water that does not enter into the wheel, but that is lost through the gaps, drag losses L_{upstr} due to the resistance encountered by the blades flowing in water upstream); 2) outflow power losses (unexploited head L_h due the the water level difference between the downstream bucket and the water level at the talrace, drag undergone by the blade during the impact into the tailrace L_t, water uplift $L_{downstr}$ downstream); 3) mechanical friction (friction at the shaft L_g due to the wheel weight, friction on canal bed L_{bed} due the the water inside the buckets that is moving over the channel bed under the wheel); 4) volumetric power losses (water losses upstream L_{Q_u}, leakages during rotation L_Q through the gaps between the blades and the channel bed); 5) power losses related to buoyancy (L_{buoy}), that tends to push the blades upwards.

Furthermore, it is worthwhile to note that inflow power losses are different from fast and slow water wheels. Inflow losses in slow wheels are related to the drag undergone by the blades moving in water upstream, and the flow kinetic energy is generally not exploited. In fast wheels inflow losses are related to the dissipation of a portion of flow kinetic energy during the impact of water flow into the blades.

Such a power loss model was applied to a fast breastshot water wheel [63], with an average discrepancy with experimental power output and efficiency of 6.7%, and compared to older power losses models (see [44] [46] [49] [50]). Further power losses related to the residual water which may be uplift by the blades over the water surface at the tailrace $L_{downstr}$ were not discussed, since they were negligible in the investigated case (and the blades were shaped in order to avoid water uplift) [63]. Garuffa (1897) did not consider power losses in the headrace and downstream [49], while Chaudy (1914) did not consider also leakages and friction at the shaft [44]. The models revealed that the most important losses in fast breastshot wheels were the impact power losses at high flow rates, while at low flow rates leakage losses through the gaps between the blades and the channel bed. Instead, historical and older theoretical model generally did not consider further power losses.

4.4. Performance improvement

Recent studies have shown that the performance of breastshot water wheels can be improved. For example, numerical works showed a performance improvement of fast breastshot wheels based on the blades design, investigating shape [65] and number [67]. In particular, it is possible to recommend, for fast breastshot wheels, a distance between two blades shorter than 2.5 times the water depth just upstream of the wheel. Instead, concerning with the blade shape, some restrictions and suggestions were discussed for fast water wheels [65].

Different inflow configurations have also been investigated for a breastshot wheel [64]. The regulation of the opening of the inflow sluice gate was described as a way to guarantee always the optimal operative conditions for a constant speed of operation, at variable flow rate. At very low flow rates the use of a weir is more recommendable [64].

5. Undershot water wheels

Breastshot and undershot water wheels are filled from the upstream side. Their behavior is very similar, with the only difference in the water entry point, that in undershot water wheels is located in the lowest portion of the wheel. Therefore, undershot water wheels can be also called low breastshot wheels. There is not a precise rule to distinguish from undershot (low breastshot) and middle breastshot wheels. Anyway, as a rule of thumb, we can suggest to consider a wheel an undershot one when the entry point of water occurs in the lowest third of the wheel. Their working behavior and principle of operation is the same of breastshot wheels.

Undershot water wheels are used for heads up to 1.5 m, with maximum efficiency of 85%. Flow rates up to 1 m^3/s per metre width are suggested from literature [27] [31]. This is in agreement with typical flow rates of some operating water wheels, that can be estimated between 0.75 m^3/s to 1.3 m^3/s per metre width (see Tab.5). Also undershot water wheels can be classified into slow and fast (see section 4). All the considerations discussed for breastshot water wheels are valid. Therefore, in the following sections, only the special and most efficient undershot water wheels will be detailed discussed.

Undershot water wheels were originally built with radial flat blades. Then, from the middle of the nineteenth century, the *Zuppinger* and *Sagebien* wheels were introduced, that represented an optimization of the radial blades wheel. The former is with curved blades designed to minimize the outflow power losses. *Sagebien* wheels instead have flat blades, designed to minimize inflow power losses [31]. *Sagebien* and *Zuppinger* wheels are generally equipped with inflow weirs, and they are not conceived to exploit the flow kinetic energy. Typical *Sagebien* and *Zuppinger* water wheels are shown in Fig.3.

5.1. Design prescriptions

In the historic literature some geometric prescriptions can be found for radial blades water wheels. Although they are not so diffused nowadays, these geometric prescriptions are anyway described in this section.

In Weisbach (1849) the diameter D was suggested to be calculated by $D = (H - h_2)/(1 - \cos\alpha)$, where H is the head difference, $h_2 = 4.4v^2/2g$ (v is the absolute flow velocity and g is the gravitational acceleration) and $\sin\alpha = \sqrt{\frac{h_2 - h_d}{h_2}}$, with h_d the tailrace water depth [47].

Weisbach suggested to calculate the number of blades n, or the peripheral distance between two blades l, by the following formulations: n=18+9.8R or l= 7(1+4d), with the bucket depth d and the radius of the wheel R in meters, with a general suggestion of l=0.25$-$0.37 m and $d = 0.37 - 0.45$ m [47]. Pacinotti (1851) recommended to use $n = 12R$, with radius generally between 2.5 to 3.5 m (30$-$42 blades) [53]. Cadolini (1835) proposed diameters of 4$-$8 m and $l = 0.28 - 0.45$ m [71] (hence 45$-$56 blades). In Fig. 12 the number of blades versus the wheel radius is also plotted for some real wheels (from Tab.5). It can be seen that the real data well fit inside the literature trends, with an interpolating equation of $n = 15.1R - 8.6$, valid for wheel radius ranging between 2 m and 4.5 m.

With regards to the blade design of *Zuppinger* and *Sagebien* water wheels, *Busquet* suggested a depth of the cells of $0.4 - 1$ m [66], while Chaudy (1896) a depth of 0.6÷0.7 m [44]. Typically, *Sagebien* water wheels have 70 to 80 blades, while *Zuppinger* wheels from 32 to 48 blades. Experiments showed that the number of blades of *Sagebien* and *Zuppinger* wheels can be lowered to 30 blades with no significant penalty in efficiency [31] [72].

The *Sagebien* wheel have generally diameters from 7.5 to 10 m, although Busquet (1906) [66] suggested a diameter of approximately 4 m and a peripheral distance between two blades of

0.35−0.4 m (thus about 32 blades). The tangential velocity was usually taken as 0.6 to 0.8 m/s, although in some cases up to 2 m/s. The rotational speed ranged from 1.5 to 2 rpm, and the flow rate per metre width of 1 m^3/s. The blades are inclined of $40° - 45°$ to the upstream surface of water [44] [73]. In 1870, a total of 63 *Sagebien* wheels were installed in 15 Départements of France [74]. *Zuppinger* water wheels have diameters of 6 to 7.5 m, a speed of rotation of 4−4.5 rpm and flow rates of up to 1.2 m^3/s per meter width [31].

5.2. Measured efficiency

Measurements at a full scale *Zuppinger* wheel indicated efficiencies of 72 to 75% [75]. Experiments showed maximum efficiencies of 85% for a wheel model of 1.8 m in diameter and head difference of 0.25 m [72], and also for a wheel model 1.2 m in diameter with 24 blades [76]. In the latter work the *Zuppinger* water wheel model was investigated by a non-intrusive velocity measurement technique, and Particle Image Velocimetry was developed for a better understanding of the flow physics around the wheel [76].

Tests have been conducted to determine the eco-compatibility of *Sagebien* water wheels for fish upstream migration [32]. A wheel in 0.9 m diameter was tested with rotational speeds of 1.2 and 2.4 rpm. The maximum efficiency was estimated in 64% at 2.4 rpm. The low efficiency was caused by the very low rotational speed (not optimal conditions); it was anyway in agreement with tests conducted from Quaranta and Müller (2017) [31].

In Quaranta and Müller (2017) *Zuppinger* and *Sagebien* wheels have been investigated and compared. The *Sagebien* wheel was tested with a modified geometry, 30 instead of the traditional 70 to 80 blades, to model a cost-effective design. It was found a maximum efficiency of 84%. The *Sagebien* wheel efficiency was less dependent from the flow rate, whilst efficiencies for the *Zuppinger* wheel had a well identified maximum and reduced for lower flow rates [31] (Fig.13). As reported in section 1.2, *Zuppinger* wheels could generate a pulsating noise, while in *Sagebien* this problem was absent, due to the better shape of the blade.

5.3. Efficiency estimation

The performance of undershot wheels can be estimated using the power losses model explained for breastshot water wheels. *Sagebien* and *Zuppinger* wheels are not conceived to exploit the flow momentum, and they are generally designed as slow water wheels. Therefore, with respect to the model presented in [63] for fast water wheels, additional considerations can be made. For what concerns with inflow power losses, the attack angle of *Sagebien* wheels blade with respect to the relative flow velocity can be considered almost zero, hence the drag power loss in L_{upstr} is minimized. Instead, the drag coefficient C_d of *Zuppinger* wheels, to evaluate drag power losses, can be considered almost $C_d = 2$, since their blades profile is near parallel to the free surface of water upstream, generating a slamming effect [31]. The kinetic energy is totally dissipated. A further consideration can be done for the uplift of water downstream. Because of the blades shape, in *Sagebien* wheels this power loss becomes appreciable especially at rotational speeds higher than the optimal ones. Note that for *Zuppinger* wheels the uplift of water is practically zero, since they are conceived with blade shape to minimize the outflow power losses.

5.4. Performance improvement

It was found that both *Sagebien* and *Zuppinger* wheels had good hydraulic characteristics. *Sagebien* wheels have the additional environmental advantage that did not generate infrasound emissions [31]. The performance of *Zuppinger* and *Sagebien* ones remained optimal also with a reduced number of blades [31][72], that is 30 instead of what was recommended in the historic literature. Optimal rotational speeds, as for breastshot wheels, can be estimated by eq.2. The noise of *Zuppinger* wheels can be reduced by using bristle elements in the paddles [36]. Instead, the use of inflow weirs can allow a better upstream water level management and the exploitation of a wider range of flow rates [77].

6. Discussion

The results here presented showed that maximum efficiency of gravity water wheels ranges from 75% to 85%. Furthermore, their installation costs and payback periods are smaller than Archimedes screws and Kaplan turbines ones (these are machines that can be used in similar hydraulic conditions), although water wheels efficiency is slightly lower. Therefore, gravity water wheels can be considered attractive and competitive hydropower converters. Their performance characteristics allow for the efficient exploitation of low head sites, also where the flow rate is variable. Water wheels can be installed both at old mill sites, and in canals for electricity generation.

The efficiency was shown to be highly dependent on wheel rotational speed for undershot and fast breastshot water wheels. Instead, the efficiency of overshot and slow breastshot wheels can be considered not affected by the wheel speed inside a wider range of rotational speeds, whereas outside of it the efficiency decreases.

The inflow power losses, related to the inflow configuration, were identified to be the most significant ones, so that they have to be considered the most important losses to be minimized. In overshot water wheels the inflow power losses are represented by volumetric water losses at the end of the conveying channel (i.e. at the inflow of the wheel). In fast breastshot water wheels the dissipation of flow kinetic energy noticeably affects the efficiency. With regards to undershot water wheels, *Sagebien* water wheels perform better than *Zuppinger* water wheels, since the former are optimized for the inflow conditions rather than for the outflow ones. Some of the presented theoretical models are accurate enough to predict the performance of water wheels, with discrepancy less than 10% from experimental results. Hence they can be used for engineering applications.

The performance of gravity wheels can be further improved. Overshot wheels efficiency can be improved by reducing volumetric losses; two optimization strategies were discussed. The former, simpler and more effective, consists in converting the energy of water that would be lost into potential energy [18], while the second strategy consists in converting energy of the lost water into kinetic energy [62]. These designs allow to extend the operational range of overshot wheels at higher flow rates. Instead, the performance of breastshot water wheels can be optimized by combining different inflow configurations, like sluice gates and overflow weirs, with the aim of maintaining the optimal efficiency also with variable flow rate [64]. The performance of undershot water wheels was identified to be optimal also using a smaller number of blades with respect to that commonly used, increasing the economic feasibility of undershot water wheels [31] [72].

Therefore, thanks to the research conducted in the last decades, it is possible to claim that:

i) rules for the geometric design of overshot water wheels were discussed in this review, so that they can be used for their design. Studies on blades design for breastshot and undershot water wheels have been performed, giving additional information for achieving an optimal design [65] [67][72] [76]. Experimental tests and numerical simulations have been performed, showing more light on the hydraulic behavior of gravity water wheels;

ii) breastshot and undershot water wheels have been studied and investigated. Two series of tests were performed for breastshot wheels [63] [69] and four for undershot wheels [31] [72] [75] [76]. Their performance is now clearer, although the number of experimental works is still lower than the number of works available for action turbines, reaction turbines and Archimedes screws. Further experimental tests, especially on middle and high breastshot water wheels would be useful;

iii) some water wheels are now being used for electricity generation, and more scientific material is available in literature. Therefore, the public image of water wheels as ancient and romantic machines is being gradually replaced. However, further research should be carried out on the electro-mechanic equipment, and its coupling with the wheel. Indeed, the electro-mechanics equipment represents the most significant difficulty in water wheels operation. The difficulty is related to the low rotational speed of water wheels and to the need of changing the rotational speed as a function of the external hydraulic conditions for undershot water wheels. Preliminary works have been conducted to improve the transmission of the rotation (the gearbox) [78] and the electric generator [79]. Instead, in [64][77], hydraulic structures like adjustable inflow weirs and sluice gates have been used to avoid the use of the variable rotational speed, hence to adapt the external conditions to the constant speed of the wheel. Therefore, the whole electro-mechanic equipment needs to be further investigated and optimized, as it is being done for Archimedes screws [80] [81];

iv) further works should also be carried out on the Hydrostatic Pressure Machine to optimize its geometric dimensions, and on the *Zuppinger* turbine wheel, that is a *Zuppinger* wheel with the buckets completely filled with water and where water enters into the buckets from the sides [17].

7. Conclusions

Gravity water wheels are hydraulic machines that mainly use water weight to produce energy. They spread considerably during the eighteenth and nineteenth century, but scientific research on their performance and design declined in the twentieth century. Nowadays, considering their high efficiency, sustainability and low costs, gravity water wheels represent attractive hydropower converters in low head sites.

In this paper, design rules were reviewed and the efficiency discussed, including theoretical models to estimate the performance; some strategies to improve the performance were also discussed.

The maximum efficiency ranges between 75% to 85%. It exhibits an almost constant trend over a wide range of operative conditions, in particular between $0.2Q_{max}$ to $1.5Q_{max}$ for overshot and slow breastshot wheels, and $0.6Q_{max}$ to Q_{max} for fast breastshot wheels, where Q_{max} is the flow rate at maximum efficiency. Recent experimental tests showed that the performance can be improved, both acting on the inflow configuration, and on the blades design, thus the optimal operational range of water wheels can be extended. The efficiency of undershot water wheels is instead more affected by the flow rate, so that they have to be designed for a given situation with more attention than what should be done for the others kinds of water wheels. Furthermore, the higher the flow velocity, the more important is the choice of the wheel rotational speed.

The only drawback of gravity water wheels is their low rotational speed, that is generally less than 10 rpm. This implies the need of high and expensive gearboxes to produce alternate electricity.

Thereby, water wheels can be considered suitable micro hydropower converters in low head sites, since they are efficient, simpler and cheaper to be installed than other turbines. However, their design must not be under evaluated.

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List of Figures

Table 1. Global installed capacity of renewable energy worldwide [5] and diffusion percentage with respect to the total installed capacity of renewable sources.

Type	Power (GW)	% on the total
Large Hydropower	860	52%
Biomass heating	250	15%
Solar cells	145	9%
Wind power	121	7%
Mini Hydropower	85	5%
Ethanol production	67	4%
Biomass power	52	3%
Geothermal heating	50	3%
Solar Photovoltaic grid connected	13	1%
Biodiesel Production	12	1%
Solar thermal power	10	1%
Ocean power	0.8	$\leq 0.1\%$
Total	1665.8	100%

Table 2. Typical exploitable head and flow rate of gravity machines. Efficiency, cost and maximum payback times are also reported. For what concerns with the costs, German costs are considered [38]. Breastshot wheels costs and payback times are considered as intermediate between overshot and undershot ones. Flow rates of water wheels are per metre width of the wheel. For comparison, Archimedes screws data are also reported.

Type	Head m	Max. Flow rate m^3/s	Max. Efficiency %	Cost €/kW	Payback time years
Overshot wheels	$3-6$	0.2	$80-85$	$3900-4300$	$7.5-8.5$
Breastshot wheels	$1-4$	$0.6-1$	$70-85$	$4000-7000$	$8-12$
Undershot wheels	≤ 1.5	1	$70-85$	$6900-8700$	$12-17$
Archimedes screw	$1-6$	8	$80-85$	7400-7800	14.4-15.4

Table 3. Scientific research performed until the beginning of the twentieth century. The kind of investigated water wheel is reported (overshot, breastshot and undershot water wheel). “T” means that theoretical works were reported, “E” means that experimental tests were conducted (generally experimental procedures were not described) and “D” means that design rules were shown.

Author	year	Institution	Country	Overshot			Breastshot			Undershot			Reference
Weisbach	1849	Academy of Freiburg	Germany	T	E	D	T	E	D	T	E	D	[47]
Pacinotti	1851	University of Pisa	Italy	T		D	T		D	T		D	[53]
Sagebien	1866	-								T	E	D	[83]
Bresse	1869	Ecole des Ponts	France	T		D	T		D	T		D	[56]
Cullen	1871	-			E	D		E	D		E	D	[54]
Bach	1886	Polytechnic Zu Stuttgart	Germany	T	E	D	T	E	D	T	E	D	[48]
Chaudy	1896	-		T	E	D	T	E	D	T	E	D	[44]
Garuffa	1897	-		T	E	D	T	E	D	T	E	D	[49]
Mueller	1899	-		T	E	D	T	E	D	T	E	D	[73]
Busquet	1906	Ecole de Lion	France	T		D	T		D	T		D	[66]
Weidner	1913	University of Wisconsin	USA	T	E	D							[61]
Church	1914	Cornell University	USA	T	E	D	T	E	D	T	E	D	[50]

Table 4. Scientific papers on gravity water wheels published in the last decades. The kind of investigated water wheel is reported (overshot, breastshot and undershot water wheels). "T" means theoretical work, "E" means experimental work and "N" means numerical simulations. All the reported papers reported also design suggestions based on the achieved results.

Authors	year	Institution	Country	Overshot	Breastshot	Undershot	Reference
Williams and Bromley	2000	Nottingham Trent University	UK	T E			[52]
Müller and Kauppert	2002-2004	Southampton University	UK	E	E	E	[27] [38]
Müller and Wolter	2004	Southampton University	UK		T E		[69]
Dubas	2005	Haute Ecole Valaisanne	France	T E			[58]
Wahyudi et al.	2013	Polytechnic of Malang	Indonesia	E			[62]
Von Harten et al.	2013	University of Stuttgart	Germany			E	[72] [75]
Pelliciardi	2015	University of Siena	Italy	T E			[84]
Quaranta and Revelli	2015	Politecnico di Torino	Italy	T E			[51]
Quaranta and Revelli	2015-2016	Politecnico di Torino	Italy		T E	T E	[63] [64]
Vidali et al.	2016	Politecnico di Torino	Italy		T E		[68]
Quaranta and Revelli	2016	Politecnico di Torino	Italy		N	N	[65] [67]
Quaranta	2017	Politecnico di Torino	Italy			N	[18]
Quaranta and Müller	2017	Politecnico di Torino-Southampton University	Italy-UK			E	[31]
Paudel et al.	2017	Darmstadt University of Applied Sciences	Germany			E	[76]

Table 5. Geometric characteristics of some gravity water wheels in operation, with their exploited head H, flow rate Q, diameter D, width b, number of blades n, rotational speed N and electrical power P_{el}. The Table also specifies the type: overshot "O", breastshot "B", *Sagebien* "S" and *Zuppinger* "Z" water wheels.

	Company/ Owner	H [m]	Q $[\frac{m^3}{s}]$	D [m]	b [m]	n $-$	N [rpm]	P_{el} [kW]	website/ reference	Type	Country
1	Smith Engineering [1]	-	-	4.1	-	24	-	-	http://www.smith-eng.co.uk/hydro/	O	US
2	Woodson's Mill (not work)	-	-	7.2	-	40	-	-	http://oldmills.scificincinnati.com/	O	US
3	Jasper City Mills	-	-	7.2	1.44	64	-	-	http://oldmills.scificincinnati.com/	O	US
4	Spring Mill	-	-	7.5	-	60	-	-	http://oldmills.scificincinnati.com/	O	US
5	Pine Run Grist Mill	-	-	4.8	0.6	40	-	-	http://oldmills.scificincinnati.com/	O	US
6	Hoover Mill	-	-	8.1	0.6	84	-	-	http://oldmills.scificincinnati.com/	O	US
7	Hopkins Old Water Mill (not work)	-	-	5.7	3	48	-	-	http://oldmills.scificincinnati.com/	O	US
8	Phoenix Mills	-	-	6	0.9	-	-	-	http://oldmills.scificincinnati.com/	O	US
9	Phoenix Mills	-	-	4.5	1.2	36	-	-	http://oldmills.scificincinnati.com/	O	US
10	Free flow 69 [2]	-	-	2	-	24	-	-	http://www.freeflow69.com/	O	UK
11	Free flow 69	-	-	3.2	-	32	-	-	http://www.freeflow69.com/	O	UK
12	Hydrowatt [3]	3	0.6	2.6	2.5	24	11	10	http://hydrowatt.de/de/en/	O	Germany
13	Hydrowatt	3	0.2	2.7	1	28	11	3.5	http://hydrowatt.de/de/en/	O	Germany
14	Hydrowatt	5.3	0.12	5	1	48	5.7	5	http://hydrowatt.de/de/en/	O	Germany
15	Hydrowatt	4.6	0.4	4.2	1.5	36	7	11	http://hydrowatt.de/de/en/	O	Germany
16	Hydrowatt	3.4	1.3	2.9	4	36	12	27	http://hydrowatt.de/de/en/	O	Germany
17	Hydrowatt	3	0.3	2.7	1	-	-	5.5	http://hydrowatt.de/de/en/	O	Germany
18	Cooperation project MAE-FAO	3.5	0.15	3	0.84	32	10	2.5	[84]	O	Nepal
19	PI Mitterfellner GMBH [4]	4.0	0.2	4.0	1	36	7-8	6.44	http://www.planing.at/	O	Austria
20	PI Mitterfellner GMBH	4.0	0.08	4	0.75	36	8	2.2	http://www.planing.at/	O	Austria
21	Ciconio mill [5,9]	-	0.058	3	2	24	-	-	[51]	O	Italy
22	Dronero mill [6,9]	-	-	3	1.3	30	-	-	http://www.mulinodellariviera.com	O	Italy
23	Mulino di Verolengo [9]	0.7	-	4	1.4	32	-	-	[63]	B	Italy
24	Mulino del Pericolo [9]	1.3	0.5	3.9	0.5	24	14	-	Pers. Comm.	B	Italy
25	Mulino di Borgo Cornalese [9]	1.85	1	3.6	1.35	36	-	11.8	Pers. Comm.	B	Italy
26	Franklin Creek Grist Mill	-	-	3.6	1.5	36	10	15	http://oldmills.scificincinnati.com/	B	US
27	Hydrowatt [3]	1	3	6.5	2.3	42	4.5	20	http://hydrowatt.de/de/en/	S	Germany
28	Patrick H. Marceau [9]	-	-	11	6	70	-	112.5	www.panoramio.com	S	France
29	Les Avins Roue [7]	-	-	9.2	-	70	-	13-18	http://coopcec.be/wcec/	S	France
30	Marie-Paule DUPUY [8,9]	-	-	7.5	-	32-40	-	-	Pers. Comm.	S	France
31	Marie-Paule DUPUY[8]	-	-	7	3	56	-	-	Pers. Comm.	S	France
32	Müller and Kauppert	1	-	6.5	2.3	-	-	0.7	[38]	Z	Germany
33	Hydrowatt [3]	2.1	1	6.5	1.2	36	4.5	12	http://hydrowatt.de/de/en/	Z	Germany
34	Hydrowatt	2	1	4.2	2.9	24	6.5	11	http://hydrowatt.de/de/en/	Z	Germany
35	Hydrowatt	1.1	4	5.5	4	30	5.5	26	http://hydrowatt.de/de/en/	Z	Germany
36	Hydrowatt	2.1	2	6	2	36	4.8	27	http://hydrowatt.de/de/en/	Z	Germany
37	Hydrowatt	2	1.5	4	2	-	-	12	http://hydrowatt.de/de/en/	Z	Germany
38	Müller, G.[9]	1.7	1.8	5	2	-	5	23	[31]	Z	Germany

[1] American company, [2] England company, [3] German company, [4] Austrian professional office, [5] water mill in Italy, [6] water mill in Italy, [7] French cooperative Condroz Energies Citoyennes, [8] Région Aquitaine Limousin Poitou-Charentes. [9] water wheel shown in Fig.4

Table 6. Blade design of overshot water wheels. *d* is the bucket depth in radial direction, *n* is the blades number, while *l* is distance between two blades. R is wheel radius, H is head and s is depth of water jet.

Author	year	number of blades [-]	distance between blades [m]	blade depth [m]	Reference
Weisbach	1849	$18 + 9R$	$7(1 + 4d)$	$0.2 - 0.35$	[47]
Pacinotti	1851	$18R$	-	-	[53]
Bresse	1869	-	$0.32 - 0.35$	$0.2 - 0.35$	[56]
Cullen	1871	$16R$	-	-	[54]
Garuffa	1897	-	$(4/3 - 3/2)s$	$1/6(H)^{1/3}$	[49]
Weidner	1913	-	-	$(1/6 - 1/4) \cdot 2.21H^{1/3}$	[61]
Ovens	1977	-	-	$0.05 < d/R < 0.26$	[57]
Paoli	2006	-	$1.25d$	-	[85]
Nuernbergk	2014	-	$0.75d + 0.1$	-	[55]

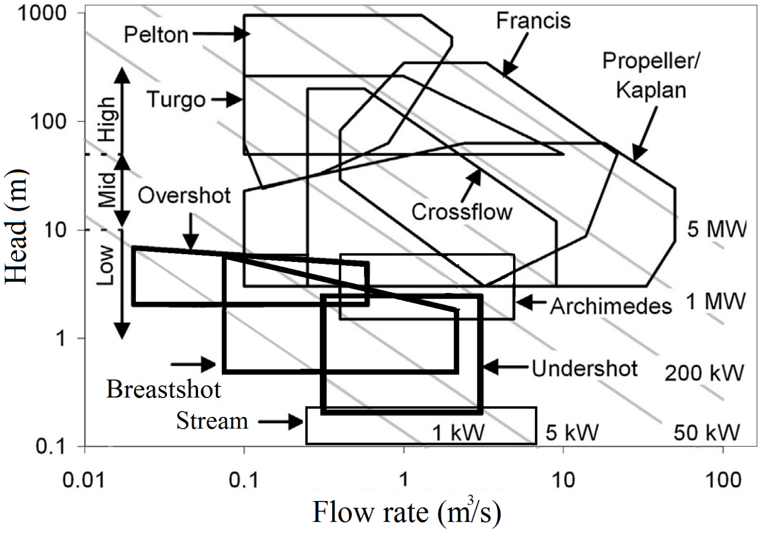
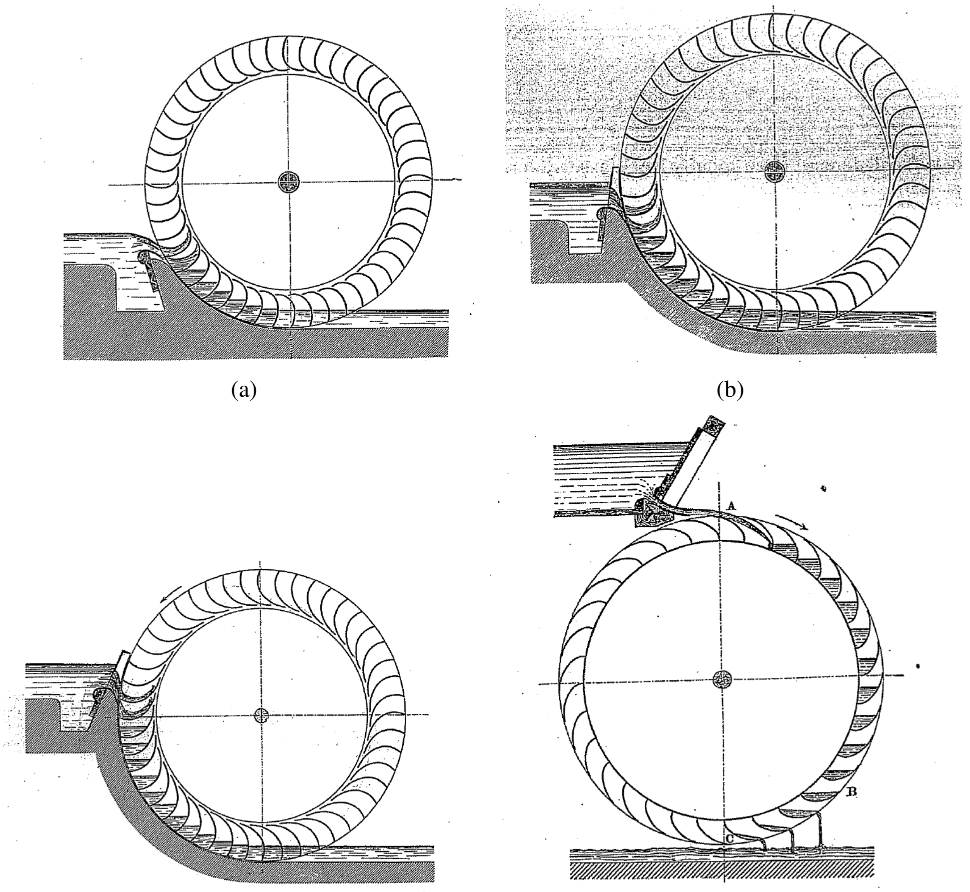


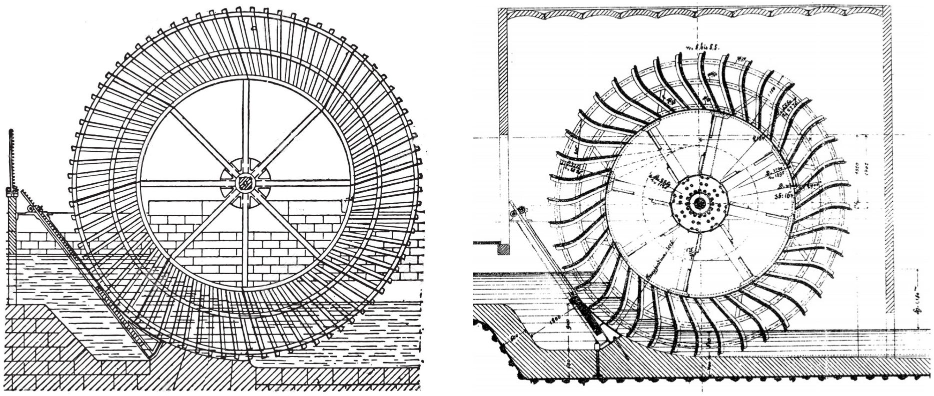
Fig. 1. Working conditions of hydropower converters (adapted from Williamson et al., 2014, [13]). Gravity water wheels (overshot, breastshot and undershot water wheels) are highlighted with a thicker line, since they were discussed in this review.



(a) (b)

(c) (d)

Fig. 2. Historic representations of gravity water wheels [48]: (a) low breastshot/undershot, (b) middle breastshot, (c) high breastshot and (d) overshot.



(a) *Sagebien water wheel* (b) *Zuppinger water wheel*

Fig. 3. Historic pictures of a *Sagebien* water wheel [82] and a *Zuppinger* water wheel [73].

(a) *Sagebien water wheel, France* (b) *Zuppinger water wheel, Germany* (c) *Sagebien water wheel, France*

(d) *Verolengo water wheel, Italy* (e) *Borgo Cornalese water wheel, Italy* (f) *Mulino del Pericolo water wheel, Italy*

(g) *Ciconio water wheel, Italy* (h) *Dronero water wheel, Italy* (i) *Barot water wheel, Italy*

Fig. 4. Water wheels in operation. The top figures are undershot water wheels, the figures at the center are breastshot water wheels, while the figures at the bottom are overshot water wheels. Representative dimensions of some water wheels are reported in Tab.5. Photo courtesy of: (a) Patrick H. Marceau, (b) prof. Müller Gerald, (c) Marie-Paule Dupuy, (d) (g) (h) research project ORME conducted by the Authors, (e) (f) Quaranta Emanuele, (i) mulino Moriena di Fenile di Campiglione, Michel Moriena.

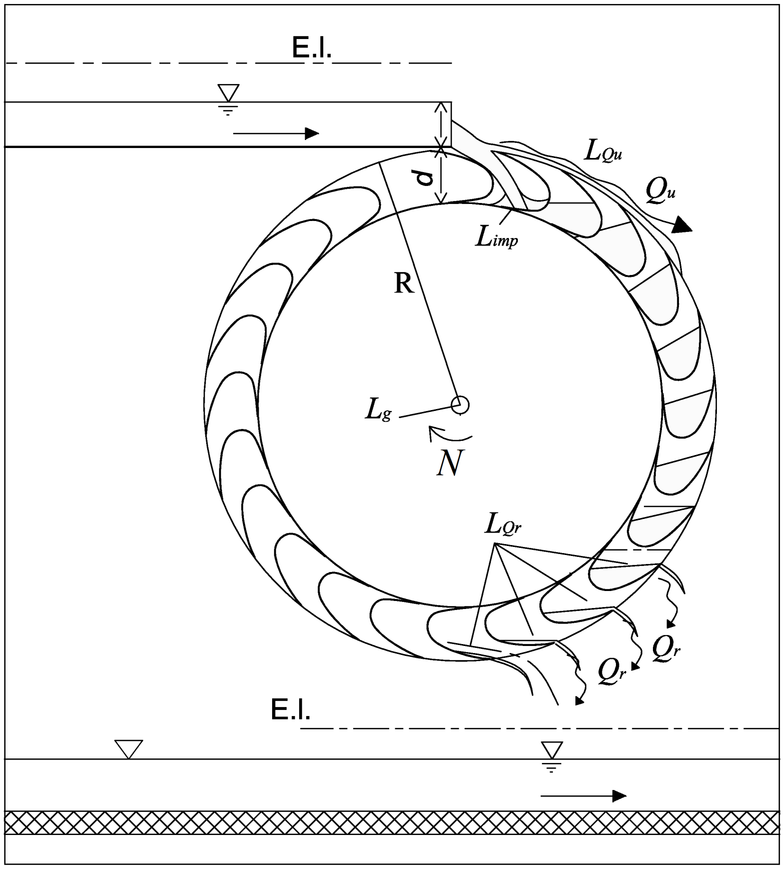


Fig. 5. Sketch of an overshot water wheel with radius R, rotational speed N, bucket depth d, lost flow rates Q_u and Q_r, and power losses L [51]. E.l. is the head energy line. Power losses are described in section 3.3.

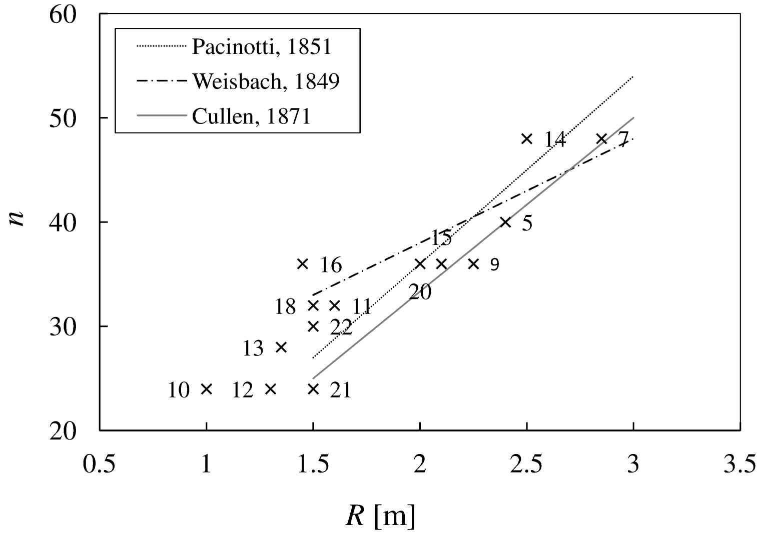


Fig. 6. The number of blades proposed by *Pacinotti* (1851) [53], *Weisbach* (1849) [47] and *Cullen* (1871) [54] as a function of the radius R (overshot water wheels). The three design laws give similar results. Data of real overshot water wheels are also depicted, with reference to the wheel number reported in Tab.5.

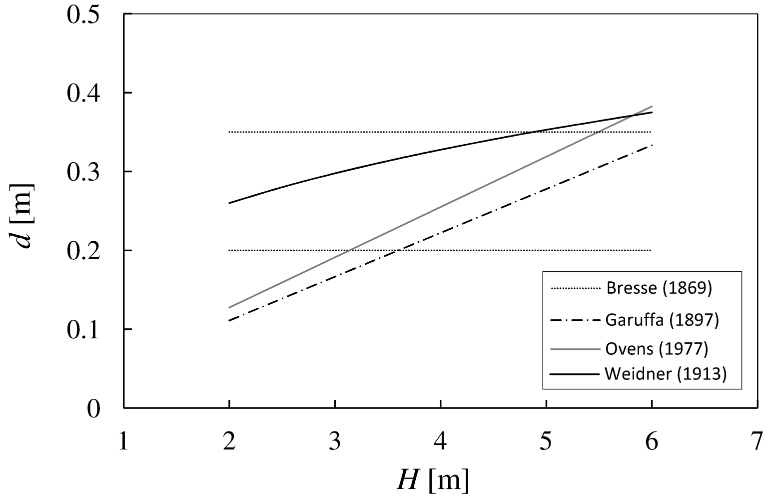


Fig. 7. The depth of the buckets proposed by *Bresse* (1869) [56] (that is the same as that proposed in *Weisbach* (1849) [47]), *Garuffa* (1897) [49], *Ovens* (1977) [57] and *Weidner* (1913) [61] as a function of the head H (overshot water wheels). In the equations where the radius/diameter appears, the value $D/H = 0.85$ is adopted. *Bresse* and *Garuffa* gave the same limit values, while *Weidner* proposed higher depths with respect to *Garuffa* and *Ovens*.

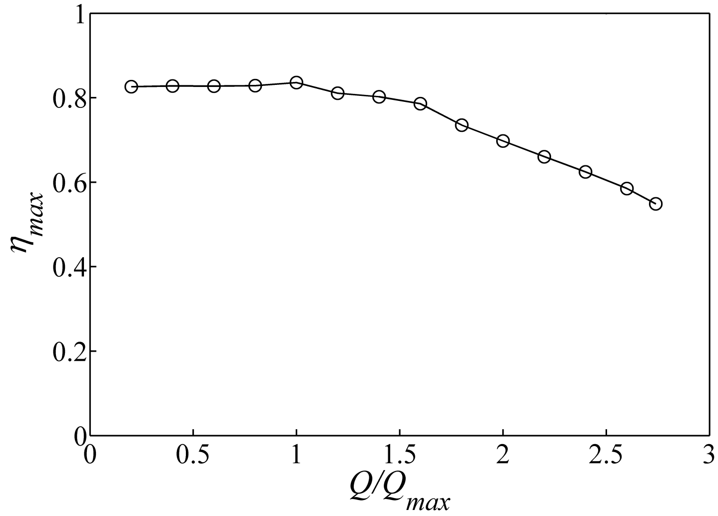


Fig. 8. Efficiency of overshot water wheels as a function of the normalized flow rate, where Q_{max} is the flow rate at maximum efficiency [51].

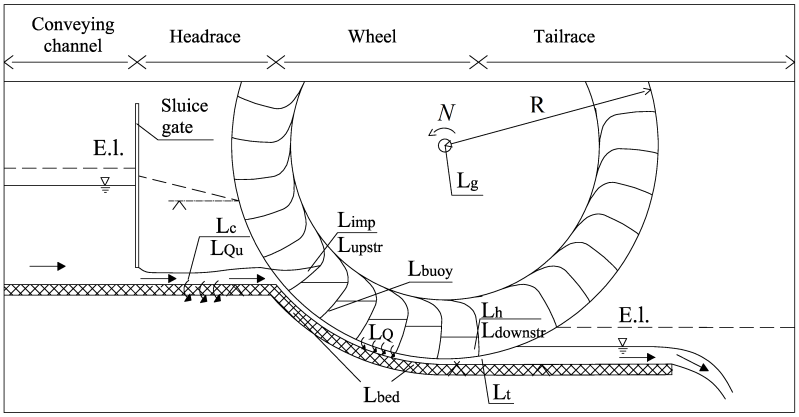


Fig. 9. Sketch of a breastshot water wheel with radius R, rotational speed N and power losses L [63]. Power losses are described in section 4.3. E.l. is the energy head line.

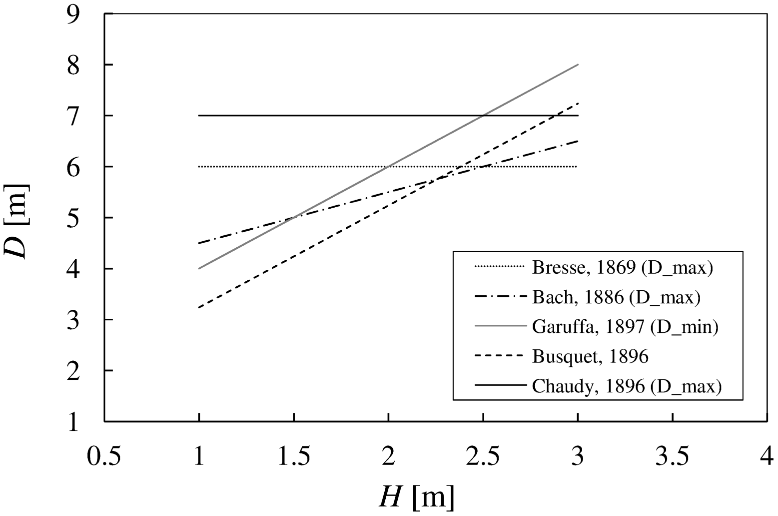


Fig. 10. The diameter proposed by *Bresse* (1869) [56], *Bach* (1886) [48], *Garuffa* (1897) [49], *Busquet* (1906) [66] and *Chaudy* (1896) [44] as a function of the head H (breastshot water wheels). A water depth $h = 0.5$ was adopted for *Busquet* formulation. *Bresse*, *Bach* and *Chaudy* proposed a maximum value, while *Garuffa* a minimum value.

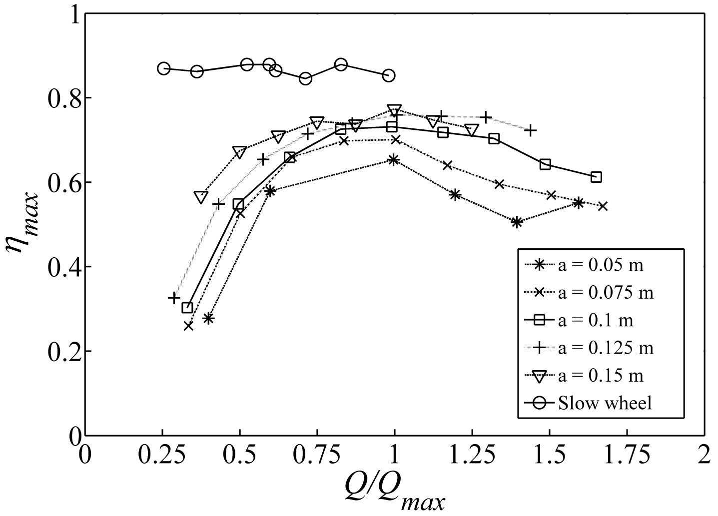


Fig. 11. Efficiency of breastshot water wheels as a function of the normalized flow rate, where Q_{max} is the flow rate at maximum efficiency. Efficiency curves of fast wheels at different sluice gate openings (a) [51] and efficiency curve for a slow wheel [69] are depicted.

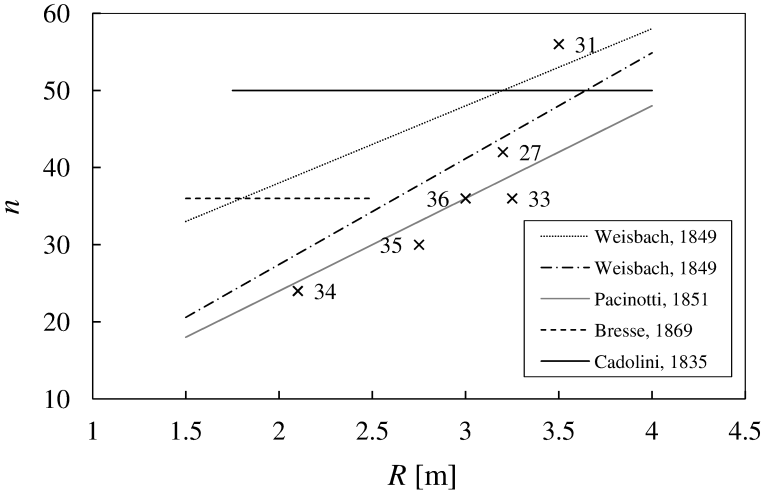


Fig. 12. The number of blades proposed by *Weisbach* (1849) [47], *Pacinotti* (1851) [53], *Bresse* (1869) [56] and *Cadolini* (1835) [71] as a function of the wheel radius (undershot water wheels). Except for *Cadolini* and *Bresse*, the proposed number of blades increases with wheel dimensions. Data of real undershot water wheels are also depicted, with reference to the wheel number reported in Tab.5.

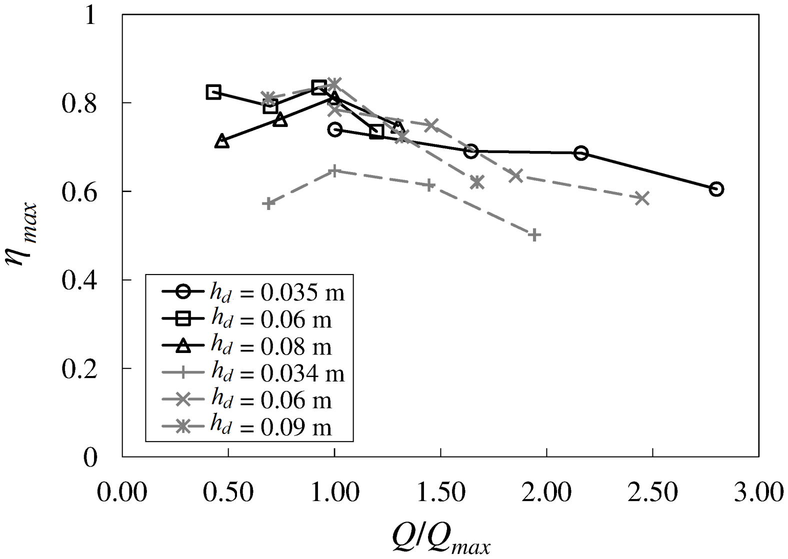


Fig. 13. Efficiency of undershot water wheels as a function of the normalized flow rate, where Q_{max} is the flow rate at maximum efficiency. Different downstream water depths (h_d) were tested, both for *Sagebien* wheels (black and full line) and *Zuppinger* wheels (gray and dotted line) [31].