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Performance of Sensor Placement Strategies Used in System Identification Based on Modal Expansion

Mahesh Murugan Jaya [1], Rosario Ceravolo[2], Emiliano Matta[3] and Luca Zanotti Fragonara [4]

[1] PhD Student, Department of Structural and Geotechnical Engineering, Politecnico di Torino, Italy, mahesh.murugan@polito.it

[2]Professor, Department of Structural and Geotechnical Engineering, Politecnico di Torino, Italy, rosario.ceravolo@polito.it

[3]Assistant Professor, Department of Architecture and Design, Politecnico di Torino, Italy, emiliano.matta@polito.it

[4]Lecturer, Centre for Autonomous and Cyberphysical Systems, Cranfield University, United Kingdom, l.zanottifragonara@cranfield.ac.uk

Abstract

Modal expansion techniques are typically used to expand the experimental modal displacements at the sensor positions to other unmeasured degrees of freedom. Since in most cases the sensors can be attached only at limited locations in a structure, an expansion is essential to assess the condition of substructures such as tanks and pipelines which are attached to this main structure. Most of the optimal sensor placement algorithms for system identification aims to reduce the correlation between the different modes thereby clearly distinguishing between closely spaced modes. In this study, the optimal sensor configuration provided by one such algorithm is studied in the context of modal expansion under the presence of modelling errors and measurement noise. A statistical sensitivity analysis is carried out to measure the correlation between the expanded mode shapes obtained from the optimal sensor positions and those corresponding to the real structure. A cantilever beam and an industrial tower are used as case studies. It is found that this correlation decreases with an increase in both the modelling errors and measurement noise. This forms the basis for a more extensive study aimed at identifying sensor locations which are more robust to modelling errors and measurement noises and thus can be used for a more reliable modal expansion.

1. Introduction

Whether in conducting an experimental test or for structural health monitoring, placement of sensors is of vital importance. The number of sensors which can be practically employed is limited by factors such as cost, availability of power, accessibility of the structure, etc. Hence, the sensors which are deployed should be placed such that they maximise the useful information. In conventional vibration-based system identification of structures, accelerometers are widely used and one of the commonly used criteria to determine their optimal position involves minimizing the correlation between the modal displacement vectors of different modes reduced to the sensor positions [(1)]. This is usually

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achieved by minimizing some scalar metric of the off-diagonal elements of the Modal Assurance Criterion (MAC)[2] matrix computed at the sensor positions.
Based on the modal displacements evaluated at these sparse sensor positions, the mode shapes can be expanded. This is important as the expanded mode shapes can be used to estimate damage. Pandey et al. [3] and Kondo and Hamamoto [4] used the curvature of mode shapes as a damage indicator. An accurate estimation of mode shapes also improves stress identification in structural members from vibration data [5]. This is also important in industrial structures wherein the condition of critical substructures such as tanks and pipelines need to be estimated based on the information provided by sensors attached to the main structure.
In vibration-based monitoring, the minimization of the off-diagonal elements of the MAC matrix is probably the most commonly adopted method for optimal sensor placement. However, to the best of the authors' knowledge, still there is no definite proof of its optimality in situations where a modal expansion is needed in the presence of modelling errors and measurement noise. In this work, a performance criterion is proposed for evaluating its effectiveness in providing the estimated mode shapes expanded to the entire structure. Using this criterion, the performance of a conventional sensor placement to expand the experimental mode shape will be evaluated in the presence of modelling errors and measurement noise. A simple cantilever beam and a more complex industrial tower are used to study the effectiveness.

2. Optimal Sensor Placement and Modal Expansion

A brief overview of the optimal sensor placement strategy and the modal expansion method used in this study is given. An algorithm for the comparison of the expanded modal displacement with that of the real structure is also presented.

2.1 Optimal location

Optimal sensor placement is a combinatorial optimization problem which involves the selection of an optimal set of sensor positions $\{A\} \in R^{a\,x\,1}$ from a set of all possible sensor positions $\{P\} \in R^{p\,x\,1}$ with $a < p$. The possible number of sensor configurations is given by

$$C_p^a = \frac{p!}{a!\,(p-a)!} \tag{1}$$

The modal displacements obtained from experimental tests (e.g., ambient vibration tests) should be linearly independent or uncorrelated in order to distinguish between closely spaced modes. The degree of correlation between modes can be quantitatively estimated using the MAC matrix. The MAC correlation between two modes i and j defined at n degrees of freedom (dof) is given by

$$MAC_{ij} = \frac{\left|\{\phi_i\}^T \{\phi_j\}\right|^2}{\left(\{\phi_i\}^T \{\phi_i\}\right)\left(\{\phi_j\}^T \{\phi_j\}\right)} \tag{2}$$

A value of MAC_{ij} close to 1 shows high correlation while a value close to 0 denote low correlation. Several other criteria exist to measure the suitability of optimal sensor positions such as the singular value decomposition of the modal matrix at the sensor positions, kinetic energy of the modes at the sensor positions, Fisher Information Matrix [6], etc. In order for the mode shapes to be uncorrelated at the a sensor positions, the MAC matrix should be close to a diagonal matrix. To ensure this, the peak off-diagonal element of the MAC matrix calculated using a numerical model is used as an optimization criterion. Thus for a number of sensors, the optimization problem is defined as follows,

$$\begin{cases} \arg\min_{\{A\}} \left(\max \left(MAC_{ij} \right) \right) & \text{for i} \neq \text{j} \\ \text{with } \{\text{A}\} \in \{\text{P}\} \end{cases} \tag{3}$$

where $\{A\}$ is the set of a sensors, $\{P\}$ denotes all possible sensor positions and $max\ (\ )$ represents the maximum value for the off-diagonal terms ($i \neq j$).

2.2 Expansion of mode shapes from sparse measurements

From the modal displacements evaluated at the sensor positions, the mode shape of the complete structure can be estimated. For this, modal expansion has to be used which can be performed in two ways: (a) through a geometric curve fitting using splines or other higher order polynomial functions without using any information from the numerical model or (b) based on the *a priori* information available from a numerical model. In this study, the mode shapes are expanded using information from the numerical model. Guyan expansion/reduction [7] is one of the first available methods for reduction/expansion of numerical models. But, due to the fact that they neglect the inertia of the unmeasured degrees of freedom, the mode shape predictions can be erroneous if significant mass are located at unmeasured degrees of freedom [8]. This method was extended to include the full equation of motion for modal expansion which resulted in more dynamically accurate methods such as the dynamic expansion method [9]. The present study uses the System Equivalent Reduction Expansion Process (SEREP) [10] which expands the mode shapes to unmeasured dofs using the complete numerical mode shapes.

Let a denote the dofs where sensor data are available and d denote all the remaining dofs. Let $\left[E_a^m \right] \in \mathrm{R}^{a\,x\,m}$ represents the experimental mode-shape coordinates estimated at the a sensor locations for m modes and $\left[\phi_n^{m'} \right] \in \mathrm{R}^{n\,x\,m'}$ denote the full mode shape matrix from the finite element model for m' number of modes. This mode shape matrix can be decomposed based on the measurement locations as $\left[\phi_a^{m'} \right] \in \mathrm{R}^{a\,x\,m'}$ and $\left[\phi_d^{m'} \right] \in \mathrm{R}^{d\,x\,m'}$ such that $n = a + d$. When the number of sensors a are more than the number of analytical modes m' used for the expansion, the expanded mode shapes can be given by,

$$\underset{n\,x\,m}{\left[\varphi_n^m \right]} = \begin{bmatrix} \varphi_a^m \\ \varphi_d^m \end{bmatrix} = \underset{n\,x\,a}{\left[T_u \right]} \underset{a\,x\,m}{\left[E_a^m \right]} \tag{4}$$

where $\left[\varphi_n^m\right] \in \mathrm{R}^{n\,x\,m}$ is the expanded mode shape matrix from the experimental data and $\left[T_u\right] \in \mathrm{R}^{n\,x\,a}$ is the transformation matrix which is calculated as follows,

$$\underset{n\,x\,a}{\left[T_u\right]} = \underset{n\,x\,m'}{\begin{bmatrix}\left[\phi_a^{m'}\right] \\ \left[\phi_d^{m'}\right]\end{bmatrix}} \underset{m'\,x\,a}{\left[\phi_a^{m'}\right]^{\dagger}} \tag{5}$$

$\left[\phi_a^{m'}\right]^{\dagger}$ represents the Moore-Penrose pseudo-inverse (left hand inverse) of $\left[\phi_a^{m'}\right]$ which is given by

$$\underset{m'\,x\,a}{\left[\phi_a^{m'}\right]^{\dagger}} = \left(\left[\phi_a^{m'}\right]^T \left[\phi_a^{m'}\right]\right)^{-1} \left[\phi_a^{m'}\right] \tag{6}$$

A transpose is missing. Please correct in the final version.

This expansion leads to smoothing of the mode-shape data at sensor locations. However, when the number of sensors is equal to that of the modes used for expansion, the pseudo-inverse can be replaced by an ordinary inverse and in this case, there will not be any smoothing of the modal displacements at the sensor locations during expansion [11].

2.3 Performance of optimal sensor placement in modal expansion

In order to evaluate the effectiveness of the conventional optimal sensor placements in expanding the experimental modal displacements to the complete structure, the following three models were used.
Model 1 – The real structural modal model,
Model 2 – The numerical model and
Model 3 – The experimental modal model (known only at the sensor positions).
The mode shapes of the real structure Φ_n^1 for all the n dofs was assumed to be equal to the numerical mode shapes Φ_n^2 modified by a modelling error ε which represents the difference between the mode shapes of the real structure and the numerical model due to the several approximations made while creating a numerical model. The mode shapes of the real structure are thus given as:

$$\Phi_n^1 = \Phi_n^2 + \varepsilon\left(\mu_\varepsilon, \sigma_\varepsilon\right) \tag{7}$$

where, $\varepsilon(\mu_\varepsilon, \sigma_\varepsilon)$ represents the error which is assumed to be normally distributed with a mean μ_ε and standard deviation σ_ε. Similarly, the experimental modal model Φ_a^3 defined in the a sensor positions was assumed to be equal to that of the real structure modified by an error η which is caused by the measurement noise and the identification errors:

$$\Phi_a^3 = \Phi_a^2 + \eta\left(\mu_\eta, \sigma_\eta\right) \tag{8}$$

where, $\eta\left(\mu_\eta, \sigma_\eta\right)$ is also assumed to be normally distributed with a mean μ_η and a standard deviation σ_η.

In order to evaluate the performance of the conventional optimal sensor placement strategy in expanding the mode shapes to unmeasured dofs, MAC matrix was computed between the expanded experimental modal model and that of the real structure as shown in Fig.1. For a fixed number of sensors a, an optimal sensor placement was found for the numerical model using equation (3). The experimental modal displacements were then expanded to all the dofs using the transformation matrix T_u calculated from the numerical model using equations (4) to (6). The correlation of the expanded modal displacements φ_n are evaluated with respect to the modal displacements from the real structure Φ_n^1 using the MAC matrix. The diagonal elements of the MAC matrix are good indicators of correlation. A Monte-Carlo simulation was performed with different values for the standard deviations σ_ε and σ_η. As performance criteria, the mean value of the peak off-diagonal element and that of the least diagonal element were calculated. Thus, $\overline{f_1} = 1 - \overline{max(MAC_{ij})}$ for $i \neq j$ and $\overline{f_2} = \overline{min(MAC_{ij})}$ for $i = j$ was computed for different values of σ_ε and σ_η.

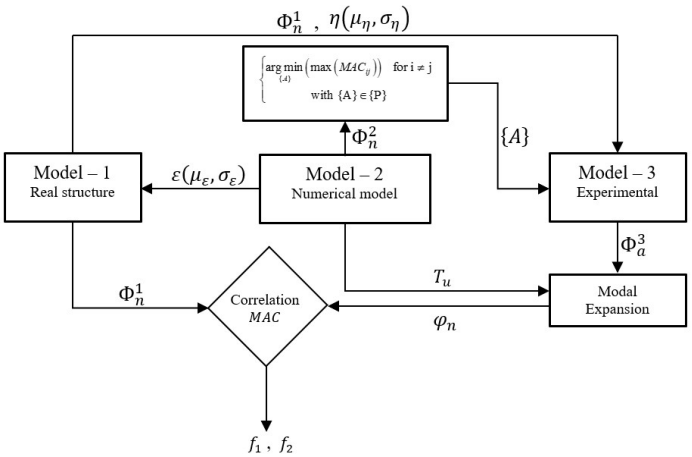


Figure 1. Algorithm for evaluating the efficiency of modal expansion

3. Performance Evaluation

The performance of the optimal sensor placement strategy in modal expansion is evaluated first using a simple cantilever model and is then extended to an industrial tower.

3.1 Cantilever beam

A 2D cantilever beam was considered. The numerical model of the beam was made using 100 2-noded beam elements with 3 dofs (translations in X and Y and rotation about Z) per node. Only the first four predominant modes of the beam in the lateral (Y) direction were considered and mode shapes were scaled such that the maximum magnitude of displacement in each mode was unity. 4 uniaxial accelerometers ($a = 4$) were supposed to be deployed on the structure, all measuring in the same direction and 20 locations were identified as possible sensor positions ($p = 20$) as shown in Fig. 2. This results in $C_{20}^4 = 4845$ possible combinations, all of which were evaluated to identify the optimal sensor locations according to equation (3). Figure 2 shows the corresponding optimal positions for keeping the 4 uniaxial accelerometers in the Y direction.

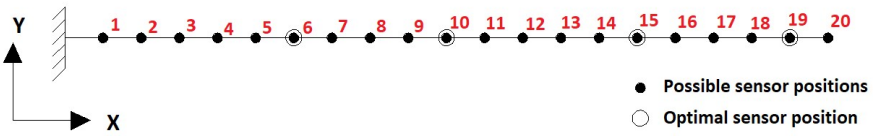
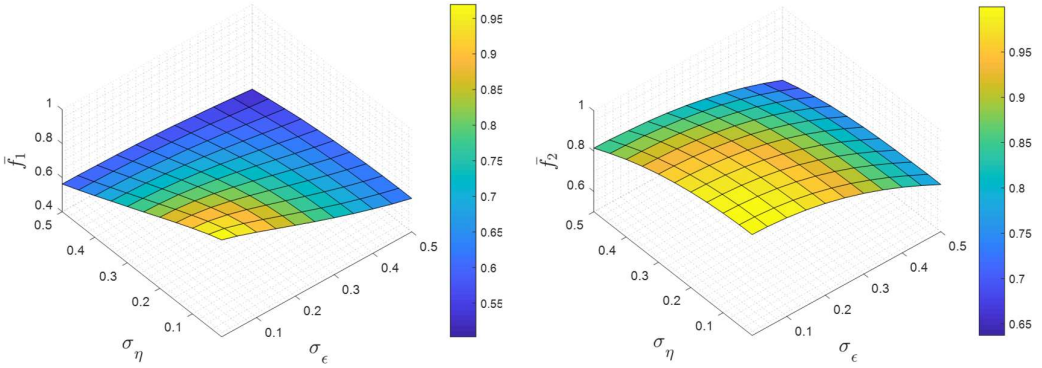


Figure 2. Cantilever beam showing the possible positions to keep uniaxial accelerometers in *Y* and the corresponding optimal configuration

Modelling error and measurement noise were simulated with zero mean ($\mu_\varepsilon = 0\,, \mu_\eta = 0$) and standard deviations σ_ε and σ_η ranging from 0.01 to 0.50. Modal expansion was performed by computing the transformation matrix T_u from the numerical model using only the corresponding 4 modes ($m = m'$). It was assumed in this preliminary study that σ_η and σ_ε is constant for all the modes. For each combination of σ_η and σ_ε, 100000 samples were generated based on a convergence study and the sample mean of the correlation metrics were calculated. It was seen that all the correlation metrics followed a similar trend. Results are shown in Figure 3, where the performance indices $\overline{f_1}$ and $\overline{f_2}$ are plotted as a function of σ_η and σ_ϵ. The mode shapes are well correlated for low values of σ_η and σ_ϵ. It should be noted that due to the approximation made in defining the modelling error to be a Gaussian noise, the mode shape of the real structure will be correlated to a certain extent with that from the numerical model depending on the amount of modelling error. Thus, a modal expansion performed using the transformation matrix from the same numerical model is expected to be in good correlation with that of the real structure when the noise level is low irrespective of sensor positions. But, with the increase in these errors, the correlation starts to decline. In the presence of high errors, there might be sensor positions which are more robust to modelling errors and measurement noise with respect to those provided by the conventional optimization adopted in this study.



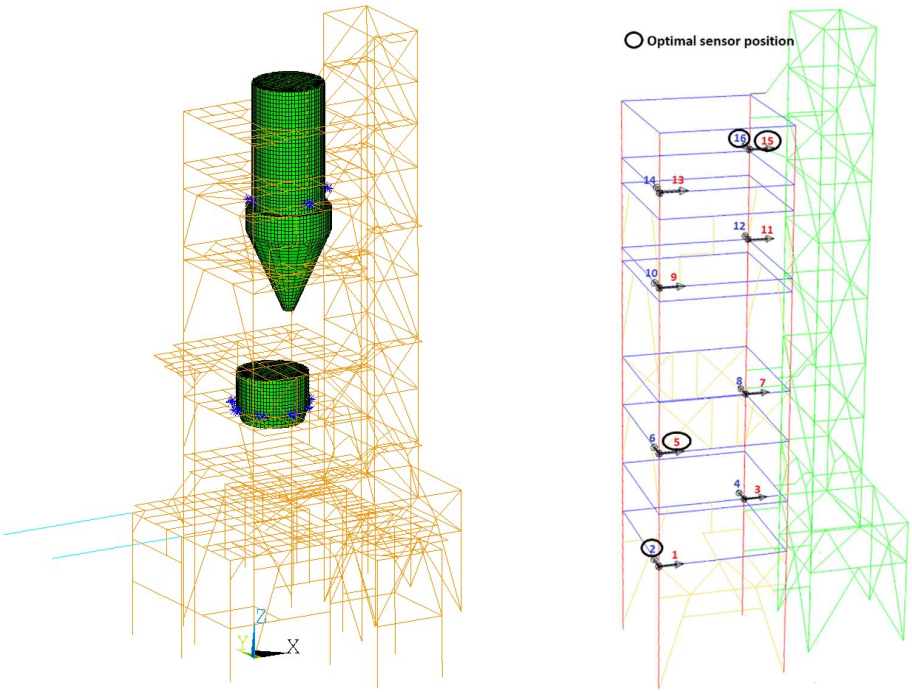
(a) **(b)**

Figure 3(a). Variation of $\overline{f_1}$ with σ_ε and σ_η and (b). Variation of $\overline{f_2}$ with σ_ε and σ_η

3.2 Industrial tower

The performance of the optimal sensor placement was also evaluated for a typical industrial tower. The structure is approximately 25 m tall with a plan dimension of 6×6.6 m. In addition, an external frame with stairs is attached to this tower. It also houses two

steel tanks at a height of 20 m and 10 m from the base. Figure 4(a) shows the finite element model of the tower along with the coordinate system. 4 uniaxial accelerometers were used to identify the first four predominant modes of the structure in X and Y directions. 16 possible locations for the placement of sensors were identified and are shown in Fig. 4(b). The optimal sensor configuration was selected after evaluating $C_{16}^{4} = 1820$ possible combinations using equation (3). Figure 4(b) also shows the corresponding optimal position. A sensitivity analysis was carried out similar to the case of the cantilever beam to evaluate the correlation between the expanded mode shapes with that from the real structure in the presence of modelling errors and measurement noise. 50000 samples were generated for each combination of σ_ε and σ_η. Figure 5 shows the variation of $\overline{f_1}$ and $\overline{f_2}$ with the standard deviation of the modelling error σ_ε and measurement noise σ_η. The results are similar to that observed for the cantilever case study.



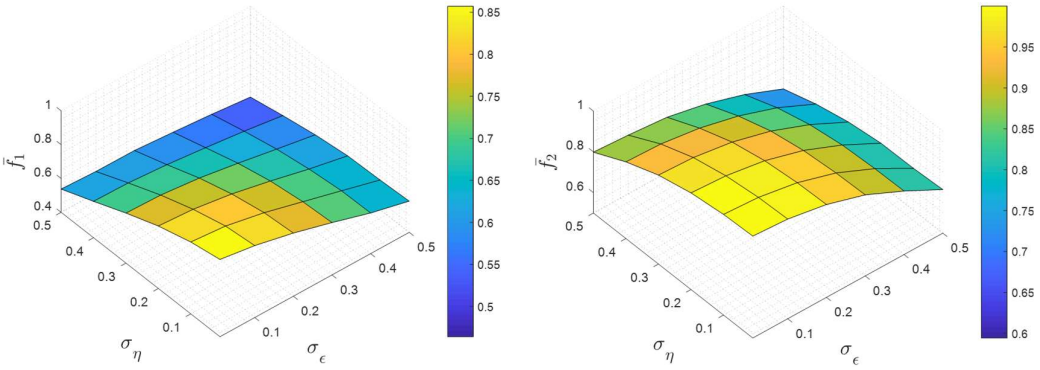
(a) **(b)**

Figure 4. (a) Finite Element model of the tower and (b) 16 possible locations for the placement of uniaxial sensors and the corresponding optimal configuration

3.3 Discussion of the results

An attempt has been made in this work to evaluate the performance of conventional optimal sensor placement strategies in terms of their capability to provide expanded identified mode shapes in good correlation with the real ones. As expected, it was found that for both the examined case studies, the correlation between the expanded experimental mode shapes and that from the real structure decreases with an increase in the modelling error and the measurement noise. More detailed studies need to be conducted to determine if other optimal sensor strategies are available which will be more robust during expansion. The presented evaluation indices are possible candidates as objective functions in the optimization problem, once a probabilistic representation of the

modelling error and the measurement noise are available. A more realistic description of modelling errors should also be considered as the currently adopted Gaussian white noise provides correlation between the mode shapes of the real structure and the numerical model.



(a) **(b)**

Figure. 5 (a) Variation of sample mean of $\overline{f_1}$ with σ_ε and σ_η and (b). Variation of sample mean of $\overline{f_2}$ with σ_ε and σ_η

4. Conclusions

The performance of one of the most commonly used optimal sensor placement strategy in operational modal analysis was assessed with respect to modal expansion. The diagonal elements of the Modal Assurance Criterion matrix evaluated between the expanded mode shapes from the experimental modal model and those from the statistically evaluated real structure was used as performance indices. The conventional optimal sensor placement strategy for modal identification which aims at minimizing the correlation between the identified modes, was found to provide good correlation between the expanded experimental modal model and that from the real structure at low modelling errors and measurement noise. However as expected, with an increase in the modelling error and measurement noise, the mode shapes expanded from the conventional sensor placement is seen to be less correlated with the real structure. Even though the conventional sensor placement criterion is highly efficient for system identification, the low robustness with respect to noise and modelling errors (and the combination of the two) can affect its performance in situations where modal expansion is of prior importance. A more detailed study needs to be conducted to evaluate if more robust sensor configurations exists which can expand the mode shapes accurately even in the presence of errors and noise.

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