RELIABILITY AND COMPONENT VULNERABILITY ANALYSIS OF CITY-SCALE NETWORKS: APPLICATION TO THE TRANSPORTATION SYSTEM OF A VIRTUAL CITY

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RELIABILITY AND COMPONENT VULNERABILITY ANALYSIS OF CITY-SCALE NETWORKS: APPLICATION TO THE TRANSPORTATION SYSTEM OF A VIRTUAL CITY

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Abstract. *Infrastructure systems are crucial for the development of communities as they provide essential services to the habitants. To improve the resilience of such systems, their intrinsic properties need to be understood and their resilience state needs be identified. In the literature, several methods to evaluate networks' reliability and resilience can be found. However, the applicability of these methods is restricted to small-size networks due to several computational limitations. In this paper, the case of large scale networks is tackled. The transportation network of a virtual city is considered as a case study. First, the road map of the city is transformed into an undirected graph. A random removal of the roads is applied until the network's failure point is reached. The network reliability is then calculated using the Destruction Spectrum (D-spectrum) approach. A Monte Carlo approach has been developed to generate failure permutations, which are necessary for the evaluation of the D-spectrum set. In addition, the Birnbaum Importance Measure (BIM) has been adopted in this study to determine the importance of the network's components. Due to the large size of the network, several computational problems have been faced. To solve the problems, two coding algorithms have been introduced in the paper to evaluate both the reliability and the BIM indexes for large scale networks. The analysis concept adopted in this study is applicable to all network-based systems such as water, gas, transportation, etc.*

1 INTRODUCTION

Transportation networks need to provide a continuous service for communities, and this necessitates a good understanding of their resilience and reliability states. For instance, understanding how the topology of the network changes under disruptive events can be fundamental in the decision making process. It could also speed up rescue operations and help in evaluating cascading effects on other interdependent networks. This paper explores mostly the reliability of large scale networks. Reliability is a very broad concept and its application is extended to all engineering fields. In general terms, network reliability can be defined as the probability of connecting the nodes of the network [1]. Other authors consider reliability as the quality of the transportation system in terms of optimal travel time, i.e. the probability that a trip between two nodes takes less than a certain time [2]. Another widely used concept is the capacity reliability, which is the probability that the network capacity can accommodate a certain traffic volume at a required level of service [3-5]. Reliability has also been studied under specific situations, such as the emergency response, using both ideas of travel time and level of service [6]. Looking at graph theory, Guidotti, Gardoni [7] have used the connectivity measures as a tool to determine whether a network is reliable or not.

In this work, the reliability of the transportation network of a large scale virtual city is evaluated. The reliability definition adopted in [8-10] is considered in this study. According to the researchers, reliability is related to the probability that some nodes, called terminals, remain connected. Thus, the system fails when the terminals are no more connected. The terminal nodes are strategic nodes with pre-defined survival probabilities assigned by the competent authorities. Knowing these probabilities helps greatly in improving the network [11]. However, it can be rather difficult to have access to such data. For this reason, a different failure criterion has been chosen in this work. Moreover, another performance parameter, the Birnbaum Importance Measure (BIM), is considered to study the behavior of the analyzed network. This parameter represents the importance of the network's components [9]; that is, components with a high BIM index are vulnerable components. The determination of the reliability and BIM indexes is relatively simple for small networks, but when applied to a large-scale road transportation network, the computational effort becomes unaffordable. To overcome the computational problems, two coding algorithms are presented in the paper.

The paper is structured as follows: a brief introduction of the performance indexes with their equations is presented in section 2. Section 3 introduces some algorithms and strategies to overcome the computational issues. Finally, a case study is presented in section 4, where the description of the network and the obtained results are provided.

2 NETWORK RELIABILITY AND COMPONENTS' IMPORTANCE

2.1 Destruction spectrum

The Destruction spectrum, or simply D-spectrum, is a representation of a network's structure and its failure definition [12]. The system in Figure 1 is used to introduce the concept of the D-spectrum. The nodes in the system are assumed reliable while the edges are unreliable; that is, only edges are subject to failure. In this example, the system's failure is defined as the loss in connectivity between the terminal nodes a and c.

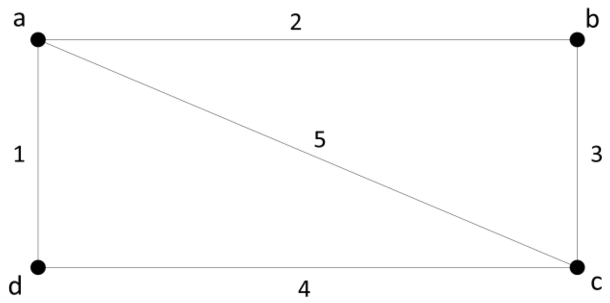

Figure 1: A simple network with two critical nodes.

The system's failure can be reached through different sequences of failing components. For instance, if the edges $(1;2;5;3;4)$ fail, the two nodes *a* and *c* become disconnected, and thus the system fails. Another permutation leading to the same result can be $(3;5;4;1;2)$. The failing component at which the system becomes down is called the anchor of the permutation. In the two permutations above, edges 5 and 4 are the anchors respectively. The total number of failure permutations in a system is *k!*, where *k* is the number of unreliable elements. After identifying all failure permutations, the D-spectrum set of the network is computed as follows:

$$D = \left\{ d_1 = \frac{x_1}{k!}, d_2 = \frac{x_2}{k!}, \ldots, d_k = \frac{x_k}{k!} \right\} \tag{1}$$

where d_i is the i^{th} component of the spectrum, x_i is the total number of permutations whose anchor's order is *i*, *k* is the total number of unreliable components. It is obvious from the equation above that the summation of all elements is 1 ($\sum_{i=1}^{k} d_i = 1$).

It is worth to note that the failure probability of each edge is not considered in the D-spectrum. One may simply consider a random removal of the edges in the sense that all edges have the same failure probability. Otherwise, a strategic edge removal can be considered but it requires additional analyses. For instance, in transportation networks, the removal of an edge (road) may be linked to the level of damage of the adjacent buildings. This requires fragility analysis to determine the level of damage that each building is subject to.

2.2 Network reliability

Generally, a network can be considered reliable when it offers a certain level of service or performance, even during emergency situations. Most of the reliability definitions that can be found in the literature deal with the concept of reliability in probabilistic terms. According to the definition of Gertsbakh and Shpungin [8], each element of the network (nodes and edges) is given a probability *p* of being available and a probability *q=1-p* of being unavailable. All these probabilities contribute in the determination of the network's reliability. The formulation used to calculate the reliability index *R(N)* is given in Eq.(2). The equation is valid when all unreliable components have the same failure probability.

$$R(N) = 1 - \sum_{i=1}^{k} y_i \frac{k! q^i p^{k-1}}{i!(k-i)!} \tag{2}$$

where y_i is the cumulative D-spectrum, given by the following equation:

$$y_i = \sum_{b=1}^{i} d_b \tag{3}$$

2.3 Components' importance

The Birnbaum Importance Measure (BIM) is a parameter that describes the importance of network's components. The vulnerability of the components is what determines their corresponding value of BIM. This measure can be a tool to identify the critical components of a network in order to strengthen them. The following equation is used to compute the BIM index of a single component *j*:

$$BIM_j = \sum_{i=1}^{k} \frac{k!(z_{i,j} q^{i-1} p^{k-i} - (y_i - z_{i,j}) q^i p^{k-i-1})}{i!(k-i)!} \tag{4}$$

where $z_{i,j} = Z_{i,j} / k!$, in which $Z_{i,j}$ is the number of permutations satisfying two conditions: (a) if the first *i* elements of the network are down, then the network is down; (b) element *j* is among the first *i* elements of the permutation. By doing some manipulation, the expression in Eq.(4) can be written in a different form, shown in Eq.(5). It is worth to note that both reliability and BIM indexes share a common factor in their equations, and this provides a unique characteristic to compute both indexes in a single operation.

$$BIM_j = \sum_{i=1}^{k} \frac{k! q^i p^{k-1}}{i!(k-i)!} \cdot \left(\frac{z_{i,j}}{q} - \frac{y_i - z_{i,j}}{p} \right) \tag{5}$$

3 METHOD: APPLICATION TO LARGE SCALE NETWORKS

Theoretical reliability analyses are not always applicable to large problems. For instance, the applications of the above mentioned equations are limited to small networks with defined number of components. This is due to the presence of *factorial* in the denominator of the spectrum set (Eq.(1)). The same problem appears when computing the reliability index *R*, although we know in advance that this is a number between 0 and 1, regardless of the network's size. In this section, we present strategies to apply the D-spectrum to large scale networks and then compute their reliability index and BIM-spectra.

3.1 D-spectrum for large scale networks: a Monte Carlo approach

We propose the use of a Monte Carlo approach to generate the failure permutations needed to compute the D-spectrum. Hereafter, the algorithm of the Monte Carlo simulation is presented.

Algorithm 1: D-spectrum

1) Define a failure criterion for the network (ex: amount of failed components, disconnected nodes, etc.).
2) Set the permutation number to n=1.
3) Choose a random number from 1 to k and store it (i.e. the chosen number represents a failed component).
4) Check if the network's failure criterion is met:
   a) if yes, stop and store the permutation and go to step 5;
   b) otherwise, choose another number from 1 to k excluding the numbers chosen in previous steps and then go again to 4.
5) Set n=n+1.

6) Repeat the steps 3-5 M times to generate M permutations.
7) Categorize the permutations according to their anchor's value (i.e., the anchor value coincides with the vector's length of the permutation).
8) Compute the D-spectrum using Eq.(1).

In case of strategic removal, the unreliable network's components should be linked with certain removal probabilities or lifetime distributions. In this case, step 3 is adjusted so that the removal process is not random.

3.2 Reliability and BIM indexes for large scale networks: Incremental calculation

Practically, computing the reliability and BIM indexes is not possible for moderate to large networks. The reason is that both numerator and denominator in the second term of the reliability index (Eq.(2)) and in the first term of the BIM index (Eq.(5)) are too large. However, it is known in advance that the reliability index is a number that ranges between 0 and 1 regardless of the network's size, and this property represents the key solution. In this section, we propose the *incremental computation* to solve the numerical problem. The proposed method can handle large numerators and denominators given that their division is a real number that is not too large. The key idea is that neither the numerator nor the denominator are allowed to exceed a certain number and yield to infinity. To go more deeply, let M denote the common term of the reliability and BIM indexes, which represents the problematic part (Eq.(6)). First, all the parameters in the M equation are collected separately ($K!;\ q^i,\ p^{k-i};\ i!;\ (k-i)!$). The parameters are then classified according to the criterion whether they increase or reduce M. The calculation starts by injecting the parameters that increase M, and whenever M reaches a defined upper limit, the terms that decreases M are injected until M reaches a lower limit. This is done repeatedly until all terms are exhausted. The calculation is done for a single i and repeated for $i=1,2,3,..,k$. More details about the numerical method are presented in algorithm 2.

$$M_i = \frac{k!q^i p^{k-i}}{i!(k-i)!} \tag{6}$$

Algorithm 2: Incremental computation

1) Define upper and lower thresholds (e.g., 10^{10} and 10^{-10}).
2) Classify the parameters in M in two categories: Category A includes the parameters that contribute in increasing M, and Category B for the parameters that contribute in decreasing M. In our case, there is only one parameter that falls under category A ($k!$), while the other parameters fall under B ($q^i,\ p^{k-i};\ i!;\ (k-i)!$).
3) Each of the parameters is written in an expanded form, as follows:
   - $k!=[1;2;3;\ldots;k]$;
   - $q^i=[q,q,q,...q]$) (i times);
   - $p^{k-i}=[p;p;p\ldots;p]$ ((k-1) times);
   - $i!=[1;2;3;\ldots;i]$;
   - $(k-i)!=[1;2;3;\ldots;k-1]$.
4) Set i = 1 and start calculating M progressively by injecting the parameters above. One can start multiplying the numbers inside the parameter $k!$ until M reaches the upper threshold (e.g. $1\times 2\times 3\times \ldots \times n > 10^{10}$). The used numbers cannot be used again and they must be removed from the set once used. Whenever M reaches the upper thresholds, the sets of the parameters in category B are injected until M reaches the lower threshold. The order of the terms is not important so the user can exploit the terms in any order until they are exhausted. The terms should be used in the same way they are in M,

so multiplication terms (*K!; q^i, p^{k-i}*) are multiplied by M while division terms (*i!; (k-i)!*) are used to divide M.

5) Repeat Step 4 for *i=1,2,3,..,k*.
6) Find $z_{i,j}$ using the permutations obtained from *Algorithm 1* and following the criteria introduced in section 2.3.
7) Compute the reliability index and BIM index as follows:

$$R = 1 - \sum_{i=1}^{k} y_i . M_i \tag{7}$$

$$BIM_j = \sum_{i=1}^{k} M_i \cdot \left(\frac{z_{i,j}}{q} - \frac{y_i - z_{i,j}}{p} \right) \tag{8}$$

4 CASE STUDY: THE TRANSPORTATION NETWORK OF A MEDIUM SIZE CITY

4.1 Network definition

The road transportation network of a virtual city has been modeled as an undirected graph (Figure 2). An undirected graph *G=(V,E)* consists of a set of vertices (or nodes), representing the intersections, together with an edge set *E*. The elements belonging to *E* are called edges or links, and represent the roads of the network. In an undirected graph every pair of nodes is connected, so each path can be passed through in both directions. Actually, a road map would be a directed graph, as the streets have a certain way. However, in emergency conditions, it is likely that respecting the directions becomes a secondary aspect.

Figure 2: The transportation network of the virtual city.

The road transportation system consists of 19614 edges connecting 15012 nodes. Mathematically, the network has been described with an adjacency matrix *A*, which is a square matrix with a side dimension equal to the number of the nodes. The elements inside *A* can be either 1 or 0. If $a_{i,j}=1$, it means that there is a connection (road) between node *i* and node *j*, while 0 means that the two nodes are not linked. Since the graph is not directed, the resulting adjacency matrix is symmetric. This matrix allows to describe and modify the topology of the network and to automate all the calculations.

4.2 Network's failure criterion

The definition of the failure criterion is strictly related to the reliability of the network. Despite its importance, there is not a unique definition for the failure criterion of a network. Gertsbakh and Shpungin [8] proposed the criterion of terminal connectivity; that is, if critical nodes are disconnected, the network becomes down. This criterion is applicable to small scale networks, but when dealing with large networks a huge computational effort would be needed to identify whether the nodes are connected or not. In this work, a simpler network failure criterion is adopted. The network is considered unavailable when at least 5% of the nodes are isolated (not connected to any edge).

4.3 Results

The methodology introduced in section 3 has been applied to the case study. Due to the large size of the analyzed network, the results are not shown using vectors or matrices, but rather using graphs.

First, *Algorithm 1* has been used to generate the failure permutations of the edges. The number of permutations considered in this case study is 3.5 million. The generated permutations have been subsequently used in the calculation of the D-spectrum. The result of the D-spectrum is shown in Figure 3. It can be clearly seen that there are only few non-zero elements in the distribution, and they are all gathered in a small range. The distribution of the D-spectrum is a perfectly normal distribution. The location of the distribution's peak depends greatly on the chosen failure criterion of the network and on the number of components forming the network. More study will be dedicated to know the reason of having a normal distribution and to identify the effect of the failure criterion and number of components on the position of the distribution's peak. Moreover, looking at the definition of the D-spectrum, the sum of all its elements is 1. This is verified in Figure 4, which shows the distribution of the cumulative D-spectrum.

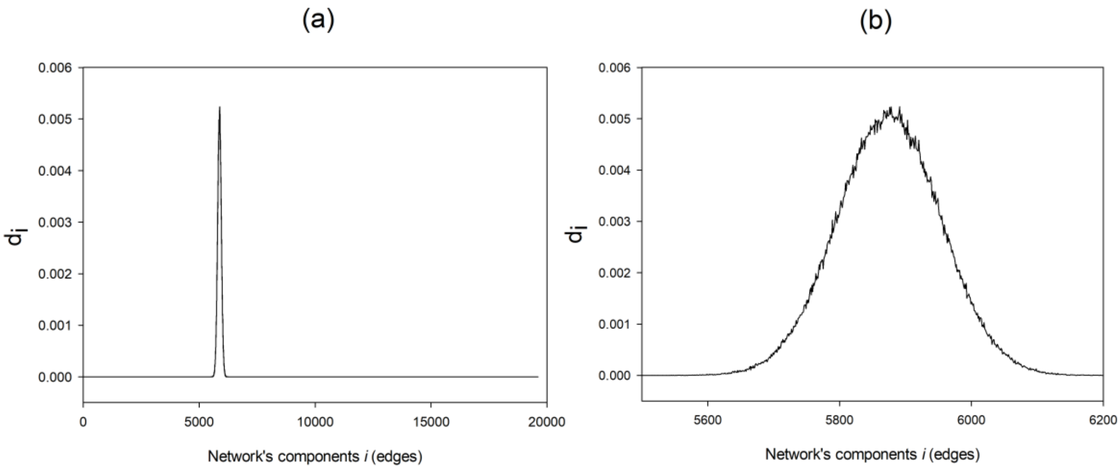

Figure 3: (a) The D-spectrum of all components the case study network; (b) a zoomed view at the distribution peak.

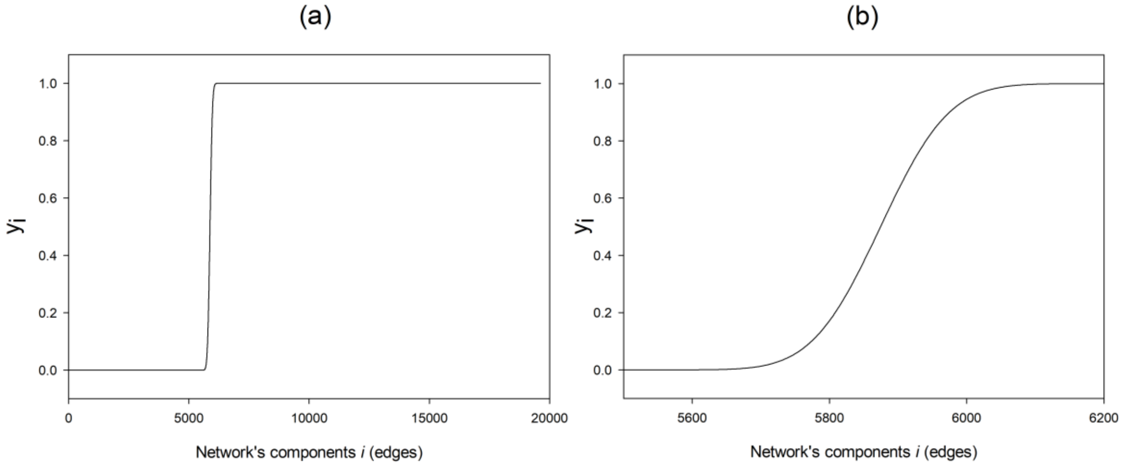

Figure 4: (a) The cumulative D-spectrum of all components the case study network; (b) a zoomed view at the transitional part.

Following the D-spectrum, the BIM index of the network's components and the network reliability index *R* have been evaluated using *Algorithm 2*. The BIM index results of the networks' components have been normalized with respect to the maximum value. Figure 5 shows the BIM results of the first 100 components. The BIM indexes of the other components range between the upper the lower bounds, 1 and 0.98 respectively. This implies that the variance in the results is very small. In fact, the importance index of the edge is ruled by the network configuration and the failure probability of the edge. In our network, we considered an equal failure probability for the edges, which was represented by the random removal process. The small difference in the importance of the edges was only due to the network's configuration.

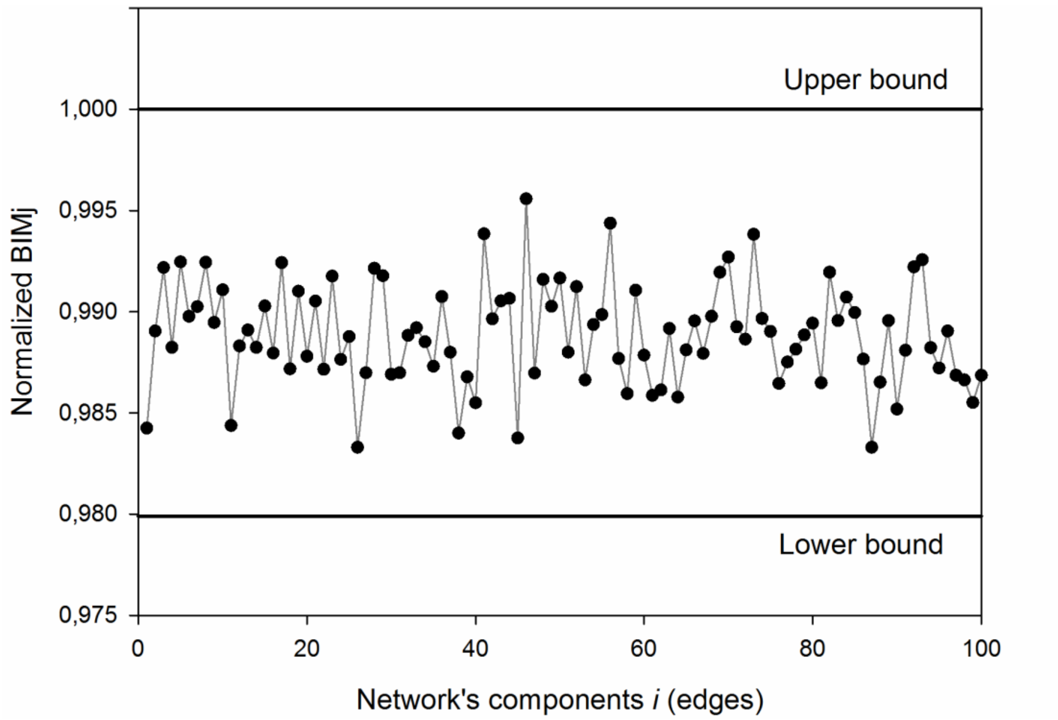

Figure 5: The BIM spectra of the network's components.

The reliability of the network was computed using Eq.(7). It is worth to mention that the network reliability depends mainly on the following factors: (a) the network size (number of components in the network k); (b) the component's failure probability q; (c) the network's failure criterion (embedded in the cumulative D-spectrum term y); (d) and the network's topography (embedded in the cumulative D-spectrum term y). The reliability of the analyzed network was found to be 46%. However, this number considers equal failure probabilities for all network components. This is rarely the case because usually the effect of disruptive events on a network system is not spatially uniform.

5 CONCLUSIONS

This paper presents a methodology to evaluate multiple performance indexes for large scale networks. In the literature, several methods to evaluate networks reliability and resilience can be found. The application of such methods to large scale networks is not feasible due to the computational complexity. In this paper, the case of large scale networks is tackled. The case study considered in this work is the transportation network of a virtual city. First, the road map of the city is transformed into an undirected graph, which consists of 15012 nodes and 19614 edges. A random removal of the edges is applied as a failure mechanism until the network's failure point is reached. The network reliability is then calculated using the Destruction Spectrum (D-spectrum) approach assuming the same failure probability for all edges. A Monte Carlo approach is used to generate failure permutations which are necessary for evaluating the D-spectrum. In addition, the network's edges have been ranked from the most to the least important by applying the Birnbaum Importance Measure (BIM). To overcome the computational obstacles, two algorithms have been presented and discussed.

The results obtained in this study are used to identify the vulnerable components of the network. The vulnerable components are the ones that should be focused on to improve the overall resilience of the infrastructure. The analysis concept adopted in this study is applicable to all network-based infrastructure systems such as water, gas, transportation, etc. Future work is geared towards replicating the analysis methodology to the case of strategic edge removal. The edge removal mechanism will be linked to the buildings' damage assuming a certain destructive event.

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