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Resilience Assessment of City-Scale Transportation Networks Using Monte Carlo Simulation

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ABSTRACT: To improve the resilience of critical infrastructure systems, their intrinsic properties need to be understood and their resilience state needs to be identified. In the literature, several methods to evaluate networks’ reliability and resilience can be found. However, the applicability of these methods is usually restricted to small-size networks. In this paper, the transportation network of a large-scale virtual city is considered as a case study. A random removal of the roads is applied simulating the network’s failure. The network reliability is then calculated using the Destruction Spectrum (D-spectrum) method and a Monte Carlo approach has been developed to generate failure permutations that are necessary for the evaluation of the D-spectrum. In addition, the Birnbaum Importance Measure (BIM) has been adopted in this study to determine the importance of the network’s components. The methodology adopted in this study can be also extended to all network-based systems. The paper also introduces resilience indicators as a soft tool to predict the performance and serviceability of transportation networks.

1 INTRODUCTION

Good physical connectivity in the urban and rural areas is essential for the economic growth of communities. Because of its intensive use of infrastructures, the transport sector is considered an important component of the economy and a common tool used for development. Transport networks need to provide a continuous service for communities, and this necessitates a good understanding of their resilience and reliability states. For instance, understanding how the topology of the network changes under disruptive events can be fundamental in the decision-making process. It could also speed up rescue operations and help in evaluating cascading effects on other interdependent networks. This paper explores mostly the reliability of large-scale networks. Reliability is a very broad concept and its application is extended to all engineering fields. In general terms, network reliability can be defined as the probability of connecting the nodes of the network (Chang & Li 2014). Other authors consider reliability as the quality of the transportation system in terms of optimal travel time, i.e. the probability that a trip between two nodes takes less than a certain time (Immers et al. 2002). Another widely used concept is the capacity reliability, which is the probability that the network capacity can accommodate a certain traffic volume at a required level of service (Chootinan et al. 2005, Chen et al. 2013, Niu et al., 2017). Reliability has so been studied under specific situations, such as the emergency response, using both ideas of travel time and level of service (Edrissi et al. 2015). Looking at graph theory, Guidotti et al. (2017) have used the connectivity measures as a tool to determine whether a network is reliable or not.

In this work, the reliability of the transportation network of a large-scale virtual city is evaluated. The reliability definition adopted in (Gertsbakh & Shpungin 2008, Gertsbakh & Shpungin 2012, Gertsbakh & Shpungin 2016) is considered in this study. According to the researchers, reliability is related to the probability that some nodes, called terminals, remain connected. Thus, the system fails when the terminals are no more connected. The terminal nodes are strategic nodes with pre-defined survival probabilities assigned by the competent authorities. Knowing these probabilities helps greatly in improving the network (Peeta et al. 2010). However, it can be rather difficult to have access to such data. For this reason, a different failure criterion has been chosen in this work. Moreover, another performance parameter, the Birnbaum Importance Measure (BIM), is considered to study the behavior of the analyzed network. This parameter represents the importance of the network’s components (Gertsbakh & Shpungin 2012); that is, components with a high BIM index are vulnerable components. The determination of the reliability and BIM indexes is relatively simple for small networks, but when ap-
applied to a large-scale road transportation network, the computational effort becomes a significant aspect to take into account.

The paper also introduces various resilience indicators of transport networks that can serve as a soft tool to predict and assess the resilience and functionality of the transportation system. The indicators are still not exhaustive and they mainly cover the physical aspect of the network only. Further work will be developed to consider organizational and managerial aspects of the network.

The paper is structured as follows: a brief introduction of the performance indexes with their equations is presented in section 2. Section 3 introduces some strategies to overcome the computational issues. A case study is presented in section 4, where the description of the network and the obtained results are provided. Finally, resilience indicators for transport networks are proposed in section 5.

2 NETWORK RELIABILITY AND COMPONENTS’ IMPORTANCE

2.1 Destruction spectrum

The Destruction spectrum, or simply D-spectrum, is a representation of a network’s structure and its failure definition (Gertsbakh & Shpungin 2009). The system in Figure 1 is used to introduce the concept of the D-spectrum. The nodes in the system are assumed reliable while the edges are unreliable; that is, only edges are subject to failure. In this example, the system’s failure is defined as the loss in connectivity between the terminal nodes a and c.

Figure 1. A simple network with two critical nodes.

The system’s failure can be reached through different sequences of failing components. For instance, if the edges (1; 2; 5; 3; 4) fail, the two nodes a and c become disconnected, and thus the system fails. Another permutation leading to the same result can be (3; 5; 4; 1; 2). The failing component at which the system becomes down is called the anchor of the permutation. In the two permutations above, edges 5 and 4 are the anchors respectively. The total number of failure permutations in a system is $k!$, where $k$ is the number of unreliable elements. After identifying all failure permutations, the D-spectrum set of the network is computed as follows:

$$D = \left\{ d_1 = \frac{x_1}{k!}, d_2 = \frac{x_2}{k!}, \ldots, d_k = \frac{x_k}{k!} \right\} \quad (1)$$

where $d_i$ is the $i^{th}$ component of the spectrum, $x_i$ is the total number of permutations whose anchor’s order is $i$, $k$ is the total number of unreliable components. It is obvious from the equation above that the summation of all elements is 1 ($\sum d_i = 1$).

It is worth to note that the failure probability of each edge is not considered in the D-spectrum. One may simply consider a random removal of the edges in the sense that all edges have the same failure probability. Otherwise, a strategic edge removal can be considered but it requires additional analyses. For instance, in transportation networks, the removal of an edge (road) may be linked to the level of damage of the adjacent buildings. This requires fragility analysis to determine the level of damage that each building is subject to.

2.2 Network reliability

Generally, a network can be considered reliable when it offers a certain level of service or performance, even during emergency situations. Most of the reliability definitions that can be found in the literature deal with the concept of reliability in probabilistic terms. According to the definition of Gertsbakh & Shpungin (2008), each element of the network (nodes and edges) is given a probability $p$ of being available and a probability $q = 1 - p$ of being unavailable. All these probabilities contribute in the determination of the network’s reliability. The formulation used to calculate the reliability index $R(N)$ is given in Equation (2). This one is valid when all unreliable components have the same failure probability.

$$R(N) = 1 - \sum_{i=1}^{k} y_i \frac{k!d_ip^{k-i}}{i!(k-i)!} \quad (2)$$

where $y_i$ is the cumulative D-spectrum, given by the following equation:

$$y_i = \sum_{b=1}^{i} d_b \quad (3)$$

2.3 Components’ importance

The Birnbaum Importance Measure (BIM) is a parameter that describes the importance of network’s components. The vulnerability of the components is what determines their corresponding value of BIM.
This measure can be a tool to identify the critical components of a network in order to strengthen them. The following equation is used to compute the BIM index of a single component:

\[
BIM_j = \sum_{i=1}^{k} \frac{k!q^i p^{k-i} - (y_i - z_{i,j})q^i p^{k-i}}{i!(k-i)!} \tag{4}
\]

where \(z_{i,j} = Z_{i,j} / k!\), in which \(Z_{i,j}\) is the number of permutations satisfying two conditions: (a) if the first \(i\) elements of the network are down, then the network is down; (b) element \(j\) is among the first \(i\) elements of the permutation. By doing some manipulation, the expression in Equation (4) can be written in a different form, shown in Equation (5). It is worth to note that both reliability and BIM indexes share a common factor in their equations, and this provides a unique characteristic to compute both indexes in a single operation.

\[
BIM_j = \sum_{i=1}^{k} \frac{k!q^i p^{k-i}}{i!(k-i)!} \left( \frac{z_{i,j}}{q} - \frac{y_i - z_{i,j}}{p} \right) \tag{5}
\]

3 METHOD: APPLICATION TO LARGE-SCALE NETWORKS

Theoretical reliability analyses are not always applicable to large problems. For instance, the applications of the above mentioned equations are limited to small networks with defined number of components. This is due to the presence of factorial in the denominator of the spectrum set (Eq. (1)). The same problem appears when computing the reliability index \(R\), although we know in advance that this is a number between 0 and 1, regardless of the network’s size. In this section, we present strategies to apply the D-spectrum to large scale networks and compute the reliability index and BIM-spectra.

3.1 D-spectrum for large scale networks: a Monte Carlo approach

We propose the use of a Monte Carlo approach to generate the failure permutations needed to compute the D-spectrum. Hereafter, the algorithm of the Monte Carlo simulation is presented.

**Algorithm:** Generate permutations and compute the D-spectrum

1) Define a failure criterion for the network (e.g. percentage of failed components, number of disconnected nodes, etc.).
2) Start a permutation counter \((n = 1)\).
3) Choose a component (edge or node) randomly from 1 to \(k\) and store it (i.e. the chosen number represents a failed component).
4) Check if the network’s failure criterion defined in step 1 is met:
   a. If yes, store the permutation and go to step 5;
   b. If no, choose another number from 1 to \(k\) excluding the numbers previously chosen and then go back to step 4.
5) Set \(n = n + 1\) and repeat the steps 3 and 4 for \(M\) times to generate \(M\) permutations.
6) Group the permutations according to their anchor’s value (i.e. the anchor value coincides with the vector’s length of the permutation).
7) Compute the D-spectrum using Equation (1).

In case of targeted removal, the network’s components should be linked with given failure probabilities or lifetime distributions. In this case, step 3 is adjusted so that the removal process is not random.

3.2 Reliability and BIM indexes for large scale networks: incremental computation

Practically, computing the reliability and BIM indexes is not possible for moderate to large networks. The reason is that both numerator and denominator in the second term of the reliability index (Eq. (2)) and in the first term of the BIM index (Eq. (5)) are too large. However, it is known in advance that the reliability index is a number that ranges between 0 and 1 regardless of the network’s size. Symbolic calculation represents an effective tool to solve such kind of numerical problems. Also, a more compact writing of the equation has been used. Defining \(M\) as shown in Equation (6), it is possible to rewrite the expressions of reliability and BIM indexes (Eqs 7-8).

\[
M_j = \frac{k!q^i p^{k-i}}{i!(k-i)!} \tag{6}
\]

\[
R = 1 - \sum_{i=1}^{k} y_i \cdot M_i \tag{7}
\]

\[
BIM_j = \sum_{i=1}^{k} M_i \cdot \left( \frac{z_{i,j}}{q} - \frac{y_i - z_{i,j}}{p} \right) \tag{8}
\]

4 CASE STUDY: THE TRANSPORTATION NETWORK OF A VIRTUAL CITY

4.1 Network definition

The road transportation network of a virtual city has been modeled as an undirected graph (Figure 2). An undirected graph \(G = (V, E)\) consists of a set of vertices (or nodes), representing the intersections, together with an edge set \(E\). The elements belonging to \(E\) are called edges or links, and represent the roads of the network. In an undirected graph every pair of nodes is connected, so each path can be passed...
through in both directions. Actually, a road map would be a directed graph, as the streets have a certain way. However, in emergency conditions, it is likely that respecting the directions becomes a secondary aspect.

![Image of transportation network of the virtual city.](image)

**Figure 2.** The transportation network of the virtual city.

The road transportation system consists of 19,614 edges connecting 15,012 nodes. Mathematically, the network has been described with an adjacency matrix $A$, which is a square matrix with a side dimension equal to the number of the nodes. The elements inside $A$ can be either 1 or 0. If $a_{ij} = 1$, it means that there is a connection (road) between node $i$ and node $j$, while 0 means that the two nodes are not linked. Since the graph is not directed, the resulting adjacency matrix is symmetric. This matrix allows to describe and modify the topology of the network and to automate all the calculations.

4.2 **Network’s failure criterion**

The definition of the failure criterion is strictly related to the reliability of the network. Despite its importance, there is not a unique definition for the failure criterion of a network. Gertsbakh & Shpungin (2008) proposed the criterion of terminal connectivity; that is, if critical nodes are disconnected, the network becomes down. This criterion is applicable to small scale networks, but when dealing with large networks a huge computational effort would be needed to identify whether the nodes are connected or not. In this work, a simpler network failure criterion is adopted. The network is considered unavailable when at least 5% of the nodes are isolated (not connected to any edge).

4.3 **Results**

The methodology introduced in section 3 has been applied to the case study. Due to the large size of the analyzed network, the results are not shown using vectors or matrices, but rather using graphs.

First, the above mentioned algorithm has been used to generate the failure permutations of the edges. The number of permutations considered in this case study is 3.5 million. The generated permutations have been subsequently used in the calculation of the D-spectrum. The result of the D-spectrum is shown in Figure 3. It can be clearly seen that there are only few non-zero elements in the distribution, and they are all gathered in a small range. The distribution of the D-spectrum is a perfectly normal distribution confirming that the assumed number of permutations was large enough to represent the problem. The location of the distribution’s peak depends greatly on the chosen failure criterion of the network and on the number of components forming the network. More study will be dedicated to know the reason of having a normal distribution and to identify the effect of the failure criterion and number of components on the position of the distribution’s peak. Moreover, looking at the definition of the D-spectrum, the sum of all its elements is 1. This is verified in Figure 4, which shows the distribution of the cumulative D-spectrum.

![Figure 3.](image)

**Figure 3.** (a) The D-spectrum of all components of the case study network; (b) a zoomed view at the distribution peak.
Following the D-spectrum, the BIM index of the network’s components and the network reliability index \( R \) have been evaluated. The BIM index results of the networks’ components have been normalized with respect to the maximum value. Figure 5 shows the BIM results of the first 100 components. The BIM indexes of the other components range between the upper the lower bounds, 1 and 0.98 respectively. This implies that the variance in the results is very small. In fact, the importance index of the edge is ruled by the network configuration and the failure probability of the edge. In our network, we considered an equal failure probability for the edges, which was represented by the random removal process. The small difference in the importance of the edges was only due to the network’s configuration.

The reliability of the network was computed using Equation (7). It is worth to mention that the network reliability depends mainly on the following factors: (a) the network size (number of components in the network \( k \)); (b) the component’s failure probability \( q \); (c) the network’s failure criterion (embedded in the cumulative D-spectrum term \( y \)); (d) and the network’s topography (embedded in the cumulative D-spectrum term \( y \)). The reliability of the analyzed network was found to be 46%. However, this number considers equal failure probabilities for all network components. This is rarely the case because usually the effect of disruptive events on a network system is not spatially uniform.

Figure 4. (a) The cumulative D-spectrum of all components of the case study network; (b) a zoomed view at the transitional part.

Figure 5. The BIM spectra of the network’s components.

5 RESILIENCE INDICATORS FOR TRANSPORTATION NETWORKS

Indicators are quantifiable variables that represent selected characteristics of resilience (Cutter 2016). Generally, individual indicators can be combined to create a single resilience index (Kammouh et al. 2017b). The index illustrates the multi-dimensional nature of resilience by aggregating multiple indicators, but also condenses its complexity into a single numeric value.

To study the resilience of a critical infrastructure, it is important to tackle the different resilience aspects of the system. For example, having a well-planned transportation network can result in a non-resilient system if the organizational and the managerial aspects are not considered. The user-behavior aspect can also contribute in the overall resilience of the analyzed system. While non-physical components are fundamental to study the resilience of transport networks, in this work only the physical aspect in considered.

The functionality of the physical aspect of transportation infrastructure has a great impact on the resilience of entire system. The physical aspect can be divided into two main parts: the inherent physical characteristics of the traffic and the network topography, which is the geometry of the network that defines the location of the links (i.e. roads, subway lines, path, bridges, etc.) and nodes (i.e. intersections, stations, etc.).
The physical characteristics of transportation infrastructure include factors like resisting strength, pavement condition, road width, number of lanes, lighting system, safety elements, etc. All of these attributes and basic elements control the proper functioning of the transport system, especially when it is under rare events, such as earthquakes, floods, hurricanes, landslides, etc. (which have long-term effect) and frequent events like traffic accidents, road maintenance (which temporarily disable intended function) (Soltani-Sobh et al. 2016).

Generally, an infrastructure with enough redundant capacity can minimize the impact of adverse events and keep the system in a relatively stable state. For example, when a car accident occurs on a road with emergency lanes or with a Variable Message Signs (VMS), travelers can use the extra lane or change to an alternative way that is suggested by the VMS before getting stuck in the traffic. Moreover, the degradation of the road environment influences the driving conditions, which affect the road capacity and result in the failure of the whole system.

In addition to the physical characteristics, the degree of betweenness and connectivity of the network system should be considered to assess the resilience of transportation networks. The concept of betweenness quantifies the number of shortest paths that pass through a node (Leu et al. 2010). As shown in Figure 6, node A has the highest betweenness among all four nodes. In a previous study, it was proved that a failure of the node with the highest betweenness can cause the maximum damage to the network because it reduces the efficiency and the survivability of the entire system (Zhang et al. 2011). Moreover, Steiglitz et al. (1969) proposed that the residual connections between nodes can increase the performance of the network. In other words, the more number of reliable passageways between an origin and a destination, the more resilient is the transportation system.

This is illustrated in Figure 6, where links 1, 2, 3 represent three alternative paths connecting the two nodes C and D. So, users would have alternative choices when one of the three routes is closed. This implies that the system has a high redundancy, which is the ability to remain (nearly) unchanged with disturbance.

Table 1 shows an extended list of resilience indicators that describe the physical system of the transportation network. Most of the indicators have been collected from the literature while some of them have been proposed by the authors. The indicators are grouped under different components representing the major aspects of the system. Each of the indicators is accompanied by a measure that allows the analytical evaluation of the indicator. Each measure is normalized with respect to a fixed quantity, the target value (TV), which is an essential quantity defined by the authority to provide a baseline to measure the corresponding indicator. Moreover, the indicators have been identified by their nature: ‘Static indicator (S)’ assigned to indicators that are not affected by the perturbation/disaster, and ‘Dynamic indicator (D)’ assigned to the indicators whose value/serviceability changes after the event.

Indicators must be weighted according to their contribution towards the resilience of the system. Kammouh et al. (2017a) introduced and exemplified three weighting methods for resilience indicators based either on the dependence tree analysis or on the spider plot analysis (Kammouh et al. 2017a, Kammouh et al., in press).

Table 1. Components and indicators of the physical system of transportation networks.

<table>
<thead>
<tr>
<th>Component/indicator</th>
<th>Measure</th>
<th>Reference</th>
<th>Nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Links/Connectors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Accessibility</td>
<td>Number of links/passageways per destination ÷ TV</td>
<td>(Ip &amp; Wang 2011)</td>
<td>D</td>
</tr>
<tr>
<td>-Road density</td>
<td>Number of alternative links between an origin and destination ÷ TV</td>
<td>(Jenelius 2009)</td>
<td>D</td>
</tr>
<tr>
<td>-Road width</td>
<td>Average width of road ÷ TV</td>
<td>(Jenelius 2009)</td>
<td>S</td>
</tr>
<tr>
<td>-Lanes of road</td>
<td>Number of lanes available ÷ TV</td>
<td>(Litman 2006)</td>
<td>D</td>
</tr>
<tr>
<td>-Link (road, track, etc.) condition</td>
<td>% links with full functionality during the event</td>
<td>*</td>
<td>D</td>
</tr>
<tr>
<td>2-Vehicles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Mode of transport</td>
<td>Number of multimode choices per destination ÷ TV</td>
<td>(Ip &amp; Wang 2011)</td>
<td>D</td>
</tr>
<tr>
<td>-Service level</td>
<td>Average speed of vehicles in normal condition ÷ TV</td>
<td>*</td>
<td>S</td>
</tr>
<tr>
<td>-Characteristics</td>
<td>Degree of preference</td>
<td>*</td>
<td>S</td>
</tr>
</tbody>
</table>

Figure 6. An illustration of the concept of connectivity and betweenness.
6 CONCLUSIONS

This paper presents a methodology to evaluate multiple performance indexes for large scale networks. In the literature, several methods to evaluate networks reliability and resilience can be found. The application of such methods to large scale networks is not feasible due to the computational complexity. In this paper, the case of large scale networks is tackled. The case study considered in this work is the transportation network of a virtual city. First, the road map of the city is transformed into an undirected graph, which consists of 15,012 nodes and 19,614 edges. A random removal of the edges is applied as a failure mechanism until the network's failure point is reached. The network reliability is then calculated using the Destruction Spectrum (D-spectrum) method assuming the same failure probability for all edges. A Monte Carlo approach is used to generate failure permutations which are necessary for evaluating the D-spectrum. In addition, the network's edges have been ranked from the most to the least important by applying the Birnbaum Importance Measure (BIM). To overcome the computational obstacles, an algorithm and calculations techniques have been presented and discussed.

The results obtained in this study are used to identify the vulnerable components of the network. The vulnerable components are the ones that should be focused on to improve the overall resilience of the infrastructure. The analysis concept adopted in this study is applicable to all network-based infrastructure systems such as water, gas, transportation, etc. Future work is geared towards replicating the analysis methodology to the case of strategic edge removal. The edge removal mechanism will be linked to the buildings’ damage assuming a certain destructive event.

The paper also proposed a list of resilience indicators for transport networks that describe the physical aspect of the system. Future studies will focus on identifying other indicators related to other resilience aspects of the transportation infrastructure.

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