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***Abstract*—This paper extends the well-established macromodeling flows based on rational fitting and passivity enforcement to the bivariate case, where the model response depends on frequency and on some additional design parameter. We propose a black-box model identification algorithm that is able to guarantee uniform stability and passivity throughout the parameter range. The resulting models, which can be cast as parameterized SPICE subnetworks, may be used to construct parameterized component libraries for design optimization, what-if analyses and fast parametric sweeps in frequency or time domain.**

I. Introduction

Macromodeling flows based on rational fitting with passivity constraints are now well established [1]. Black-box macromodels provide compact reduced-order equivalents of complex interconnect or electromagnetic structures, thus enabling very fast and reliable frequency- or time-domain simulations using off-the-shelf circuit solvers. One of the key reasons for this success is the availability of robust model identification algorithms, which are able to enforce model stability and passivity while matching with excellent accuracy the response of the original system.

Bivariate and more generally multivariate macromodeling is a very interesting approach for making the compact models scalable and flexible, by including in the model equations the explicit closed-form dependence on some additional design parameter, such as a geometrical size, an electrical parameter, or even temperature or bias in case of linearized models of active devices [2]. The availability of parameterized macromodels would allow fast design optimization, what-if analyses, and parametric sweeps based on reduced-order equivalents, thus avoiding the need to regenerate the model for every geometrical configuration that needs to be evaluated during a design process.

Several approaches have been proposed for the multivariate extension of standard macromodeling flows [3]–[7]. Unfortunately, none of the existing solutions has been proven to be robust enough to grant routine applicability in industry design flows. With this paper, we propose an algorithm for the extraction of bivariate macromodels from sets of scattering responses obtained from some electromagnetic solver. The proposed algorithm is able to guarantee uniform stability and passivity throughout the parameter range, using a robust formulation. Various examples confirm the excellent numerical properties of the produced models, and confirm that the proposed scheme has a very good potential for opening new scenarios in automated design flows.

II. Formulation

Let us consider an interconnect or a component with P electrical ports, whose response depends on frequency $s = \mathrm{j}\omega$ and on some additional parameter $\vartheta \in \Theta \subset \mathbb{R}$. As illustrative examples, we may consider ϑ to be the width of a signal conductor, or the permittivity of a dielectric substrate, or the length of a via stub. We denote as $\breve{\mathbf{H}}(s;\vartheta)$ the "true" scattering matrix of the structure, which can be computed, e.g., via an electromagnetic field solver at a finite set of points (s_k, ϑ_m) in the frequency and parameter space. Our objective is to construct a reduced-order model whose scattering response $\mathbf{H}(s;\vartheta)$ matches this data through

$$\mathbf{H}(s_k;\vartheta_m) \approx \breve{\mathbf{H}}(s_k;\vartheta_m), \quad k = 1,\ldots,\bar{k},\ m = 1,\ldots,\bar{m}. \tag{1}$$

In addition, we seek a procedure that is able to enforce uniform stability and passivity of the model throughout the parameter range $\forall\vartheta \in \Theta$, which can be summarized in the following Bounded Realness conditions [9]

1) $\mathbf{H}(s;\vartheta)$ regular for $\mathrm{Re}\{s\} > 0$,
2) $\mathbf{H}^*(s;\vartheta) = \mathbf{H}(s^*;\vartheta)$,
3) $\mathbf{I}_P - \mathbf{H}^{\mathsf{H}}(s;\vartheta)\mathbf{H}(s;\vartheta) \geq 0$ for $\mathrm{Re}\{s\} > 0$,

where $^{\mathsf{H}}$ is the Hermitian transpose, and $\mathbf{I}_P$ is the identity matrix of size P. Condition 1 implies stability, condition 2 implies a real impulse response, and condition 3 implies passivity.

As proposed in [3], [4], we adopt the following model structure

$$\mathbf{H}(s;\vartheta) = \frac{\mathsf{N}(s,\vartheta)}{\mathsf{D}(s,\vartheta)} = \frac{\sum_{n=0}^{\bar{n}}\sum_{\ell=1}^{\bar{\ell}} \mathbf{R}_{n,\ell}\,\xi_\ell(\vartheta)\,\varphi_n(s)}{\sum_{n=0}^{\bar{n}}\sum_{\ell=1}^{\bar{\ell}} r_{n,\ell}\,\xi_\ell(\vartheta)\,\varphi_n(s)}, \tag{2}$$

where $\mathbf{R}_{n,\ell} \in \mathbb{R}^{P\times P}$ and $r_{n,\ell} \in \mathbb{R}$ are the real-valued model coefficients. Two separate sets of basis functions are used in (2). Frequency dependence is captured by the standard partial fraction basis functions adopted in Vector Fitting: $\varphi_0(s) = 1$ and $\varphi_n(s) = (s - q_n)^{-1}$ for $n > 0$, where q_n are fixed "basis poles", which are either real or occur in complex conjugate pairs (so that condition 2 above is automatically satisfied [1], [8]). Parameter dependence is embedded through the basis functions $\xi_\ell(\vartheta)$ (in this work, first-kind Chebychev polynomials). As a result, both model poles (the zeros of the denominator $\mathsf{D}(s,\vartheta)$) and residues are parameter-dependent. Due to this fact, a direct least-squares fit based on (1) is not able to guarantee that the model poles are stable, or that the model is passive for any arbitrary parameter value ϑ in the range of interest.

Let us assume first that the model is stable $\forall\vartheta$ and that we want to check and enforce its uniform passivity. For any fixed ϑ, we can construct the Hamiltonian matrix $\mathbf{M}(\vartheta)$ or the Skew-Hamiltonian/Hamiltonian (SHH) pencil $(\mathbf{M}(\vartheta), \mathbf{K})$ depending on the preferred state-space or descriptor realization [1], [10], and find the corresponding set of (finite) eigenvalues $\{\mu_i(\vartheta)\}$. We then define the (continuous) function

$$\psi(\vartheta) = \min_i \frac{1}{\rho(\vartheta)} |\operatorname{Re}\{\mu_i(\vartheta)\}| \tag{3}$$

where ρ is the maximum eigenvalue magnitude. This function can be interpreted as the normalized distance of the Hamiltonian eigenspectrum from the imaginary axis. If $\psi(\vartheta) = 0$, then there exist some purely imaginary eigenvalues $\mu_i(\vartheta) = \mathrm{j}\omega_i(\vartheta)$ whose frequencies $\omega_i(\vartheta)$ delimit the frequency bands where the model is locally non-passive [1]. Conversely, if $\psi(\vartheta) > 0$, then there are no imaginary eigenvalues and the model is locally passive for that value of ϑ.

We perform a bivariate passivity check by running an adaptive sampling process on $\psi(\vartheta)$, supported by the observation that if $\psi(\vartheta) > 0$, a small perturbation $\vartheta + \delta\vartheta$ will lead to a small perturbation on the Hamiltonian eigenspectrum and on consequently on $\psi(\vartheta + \delta\vartheta)$. A simple adaptive bisection process starting from an initial set of samples $\hat{\vartheta}_m$ uniformly distributed in Θ is setup, with the objective of tracking all the subintervals where $\psi(\vartheta) = 0$. Once the bisection is complete, we have a precise knowledge of all areas in the frequency-parameter plane (ω, ϑ) where the model is not passive. Local subsampling within these areas leads to the points $(\bar{\omega}_\nu, \bar{\vartheta}_\nu)$ where the largest singular value $\bar{\sigma}_\nu$ of the model response $\mathbf{H}(\mathrm{j}\bar{\omega}_\nu, \bar{\vartheta}_\nu)$ attains the local maximum. We then setup a singular value perturbation scheme that perturbs the model (numerator) coefficients

$$\mathbf{R}_{n,\ell} \;\rightarrow\; \mathbf{R}_{n,\ell} + \Delta\mathbf{R}_{n,\ell} \tag{4}$$

and corrects this singular value to be less than one. One should note that this scheme is a straightforward extension to the bivariate case of the original singular value perturbation scheme documented in [11]. Full details of the adaptive sampling process and related bivariate singular value perturbation are available in [12].

We now address condition 1 above, related to stability, which implies that all zeros of the denominator $\mathsf{D}(s, \vartheta)$ are stable. Let us assume that we are able to enforce $\mathsf{D}(s, \vartheta)$ to be a Positive Real (PR) function (we recall that PR conditions [9] are similar to the BR conditions above, but with condition 3 replaced by $\mathbf{H}(s; \vartheta) + \mathbf{H}^{\mathsf{H}}(s; \vartheta) \geq 0$ for $\operatorname{Re}\{s\} > 0$). Since $\mathsf{D}(s, \vartheta)$ is scalar, this reduces to enforcing the real part of $\mathsf{D}(s, \vartheta)$ to be nonnegative. In practice, for enhanced numerical robustness, we formulate a strict PR constraint as

$$\operatorname{Re}\{\mathsf{D}(s, \vartheta)\} > 0 \quad \text{for} \quad \operatorname{Re}\{s\} > 0, \quad \forall\vartheta \in \Theta. \tag{5}$$

If (5) holds true, then the inverse of the denominator $\mathsf{D}^{-1}(s, \vartheta)$ is also a PR function, without poles for $\operatorname{Re}\{s\} > 0$. The

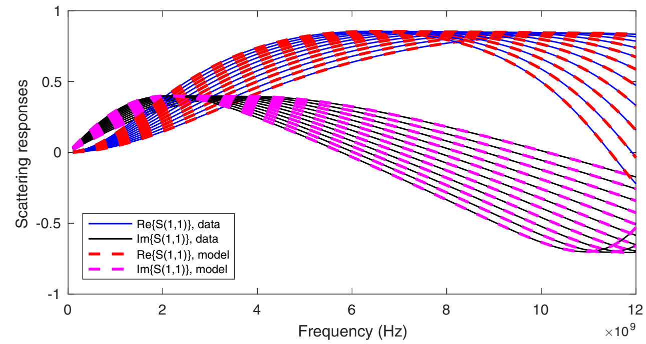

Fig. 1. Comparison between scattering responses of the passive parameterized model, evaluated at the same parameter values available in the raw dataset. Only one half (odd-numbered) of these responses were used for model identification, the other responses were used for validation only.

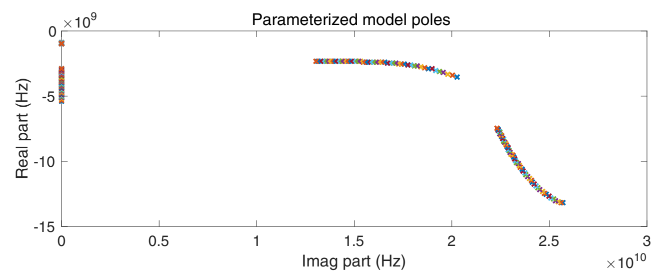

Fig. 2. Sweep of the parameterized model poles (refined by a factor 4 with respect to the raw parameter samples used for model identification). All poles are stable.

poles of $\mathsf{D}^{-1}(s, \vartheta)$ are the zeros of $\mathsf{D}(s, \vartheta)$ and correspond to the parameter-dependent poles of our bivariate model. We conclude that (5) is a sufficient (although not necessary) condition for the uniform stability of the model. This condition is enforced by applying the above-described passivity enforcement process to the denominator $\mathsf{D}(s, \vartheta)$ alone, considered as an immittance (admittance or impedance) function so that PR is equivalent to passivity. The process is further facilitated by starting with an almost-stable $\mathsf{D}(s, \vartheta)$, obtained by embedding the inequality constraint (5) in the initial model identification stage, here formulated through a standard Generalized Sanathanan-Koerner iteration [13]. More details on the proposed stability-constrained model construction are available in [14], where also the parameterized SPICE synthesis of the bivariate model is discussed.

III. Numerical results

We illustrate the performance of proposed parameterized macromodeling scheme by applying it to a 2-port single-layer 1.5-turn integrated square inductor, parameterized by its sidelength $\vartheta \in \Theta = [1.02, 1.52]$ mm. Original scattering responses were computed through a field solver over $\bar{k} = 477$ samples covering a frequency band up to 12 GHz (courtesy of Prof. Madhavan Swaminathan, Georga Institute of Technology, Atlanta, USA) over $\bar{m} = 11$ linearly spaced samples in the parameter space. Only the 6 (odd-indexed) parameter samples were used for model identification, while the remaining 5 were

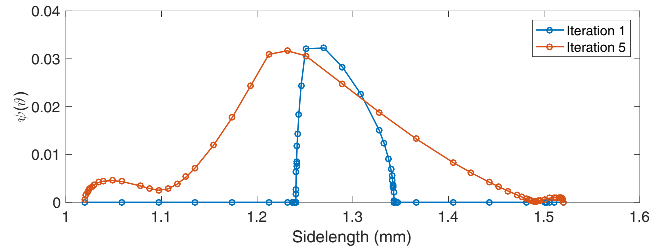

Fig. 3. Hamiltonian spectral distance $\psi(\vartheta)$ from the imaginary axis, plotted for the initial model (Iteration 1) and the final model after passivity enforcement (Iteration 5). The final model is passive, since $\psi(\vartheta) > 0$ for all ϑ in its range.

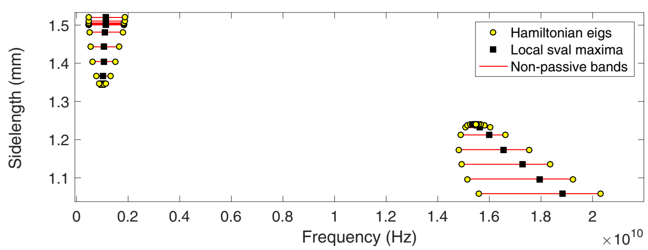

Fig. 4. Results of passivity check performed in the initial model. Yellow dots depict the frequencies of the purely imaginary Hamiltonian eigenvalues obtained from the SHH pencil at the corresponding parameter values (the latter resulting from the adaptive sampling on $\psi(\vartheta)$, see Fig. 3. Red lines highlight the frequency bands where passivity violations are located. Black dots are the local singular value maxima computed in the non-passive bands. These are the points that are perturbed in the passivity enforcement loop.

used for self-validation. Model identification using $\bar{n} = 6$ poles and degree $\ell = \{3, 2\}$ polynomials for parameterization of numerator and denominator, respectively, led to a worst-case approximation error between final passive model and original data of $\varepsilon = 0.7 \times 10^{-3}$ among all responses and all frequency points. Only 1.8 s were needed for initial model estimation, while passivity enforcement required 18 s, mostly spent on iterative computation of Hamiltonian eigenvalues.

Figure 1 compares the model responses to the original data, confirming the excellent accuracy throughout the parameter range, whereas Fig. 2 confirms that the model is uniformly stable by plotting the poles trajectories obtained by a fine sweep of the parameter in its range, and superimposing all poles in the same panel. We remark that uniform stability does not require the explicit determination of the poles at any parameter value, since it is enforced implicitly by the PR constraint on the model denominator.

Figure 3 (blue line) depicts the normalized distance of the Hamiltonian eigenvalues from the imaginary axis $\psi(\vartheta)$ at the initial passivity enforcement step (iteration 1), showing that there are two subintervals where this distance vanishes, indicating the presence of passivity violations. Such violations are depicted in Fig. 4, which collects those imaginary Hamiltonian eigenvalues arising from the adaptive parameter bisection loop (yellow dots), the corresponding passivity violation regions (obtained by slices in the parameter range, the red lines), and the location of the local singular value maxima that are perturbed (black squares). After 5 iterations all imaginary eigenvalues are removed, all singular values are uniformly below one, and the Hamiltonian spectral distance $\psi(\vartheta)$ is uniformly positive (red line in Fig. 3).

IV. Conclusion

This paper introduced a novel algorithm for the extraction of black-box bivariate models from a set of sampled scattering responses over frequency and parameter space, obtained by a field solver. Assuming a Generalized Sanathanan-Koerner form, and leveraging on a strong theoretical result that relates model stability to the positive realness of its denominator, our scheme is able to enforce global (uniform) stability by embedding positive realness constraints in the model identification step. Model passivity is then enforced by an adaptive-sampling driven perturbation of its coefficients, based on singular value perturbation.

The proposed scheme provides the first truly black-box algorithm that is able to guarantee stability and passivity in addition to a well-controlled accuracy. The resulting parameterized models are easily cast as parameterized SPICE subcircuits, thus enabling component library generation for automated design optimization and fast parametric sweeps.

References

[1] S. Grivet-Talocia and B. Gustavsen, *Passive Macromodeling: Theory and Applications*. New York: John Wiley and Sons, 2016.

[2] S. B. Olivadese, G. Signorini, S. Grivet-Talocia, P. Brenner, "Parameterized and DC-compliant small-signal macromodels of RF circuit blocks," *IEEE Trans. CPMT*, vol. 5, pp. 508-522, April 2015.

[3] P. Triverio, S. Grivet-Talocia, and M.S. Nakhla, "A Parameterized Macromodeling Strategy with Uniform Stability Test," *IEEE Trans. on Adv. Packaging*, vol. 32, no. 1, pp. 205-215, Feb. 2009.

[4] D. Deschrijver, T. Dhaene and D. De Zutter, "Robust Parametric Macromodeling Using Multivariate Orthonormal Vector Fitting," *IEEE Trans. MTT*, vol. 56, no. 7, pp. 1661–1667, July 2008.

[5] E. R. Samuel, L. Knockaert, F. Ferranti, and T. Dhaene, "Guaranteed Passive Parameterized Macromodeling by Using Sylvester State-Space Realizations", *IEEE Trans. MTT*, vol. 61, no. 4, pp. 1444-1454, Mar. 2013.

[6] F. Ferranti, T. Dhaene, and L. Knockaert, "Compact and Passive Parametric Macromodeling Using Reference Macromodels and Positive Interpolation Operators," *IEEE Trans. CPMT*, vol. 2, no. 12, pp. 2080–2088, Dec. 2012.

[7] F. Ferranti, L. Knockaert, and T. Dhaene, "Passivity-Preserving Parametric Macromodeling by Means of Scaled and Shifted State-Space Systems," *IEEE Trans. MTT*, vol. 59, no. 10, pp. 2394–2403, Oct. 2011.

[8] B. Gustavsen, A. Semlyen, "Rational approximation of frequency domain responses by vector fitting", *IEEE Trans. Power Del.*, vol. 14, no. 3, pp. 1052-1061, July, 1999.

[9] B. D. O. Anderson and S. Vongpanitlerd. *Network analysis and syntesis*. Prentice-Hall, 1973.

[10] Z. Zhang and N. Wong, "Passivity Check of S -Parameter Descriptor Systems via S-Parameter Generalized Hamiltonian Methods," in *IEEE Trans. Adv. Packag.*, vol. 33, no. 4, pp. 1034-1042, Nov. 2010.

[11] S. Grivet-Talocia and A. Ubolli, "A comparative study of passivity enforcement schemes for linear lumped macromodels," *IEEE Trans. Adv. Packag.*, vol. 31, pp. 673–683, Nov 2008.

[12] S. Grivet-Talocia, "A Perturbation Scheme for Passivity Verification and Enforcement of Parameterized Macromodels", to appear in *IEEE Trans. CPMT*, 2017.

[13] C. K. Sanathanan and J. Koerner, "Transfer function synthesis as a ratio of two complex polynomials," *IEEE Trans. Aut. Control*, vol. 8, no. 1, pp. 56–58, Jan. 1963.

[14] S. Grivet-Talocia, R. Trinchero, "Behavioral, Parameterized and Broadband Modeling of Wired Interconnects with Internal Discontinuities", to appear in *IEEE Trans. EMC*, 2017.