Engineering characteristics prioritisation in QFD using ordinal scales: a robustness analysis

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Abstract

Quality Function Deployment (QFD) is a management tool used for the design of new products/services and the related production/supply processes, which includes several operative phases, starting from the collection of the Customer Requirements (CRs) till the definition of the procedures for the quality control of the designed production/supply processes. The goal of the first phase is to translate the CRs into measurable Engineering Characteristics (ECs) of the new product/service and prioritize them, basing on their relationships with CRs and the related importances. To this purpose, the current scientific literature encompasses several alternative approaches (among them, the most used is the Independent Scoring Method - ISM) in most of which cardinal properties are arbitrarily attributed to data collected on ordinal scales.

This paper describes and discusses a new approach based on ME-MCDM (Multi Expert / Multiple Criteria Decision Making) techniques, which do not require any debatable ordinal to cardinal conversion. The theoretical principles of the method are presented and tested through some application examples related to a well-known case study reported in the scientific literature. A robustness analysis is also carried out.

Keywords: Quality Function Deployment, ordinal scale, Independent Scoring Method, MCDM operators.

1 Introduction

Quality Function Deployment (QFD) is a practical and effective tool for structuring the design activities for a new product/service and the related production/supply process, according to the real exigencies of customers [Akao, 1988; Franceschini, 2001]. Due to its practicality and effectiveness, QFD is universally recognized as a strategic approach to pursue customer satisfaction. The large diffusion of this tool is also proved by the large amount of scientific literature produced over the years [Carnevalli and Cauchick Miguel, 2008].

Many empirical studies demonstrated that the correct implementation of QFD may bring significant improvements in the development of products/services, including earlier and fewer design modifications, fewer start-up issues, improved cross-functional communications, improved product/service quality, reduced time and cost for product/service development, etc. [Biren, 1998; Chan and Wu, 2002.a, 2002.b; Lager, 2005; Carnevalli and Cauchick Miguel, 2008].

From a procedural point of view, QFD is based on four phases, which deploy Customer Requirements (CRs) throughout a structured planning process of the product/service of interest [Akao, 1988]. Each phase is supported by a specific matrix, which establishes a relationship between variables of different nature. A schematic structure of these four phases and the relevant matrices are reported in Fig. 1 [Akao, 1988; Franceschini, 2001].

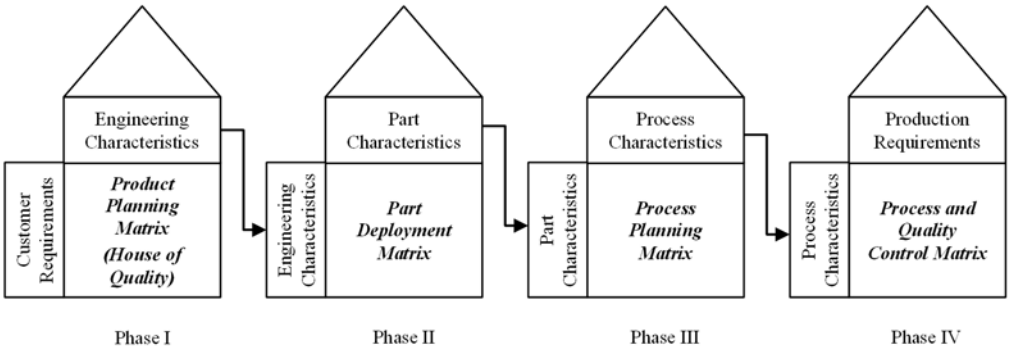

Figure 1. Scheme of the four phases of QFD. Adapted from [Lager, 2005].

In the first phase, CRs are related to a set of Engineering Characteristics (ECs) of the product/service. In the second phase, ECs are associated with a set of critical part characteristics, through the so-called Part Deployment Matrix. Then, the Process Planning Matrix relates the critical part characteristics to the relevant production processes. Finally, the Process and Quality Control Matrix defines suitable quality control parameters and methods to monitor the production process. All the phases are managed by a cross-functional team of experts (the QFD team).

Special attention is given to Phase I, characterized by the construction of the so-called Product Planning Matrix, or House of Quality (hereafter abbreviated as HoQ). As well as defining and prioritizing the CRs, the double goal of this phase is turning the CRs into the ECs and obtaining a prioritization of these, considering (i) their relationships with CRs and (ii) the importance of the related CRs. This phase is fundamental for the success of QFD implementation [Franceschini 2001; Tontini 2007; Li, Tang et al. 2009; Li, Tang et al. 2010], as errors at this stage can propagate throughout the subsequent phases.

With reference to Fig. 2, the construction of the HoQ can be broadly structured into the following ten steps (for details, see Franceschini et al., 2014):

1. Definition of the CRs for the product/service concerned.
2. Prioritization of the CRs.
3. Analysis of the competitors' position (Comparative Benchmarking).
4. Correction of the CRs rating by considering the perception of competitors positioning and according to organization strategic considerations.
5. Identification of the ECs related to the defined CRs.
6. Construction of the Relationship Matrix.
7. Analysis of the correlation among ECs.
8. Prioritization of ECs.
9. Technical comparation with the competitors (Technical Benchmarking).
10. Setting of the technical targets for the new product/service.

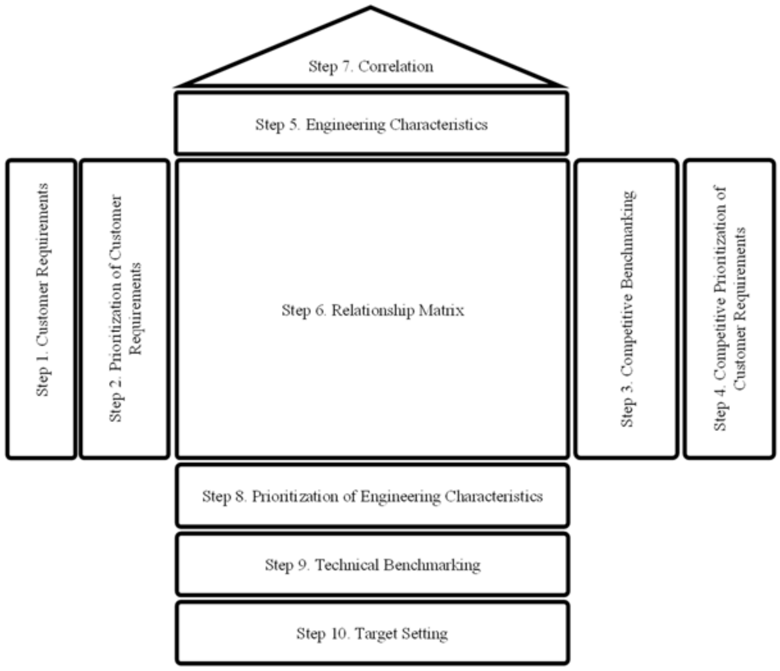

Figure 2. Main steps of House of Quality [Franceschini et al., 2014].

The focus of the present paper is on Step 8, which is aimed at prioritizing the ECs. To this purpose, several approaches are possible. The traditional method is the Independent Scoring Method (ISM) [Akao 1988; Franceschini, 2001], which combines the importances of CRs and the data contained in the Relationship Matrix. The ISM can be subdivided in two operative steps. In the first step, the Relationship Matrix is turned into a cardinal matrix, according to an arbitrary convention: a typical approach is to define the ordinal relationships between CRs and ECs on four levels (i.e., absent, weak, medium and strong relationship, typically expressed by symbols, such as: empty cell, ∆, ○ and ●) and encode them into four numerical coefficients, respectively 0, 1, 3 and 9. In the second step, the importance of each EC is evaluated through a weighted sum of the importances of CRs and the encoded Relationship Matrix coefficients, according to the following model [Akao 1988]:

$$w_j = \sum_{i=1}^{n} d_i \cdot r_{ij} \qquad (1)$$

where:

w_j is the importance of the j-th EC ($j = 1...m$),

d_i is the importance of the i-th CR ($i = 1...n$),

r_{ij} is the coefficient (0, 1, 3 or 9) corresponding to the relationship between the i-th CR and the j-th EC.

The most critical aspect of this process is that ordinal data are "promoted" to cardinal data, relying on two controversial assumptions [Franceschini et al., 2005; Van de Poel, 2007]:

- The importance of each CR, actually expressed on an *ordinal scale*, is artificially encoded in the form of a number, expressed on a *cardinal scale* (i.e., *interval* or *ratio scale*) [Wasserman, 1993; Franceschini and Rupil, 1999; Franceschini, 2001].

- In the same way, the (qualitative) ordinal relationships between CRs and ECs, typically expressed by symbols with only ordinal properties, are arbitrarily converted in numbers (hence, assuming cardinal properties) [Akao, 1988; Franceschini, 2001].

These two assumptions may generate many different issues regarding the resulting prioritization of the ECs and its interpretation. First of all, the numerical codification of both the importances and the relationships is completely arbitrary.

For the importances, a linear 5-level scale (from 1 to 5) is typically used, but there is no restriction to use scales with other levels (such as, for example, from 0 to 4, or from 2 to 10 with step 2) or with non-linear graduation (such as, for example, power, exponential and so on). Hence, the alternative use of scales with different codifications and graduations, applied to the same sets of CRs and ECs, will produce different results and may drive the QFD team to controversial conclusions [Franceschini et al., 2005].

For the relationships, a non-linear 4-level scale, with a power base 3 graduation, is used, but, again, there is no restriction a priori to use different codifications. The consequence is again that those codifications, alternatively applied to the same case, will produce different results and will drive to controversial conclusions [Franceschini et al., 2005].

An example of those controversial outcomes is reported in Sect. 3.

For those reasons, in order to overcome these two assumptions, several alternative techniques have been proposed in the scientific literature; e.g., Multi Criteria Decision Making (MCDM) techniques, Borda's method, techniques based on pairwise comparisons, techniques based on fuzzy logic, hybrid methods, etc. [Franceschini and Rossetto, 1995; Dym and Wood, 2002; Han et al., 2004; Yan et al., 2013; Franceschini et al. 2014; Chen and Chen, 2014; Jin et al., 2014 ; Chun-Chieh et al., 2014 ; Iqbal et al., 2015; Jianga et al., 2015,].

This paper proposes an alternative method to prioritize ECs, which overcomes the aforementioned assumptions. The method is able to deal with data expressed on *ordinal scales*, with no need to "promote" them to data expressed on *interval* or *ratio scales* [Roberts, 1979]. Being inspired by a technique proposed by Yager and Filev (1994) for multi-criteria decision-making problems, the new method can be classified as a ME-MCDM (Multi Expert / Multiple Criteria Decision Making) technique.

From a technical point of view, the method (i) extends the logic of the Boolean operators *Min* and *Max* to multilevel *ordinal scales* and (ii) uses the importances of CRs as linguistic quantifiers for weighting the impact of the relationship coefficients [Yager and Filev, 1994]. The final result is a prioritization of the ECs, in the form of a rank-ordering.

The remainder of this paper is organized into three sections. Sect. 2 presents a conceptual and formal description of the new method, focusing on its advantages and limitations. Some practical examples are reported and discussed in Sect. 3. Sect. 4 discusses the new method, focusing the attention on its implications, robustness, limitations and possible future developments.

2 The proposed method

EC prioritization is aimed at selecting the ECs with a stronger impact on the most important CRs [Akao, 1988; Franceschini, 2001]. However, this prioritization should not alter the properties of the original data (i.e., CR importances and Relationship Matrix coefficients, both defined on *ordinal scales*) [Franceschini et al., 2014].

The proposed method is able to deal with ordinal data, with no need to introduce an artificial numerical conversion. As anticipated, it can be classified as a ME-MCDM (Multi Expert / Multiple Criteria Decision Making) technique [Yager, 1993].

The use of *ordinal scales* raises an important issue: while the distance between two elements is defined on *cardinal scales* (hence, sum and product operators may be applied), this is no longer true for *ordinal scales* [Roberts, 1979]. For this reason, the ISM and other prioritization techniques are rather questionable.

The proposed method is inspired by the work of Bellman and Zadeh (1970), lately "enriched" by Yager and Filev (1994) for the solution of MCDM problems. In the specific case of the QFD, the EC prioritization can be considered as a special decision-making problem: precisely, the CRs represent the decision *criteria* and the ECs represent the *alternatives* [Yager and Filev, 1994]; finally, the Relationship Matrix coefficients can be interpreted as *assessments* of each j-th EC (EC_j), according to each i-th CR (CR_i). The proposed method carries out an overall synthesis of these "assessments", considering the CR importances as *weights* of the criteria.

The approach can be organized in four steps:

i) Definition of the scale levels for the importances associated with each i-th CR_i, ($i = 1...n$) and for the Relationship Matrix coefficients (r_{ij}) between CR_i and EC_j ($j = 1...m$).

   For simplicity, it is assumed that the importance associated with each CR is defined on an *ordinal scale*, with the same number of levels of the scale used for representing the Relationship Matrix coefficients. It will be shown later on that the method may be extended to scales with different number of levels.

   Tab. 1 is a *correspondence map* between CR importances and Relationship Matrix coefficients, expressed on a 5-level *ordinal scale* ($s = 5$).

Scale level	**CR importance (d_i)**	**Importance value**	**Relationship coefficient (r_{ij})**	**Symbol**
L_1	not important	1	no relationship	(empty cell)
L_2	weakly important	2	weak relationship	□
L_3	moderately important	3	medium relationship	■
L_4	important	4	strong relationship	○
L_5	very important	5	very strong relationship	●

Table 1. Correspondence map between CR importances and Relationship Matrix coefficients, expressed on a 5-levels *ordinal scale* ($s = 5$).

ii) Data collection and construction of the Relationship Matrix.

iii) Implementation of the EC_j prioritization model:

$$w_j = \underset{i=1...n}{Min}\left\{Max\left[Neg\left(d_i\right), r_{ij}\right]\right\} \qquad (2)$$

where:

w_j is the calculated importance of the j-th EC ($j = 1...m$),

d_i is the importance of the i-th CR ($i = 1...n$),

r_{ij} is the Relationship Matrix coefficient between CR_i and EC_j,

Min is the Minimum operator,

Max is the Maximum operator,

$Neg(d_i)$ is the negation operator, defined as [Yager, 1993]:

$$Neg(L_k) = L_{s-k+1} \tag{3}$$

where L_k is the k-th level of the evaluation scale ($k = 1...s$).

It is worth noting that the resulting w_j values are defined on the same (s-level) *ordinal scale*, utilized for rating the CR importances and the r_{ij} coefficients.

iv) Determination of the EC prioritization, based on the weights calculated using Eq. (2). If two or more ECs have the same w_j, a more refined selection can be obtained through a further indicator (Tie-break indicator):

$$T(\text{EC}_j) = Dim\left[\text{A}(\text{EC}_j)\right] \tag{4}$$

where the operator $Dim\left[\text{A}(\text{EC}_j)\right]$ gives the number of elements contained in the set $\text{A}(\text{EC}_j)$, with $\text{A}(\text{EC}_j) = \left\{\text{CR}_i \mid r_{ij} > w_j\right\}$.

This represents a refined investigation for estimating the dispersion in the resulting EC importance. Basically, $T(\text{EC}_j)$ is the count of the CRs with relatively high r_{ij} coefficient (with respect to the EC importance value), related to the j-th EC. The meaning of $T(\text{EC}_j)$ will be clarified in Sect. 3 by several practical examples.

Considering ECs with the same w_j, those with higher values of $T(\text{EC}_j)$ can therefore be considered as the most important and the EC ordering can be refined.

In other terms, the rationale of the procedure is to consider those ECs with strong relationships with the most important CRs, as the most important ones. When two or more ECs have the same weight, a refined selection is performed using the $T(\text{EC}_j)$ indicator.

From Eq.(2), it is possible to observe that low-importance CRs have little effect on the importance (w_j) of a generic j-th EC. In fact, a CR with little importance entails a low importance rating L_k and therefore a high value of the negation of this value. Then, applying the Max operator, the highest value between the negation of the importance and the relationship coefficient is selected. For a given EC, all the values related to the whole set of CRs are computed. Then, the Min operator extracts the smallest of these values. In this way, all the contributions from CRs with little importance are automatically cut off.

The result of the application of Eq. (2) is a balanced tradeoff between high-value relationship coefficients, related to the CRs with low importance, and low-value relationship coefficients, related to CRs with high importance.

It can be demonstrated that the model in Eq.(2) satisfies the properties of Pareto optimality, independence to irrelevant alternatives, positive association of individual scores with overall score and symmetry [Arrow and Rayanaud, 1986; Yager, 1993].

An essential feature of this approach is that there is no need for numeric values and it does not force undue precision on the experts of the QFD team.

3 Robustness analysis of the method and application examples

For the purpose of example, consider the design of a new model of a climbing safety harness. This example is inspired by a case study present in the scientific literature and may therefore represent a helpful benchmark for the application of the proposed method [Hunt, 2013; Franceschini et al., 2014].
The CRs and ECs, identified by customer interviews and a technical analysis by the QFD team, are reported in Tabs. 2 and 3 respectively. While the codification of CR importances and Relationship Matrix coefficients is reported in Tab. 4.

Customer Requirements (CRs)		
Usability	Easy to put on	CR_1
	Confortable when hanging	CR_2
	Fits over different clothes	CR_3
	Accessible gear loops	CR_4
Performance	Does not restrict movement	CR_5
	Lightweight	CR_6
	Safe	CR_7
	Attractive	CR_8

Table 2. CRs for the design of a new model of a climbing safety harness [Hunt, 2013].

Engineering Characteristics (ECs)	
Meets safety standards	EC_1
Harness weight	EC_2
Webbing strength	EC_3
No. of clours	EC_4
No. of sizes	EC_5
Padding thickness	EC_6
No. of buckless	EC_7

Table 3. ECs for the design of a new model of a climbing safety harness [Hunt, 2013].

CR importance (d_i)	**Importance value**
not important	1
weakly important	2
moderately importante	3
important	4
very important	5

(a)

Relationship coefficient (r_{ij})	**Symbol**	**Value**
no relationship	(empty cell)	0
weak relationship	Δ	1
medium relationship	○	3
strong relationship	●	9

(b)

Table 4. Standard codification of CR importances and Relationship Matrix coefficients for the design of a new model of a climbing safety harness [Hunt, 2013].

According to the codifications reported in Tab. 4, the Relationship Matrix and the EC prioritization obtained with the ISM (see Eq. (1)) are reported in Fig. 3.

Customer Requirements		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
			Engineering Characteristics						
	CR_1	2					○		●
	CR_2	5					○	●	○
	CR_3	1					○	○	●
	CR_4	3							
	CR_5	5		○			○	●	○
	CR_6	3		●	○			○	Δ
	CR_7	5	●	○	●				
	CR_8	2		Δ		●		Δ	Δ
		w_j	45	59	54	18	39	104	62

Figure 3. Relationship Matrix and the EC prioritization obtained with the ISM for the design of a new model of a climbing safety harness, using standard codification of importances and relationship coefficients [Hunt, 2013].

The resulting EC ranking is:

$$\mathrm{EC}_6 \succ \mathrm{EC}_7 \succ \mathrm{EC}_2 \succ \mathrm{EC}_3 \succ \mathrm{EC}_1 \succ \mathrm{EC}_5 \succ \mathrm{EC}_4$$

where "$\succ$" is the symbol of *strict preference*.

In order to demonstrate that one of the main criticalities of the ISM is the arbitrary numeric codification of the CR importances and the Relationship Matrix coefficients, the same case reported in Fig. 3 is reconsidered by using the non-standard codifications reported in Tab. 5, which are different from the standard approach, but can be equally used according to QFD principles.

CR importance (d_i)	Importance value
not important	0
weakly important	1
moderately importante	2
important	3
very important	4

(a)

Relationship coefficient (r_{ij})	Symbol	Value
no relationship	(empty cell)	0
weak relationship	Δ	1
medium relationship	○	2
strong relationship	●	3

(b)

Table 5. Non-standard codification of CR importances and Relationship Matrix coefficients for the design of a new model of a climbing safety harness.

Customer Requirements		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
			Engineering Characteristics						
	CR_1	1					○		●
	CR_2	4					○	●	○
	CR_3	0					○	○	●
	CR_4	2							
	CR_5	4		○			○	●	○
	CR_6	2		●	○			○	Δ
	CR_7	4	●	○	●				

	CR_8	1		Δ		●		Δ	Δ

w_j	12	23	16	3	18	29	22

Figure 4. Relationship Matrix and the EC prioritization obtained with the ISM for the design of a new model of a climbing safety harness, usiong the non-standard codifications reported in Tab. 5.

In this case, the resulting EC ranking is:

$$EC_6 \succ EC_2 \succ EC_7 \succ EC_5 \succ EC_3 \succ EC_1 \succ EC_4,$$

which is significatively different from the one obtained with the standard codifications.

This shows that the arbitrary codification from *ordinal scales*, on which are expressed CR importances and Relationship Matrix coefficients, to *cardinal scales* may introduce a distortion in the final results and, hence, drive to contradictory decision by the QFD team.

In the following sub-sections some examples of application of the proposed method as an alternative of the ISM are reported, showing that there is no need of any artificial cardinal codification of the collected ordinal data.

Furthermore, according to the theoretical principles of the QFD approach, scales with a different number of levels may be used both for the CR importances and for the relationship coefficients [Akao 1988; Franceschini, 2001]. The only requirement is that the number of levels of the used scale must comply with the human evaluator ability of discernment (i.e. comparing the evaluator to a measurement instrument, his/her *resolution*). However, since the choice of s (i.e. the number of levels of the *ordinal scale*, in which CR importances d_i and r_{ij} values are defined) may impact on the results of the HoQ analysis, some distinct situations will be analyzed and discussed (*ordinal scale* robustness analysis).

For each of these situations, the CR importances (d_i) and the (r_{ij}) coefficients of the Relationship Matrix are defined by the QFD team.

3.1 Case of 3-level scale

As a first application example, a case with a 3-level scale both for importances and relationship coefficients is reported hereinafter. This scale is obtained by associating some of the contiguous levels reported in Tab. 4 in order that the obtained 3-level scale is coherent with the original 5-level and 4-level ones. Even if this type of scales is aligned with the human discernment ability, the results will show the poor discrimination capability of the method.

According to the case study reported in Sect. 2 and using the correspondence map in Tab.6 (with $s = 3$), the Relationship Matrices reported in Figs. 5 and 6 are obtained. Note that, referring to Tab. 4, “not important” and “weakly important” have been coded with the same scale level, the same for “moderately important” and “important”, and the same for “no relationship” and “weak relationship”.

Scale level	CR importance (d_i)	Importance value	Relationship coefficient (r_{ij})	Symbol
L_1	not (or weakly) important	1	no (or weak) relationship	(empty cell)
L_2	(moderately or) important	2	medium relationship	○
L_3	very important	3	strong relationship	●

Table 6. Correspondence map between CR importances and relationship matrix coefficients, expressed on a 3-levels *ordinal scale* ($s = 3$), obtained by associating some of the contiguous levels reported in Tab. 4.

			Engineering Characteristics						
		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer Requirements	CR_1	1					○		●
	CR_2	3					○	●	○
	CR_3	1					○	○	●
	CR_4	2							
	CR_5	3		○			○	●	○
	CR_6	2		●	○			○	
	CR_7	3	●	○	●				
	CR_8	1				●			

Figure 5. Relationship Matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tabs. 2, 3 and 6.

			Engineering Characteristics						
		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer Requirements	CR_1	L_1	L_1	L_1	L_1	L_1	L_2	L_1	L_3
	CR_2	L_3	L_1	L_1	L_1	L_1	L_2	L_3	L_2
	CR_3	L_1	L_1	L_1	L_1	L_1	L_2	L_2	L_3
	CR_4	L_2	L_1	L_1	L_1	L_1	L_1	L_1	L_1
	CR_5	L_3	L_1	L_2	L_1	L_1	L_2	L_3	L_2
	CR_6	L_2	L_1	L_3	L_2	L_1	L_1	L_2	L_1
	CR_7	L_3	L_3	L_2	L_3	L_1	L_1	L_1	L_1
	CR_8	L_1	L_1	L_1	L_1	L_3	L_1	L_1	L_1

Figure 6. "Transformed" Relationship Matrix, obtained from that in Fig. 5, when using a 3-level *ordinal scale* for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tabs. 2, 3 and 6.

According to Eq. (3), the negations for the levels of a 3-level *ordinal scale* are:

$$Neg(L_1) = L_3, \qquad Neg(L_2) = L_2, \qquad Neg(L_3) = L_1.$$

Hence, the importance of EC_1 may be calculated using Eq. (2), as follows:

$$w_1 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i1}\right]\right\} =$$
$$= Min\left\{\begin{matrix} Max\left[Neg\left(L_1\right),L_1\right], Max\left[Neg\left(L_3\right),L_1\right], Max\left[Neg\left(L_1\right),L_1\right], Max\left[Neg\left(L_2\right),L_1\right], \\ Max\left[Neg\left(L_3\right),L_1\right], Max\left[Neg\left(L_2\right),L_1\right], Max\left[Neg\left(L_3\right),L_3\right], Max\left[Neg\left(L_1\right),L_1\right] \end{matrix}\right\} =$$
$$= Min\left\{\begin{matrix} Max\left[L_3,L_1\right], Max\left[L_1,L_1\right], Max\left[L_3,L_1\right], Max\left[L_2,L_1\right], \\ Max\left[L_1,L_1\right], Max\left[L_2,L_1\right], Max\left[L_1,L_3\right], Max\left[L_3,L_1\right] \end{matrix}\right\} =$$
$$= Min\left\{L_3, L_1, L_3, L_2, L_1, L_2, L_3, L_3\right\} = L_1$$

The importances for the other ECs may be computed in the same way, obtaining the following results:

$$w_2 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i2}\right]\right\} = L_1$$
$$w_3 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i3}\right]\right\} = L_1$$
$$w_4 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i4}\right]\right\} = L_1$$
$$w_5 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i5}\right]\right\} = L_1$$
$$w_6 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i6}\right]\right\} = L_1$$
$$w_7 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i7}\right]\right\} = L_1$$

In this specific case, all the ECs obtain the same importance, hence the resulting ranking for the first part of the proposed procedure is:

$$\text{EC}_1 \approx \text{EC}_2 \approx \text{EC}_3 \approx \text{EC}_4 \approx \text{EC}_5 \approx \text{EC}_6 \approx \text{EC}_7$$

where symbol "$\approx$" denotes the *indifference* relationship.
This "flattening effect" is mainly due to the low discriminating power of the algorithm used in the first part of the proposed approach (see Eq. (2)), when using scales with a small number of levels. However, a better discrimination of the ECs can be obtained, refining the analysis by means of the $T(\text{EC}_j)$ indicators:

$$T(\text{EC}_1) = Dim\left[\text{A}(\text{EC}_1)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i1} > w_1\right\}\right] = Dim\left[\left\{\text{CR}_7\right\}\right] = 1$$
$$T(\text{EC}_2) = Dim\left[\text{A}(\text{EC}_2)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i2} > w_2\right\}\right] = Dim\left[\left\{\text{CR}_5, \text{CR}_6, \text{CR}_7\right\}\right] = 3$$
$$T(\text{EC}_3) = Dim\left[\text{A}(\text{EC}_3)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i3} > w_3\right\}\right] = Dim\left[\left\{\text{CR}_6, \text{CR}_7\right\}\right] = 2$$
$$T(\text{EC}_4) = Dim\left[\text{A}(\text{EC}_4)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i4} > w_4\right\}\right] = Dim\left[\left\{\text{CR}_8\right\}\right] = 1$$
$$T(\text{EC}_5) = Dim\left[\text{A}(\text{EC}_5)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i5} > w_5\right\}\right] = Dim\left[\left\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5\right\}\right] = 4$$
$$T(\text{EC}_6) = Dim\left[\text{A}(\text{EC}_6)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i6} > w_6\right\}\right] = Dim\left[\left\{\text{CR}_2, \text{CR}_3, \text{CR}_6, \text{CR}_7\right\}\right] = 4$$
$$T(\text{EC}_7) = Dim\left[\text{A}(\text{EC}_7)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i7} > w_7\right\}\right] = Dim\left[\left\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5\right\}\right] = 4$$

The refined ranking of the ECs is:

$$\text{EC}_5 \approx \text{EC}_6 \approx \text{EC}_7 \succ \text{EC}_2 \succ \text{EC}_3 \succ \text{EC}_1 \approx \text{EC}_4,$$

where symbols "$\succ$" and "$\approx$" denote the *strict preference* and *indifference* relationship respectively.

Using this 3-level scale the result is partially in agreement with the ISM approach. The main inconvenience is due to the fact that the proposed method has a poor discrimination power when working with scales with a small number of levels [Franceschini et al., 2005]. In general, working with s-level *ordinal scale*, the final ordering of the ECs cannot be expressed on more than s ordered categories.

3.2 Case of 10-level scale

In this second example, starting again from the codification of Tab. 4, a 10-level scale is used for both importances and relationship coefficients. That means that the human evaluator ability of discernment is pushed to its extreme by splitting in further detail the levels reported in Tab. 4, in order that the obtained 10-level scale is coherent with the original 5-level and 4-level ones.

According to the case study reported in Sect. 2 and using the correspondence map in Tab.7 (with $s = 10$), the Relationship Matrices reported in Figs. 7 and 8 are obtained.

Scale level	**CR importance (d_i)**	**Importance value**	**Relationship coefficient (r_{ij})**	**Symbol**
L_1	not important	1	no relationship	(empty cell)
L_2	…	2	…	◇
L_3	…	3	…	⊕
L_4	moderately important	4	medium relationship	◆
L_5	…	5	…	□
L_6	…	6	…	⊠
L_7	important	7	strong relationship	■
L_8	…	8	…	○
L_9	…	9	…	⊗
L_{10}	very important	10	very strong relationship	●

Table 7. Correspondence map between CR importances and relationship coefficients, expressed on a 10-level *ordinal scale* ($s = 10$) obtained by splitting in further detail the levels reported in Tab. 4.

		d_i	Engineering Characteristics						
			EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer Requirements	CR_1	3	◇	◇			■	◇	○
	CR_2	9		◇			■	●	□
	CR_3	1	◇			◇	■	□	○
	CR_4	5	◇						◇
	CR_5	10	◇	⊠			■	○	□
	CR_6	6	◇	●	⊠			■	◆
	CR_7	10	●	□	⊗		◇	◇	
	CR_8	3		⊕		●	◇	⊕	◆

Figure 7. Relationship Matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tabs. 2, 3 and 7.

		d_i	Engineering Characteristics						
			EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer	CR_1	L_3	L_2	L_2	L_1	L_1	L_7	L_2	L_8
	CR_2	L_9	L_1	L_2	L_1	L_1	L_7	L_{10}	L_5
	CR_3	L_1	L_2	L_1	L_1	L_2	L_7	L_5	L_8

	CR_4	L_5	L_2	L_1	L_1	L_1	L_1	L_1	L_2
	CR_5	L_{10}	L_2	L_6	L_1	L_1	L_7	L_8	L_5
	CR_6	L_6	L_2	L_{10}	L_6	L_1	L_1	L_7	L_4
	CR_7	L_{10}	L_{10}	L_5	L_9	L_1	L_2	L_2	L_1
	CR_8	L_3	L_1	L_3	L_1	L_{10}	L_2	L_3	L_4

Figure 8. “Transformed” Relationship Matrix, obtained from that in Fig. 7, when using a 10-level *ordinal scale* for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tabs. 2, 3 and 7.

According to Eq. (3), the negations for the levels of a 10-level *ordinal scale* are:

$$\begin{array}{lllll} Neg\left(L_1\right)=L_{10}, & Neg\left(L_2\right)=L_9, & Neg\left(L_3\right)=L_8, & Neg\left(L_4\right)=L_7, & Neg\left(L_5\right)=L_6, \\ Neg\left(L_6\right)=L_5, & Neg\left(L_7\right)=L_4, & Neg\left(L_8\right)=L_3, & Neg\left(L_9\right)=L_2, & Neg\left(L_{10}\right)=L_1. \end{array}$$

According to Eq. (2), the importances related to each of the 7 ECs are:

$$w_1 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i1}\right]\right\} = L_2$$
$$w_2 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i2}\right]\right\} = L_2$$
$$w_3 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i3}\right]\right\} = L_1$$
$$w_4 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i4}\right]\right\} = L_1$$
$$w_5 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i5}\right]\right\} = L_2$$
$$w_6 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i6}\right]\right\} = L_2$$
$$w_7 = \underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i7}\right]\right\} = L_1$$

The resulting ranking for the first part of the method is:

$$\mathrm{EC}_1 \approx \mathrm{EC}_2 \approx \mathrm{EC}_5 \approx \mathrm{EC}_6 \succ \mathrm{EC}_3 \approx \mathrm{EC}_4 \approx \mathrm{EC}_7$$

In general, it has been demonstrated by the scientific literature that, when increasing s, the “flattening effect” produced by Eq. (2) tends to disappear and the discrimination power of the resulting ranking tends to increase [Franceschini et al., 2005]. Also, if the number of ECs is higher than s, the maximum level of discrimination is exactly equal to s. On the other hand, scales with too many levels may be difficult to interpret by respondents and QFD team members. For this reason, the scientific literature often suggests not to exceed 5 levels [Franceschini and Rupil, 1999; Franceschini, 2001].

Furthermore, the “flattening effect” is strictly correlated to the structure of the Relationship Matrix and appears when the number of CRs is large. In fact, it is implicit that, in case of a great number of CRs, the possibility that, for each EC, at least one CR with the highest level of importance (i.e. L_{MAX}) is associated to a relationship coefficient (r_{ij}) scoring the lowest level of the scale (i.e. L_1) is very high. As a consequence, Eq. (2) produces a value of w_j on the same lowest level L_1.

Again, the $T(EC_j)$ indicator may be calculated in order to refine the EC ordering:

$$T(\mathrm{EC}_1) = Dim\left[\mathrm{A}(\mathrm{EC}_1)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i1} > w_1\right\}\right] = Dim\left[\left\{\mathrm{CR}_7\right\}\right] = 1$$
$$T(\mathrm{EC}_2) = Dim\left[\mathrm{A}(\mathrm{EC}_2)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i2} > w_2\right\}\right] = Dim\left[\left\{\mathrm{CR}_5, \mathrm{CR}_6, \mathrm{CR}_7, \mathrm{CR}_8\right\}\right] = 4$$
$$T(\mathrm{EC}_3) = Dim\left[\mathrm{A}(\mathrm{EC}_3)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i3} > w_3\right\}\right] = Dim\left[\left\{\mathrm{CR}_6, \mathrm{CR}_7\right\}\right] = 2$$
$$T(\mathrm{EC}_4) = Dim\left[\mathrm{A}(\mathrm{EC}_4)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i4} > w_4\right\}\right] = Dim\left[\left\{\mathrm{CR}_3, \mathrm{CR}_{10}\right\}\right] = 2$$
$$T(\mathrm{EC}_5) = Dim\left[\mathrm{A}(\mathrm{EC}_5)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i5} > w_5\right\}\right] = Dim\left[\left\{\mathrm{CR}_1, \mathrm{CR}_2, \mathrm{CR}_3, \mathrm{CR}_5\right\}\right] = 4$$
$$T(\mathrm{EC}_6) = Dim\left[\mathrm{A}(\mathrm{EC}_6)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i6} > w_6\right\}\right] = Dim\left[\left\{\mathrm{CR}_2, \mathrm{CR}_3, \mathrm{CR}_5, \mathrm{CR}_6, \mathrm{CR}_8\right\}\right] = 5$$
$$T(\mathrm{EC}_7) = Dim\left[\mathrm{A}(\mathrm{EC}_7)\right] = Dim\left[\left\{\mathrm{CR}_i \mid r_{i7} > w_7\right\}\right] = Dim\left[\left\{\mathrm{CR}_1, \mathrm{CR}_2, \mathrm{CR}_3, \mathrm{CR}_4, \mathrm{CR}_5, \mathrm{CR}_6, \mathrm{CR}_8\right\}\right] = 7$$

The resulting refined ranking is:

$$\left(\mathrm{EC}_6 \succ \mathrm{EC}_2 \approx \mathrm{EC}_5 \succ \mathrm{EC}_1\right) \succ \left(\mathrm{EC}_7 \succ \mathrm{EC}_3 \approx \mathrm{EC}_4\right)$$

Even if the Relationship Matrix in Fig. 7 is consistent with that in Fig. 5 (coefficients and CR importances in Fig. 7 are obtained by splitting those in Fig. 3 in a further detail), some significant rank reversals of the ECs are observed. See, for example, EC_1 and EC_7.

This rank reversal is intrinsically due to the increase of the number of scale levels. It is not a peculiarity of this method, it may happen also using more “traditional” approaches, such as, for example, ISM [Franceschini, 2001].

3.3 Case of 5-level scale

This case considers the situation in which both the CR importances d_i and r_{ij} coefficients are expressed on a 5-level *ordinal scale* ($s = 5$) (see Tab. 1), obtained by splitting in further detail the levels reported in Tab. 4 for relationship coefficients without losing the coherence with the original scale.

This number of scale levels seems to represent a good compromise between the previous two cases. According to the case study reported in Sect. 2, the related Relationship Matrices are reported in Figs. 9 and 10.

Customer Requirements		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
			Engineering Characteristics						
	CR_1	2					○		○
	CR_2	5					○	●	○
	CR_3	1					○	■	○
	CR_4	3							
	CR_5	5		■			○	○	■
	CR_6	3		●	■			○	□
	CR_7	5	●	■	○				
	CR_8	2		□		●		□	□

Figure 9. Relationship Matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tabs. 1, 2 and 3.

			Engineering Characteristics						
		d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer Requirements	CR_1	L_2	L_1	L_1	L_1	L_1	L_4	L_1	L_4
	CR_2	L_5	L_1	L_1	L_1	L_1	L_4	L_5	L_4
	CR_3	L_1	L_1	L_1	L_1	L_1	L_4	L_3	L_4
	CR_4	L_3	L_1	L_1	L_1	L_1	L_1	L_1	L_1
	CR_5	L_5	L_1	L_3	L_1	L_1	L_4	L_4	L_3
	CR_6	L_3	L_1	L_5	L_3	L_1	L_1	L_4	L_2
	CR_7	L_5	L_5	L_3	L_4	L_1	L_1	L_1	L_1
	CR_8	L_2	L_1	L_2	L_1	L_5	L_1	L_2	L_2

Figure 10. “Transformed” Relationship Matrix, obtained from that in Fig. 9, when using a 5-level *ordinal scale* for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tabs. 1, 2 and 3.

According to Eq. (3), the negations of a 5-level *ordinal scale* are:

$$Neg\left(L_1\right)=L_5, \qquad Neg\left(L_2\right)=L_4, \qquad Neg\left(L_3\right)=L_3, \qquad Neg\left(L_4\right)=L_2, \qquad Neg\left(L_5\right)=L_1.$$

Hence, according to Eq. (2), the following EC importances are obtained:

$$w_1=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i1}\right]\right\}=L_1$$
$$w_2=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i2}\right]\right\}=L_1$$
$$w_3=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i3}\right]\right\}=L_1$$
$$w_4=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i4}\right]\right\}=L_1$$
$$w_5=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i5}\right]\right\}=L_1$$
$$w_6=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i6}\right]\right\}=L_1$$
$$w_7=\underset{i=1...8}{Min}\left\{Max\left[Neg\left(d_i\right),r_{i7}\right]\right\}=L_1$$

The resulting ranking for the first part of the proposed procedure is therefore:

$$\text{EC}_1 \approx \text{EC}_2 \approx \text{EC}_3 \approx \text{EC}_4 \approx \text{EC}_5 \approx \text{EC}_6 \approx \text{EC}_7$$

Applying Eq. (4), the resulting $T(\text{EC}_j)$ values are:

$$T(\text{EC}_1)=Dim\left[\text{A}(\text{EC}_1)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i1}>w_1\right\}\right]=Dim\left[\left\{\text{CR}_7\right\}\right]=1$$
$$T(\text{EC}_2)=Dim\left[\text{A}(\text{EC}_2)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i2}>w_2\right\}\right]=Dim\left[\left\{\text{CR}_5,\text{CR}_6,\text{CR}_7,\text{CR}_8\right\}\right]=4$$
$$T(\text{EC}_3)=Dim\left[\text{A}(\text{EC}_3)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i3}>w_3\right\}\right]=Dim\left[\left\{\text{CR}_6,\text{CR}_7\right\}\right]=2$$
$$T(\text{EC}_4)=Dim\left[\text{A}(\text{EC}_4)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i4}>w_4\right\}\right]=Dim\left[\left\{\text{CR}_8\right\}\right]=1$$
$$T(\text{EC}_5)=Dim\left[\text{A}(\text{EC}_5)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i5}>w_5\right\}\right]=Dim\left[\left\{\text{CR}_1,\text{CR}_2,\text{CR}_3,\text{CR}_5\right\}\right]=4$$
$$T(\text{EC}_6)=Dim\left[\text{A}(\text{EC}_6)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i6}>w_6\right\}\right]=Dim\left[\left\{\text{CR}_2,\text{CR}_3,\text{CR}_5,\text{CR}_6,\text{CR}_8\right\}\right]=5$$
$$T(\text{EC}_7)=Dim\left[\text{A}(\text{EC}_7)\right]=Dim\left[\left\{\text{CR}_i \mid r_{i7}>w_7\right\}\right]=Dim\left[\left\{\text{CR}_1,\text{CR}_2,\text{CR}_3,\text{CR}_5,\text{CR}_6,\text{CR}_8\right\}\right]=6$$

The refined ranking is:

$$\text{EC}_7 \succ \text{EC}_6 \succ \text{EC}_2 \approx \text{EC}_5 \succ \text{EC}_3 \succ \text{EC}_1 \approx \text{EC}_4$$

Even if the Relationship Matrix is consistent with those in Figs. 3 and 5, a few significant rank reversals can be observed.

3.4 Case of scales with a different number of levels

In typical QFD applications, CR importances and relationship coefficients are defined on not-identical *ordinal scales*. Precisely, CR importances are usually evaluated on a 5-level scale, while r_{ij} coefficients on a 4-level scale (see Tab. 4) [Akao, 1988; Franceschini, 2001].

In this case, the aggregation method proposed in Eq. (2) cannot be applied [Yager and Filev, 1994]. However, a practical approximated solution may be obtained by merging two or more contiguous levels of the *ordinal scale* with the largest number of levels into one, or introducing one or more "dull" levels in the *ordinal scale* with the lowest number of levels; this second option is implemented in the example in Tab. 8. It must be remarked that this approach leaves a certain discretionary power to the QFD team, in choosing the scale levels to be adjusted; however, the suggested "adjustment" does not alter the ordinal relationships between the objects represented on the initial *ordinal scale*(s) [Roberts, 1979].

Scale level	**Relationship coefficient (r_{ij})**	**Symbol**
L_1	no relationship	(empty cell)
L_2 (dull)	N/A	N/A
L_3	weak relationship	Δ
L_4	medium relationship	○
L_5	strong relationship	●

Table 8. Example of a possible correspondence map of the relationship coefficients evaluated on a symbolic 4-level *ordinal scale* ($s=4$).

Referring again to the case study reported in Sect. 2 and considering the mappings in Tab. 1 and in Tab. 8, the Relationship Matrices reported in Figs. 11 and 12 are obtained.

Customer Requirements	d_i	EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
CR_1	2					○		●
CR_2	5					○	●	○
CR_3	1					○	○	●
CR_4	3							
CR_5	5		○			○	●	○
CR_6	3		●	○			○	Δ
CR_7	5	●	○	●				
CR_8	2		Δ		●		Δ	Δ

Figure 11. Relationship Matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tabs. 1, 2, 3 and 8.

		d_i	Engineering Characteristics						
			EC_1	EC_2	EC_3	EC_4	EC_5	EC_6	EC_7
Customer Requirements	CR_1	L_2	L_1	L_1	L_1	L_1	L_4	L_1	L_5
	CR_2	L_5	L_1	L_1	L_1	L_1	L_4	L_5	L_4
	CR_3	L_1	L_1	L_1	L_1	L_1	L_4	L_4	L_5
	CR_4	L_3	L_1	L_1	L_1	L_1	L_1	L_1	L_1
	CR_5	L_5	L_1	L_4	L_1	L_1	L_4	L_5	L_4
	CR_6	L_3	L_1	L_5	L_4	L_1	L_1	L_4	L_3
	CR_7	L_5	L_5	L_4	L_5	L_1	L_1	L_1	L_1
	CR_8	L_2	L_1	L_3	L_1	L_5	L_1	L_3	L_3

Figure 12. "Transformed" Relationship Matrix, obtained from that in Fig. 11, when using a 5-level *ordinal scale* for CR importances and a 4-level one for relationship coefficients. For details on symbols/abbreviations, see Tabs. 1, 2, 3 and 8.

The negations of a 5-level *ordinal scale* are reported in the example in Sect. 3.3.
By applying Eq. (2), the following EC importances are obtained:

$$w_1 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i1}\right]\right\} = L_1$$
$$w_2 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i2}\right]\right\} = L_1$$
$$w_3 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i3}\right]\right\} = L_1$$
$$w_4 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i4}\right]\right\} = L_1$$
$$w_5 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i5}\right]\right\} = L_1$$
$$w_6 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i6}\right]\right\} = L_1$$
$$w_7 = \underset{i=1\ldots8}{Min}\left\{Max\left[Neg\left(d_i\right), r_{i7}\right]\right\} = L_1$$

The resulting ranking for the first part of the proposed procedure is:

$$\text{EC}_1 \approx \text{EC}_2 \approx \text{EC}_3 \approx \text{EC}_4 \approx \text{EC}_5 \approx \text{EC}_6 \approx \text{EC}_7$$

Using Eq. (4), the $T(\text{EC}_j)$ indicators may be calculated as:

$$T(\text{EC}_1) = Dim\left[\text{A}(\text{EC}_1)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i1} > w_1\right\}\right] = Dim\left[\left\{\text{CR}_7\right\}\right] = 1$$
$$T(\text{EC}_2) = Dim\left[\text{A}(\text{EC}_2)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i2} > w_2\right\}\right] = Dim\left[\left\{\text{CR}_5, \text{CR}_6, \text{CR}_7, \text{CR}_8\right\}\right] = 4$$
$$T(\text{EC}_3) = Dim\left[\text{A}(\text{EC}_3)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i3} > w_3\right\}\right] = Dim\left[\left\{\text{CR}_6, \text{CR}_7\right\}\right] = 2$$
$$T(\text{EC}_4) = Dim\left[\text{A}(\text{EC}_4)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i4} > w_4\right\}\right] = Dim\left[\left\{\text{CR}_8\right\}\right] = 1$$
$$T(\text{EC}_5) = Dim\left[\text{A}(\text{EC}_5)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i5} > w_5\right\}\right] = Dim\left[\left\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5\right\}\right] = 4$$
$$T(\text{EC}_6) = Dim\left[\text{A}(\text{EC}_6)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i6} > w_6\right\}\right] = Dim\left[\left\{\text{CR}_2, \text{CR}_3, \text{CR}_5, \text{CR}_6, \text{CR}_8\right\}\right] = 5$$
$$T(\text{EC}_7) = Dim\left[\text{A}(\text{EC}_7)\right] = Dim\left[\left\{\text{CR}_i \mid r_{i7} > w_7\right\}\right] = Dim\left[\left\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5, \text{CR}_6, \text{CR}_8\right\}\right] = 6$$

The refined ranking is:

$$\text{EC}_7 \succ \text{EC}_6 \succ \text{EC}_2 \approx \text{EC}_5 \succ \text{EC}_3 \succ \text{EC}_1 \approx \text{EC}_4$$

This result is not different from that obtained in Sect. 3.3.

3.5 Comparison of the obtained results

Tab. 9 reports a synthesis of the results obtained in the examples of Sects. 3.1 to 3.4.

Method	Scale for d_i	Scale for r_{ij}	Number of levels (s) for d_i	Number of levels (s) for r_{ij}	Ranking
ISM	Linear	Power base 3	5	4	$EC_6 \succ EC_7 \succ EC_2 \succ EC_3 \succ EC_1 \succ EC_5 \succ EC_4$
ISM	Linear	Linear	5	4	$EC_6 \succ EC_2 \succ EC_7 \succ EC_5 \succ EC_3 \succ EC_1 \succ EC_4$
MCDM	Ordinal	Ordinal	3	3	$EC_5 \approx EC_6 \approx EC_7 \succ EC_2 \succ EC_3 \succ EC_1 \approx EC_4$
MCDM	Ordinal	Ordinal	10	10	$(EC_6 \succ EC_2 \approx EC_5 \succ EC_1) \succ (EC_7 \succ EC_3 \approx EC_4)$
MCDM	Ordinal	Ordinal	5	5	$EC_7 \succ EC_6 \succ EC_2 \approx EC_5 \succ EC_3 \succ EC_1 \approx EC_4$
MCDM	Ordinal	Ordinal	5	4	$EC_7 \succ EC_6 \succ EC_2 \approx EC_5 \succ EC_3 \succ EC_1 \approx EC_4$

Table 9. Comparison of the results obtained in the examples of Sects. 3.1 to 3.4.

Even if the aim of the examples here reported is to show the applicability of the proposed approach (MCDM), from the comparison in Tab. 9, some peculiarity of the method, already demonstrated in the scientific literature [Franceschini et al., 2005; Yager and Filev, 1994], are evident:

- Using the ISM, based on *cardinal scales*, the final ranking may change if the scale levels are encoded in different ways. This does not happen with the MCDM approach, which is based on *ordinal scales* and considers only the relative positions of the scale levels.
- Changing the number of levels of the scales (in order that different scales remain each other coherent) the final result may change using either approach.
- 3-level scales may reduce the discrimination power of the method and introduce a “flattening effect” in the final ranking.
- 10-level scales may increase the discrimination power of the method, but the final results heavily depend on the human evaluator ability of discernment. Typically, respondents and QFD team members have difficulty in using scales with too many levels.
- Furthermore, the discrimination power of the method is also related to the structure of the Relationship Matrix and the values of the importances. For that reason, the “flattening effect” may appear also with scales characterized by a high number of levels.
- As suggested in the scientific literature, 5-level scales are the appropriate trade-off between discrimination power of the method and human evaluator ability of discernment.
- Despite the difficulty in splitting scales with a low number of levels into scales with a high number levels, the final results are substantially coherent (ordinal scale robustness).

4 Conclusions

In the paper, a new method for EC prioritization in QFD have been presented and discussed. The approach is consistent with the ordinal features of the linguistic scales used for representing the CR importances and Relationship Matrix coefficients.

Even if the simplicity of application of the new method is comparable to that of the traditional approach (i.e., the ISM), it is able to aggregate data evaluated on *ordinal scales*, overcoming controversial assumptions of data cardinality and avoiding any arbitrary and artificial "scalarization" of the data. It can be also implemented in situations in which both CR importances and Relationship Matrix coefficients are rated on different *ordinal scales*.

On the other hand, in the practical applications, some limitations may not be ignored. In fact, the method may generate a "flattening effect" when applied to scales with a small number of levels. This may apparently encourage the use of scales with a large number of levels (e.g., 10 or more). However, scales with too many levels may be difficult to interpret for respondents and QFD team members. The scientific literature suggests that using a 5-level scale can be an acceptable compromise [Franceschini and Rupil, 1999; Franceschini, 2001; Franceschini et al., 2005]. It is also remarked that the aforementioned "flattening effect" can also occur when the number of CRs and/or ECs is large. Furthermore, due to then use of a s -level *ordinal scale*, the final ordering of the ECs cannot be expressed on more than s ordered categories.

Finally, when both CR importances and r_{ij} coefficients have high values, the method tends to flatten the importance values (w_j) upwards, for all the ECs. This is coherent with the aim of the method, since it indicates that several ECs are important and should not be neglected by designers. Similarly, when CR importances have high values and r_{ij} coefficients have low values, the method tends to flatten all the computed EC importances downwards.

Future research will be addressed to deeply analyze the mathematical properties of the method and perform a structured comparison between the proposed method and other possible approaches (different from ISM) for prioritizing the ECs.

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