Spatial network investigation of wall turbulence

Giovanni Iacobello[1], Stefania Scarsoglio[1], Hans Kuerten[2], and Luca Ridolfi[3]

[1] Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Turin, Italy
[2] Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands,
[3] Department of Environmental, Land and Infrastructure Engineering, Politecnico di Torino, Turin, Italy
giovanni.iacobello@polito.it

1 Introduction

In the last decades, complex networks have been exploited in a wide range of applications, with an increasing interest to physical and engineering problems. Specifically, the study of problems related to fluid flows mainly concerns two-phase flows [1] and geophysical flows [2], with particular attention to time-series mapping into networks (e.g., visibility [3]). In this work, we propose a correlation network-based investigation [4–7] of a turbulent channel flow, with the aim to spatially characterize the flow dynamics [8], introducing a novel statistical approach to wall turbulence analysis.

2 Database Description and Methods

The data [9] were extracted from a direct numerical simulation of a fully developed turbulent channel flow at $Re_\tau = Hu_\tau/\nu = 180$, where $H = 1$ is the half-channel height, $\nu = 1/180$ is the kinematic viscosity and $u_\tau = 1$ is the friction velocity. The physical domain is $(4\pi H \times 2H \times 4/3\pi H)$ with a grid resolution of $(N_x \times N_y \times N_z) = (576 \times 191 \times 288)$, where (x, y, z) are the streamwise, wall-normal and spanwise directions, respectively. The velocity field was computed and data were acquired for 5000 time samples, with a time-step $\Delta t = 2.510^{-4}$. Firstly, we selected $N'_x = 144$ equally spaced grid points in the x direction and $N'_z = 150$ consecutive grid points in the $z-$direction. We then assigned a node to each selected grid point, resulting in $n = (144 \times 191 \times 150) = 4125600$ nodes. The Pearson correlation coefficients based on the streamwise velocity component time-series were evaluated for each pair of nodes. Although such procedure can also be carried out for other physical quantities, the streamwise velocity is one of the most significant variables to characterize turbulent channel flows. A spatial network was then built, where links are active if the absolute value of the correlation coefficient is greater than or equal to a suitable threshold, τ, which was here set equal to 0.85. A high value of τ was chosen to highlight the strongest spatial correlations and to have a manageable number of links. In order to take into account the non-uniform spacing of the grid in the wall-normal direction, we assigned to each node a weight equal to the volume, V_j, of that node. Specifically, we defined the *volume-weighted connectivity* [10] of node i as $VWC(i) = \sum_j^N a_{ij} V_j / V_{tot}$, where a_{ij} are the entries of the adjacency matrix (with $a_{ii} = 1$), and V_{tot} is the total volume of the domain. $VWC(i)$ ranges between zero and one, and represents the fraction of volume to which the node i is connected.

3 Results

In Fig. 1(a) we highlight the nodes with high VWC values; in particular, we say that a node has a high VWC value if its cumulative probability, defined as $P(VWC) = \sum_{VWC'=VWC}^{\infty} p(VWC')$, satisfies $P(VWC) \leq 10^{-2}$ (corresponding to the 99th percentile), where $p(VWC)$ is the VWC probability distribution [11]. The nodes connected to a higher fraction of volume tend to group into clusters elongated in the streamwise direction, x, representing regions of high kinematic coherence, i.e. nodes with stronger spatial connections. Such clusters of *hubs*, as also evidenced in Fig. 1(b), are present both close to the walls (i.e., at $y^+ = 0, 360$) and at the center of the channel (i.e., at $y^+ = 180$), although the dynamics leading to the connections change at different wall-normal distances. It is interesting in Fig. 1(b), the nesting behavior of high VWC nodes.

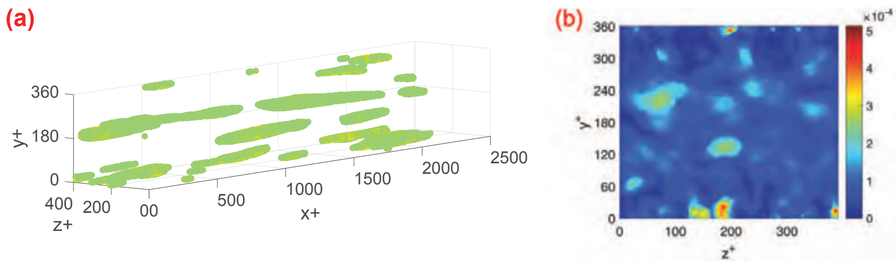

Fig. 1. *(a)* View of nodes with VWC in the 99th percentile. *(b)* Planar section at $x^+ = 2000$; the colorbar refers to the full range of VWC values. The axes are reported in wall-units, i.e. $(x^+, y^+, z^+) = (x, y, z) \cdot u_\tau / \nu$.

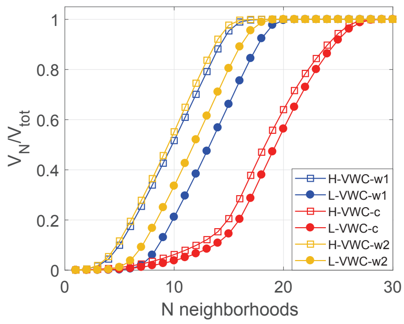

Fig. 2. The fraction of volume of the first N neighborhoods for three pairs of source-nodes. Notation: H/L-VWC, high/low VWC values; w1/w2, walls 1 and 2; c, center of the channel.

To have a better spatial characterization, the location of progressive neighbors of nodes at different VWC is investigated next. By doing so, it is possible to understand how different regions in the channel are connected. To this end, we analyzed the *successive neighborhoods* [12] of selected *source-nodes*, at different wall-normal distances.

We considered the N *cumulative neighborhoods* as the union of the first N neighborhoods. In order to illustrate the differences in the progressive patterns of the cumulative neighborhoods for different source-nodes, we selected three pairs of representative nodes — two pairs close to the walls and one at the center —, each one with a high and a low VWC value. Fig. 2 shows the behavior of the the fraction of the total volume occupied by the nodes in the first N neighborhoods for the three pairs of nodes. Starting from the source-nodes close to the wall, the expansion of the neighborhoods is clearly much faster than the expansion of the neighborhoods of source-nodes at the center of the channel, independently of the VWC values. However, at a fixed y^+, the expansion of the neighborhoods is slower for nodes with low VWC. In this context, an important role is played by nodes connected with negative correlation links, since they are generally less frequent and could suggest the presence of particular kinematic dynamics.

Summary. The present analysis can highlight spatial relations among different regions in a novel view that relies on the network formalism. In particular, the presence of clustered and elongated groups of highly connected nodes at various wall-normal distances indicates a spatial coherence related to the streamwise velocity correlations. The analysis of progressive neighborhoods, feasible only through a network approach, offers useful information on the spatial propagation of high-correlation patterns. Therefore, the proposed approach can provide new insights into the spatial characterization of complex systems such as turbulent flows, which deserves additional extensive investigation.

References

1. Z.K. Gao, W.X. Wang, N.D. Jin: Nonlinear Analysis of Gas-Water/Oil-Water Two-Phase Flow in Complex Networks, Springer Press (2014)
2. Ser-Giacomi, E., Rossi, V., Lpez, C., Hernndez-Garca, E.: Flow networks: A characterization of geophysical fluid transport. Chaos, 25(3), 036404 (2015)
3. Lacasa, L., Luque, B., Ballesteros, F., Luque, J., Nuno, J. C.: From time series to complex networks: The visibility graph. P. Natl. Acad. Sci., 105(13), 4972-4975 (2008)
4. Massara, G. P., Di Matteo, T., Aste, T.: Network filtering for big data: triangulated maximally filtered graph. J. of complex Networks, 5(2), 161-178 (2016)
5. Mantegna, R. N.: Hierarchical structure in financial markets. Eur. Phys. J. B, 11(1), 193-197 (1999)
6. Tumminello, M., Aste, T., Di Matteo, T., Mantegna, R. N.: A tool for filtering information in complex systems. P. Natl. Acad. Sci. USA, 102(30), 10421-10426 (2005)
7. Aste, T., Di Matteo, T.: Dynamical networks from correlations. Physica A, 370(1), 156-161 (2006)
8. Scarsoglio, S., Iacobello, G., Ridolfi, L.: Complex networks unveiling spatial patterns in turbulence. Int. J. Bifurcat. Chaos, 26(13), 1650223 (2016)
9. Computational resources were provided by HPC@POLITO (http://www.hpc.polito.it) and SURFsara-Cartesius (https://userinfo.surfsara.nl/)
10. Donges, J. F., Zou, Y., Marwan, N., Kurths, J.: Complex networks in climate dynamics. Eur. Phys. J.Spec. Top. 174(1),157–179, (2009)
11. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., Hwang, D. U.:Complex networks: Structure and dynamics. Phys. Rep., 424(4), 175-308(2006)
12. McAuley, J. J., da Fontoura Costa, L., Caetano, T. S.: Rich-club phenomenon across complex network hierarchies. Appl. Phys. Lett., 91(8), 084103 (2007)