Constitutive laws for metal friction

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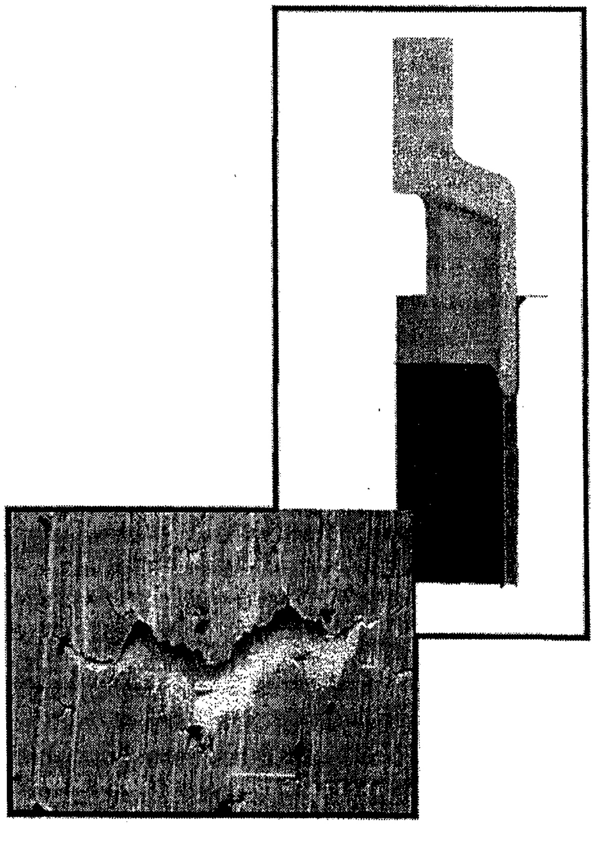

LAMIH - Laboratoire d'Automatique, de Mécanique et d'Informatique industrielles et Humaines
Université de Valenciennes et du Hainaut Cambrésis, Le Mont Houy, F-59313 Valenciennes Cedex 9
Phone: +33 3 27 51 14 11, Fax: +33 3 27 51 13 17, www.univ-valenciennes.fr/LAMIH

Constitutive law for metal friction

G. Zavarise * — A. Nardin ** — B. A. Schrefler**

**Dept. Structural and Geotechnical Engineering - The Turin Polytechnic*
Corso Duca degli Abruzzi,24 – 10129 Torino, Italy
zavarise@athena.polito.it

*** Department* of *Structural and Transportation Engineering - University of Padua*
Via F. Marzolo, 9 – 35131 Padova, Italy
nardin@caronte.dic.it
bas@caronte.dic.it

ABSTRACT: *In this paper we present a method for modeling both the large plastic deformations and the frictional phenomena. The strategy is based on a discrete elements technology, i.e. a collection of rigid spheres with a suitable cohesive contact formulation is used to model the continuum. Sliding and wear between surfaces are also taken into account using a similar method. The strategy presents interesting characteristics both for forging and deep drawing processes. The formulation is based on extension of general concepts of contact as a unilateral constraint condition. For such purpose constitutive laws have been implemented in the node-to-segment contact formulation within the framework of the penalty method..*

KEY WORDS: *Discrete Elements, Cohesive Contact, Friction, Wear, Contact Algorithms*

1. Introduction

The field of constitutive laws for friction is actually one of the most interesting within the framework of enhanced formulations in contact mechanics. In fact, friction and wear effects present relevant costs for the industry and the society. The problem involves several interesting aspects both on the theoretical and on the applied side. Due to the involved non-linearities, the numerical analysis of such phenomena is very complex and the required computational effort is quite large.

In this paper we present a strategy for modeling both the large plastic deformations and the frictional phenomena. The strategy is based on a discrete elements methodology, i.e. a collection of rigid spheres with a suitable cohesive contact formulation is used to model the continuum [1] [2]. The basic idea is to concentrate at the contact level the real mechanical behaviour of the material. The goal is achieved by extending the general concept of contact as a unilateral constraint condition, through a suitable constitutive law. For such purpose constitutive laws have been implemented in the node-to-segment contact formulation within the framework of the penalty method [3]. In this case the penalty parameter is not simply a constant based on numerical necessities (e.g. to avoid ill conditioning of the stiffness matrix), but it is transformed into a non-linear function through the constitutive law itself. The main advantage of this approach lies in the fact that classical remeshing and crack propagation problems of a continuum discretization are avoided. In this a way any tool penetration, crack propagation, change of shape will be transformed into a series of contact opening and/or sliding. This approach permits to simulate wear, friction and chip formation trough the partial removal of spheres from the surfaces. Moreover this model can reproduce the shear band localization. The strategy consists of the following tasks: continuum model definition; original friction law formulation; wear law definition. Regarding the interfacial friction of metals some theories were developed [4], [5] to formulate the friction effects by modelling the interaction of microscopic asperities. In this sense it is necessary to characterize the friction law between the asperities. In order to carry out the proposed strategy a "macro" and a "micro" levels has to be established. In the micromechanical model the mutual contact interaction between two spheres is studied (see Figure 1.a). An elasto-plastic frictional contact law is implemented to simulate the real behaviour of the materials at macroscopic level. In the macromechanical model the behavior of a random array of spheres is described. In [6] it has been shown that with appropriate assumption at microscopic level the macroscopic behaviour is in fact elasto-plastic both for a continuum and spheres collection model. In this work we have focused our attention on the micromechanical model. This model is based on the classical concept of the elasticity and the plasticity theories, with suitable modifications. The framework for the plastic behaviour of the material consists of a failure criterion; a rate-independent elasto-plastic flow rule for the normal and the tangential force and a non-linear yield criterion. Due to the discretization strategy and the intrinsic characteristic of the contact material surfaces, the resulting constitutive law can be very complex. All these factors heavily depend on the metals considered, hence experimental characterizations have been collected and comparisons between experimental and numerical results have been performed.

2. The constitutive model of the continuum

The discrete approach to the metal modelling problem is formulated within the framework of the finite element method, and it has been implemented into the FEAP code [7]. Due to the fact that the metal was modelled as a close random pack of rigid spheres and that the FEM contact formulation needs some specific element, a contact force-displacement law was developed to relate the contact force acting between two spheres to their relative displacement. At this stage we focus our attention at the microscopic level. We define a micromechanical model composed of two spheres interacting between them trough a nonlinear elasto-plastic contact relation. For this purpose we adopt two potential functions: one for the normal and another for the tangential stress-strain relation. The basic assumption is that the total strain increments can be divided into elastic and plastic components

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \qquad [1]$$

The elastic strain increments are considered linear, and are calculated from the classical Hooke's law. As experimentally observed, this approximation can be widely used for metals. The elastic stress-strain relationship, which is a first component of the considered constitutive law, is applied to both the directions and it can be expressed as follows

$$\sigma_i(F_i) = E_i \cdot \left(\varepsilon_i(g_i) - \varepsilon^{p}{}_i(g^{p}{}_i)\right), \qquad i = n, t \qquad [2]$$

where σ is the normal or tangential stress; ε is the total strain; ε^p is the plastic component of the total strain; F is the contact force; g is the total relative displacement of the contact points; g^p is the plastic component of the relative displacement, n, t are the normal and tangential direction; E_i are the elastic parameters, which characterize the elastic properties of the metal. Moreover the expression for the shear modulus is derived from experimental observations, and it depends directly on the normal applied stress as follows

$$G = (\sqrt[3]{|F_N|} - R)m \qquad [3]$$

where F_N is the applied normal stress; R is a constant which depends on the peak stress value derived from experimental test; m is a constant parameter derived from experimental test. The determination of the value of R and m depends on the material considered.

2.1. Flow Rule

The plastic strain increments for both the directions, i.e. the normal and the tangential strain, are calculated from the following flow rule

$$\boldsymbol{\varepsilon}^p : [0,T] \to \Re \qquad \overset{*}{\varepsilon^p} = \frac{\partial}{\partial t}\varepsilon^p \qquad [4]$$

where T is the time parameter. The plastic increment can be completely described for any admissible state of stress $\sigma \in E_\sigma$ from the classical relation $\overset{*}{\varepsilon^p} = \gamma \cdot sign(\sigma)$ with $\gamma \geq 0$ the absolute value of the slip rate; $\overset{*}{\varepsilon^p}$ the time derivative of the plastic strain (normal or tangential); σ the stress in the normal or tangential direction. In the non-linear formulation of plasticity, the variation of γ is non-linear and it is calculated trough the consistency parameter $\Delta\gamma$ as shown below.

2.2. Yield Function

The chosen yield function is a general law, both for the normal and the tangential direction, which underlines separately the elastic and the plastic material behavior

$$f(\boldsymbol{\sigma},\alpha) = |\,\boldsymbol{\sigma}\,| - \left(\sigma_y + K\alpha\right) \qquad [5]$$

The first part of the equation represents the elastic range of the material $\sigma = E \cdot \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p\right)$. The second part considers the plastic flow: $\sigma_y > 0$ delimits the flow stress; K is the plastic hardening (or softening) modulus; α is the hardening (softening) internal variable, which is a non negative function of the amount of plastic flow, $\alpha : \left[0, T\right] \rightarrow \Re$. From this general criterion it is possible to obtain some well-know criteria such as the Hubert-von Mises one.

For our purposes, starting from eq. [5], a more general non-linear expression was developed

$$f(\sigma,\alpha) = |\,\sigma\,| - \left[\sigma_y + H(\alpha)\right] \qquad [6]$$

where $H(\alpha)$ is a generic function of the plastic strain and it is expressed as a function of the softening or hardening parameter α. The dependence of the tangential on the normal one is expressed by applying a two-dimensional failure criterion to the tangential law. Within the solution process the two-dimensional yield surface is used to establish the elastic and plastic range of tangential forces at fixed value of normal forces (Figure 1.b). The shape of the yield surface depends on the type of criterion adopted, and then the elastic range for the tangential force depends on the shape of the yield surface and the characteristics of the plastic function. For the treated cases it is assumed that the stress field at the contact point can be expressed only with unidirectional normal and tangential components, in accordance to normal or tangential component of the contact force. The kinematically admissible directions for the contact points impose the characteristics of the stresses, i.e. the normal and tangential stresses are oriented, respectively toward the normal and tangential directions of the contact point.

(a) **(b)**

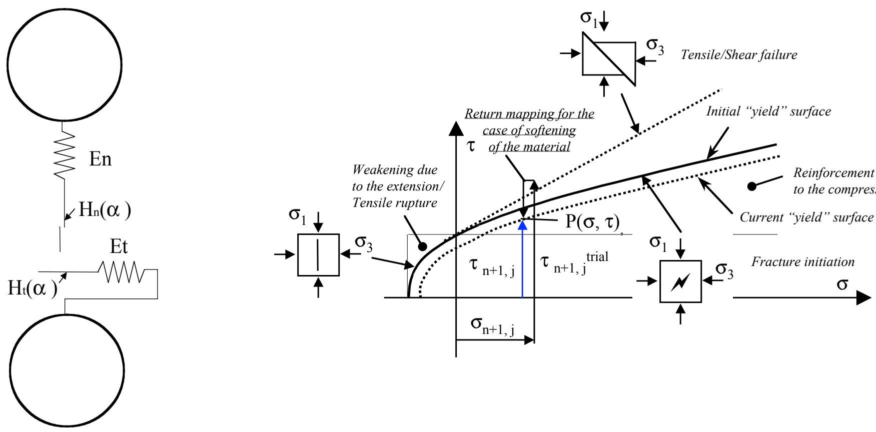

Figure 1. *Micromechanical model schematisation (a). Yield surfaces constitutive law (b).*

3. Friction law

An original two-parameters model of failure and friction has been developed in [8]. This formulation is based on experimental tests and on the basic assumption of the classical strength criteria such as Von Mises, Hoek-Brown and Mohr-Coulomb [9]. In the proposed criterion the slip between surfaces is assumed to occur when the shear stress, τ, on the plane of contact reaches a so-called critical value. This value depends non-linearly upon the normal stress in the same plane. The basic concept of the frictional criterion is that the shear strength is given by two parts varying with the normal stress: an elastic "cohesive" part and a frictional, plastic part. The shear strength which develops on the plane of friction between materials, can be mathematically expressed as follows

$$\tau = f\left(\varepsilon_t, \varepsilon_t^p\right) + f^1\left(\sigma_n, H\right) \qquad [7]$$

where τ is the tangential critical stress; σ_n is the applied normal stress; ε_t and ε_t^p are, respectively, the total tangential strain and the plastic tangential strain; H is a general non-linear function of plasticity. The cohesive part of the formulation is non-linear elastic as far as the shear modulus is concerned: in fact it was experimentally noticed that for metals, the slope of the shear-normal stresses relationship changes when the normal applied stress increases or decreases. Moreover it was found that there is a further contribution which affects the real value of the shear modulus of the material. In this way it is possible to explain the strong dependency on the normal applied load. The plastic hardening (or softening) frictional behaviour starts after τ has reached the limit flow stress value, which is a function of the normal applied stress. As experimentally observed, if the normal applied stress increases, also the tangential peak stress increases. Due to the previous considerations eq. [7] can be more specifically rewritten as

$$\tau = G(\sigma_n) \cdot (\varepsilon_t - \varepsilon_t^p) - (n(\sigma_n) + H^1(\alpha)) \qquad [8]$$

where G is the shear modulus of the material and it depends on the normal applied stress; α is the so called internal hardening variable; $n(\sigma_n)$ is the parameter which delimits the flow range. The parameter $n(\sigma_n)$ could be tuned differently for the tangential stress in the continuum and the tangential friction stress developed in the contact surfaces of two bodies. In this case it is represented as a function of the normal stress and is expressed trough two terms

$$n(\sigma_n) = \eta \cdot (\sigma_n + c) \qquad \text{for the continuum}$$
$$n(\sigma_n) = \eta \cdot \left(\sqrt[3]{\sigma_n} + c\right) \qquad \text{for the friction} \qquad [9]$$

This formulation of $n(\sigma_n)$ was chosen in order to reproduce the plastic behaviour of a metal and the friction between a metal and another material, where the shear strength increase with the normal applied stress. The material parameters η and c are constant during the loading process: η is the parameter that gives a contribution to the frictional behavior of the material. Moreover this parameter characterizes also the angle of the shear failure plane for biaxial test: the greater the numerical value of η, the greater is this angle; c is a constant correlated with the cohesive properties of the material.

4. Wear law

Starting from basics of [10] we model the wear with the classical wear law of Archard, stating that the wear rate is proportional to the contact normal force, F_N, and the slip velocity, v_t

$$\overset{*}{w} = k \frac{F_N v_t}{H} \qquad [10]$$

where H is the hardness of the worn surfaces and k is a dimensionless parameter. The Archard law was derived for adhesive wear, but it can be extended to the abrasive wear, as shown from Rabinowicz.

Another interesting strategy that can be used is given by the Lim and Ashby wear equations and wear maps. Regardless from the adopted law, the effect of wear can be taken into account by modelling the parts close to the surface with tiny spheres, and breaking the connections of some spheres. The volume of the disconnected spheres will correspond to the worn material.

5. Numerical examples

With the outlined model it is possible to define frictional behaviour of metals taking into account the increase of the peak shear load of the material when the normal stress increases. In Figure 3 the friction between two bodies of metal is shown. In this numerical example the most important parameters were tuned, in order to obtain a useful macromechanical model.

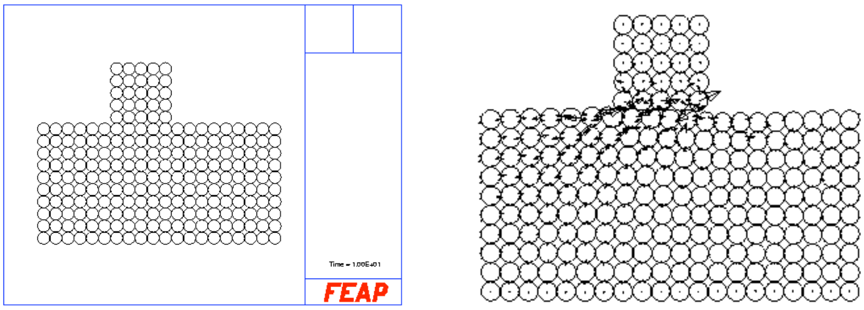

Figure 3. *Friction simulation for metal with macromechanical model; initial condition, and reaction contact forces after 10 steps .*

The plastic function $H(\alpha_t)$ and the strength parameter were tuned to the following values in order fit the experimental results: $H(\alpha_t) = b_1 \cdot \alpha_t^2 + b_2 \cdot \alpha_t + b_3$, $b_1 = 0.0$; $b_2 = 15000$; $b_3 = 0.0$, $\sigma_c = 365 \times 10^6\ N/m^2$, $\sigma_t = 365 \times 10^6\ N/m^2$. The comparisons between experimental results and numerical simulations show a good correspondence. Figure 4 describes the value of the maximum value of F_T by varying the normal applied force F_N, and is compared with the Mohr-Coulomb criterion (dotted line). The developed law is very similar to the real metal behaviour.

(a) **(b)**

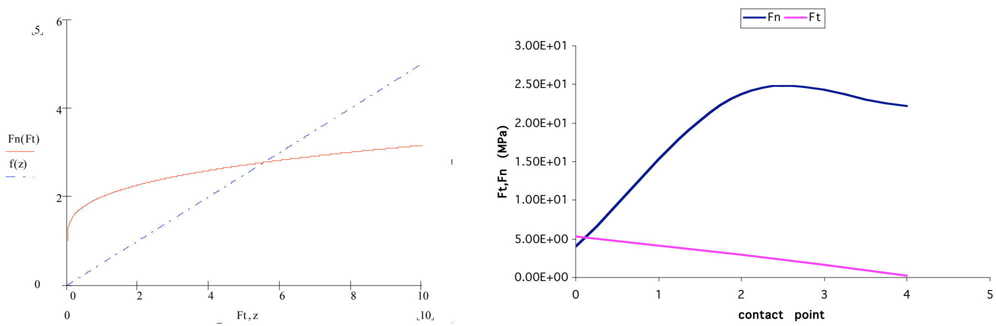

Figure 4. (a)*Maximum value of tangential force versus normal applied force; (b) normal and tangential forces at different contact points.*

6. Conclusions

The proposed method has shown very attractive characteristics to study the wear and fracture of the continuum metals. Moreover the discrete element approach permits to overcome the numerical problems derived from the non-linearities due to the metal friction and wear.

The frictional law adopted presents some innovative aspects, which must be further developed. For this purpose we adopt two different potential functions: one for the normal and another for the tangential stress-strain relation. The use of two different yield criteria involves a greater number of parameters. The approach of the new law captures effectively the real behaviour of materials, and the obtained law is adaptable to different types of metal just changing and tuning some specific parameters. A better definition of the interfacial friction and especially of the interaction asperities has to be developed, to characterize the adhesive and abrasive wear behaviour of metal forming.

7. References

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