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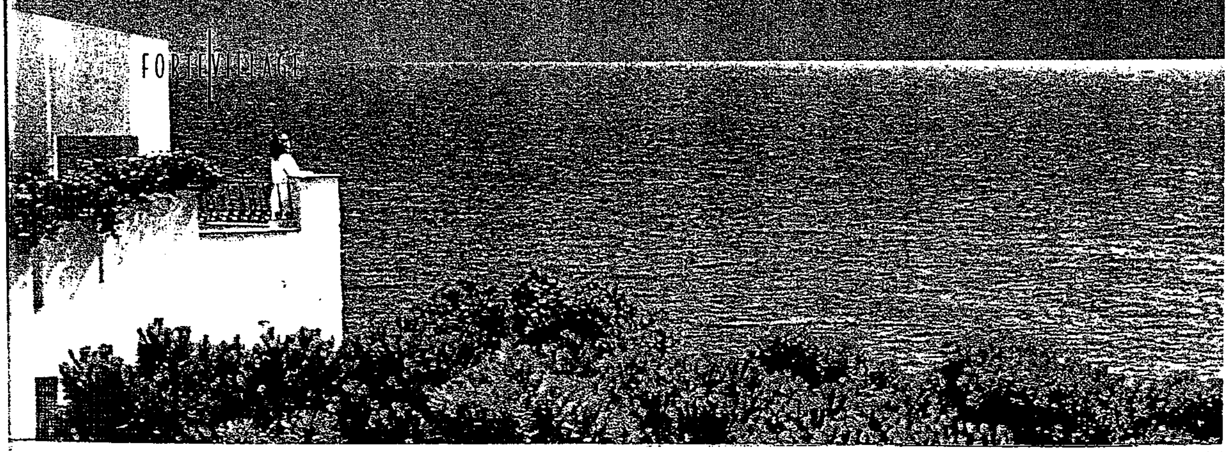

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Augmentation through linear regression in contact mechanics

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Abstract.

The paper is focused onto a fast, non-conventional, augmentation scheme. The basic starting consideration concerns the fact that a converged contact problem that requires an improvement of the contact constraints enforcement becomes usually a very smooth problem with respect to the original one. Hence an estimate of the correct value of the contact forces can be carried out using linear regression or interpolation methods. Despite the fact that special care has to be used to stabilize the prediction, a very fast convergence rate can be obtained. With respect to a previously proposed formulation, the current enhanced one is able to deal quite well with the cited instability within the iterative solution process. The resulting convergence path is hence very smooth, and the convergence rate is superlinear. It has to be remarked that the method is also quite insensitive to the penalty parameter. Several numerical tests have been carried out to check the characteristics, and the most interesting results are here shown.

1. Introduction

Penalty methods are widely used in contact mechanics. Among the most important drawbacks, ill conditioning and solution error in the constraints enforcements have to be cited. Considering the last one, it could result into a non acceptable penetration of the contacting surfaces. This error can be reduced increasing the penalty parameter, but this choice usually generates ill-conditioning problems. The most common way to overcome the problem concerns the employment of augmentation techniques. The target of augmented schemes concerns the improvement of penalty solutions at very low cost.

From the practical point of view augmented schemes try to get the best from the two most popular methods, i.e. Penalty and Lagrangian Multiplier. Briefly speaking, penalty is a very simple method that provides in any case a solution that is affected by an error. Hence constraint conditions are satisfied in approximated way. Contact forces in this case are given as a function of the penetration. On the contrary, the Lagrangian Multiplier method provides an "exact" solution, treating contact forces as new, additional unknowns. Augmentation schemes try to move the penalty solution toward the Lagrangian Multiplier one by building a set of contact forces computed as a function of the penalty solution. This results in a two-step scheme, in which the penalty solution is computed first, and then used to update a Lagrangian-Multiplier-like set of contact forces. The efficiency of classical augmentation schemes strongly depends on the penalty value. The drawback of the method is related to the linear convergence rate of the update scheme.

For more details, a detailed theoretical background can be found in [1-3]. Specific applications to contact mechanics within the framework of Finite Element Methods can be found in [4-7].

2. Problem characteristics

In trying to improve classical augmentation schemes we can benefit from the context of the problem. Our framework is not anymore a purely theoretical, mathematical one. We are focused on solid mechanics problems within the framework of the Finite Element Method. In such a context, using an engineeristic approach there exist some peculiarities that can be taken into account. A generic contact mechanics problem is usually a problem hard to be solved. Often at the start there is no knowledge about the amount of contact area that will come into contact. Contact force may vary within a wide range and several other nonlinearities could be involved. Within this context the iterative solution requires robust algorithms to converge. For this reasons there are no heuristic methods till now proposed to improve the efficiency of the mathematical ones.

A completely different scenario takes usually place when augmentation schemes are applied. In this case a first solution has been achieved. It is affected by too large penetrations in the active contact area, but this means also that the achieved solution is very close to the correct one. Hence to improve the solution quality is a very easy task compared to the achievement of the first problem solution. This step is hence much more smooth and easy with respect to the previous one. I this context it is reasonable to check if perhaps a non classical, heuristic scheme could give some help in predicting the final solution of the contact forces. This strategy has been outlined in [8]. In such paper a simple interpolation scheme has been used to predict the correct value of the contact forces. A simple comparison about the classic augmentation scheme and the interpolation one for a 1-D problem is depicted in Figure 25-1.

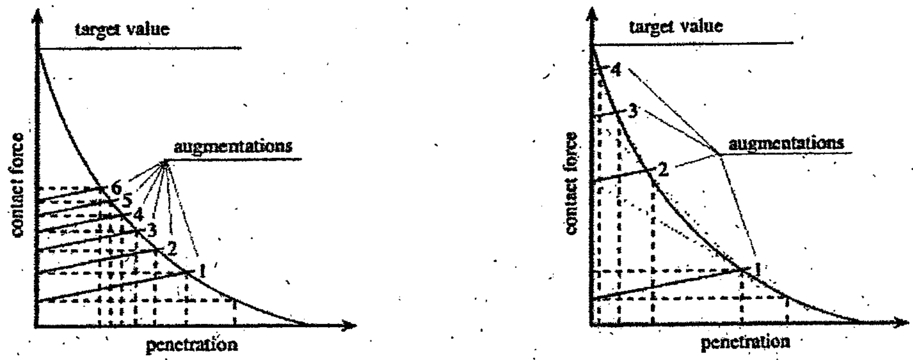

Figure 32-1. Classic and interpolated augmentation scheme.

The method has shown very interesting characteristics, with a ratio about one to ten concerning the number of augmentations required to minimize the penetration. However the minimization of the normalized penetration norm, defined as

$$\hat{\Re}_a = \frac{\Re_a}{\Re_1} \qquad (1)$$

where R_i is the penetration norm computed before performing the i^{th} augmentation

$$\Re_i = \frac{\sqrt{\sum_{i=1}^{n.\,active\ contacts} \left(g_i^j\right)^2}}{n.\,active\ contacts} \qquad (2)$$

presented a non-monotone behavior. The target is then to improve the above cited method by trying to get a more smooth path.

We fully concentrate in this task, and to do this we briefly summarize the mathematical problem description.

$$\begin{cases} \overline{\pi}(\mathbf{x}) \rightarrow \min \\ \mathbf{b}(\mathbf{x}) = \mathbf{0} \\ \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \end{cases} \qquad (3)$$

where x indicates the unknowns, and the functional p (x) describes the elastic potential associated to the continuum. The constraint set b(x) = 0 denotes the classical boundary conditions while the set g(x) £ 0 collects the unilateral contact constraints. Thisproblem is then solved by the minimization of the functional

$$\left[\pi(\mathbf{x}) + \frac{1}{2} c \|\mathbf{g}(\mathbf{x})\|^2 + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x})\right] \rightarrow \min \qquad (4)$$

where **x** indicates the unknowns, and the functional $\overline{\pi}(x)$ describes the elastic potential associated to the continuum. The constraint set b(x) = 0 denotes the classical boundary conditions while the set $g(x) \leq 0$ collects the unilateral contact constraints. This problem is then solved by the minimization of the functional

where c is the penalty parameter and λ^T is the set of augmented forces. It has to be remarked that such forces are here computed through augmentation, they are not new unknowns like in the Lagrangian Multiplier method. A complete outline of the problem can be found in [8].

3. The Linear Regression concept

The problem characteristics outlined in the previous sections permit to perform a step forward with respect to the interpolation method proposed in [8]. From the practical point of view by performing the Newton's iterations till convergence before each augmentation step gives us a collection of equilibrated solutions, S

$$S_1 = \left(\hat{\lambda}_1, \hat{g}_1\right),\ S_2 = \left(\hat{\lambda}_2, \hat{g}_2\right),\ \ldots\ldots,\ S_i = \left(\hat{\lambda}_i, \hat{g}_i\right) \quad (5)$$

†

where $\hat{\lambda}_a$ a are the Lagrangian Multipliers set computed before performing the a^{th} augmentation; $\hat{g}_a$ are the related penetration set and a is the augmentation index.

For each node, p, in contact we can now consider an hyperspace where to place the achieved solution points, P_a^p

$$P_1^p = \left(\hat{\lambda}_1^p, \hat{g}_1\right),\ P_2^p = \left(\hat{\lambda}_2^p, \hat{g}_2\right),\ P_3^p = \left(\hat{\lambda}_3^p, \hat{g}_3\right),\ \ldots\ldots \quad (6)$$

The point coordinates are given by the augmented contact force on the node, and the penetrations of all the active contacts. These points belong to an ideal hypersurface describing the nodal contact force in p as a function of the penetrations.

$$\hat{\lambda}^p = f^p(\hat{g}) \quad (7)$$

Of course the contact force presents a dependence on the penetrations that varies from node to node. The strongest dependence takes place with the penetration of the node itself. Then the dependence on the surrounding becomes more and more weak with increasing the distance from the node p; see also [8] for more detail in this concept. A minimal dependence can hence be set as

$$\hat{\lambda}^p = f^p\left(\hat{g}^p\right) \quad (8)$$

This choice is labeled as "uncoupled", i.e. for each node we assume that the augmented contact force depends only on the related nodal penetration, as depicted also in Figure 25-1. From the experience gained in [8] we can say that the considearion of two or max 4 surrounding nodes is usually enough to get a good prediction. Hence for each node, with respect to (7) we consider a subset of the penetrations as independent variables

$$\hat{\lambda}^p = f^p\left(\hat{g}^{p-1}, \hat{g}^p, \hat{g}^{p+1}\right) \qquad (9)$$

$$\hat{\lambda}^p = f^p\left(\hat{g}^{p+2}, \hat{g}^{p-1}, \hat{g}^p, \hat{g}^{p+1}, \hat{g}^{p+2}\right) \qquad (10)$$

These kind of choice is labeled as "coupled". Regardless from the chosen dependence we can say that the solution of the problem is related to a zero penetration.

$$\lambda^p = f^p(0) \qquad (11)$$

With this respec, due to the outlined characteristics, t the hyperplane strategy outlined in[8] can be enhanced considering the problem as a regression one. Several algorithms are available within the family of linear regression methods. It has to be observed that only for the uncoupled scheme we can use simple linear regression methods. In general multiple linear regression methods have to be considered for coupled schemes.

We started considering first the classic multiple linear regression scheme. For each augmented force estimate an hyperplane equation is obtained using the least square criterion

$$\lambda^p = \beta_0 g_1^p + \beta_1 g_2^p + \beta_2 g_3^p + \ldots + \beta_k g_k^p + \varepsilon_i \qquad (12)$$

where b are the regression coefficients,e are independently distributed normal errors each with mean zero and k is the total number of penetrations, i.e. of contact nodes considered. Results computed using this model were not satisfactory. This is due to the fact that least squares methods have desirable characteristics when the errors are normally distributed. In other cases a poor estimator can be obtained. The data set (6), used to build the regression,are obviously associated to different errors. We can simply consider the penetration norm (1) as an indicator of such associated error to easily get the point. From a general point ofview a data set with a small penetration norm is much more reliable than another set with a bigger one. For this reason we used a linear regression model with the least absolute values criterion. The criterion satisfied is the minimization of the sum of the absolutevalues of the deviations of the observed response from the fitted one. Hence the regression coefficient estimate minimize

$$\sum_{i=1}^{k} \left|\hat{\lambda}_i^p - \lambda_i^p\right| \qquad (13)$$

This criterion has shown a really good behavior with respect to the previous one.

4. Numerical setup and testing

The proposed method has been implemented in the finite element code FEAP [9] (courtesy of Prof. R.L. Taylor). A suitable collection of routines has been developed to collect and organize the data taking into account available options. The linear regression problem has been solved by linking the IMSL scientific library to the FEAP code.To compare the performances a test problem with an elastic block on elastic foundation, previously solved in [8], has been used. The material model is characterized by an extension of small strain linear elasticity to finite deformations as described by Simo (1992). The dimensions of the block, and of the foundation are, respectively,4x2 and 4.6x1. The material is characterized by an elastic modulus equal to 10^4 and a Poisson's ratio of 0.48. Vertical displacements on the top of the block and both horizontal and vertical displacements at the bottom of the foundation are restrained. A vertical displacement is then imposed on the left and right side of the top of the block to generate a strongly non-uniform contact pressure distribution along the contact area. The initial and deformed geometry at convergence is depicted, respectively in Figure 25-2 and Figure 25-3.,

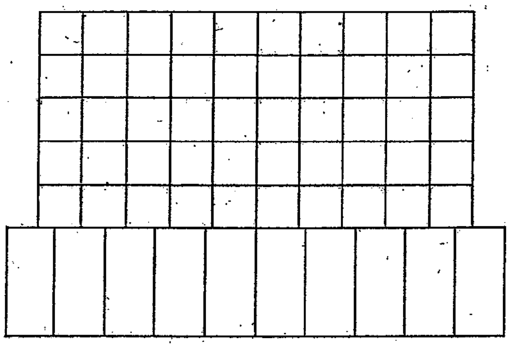

Figure 32-2. Problem geometry and discretization.

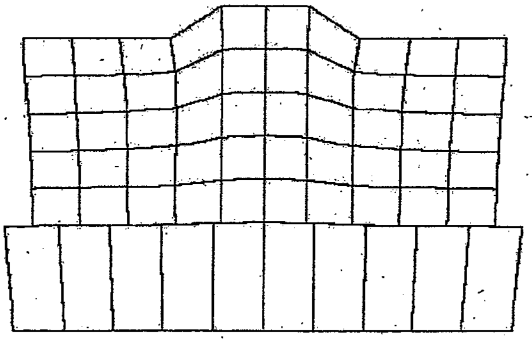

Figure 32-3. Geometry at convergence.

The results with the linear interpolation schemes obtained in [8] are depicted in Figure 25-4. Here the classical augmentation scheme is compared with the uncoupled interpolation one and with the coupled with 3 and 5 nodes in total. Results obtained with the regression scheme are depicted in Figure 25-5.

A comparison of the diagrams clearly show that there is almost no difference for the uncoupled case. Both the direct interpolation and the regression method present a very close, non-monotone path. On the contrary, a remarkable difference takes place when coupling nodes are involved. In this case the regression scheme is able to minimize the penetration norm in a much more smooth way. Also, a value close to 10^{-16} is obtained with 20 augmentations, instead of 40. The regression scheme seems hence a natural step forward of the linear interpolation one. These preliminary results have been confirmed also with several other test cases. For all the examples we got the same general trend.

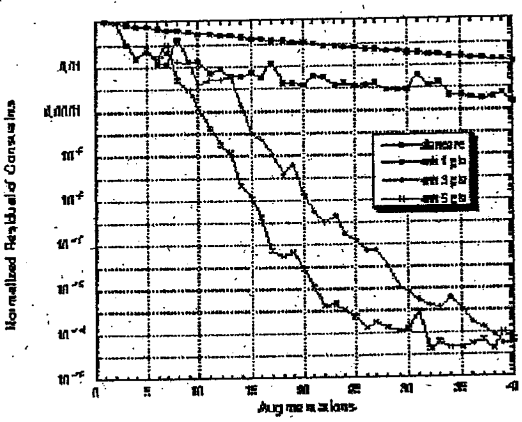

Figure 32-4. Penetration norm minimization– interpolation scheme [8].

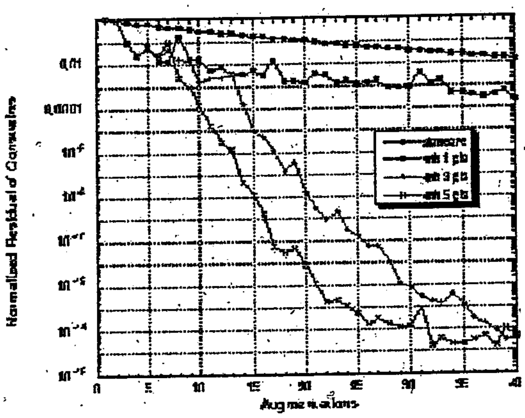

Figure 32-5. Penetration norm minimization – regression scheme.

5. Conclusions

The basic idea here outlined deals with a technique to predict the Lagrangian Multipliers in contact mechanics problems. The paper presents a step forward with respect to theinterpolation technique presented in [8]. The new prediction is based on regression methods. The unusual coupling between finite elements and regression models seems able to remarkably increase the performances of the classical augmentation scheme.

6. Acknowledgements

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